

REPORT DOCUMENTATION PAGE

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14. ABSTRACT This project, joint with Antoinette Tordesillas of University of Melbourne (Proposal 58763-EV), investigated the behavior of the upper soil surface and its response to applied loads including vehicles and massive objects. This final report is the fourth for this project. It includes material on overall objectives and approaches that have been used. Note that this report focuses on the experimental aspects of this combined theory/experimental study. The theoretical aspects are emphasized in the corresponding report from Prof. Tordesillas. The regime of interest is the so called liquid-solid (or jamming) transition which is highly relevant to at least two key aspects of the Army's				
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Report Title

Final Report: Topology, Structure and Functionality: Analysis, Modelling and Experimentation of Dense Granular Deformation in 2D and 3D

ABSTRACT

This project, joint with Antoinette Tordesillas of University of Melbourne (Proposal 58763-EV), investigated the behavior of the upper soil surface and its response to applied loads including vehicles and massive objects. This final report is the fourth for this project. It includes material on overall objectives and approaches that have been used. Note that this report focuses on the experimental aspects of this combined theory/experimental study. The theoretical aspects are emphasized in the corresponding report from Prof. Tordesillas. The regime of interest is the so-called liquid-solid (or jamming) transition which is highly relevant to at least two key aspects of the Army's operations: off-road vehicle mobility and soil stability. Key aspects of this transition involve force transmission, energy dissipation, and kinematics in granular assemblies.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper

09/02/2013 17.00 A. H. Clark, R. P. Behringer. Granular impact model as an energy-depth relation, EPL (Europhysics Letters), (03 2013): 0. doi: 10.1209/0295-5075/101/64001

09/02/2013 18.00 David M. Walker, Gary Froyland, Jie Zhang, Robert P. Behringer, Antoinette Tordesillas. Transition dynamics and magic-number-like behavior of frictional granular clusters, Physical Review E, (07 2012): 0. doi: 10.1103/PhysRevE.86.011306

09/02/2013 19.00 Qun Lin, Jie Zhang, R.P. Behringer, Jingyu Shi, Antoinette Tordesillas. Structural stability and jamming of self-organized cluster conformations in dense granular materials, Journal of the Mechanics and Physics of Solids, (02 2011): 0. doi: 10.1016/j.jmps.2010.10.007

09/02/2013 20.00 David M. Walker, Antoinette Tordesillas, Colin Thornton, Robert P. Behringer, Jie Zhang, John F. Peters. Percolating contact subnetworks on the edge of isostaticity, Granular Matter, (02 2011): 0. doi: 10.1007/s10035-011-0250-y

09/02/2013 21.00 T. S. Majmudar, A. Tordesillas, R. P. Behringer, J. Zhang. Statistical properties of a 2D granular material subjected to cyclic shear, Granular Matter, (03 2010): 0. doi: 10.1007/s10035-010-0170-2

09/02/2013 22.00 Joshua A. Diksman, Robert P. Behringer, Jie Ren. Reynolds Pressure and Relaxation in a Sheared Granular System, Physical Review Letters, (01 2013): 0. doi: 10.1103/PhysRevLett.110.018302

09/02/2013 23.00 Dapeng Bi, Jie Zhang, R. P. Behringer, Sumantra Sarkar, Bulbul Chakraborty. Origin of Rigidity in Dry Granular Solids, Physical Review Letters, (08 2013): 0. doi: 10.1103/PhysRevLett.111.068301

09/02/2013 24.00 Jie Zhang, Robert Behringer, Antoinette Tordesillas. Buckling force chains in dense granular assemblies: physical and numerical experiments, Geomechanics and Geoengineering, (03 2009): 0. doi: 10.1080/17486020902767347

TOTAL: 8

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received

Paper

- 09/02/2013 12.00 Joshua Dijksman, Jie Ren, Robert P. Behringer. Homogeneity and packing structure of a 2D sheared granular system,
POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media. 08-JUL-13, Sydney, Australia. : ,
- 09/02/2013 13.00 Somayeh Farhadi, Robert P. Behringer, Alex Zihao Zhu. Slow dynamics for elliptical particles under continuous shear and cyclic compression,
POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media. 08-JUL-13, Sydney, Australia. : ,
- 09/02/2013 14.00 Joshua A. Dijksman, Hu Zheng, Robert P. Behringer. Imaging soft sphere packings in a novel triaxial shear setup,
POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media. 08-JUL-13, Sydney, Australia. : ,
- 09/02/2013 15.00 R. P. Behringer, Joshua Dijksman, Jie Ren, Jie Zhang, Trushant Majmudar, Bulbul Chakraborty, Daipeng Bi, Antoinette Tordesillas. Jamming and shear for granular materials,
POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media. 08-JUL-13, Sydney, Australia. : ,
- 09/02/2013 16.00 Hu Zheng, Joshua A. Dijksman, Robert P. Behringer. Novel experimental apparatus for granular experiments on basal friction,
POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media. 08-JUL-13, Sydney, Australia. : ,

TOTAL:

5

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

- 08/31/2012 6.00 Dapeng Bi, Jie Zhang, Bulbul Chakraborty , R. P. Behringer. Jamming by shear, Nature (12 2011)
- 08/31/2012 7.00 Max Bi, Jie Zhang, Bulbul Chakraborty, R. P. Behringer. Jamming by Shear--Supplementary Information, Nature (12 2011)
- 08/31/2012 8.00 Jie Ren, Joshua A. Dijksman, Robert P. Behringer. Reynolds Pressure and Relaxation in a Sheared Granular System, Physical Review Letters (submitted) (07 2012)
- 08/31/2012 9.00 Antoinette Tordesillas, David M. Walker, Gary Froyland, Jie Zhang, Robert P. Behringer. Transition Dynamics of Frictional Granular Clusters, Physical Review E (07 2012)
- 08/31/2012 10.00 D. M. Walker, A. Tordesillas, C. Thornton, R. P. Behringer, J. Zhang, J. F. Peters. Percolating contact subnetworks on the edge of isostaticity, Granular Matter (02 2011)
- 08/31/2012 11.00 Jie Ren, Joshua A. Dijksman, Robert P. Behringer. Linear shear in a model granular system, CHAOS (12 2011)

TOTAL: **6**

Number of Manuscripts:

Books

Received Book

TOTAL:

Received

Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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FTE Equivalent:
Total Number:

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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FTE Equivalent:
Total Number:

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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FTE Equivalent:
Total Number:

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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FTE Equivalent:
Total Number:

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):.....

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:.....

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME

PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

TERRESTRIAL SCIENCES PROJECT PROGRAM REVIEW REPORTING
2014 Final Project Report:
Topology, Structure and Functionality:
Analysis, Modelling and Experimentation of Dense Granular Deformation in 2D and 3D

Proposal Number 58533-EV

Robert P. Behringer
Department of Physics
Duke University

Objectives

This project, joint with Antoinette Tordesillas of University of Melbourne (Proposal 58763-EV), investigated the behavior of the upper soil surface and its response to applied loads including vehicles and massive objects. This final report is the fourth for this project. It includes material on overall objectives and approaches that have been used. Note that this report focuses on the experimental aspects of this combined theory/experimental study. The theoretical aspects are emphasized in the corresponding report from Prof. Tordesillas. I indicate specific contributions, such as papers published or submitted, over the period of this project.

The regime of interest is the so-called liquid-solid (or jamming) transition which is highly relevant to at least two key aspects of the Army's operations: off-road vehicle mobility and soil stability. Key aspects of this transition involve force transmission, energy dissipation, and kinematics in granular assemblies. The project objectives are to:

- Identify and apply completely new structural measures to characterize granular deformation and failure in experiments and simulations in fully 3D as well as 2D systems
- Develop structural mechanics models of self-organized structures that govern bulk rheology; develop new continuum descriptions, as well as fast discrete cellular automata and lattice models, that incorporate the findings from these studies
- In particular, understand the properties and dynamics of granular states in the shear jamming regime.
- Extend all facets of the investigations to non-spherical particles, starting with ellipses, and then more complex particle shapes
- Probe the effects of grain polydispersity

Approach

Experiments were carried out at Duke University which exploit the special properties of two kinds of granular systems: photoelastic materials using quasi-two-dimensional (2D) particles, and transparent fully three-dimensional (3D) grains consisting of hydrogels that can be characterized with laser scanning techniques. These materials, plus the expertise developed by the PI and his group, allow for the detailed particle-scale determination of all physically relevant properties of granular systems at the grain scale. Specifically, for every photoelastic particle, we measure displacements, rotations and all contact forces. This approach allows experimental information at all scales within a granular material. For our new 3D systems, we determine particle positions, contacts, and contact forces, so that for the first time, it is possible to carry out precise

characterizations of fully 3D granular systems subject to a wide range of strain protocols. From the experimental measurements, it is possible to compute stresses and other quantities that are relevant at the macroscopic scale, which are in turn important to vehicle mobility and other important issues for the Army. To our knowledge, the Duke lab is the only facility anywhere in the world which is capable of this combination of measurements. Details of the new 3D approach are discussed below. For the moment, I simply note that there are two key aspects to the functionality of the experiment. One is the basic apparatus, which involves laser scanning with index matched surrounding fluid, and the other is the processing of the scans to reconstruct the actual 3D particles, their contacts, and the amount of particle deformation at the contacts.

Experiments are carried out in several specific settings which are of relevance to the objectives above:

- **Failure under shear**--Perhaps the most important failure mode for granular materials occurs when they are subject to shear. This is important in the case of off-road vehicles, and also for a granular/soil surface that is loaded by heavy objects. A set of experiments, in coordination with the theoretical and DEM studies of Antoinette Tordesillas focuses on the multiscale nature of failure under shear. In this case, the experiments exploit the special properties of particles that are made from a photoelastic material. The particles are contained in either a biaxial device whose purpose is to provide highly controlled shear or other stresses, in a special device that provides cyclic simple shear strain, or in a granular Couette device. During the course of any of these macroscopic stress protocols, we measure all physical properties, such as contact forces, particle motion, etc. A particularly important finding from this work is the observation that loosely packed unstable soils can be stabilized, i.e. jammed, by the application of shear. This is a quite new finding that requires a rethinking of the widely accepted picture of jamming, i.e. the way that granular or granular-like systems can transition between fluid-like and solid-like behavior. We have carried out extensive experiments on cyclic shear—that is, a system of photoelastic particles is repeatedly sheared, first in one direction, and then in the reverse direction. We are particularly interested in the topology of the resulting network of contacts. These networks and their topological properties are the key focus of the combined Melbourne-Duke project. Cyclic shear also allows us to probe large ensembles of states, understand the nature of the shear jammed states, and characterize the dynamics of particle motion, including diffusion and glass-like slowing down.
- **Three Dimensional scanner experiments**--As noted already above, we have developed a device that allows us to characterize the grain-scale response of systems in 3D to applied strains/stresses. We have carried out uniaxial and shear strain protocols on this type of system, where we have demonstrated our ability to accurately relate microscopic measurements to global measurements, and to resolve a long-standing issue regarding the stress-strain response of granular assemblies. Several papers have been published, submitted for publication or are in progress from these experiments. In particular, our coordinated effort with the Melbourne group has led to one submitted and one pending paper from the results of our 3D systems.
- **Traction on a granular surface**--A key issue involves traction and the response of a granular material to a moving object on its surface. A set of experiments involves pulling an object across the surface of a layer of photoelastic particles. Simultaneous measurements are made of the pulling force and the photoelastic response of the granular layer. It is then possible to correlate the slipping behavior of the object to the internal response of the granular material. Since the material is primarily being subject to shear stress/strain in this type of experiment, there is a useful connection to studies of cyclic shear discussed above. This work involves interesting parallels between the nature of sliding and friction from the atomic to the tectonic scale.
- Identifying order in shear granular systems--We have shown that mathematically, it is possible to map the shear state of a grain, as characterized by its force moment tensor (essentially, the grain scale shear stress) onto a vector field. This is likely to have important consequences for understanding the spontaneous ordering of the shear stress, and its spatial fluctuations. For instance, the vector

representation of the shear state of one grain, has a well defined scalar product with the corresponding state vector of any other grain, in much the same way as quantum spins of magnetic particles. It is then possible to bring to bear all the machinery developed for characterizing order within near-critical thermal systems to our granular systems.

- **Other related work, supported from non-ARO sources**—Several other experiments are or will be carried out, exploiting the photoelastic (2D) and hydrogel (3D) particles used above. One set of experiments that is particularly relevant to the US Army involves the impact of a heavy object on the surface of a granular material. This project is supported by DTRA, and we have accumulated substantial data sets and a number of novel results. (Four papers on this work have been published recently, and other papers should appear soon.) The data has helped inform the present project, and it is relevant to NASA-related needs to understand Lunar and planetary regoliths.
- **Coordination with computer simulations and analysis**—Parallel simulations are being carried out by A. Tordesillas of the University of Melbourne, Australia. This work also involves the development of new types of continuum or mixed continuum—discrete models which are based on the experiments and simulations.

Significance and Army Value

The Army uses an array of vehicles that must move over or rest on the soil—i.e. a granular material. Specifically, the Army is concerned with off-road vehicle mobility. In addition, the penetrability of soils by an impacting object is of great significance, and is not well understood currently. In other US-related activities, NASA missions to the Moon and to Mars will depend crucially on the ability to predict the response of granular materials, and to ascertain the stability of soils. Here, a particularly important issue concerns soil stability and erosion due to retro rockets during moon or other planetary landings.

This project provided key insights into the nature of granular stability and failure. A key goal was to evolve models which are effective predictors of soil stability and which can be used in the design and possibly use of off-road vehicles. A second goal was to understand the nature of soil penetration by a moving object at both vehicle and projectile speeds.

Accomplishments, Activities, and Publications

- With Antoinette Tordesillas David Walker, Gary Froyland, Jie Zhang, we have applied novel ideas from network theory, in combination with the stability criterion from the previous point, to characterize the dynamical evolution of the force network in granular materials. The paper: *Transition Dynamics of Frictional Granular Clusters, has appeared in Phy. Rev. E 86, 011306 (2012)*. We have applied the basic principles of this work in the recently published *A complex systems analysis of stick-slip dynamics of a laboratory fault, Chaos 24, 013132 (2014)*
- Based on preliminary ARO-supported work with Antoinette Tordesillas, my group in collaboration with workers at Brandeis (Bulbul Charabarty and Max Bi (both of Brandeis University). Duke group member Jie Zhang is now assistant professor at STJU in Shanghai, and I) discovered and characterized **Shear Jamming**, a new way of jamming in granular materials, due to the effects of shear. This requires a very significant rethinking of what is known as the jamming diagram. The nature of jamming/and the reverse process of unjamming are of considerable importance to many practical processes involve granular stability, mobility of vehicles on off-road terrains, and flows, such as those in a hopper, that are subject to stoppage of flow. I discuss this important finding below, and note here that our key work on this

topic appeared in *Statistical properties of a 2D granular material subjected to cyclic shear*, J. Zhang, A. Tordesillas, and R. Behringer, *Granular Matter* DOI 10:1007/s10035-010-01-70-02 (2010), and in *Jamming by Shear*, M. Bi, J. Zhang, B. Chakraborty and R. Behringer, *Nature* 480, 355 (2011). The work in *Nature* was featured in a *Nature News & Views* piece, *Nature*, 480, 325 (2011) and a *Nature Physics News & Views* piece, *Nature Physics* 8 (January) (2012). The work in Granular Matter, above, was reviewed in Zhang et al. *Soft Matter* 6, 2892 (2010).

- The paper *Linear Shear in a Model Granular System*, by J. Ren, J. Dijksman, and R. Behringer, *Chaos* 21, 041105 (2011) represents a winning entry in the American Physical Society March Meeting, 2011, for the Gallery of Images.
- We have characterized the evolution of network properties under shear, as described in the paper *Percolation of Contacting Subnetworks on the Edge of Isostaticity*, D. Walker, A. Tordesillas, C. Thornton, R. Behringer, J. Zhang, and J. Peters, *Granular Matter* 13, 233—240 (2011).
- In collaboration with Antoinette Tordesillas, and using our photoelastic experiments, we have developed and tested a new stability method for characterizing granular materials, specifically at the level of clusters: Antoinette Tordesillas, Qun Lin, Jie Zhang, R. P. Behringer and Jingyu Shi, *Journal of the Mechanics and Physics of Solids* 59, 265–296 (2011). This paper outlines some of the key techniques that we will use throughout the present project.
- Percolating contact subnetworks on the edge of isostaticity, D.M. Walker, A. Tordesillas, C. Thornton, R. P. Behringer, J. Zhang, and J. F. Peters, *Granular Matter* 13 233-240 (2011).
- With post-doc Joshua Dijksman and Ph.D. student Jie Ren, we probed the evolution of forces, stresses, and networks in a special shear apparatus that allows us to apply simple shear over many strain cycles without creating shear bands, i.e. localized inhomogeneities in the density and strain. We have developed a clear picture of the effect of cyclic shear on granular systems near jamming. We showed earlier that the nature of jamming for frictional grains, is substantially different from what has been reported for frictionless grains. The previous picture of jamming (see Figure 2, below) was that jamming occurred at a lowest density, expressed in terms of the packing fraction, ϕ , via an isotropic stress state. This lowest $\phi = \phi_J$ (≈ 0.84 in 2D) is referred to as the jamming density. We showed that for frictional grains, jamming could be induced below ϕ_J through the application of shear strain, starting from a stress-free state. To put this into perspective, we can think of ϕ_J as corresponding to random close packing (rcp) and the lowest density, ϕ_S (≈ 0.78 in 2D) for which shear jamming to occur as corresponding to random loose packing (rlp). Continuing work supported by this project has shown that cyclic shear in the region between ϕ_S and ϕ_J produces a series of stress states that resemble an activated process, but in a space of stresses (not energies/temperatures). This is a highly novel finding that was reported in *Phys.Rev. Lett.* 110, 018302 (2013), and discussed in more detail below. Related aspects of this work are described in *Homogeneity and packing structure of a 2D sheared granular system*, *Powders and Grains*, 2013, AIP Conf. Proc. 1542, 527 (2013). From these measurements we deduced a new property for sheared systems at constant volume, the ‘Reynolds coefficient’ = R , that relates the pressure increase to the applied shear strain in the shear jamming regime. Specifically, the pressure, P , varies with the shear strain, γ , as $P = R\gamma^2$, which implies the absence of linear elasticity in the shear jamming regime. R is only a function of density, and it appears to have a strong power-law divergence at the isotropic jamming point. The cyclic shear experiments have led to the discovery of a novel convective state. The discovery of this state was only possible because our special apparatus allows us to carry out extended shear over many cycles without developing shear bands.
- We have pursued ways to represent shear jamming in terms of an order-disorder transition. With collaborator Bulbul Chakraborty at Brandeis University, we have shown that one way of representing the onset of order due to shear jamming involves a representation in

terms of force tiles, a concept that was initially pioneered by Maxwell in the 19th century. This work has recently appeared in [Phys. Rev. Lett. 111, 068301 \(2013\)](#).

- We have developed new experimental approaches which remove the effects of friction between the particles and the base of the experiment, as described in *Shear Jamming in Granular Experiments without Basal Friction*, Hu Zheng, Joshua A. Dijksman and R. P. Behringer, [Europhys. Lett. 107, 34006 \(2014\)](#).
- We have explored a novel mapping of the local (particle-scale) shear stress state onto a vector order parameter that is reminiscent of a magnetic spin. This mapping allows us to apply sophisticated techniques from statistical mechanics to explore spatio-temporal correlations as well as system-wide properties that are analogous to more conventional magnetic materials.
- The cyclic shear experiments also have allowed us to characterize shear-driven diffusion near the shear jamming transition. A paper is nearing completion on this (Dijksman et al. 2014).
- In different experiments, we have explored the effect of particle shape on the near-jamming properties of granular materials. This work has consisted of several types of experiments, including Couette shear (e.g. steady shear over large shear strains), and cyclic isotropic compression. We contrasted two particle shapes, namely elliptical particles and disks. Perhaps the most important difference between the response for different particle shapes was the fact that elliptical particles have extremely slow relaxation modes that are tied to the slow reorientation of the director (i.e. long axis). This work has resulted in two published papers,: Dynamics of Sheared Ellipses and Disks: Effects of Particle Shape, [Phys. Rev. Lett. 112, 1248301 \(2014\)](#) and Slow dynamics for elliptical particles under continuous shear and cyclic compression Somayeh Farhadi, Robert P Behringer, and Alex Zihao Zhu, [Powders and Grains, 2013, AIP Conf. Proc. 1542, 879 \(2013\); doi: 10.1063/1.4812072](#). A third paper has been submitted for publication. These studies formed part of the thesis of Somayeh Farhadi, who is now pursuing post-doctoral studies.
- In related experiments, supported by DTRA, we have characterized the granular dynamics involved in the impact of high speed objects on granular beds. This work has appeared in [Phys. Rev. Lett. 109, 238302 \(2012\)](#) and [Europhysics. Lett. 101, 64001 \(2013\)](#). The first of these papers was the subject of a Focus Story (Physics **5**, 137 (2012)) and was also featured in Nature (Research Highlights, Nature **492** 315 (2012)). More recent work has appeared in *Granular Impact Model as an Energy-Depth Relation*, Abraham Clark and R. P. Behringer, [Europhys. Lett. 101, 64001 \(2013\)](#)., *Granular Impact Dynamics: Fluctuations at Short Time-Scales*, Abram H. Clark, Lou Kondic, and R. P. Behringer, [Powders and Grains, 2013, AIP Conf. Proc. 1542, 445 \(2013\); doi: 10.1063/1.4811963](#), *Collisional Model for Granular Impact Dynamics*, Abram, H. Clark, Alec J. Petersen, R. P. Behringer, [Phys. Rev. E 89,012201 \(2014\)](#), *Nonlinear Sound During Granular Impact*, Abram H. Clark, Alec J. Petersen, Lou Kondic, and Robert P. Behringer, submitted (2014)
- We have developed highly novel experimental and computational techniques which allow us to characterize fully three-dimensional particle/granular systems. This work is described below. It has resulted in two papers: *Spanning the Scales of Granular Materials: Microscopic Force Imaging*, Nicolas Brodu, Joshua A. Dijksman and R. P. Behringer, submitted (2014), and *Imaging Soft Sphere Packings in a Novel Triaxial Shear Setup*, JoshuaA. Dijksman, Hu Zheng, and Robert P. Behringer, [Powders and Grains, 2013, AIP Conf. Proc.1542, 457 \(2013\); doi: 10.1063/1.4811966](#).
- The invited review article: Statistical Properties of Granular Materials near Jamming, R. P. Behringer, Daping Bi, Bulbul Chakraborty, Abram Clark, Joshua Dijksman, Jie Ren and Jie Zhang, [J. Stat. \(2014\)](#) reviews a significant part of research relevant to this project.
- We have explored structure and networks in Structural templates of disordered granular media, David M. Walker, Anoinette Tordesillas, Andrew Druckrey, Jie Zhang, R. P. Behringer, Edward Ando, Gioacchino Viggiani and Khalid Alshibli, submitted.

- In related work, we have probed the nature of granular stick-slip, and analogies to similar behavior for solid materials: Stick-Slip and the *Transition to Steady Sliding in a 2D Granular Medium and a Fixed Particle Lattice*, J. Krim, Peidong Yu, and R. P. Behringer, *Pure and Appl. Geophys.* **168**, 2259–2275 (2011).
- I was author/co-author on two review articles, one on Granular Packings, and the other on Avalanches, that will become part of an Encyclopedia of Granular Materials, edited by Scott Franklin and Mark Shattuck (to appear, 2014).
- The various cyclic shear experiments formed the basis for the Ph.D. thesis of Jie Ren. During the course of this work, she was awarded Duke University's Fritz London Graduate Fellowship Award in 2012. She completed her Ph.D. thesis in 2013 and is now employed by Merck, in a research position that will apply her skills at granular experiment and modeling to important issues related to pharmaceutical processing.
- PI R. Behringer was awarded the American Physical Society Jesse W. Beams Award for his work in granular materials.
- The PI was elected to the chair line of a new American Physical Society Topical Group on the Physics of Climate
- As noted, the novel exhibit (shown in the image below) using our photoelastic techniques was part of an overall exhibit at the Mueseum of Science and Industry in Chicago that received a Gold medal.
- Since September 2010: approximately 40 Invited talks, in addition to various contributed talks for professional meetings, for example March APS Meetings, APS Division of Fluid Dynamics meetings, Gordon conferences, etc.
- For instance,during July 2013, the PI was invited to give three plenary talks in 2013 at: 1) Powders and Grains, 2013, Sydney Australia, 2) The Physics of Glassy and Granular Materials, satellite meeting of STATPHYS25, Kyoto, Japan, 3) STATPHYS25, Seoul, South Korea, and two invited talks at universities in Shanghai, China, namely, SJTU, and Tongji University.
- The PI was co-organizer of a semester-long program during the Fall of 2014 at the Kavli Institute for Theoretical Physics, on out-of-equilibrium solid systems.
- The PI organized an international Pan-American scientific program on granular materials in La Plata, Argentina in August, 2014.
- The PI is co-founder and Editor-in-Chief (with Hans Herrmann and Stefan Luding) of the journal *Granular Matter*.
- The PI, with Profs. Lou Kondic, Konstantin Mischaikov, and Corey O'Hern, organized a mini-symposium on granular materials at the European Solid Mechanics Congress, held in Graz, Austria, July, 2012.
- The PI organized a special collaboration between granular modelers and experimentalists to make direct comparisons between experiment and simulation of granular flows. Supported by NSF-CBET and IFPRI.
- The PI co-organized a three week meeting at the Aspen Center for Physics on Fluctuations and Response in Granular Materials, May-June, 2012.
- The PI was chief Organizer of the international dynamical systems meeting *Dynamics Days* in January, 2011.

Details of experiments and key experimental results

I. Cyclic simple shear experiments

These experiments use a specially designed and unique experiment that allows us to apply *uniform* shear to a 2D sample of photoelastic disks, as in the figure below.

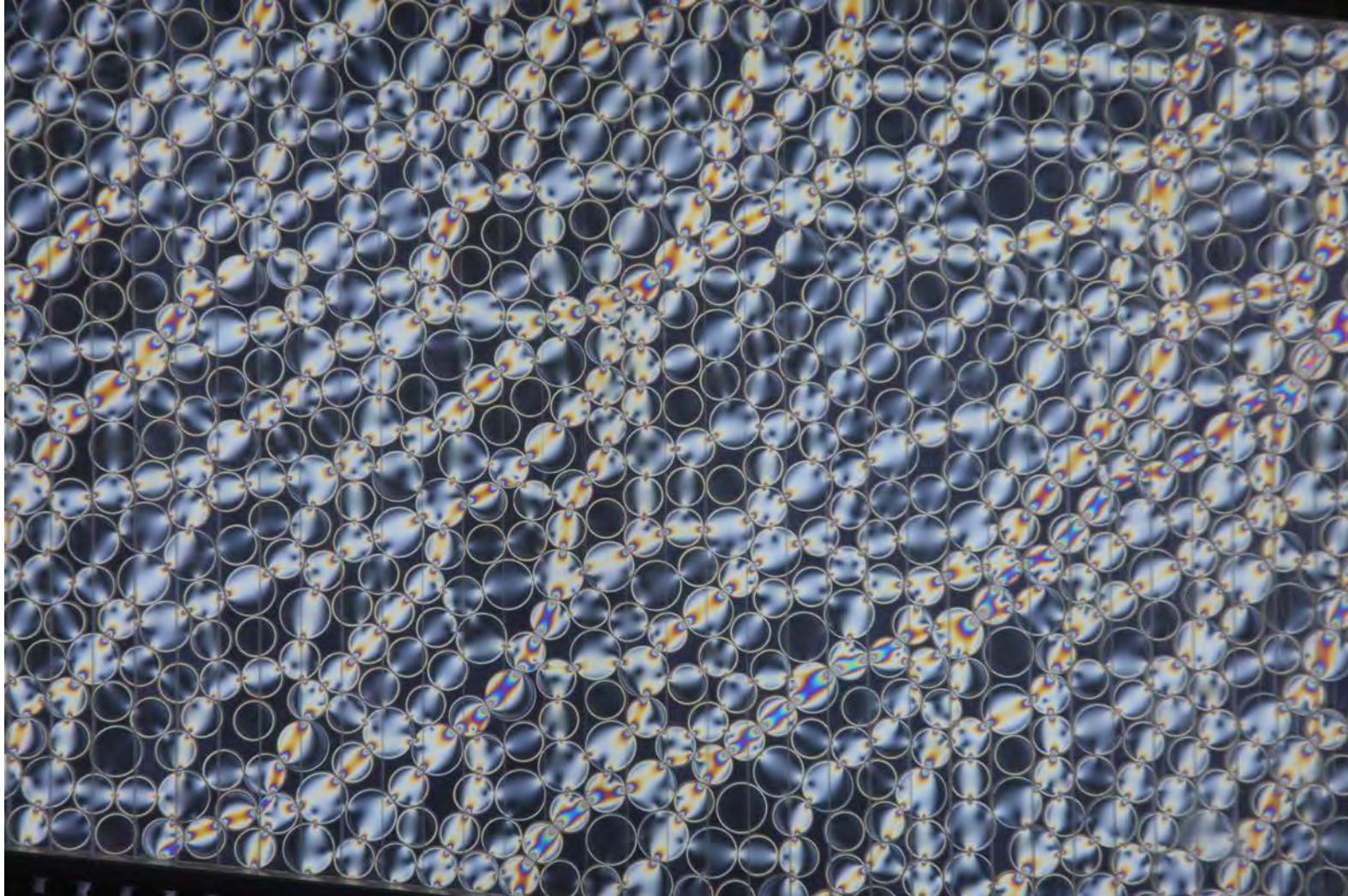


Figure 1: Photoelastic image from cyclic shear experiment-a shear jammed anisotropic state. Brighter particles carry larger force. See text for explanation of shear jamming.

In this photoelastic image, particles that appear brighter experience greater mean forces. Particularly evident is the force network sometimes referred to as ‘force chains’ that are indicative of jamming, in this case, jamming that is induced by shear. A jammed system is one that is solid, i.e. it has non-zero mechanical response to applied stress or strain.

Recent work on shear

A particularly important aspect of recent work has been the discovery and characterization of shear jamming. The figure below contrasts what was previously the understanding of the jamming diagram (a), and our newly discovered jamming diagram (b) that applies for frictional systems.

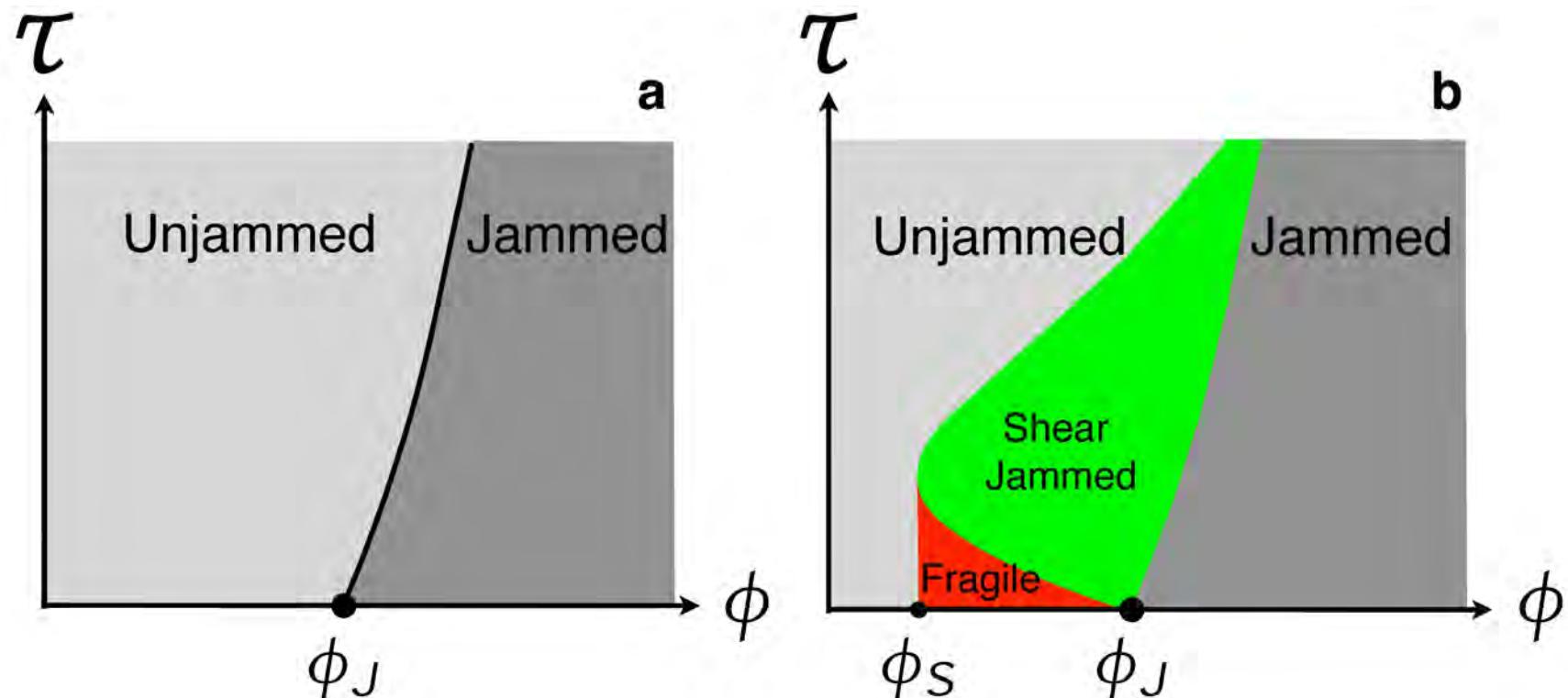


Figure 2: Sketches of (a) shear jamming diagram, following Liu and Nagel, and (b) Shear jamming diagram that we have discovered recently (for frictional granular materials). Fragile and shear jammed states are described in the text.

Note that the transition from unjammed to jammed states, or vice-versa in particulate systems refers to a transition from a relatively fluid-like state to one that is solid-like, hence mechanically stable. In the previous understanding of jamming, part (a) of the above

figure—due to Liu and Nagel (Nature, 1998), states at low shear stress, τ , and high enough density, or equivalently packing fraction, ϕ , are jammed. These states are to the right of the solid line in part (a). It is possible to unjam the system by reducing the density, or by applying large enough shear stress. Note that in this picture, there is a lowest $\phi = \phi_J$ such that the stress (all components) vanishes below this density in all cases. In our newly discovered jamming diagram, we find that it is possible to find zero-stress states below this density, and then to apply shear strain, with the result that the system arrives at a jammed anisotropic state. The fragile states in this diagram correspond to states that are highly anisotropic (i.e. have very long force chains, see images below) and are stable in one direction, corresponding to the force chains, but unstable, hence fragile in the direction transverse to the force chains (e.g. Figure 1). States in the green shear jammed regions are mechanically stable overall. Our recent work has focused on response and dynamics in the region covered by the red and green parts of the new jamming diagram. Specifically, in our special shear apparatus, we can produce shear-band-free dynamics for systems of quasi-2D particles. From these systems, we can experimentally determine complete particle motion, and particle-scale tensoral stresses. In the shear jamming regime, the pressure grows essentially as the square of the shear strain, γ , as shown in the figure below. Specifically, we find that $P = R(\phi) \gamma^2$, and R diverges at what appears to be ϕ_J . Note that only the density determines R below the isotropic jamming point.

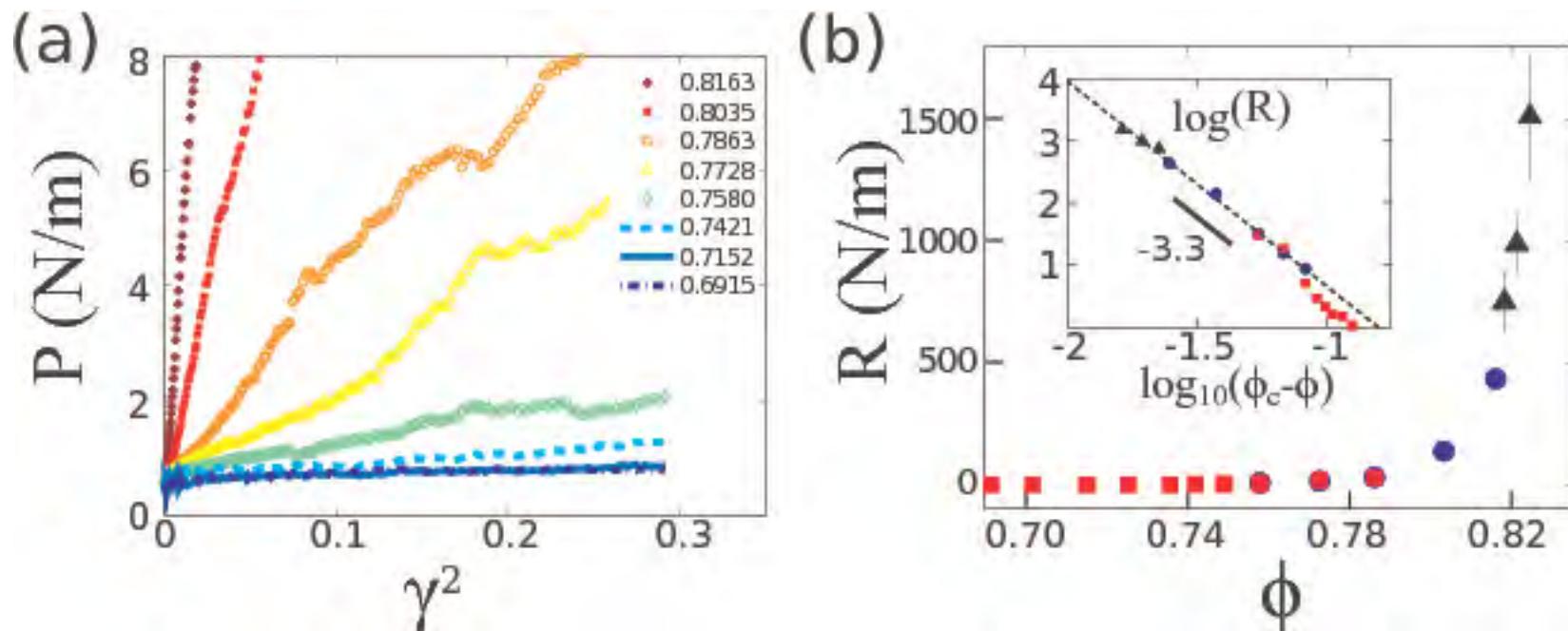


Figure 3: Pressure vs. square of shear strain (a) and corresponding slope, R , vs. packing fraction for simple shear in the shear jamming regime.

Under symmetric cyclic shear strain, there is rapid relaxation of the pressure and shear stress to limit cycle behavior, as in parts (a) and (b) of the figure below. However, if the cycles of shear strain are applied about a point that does not initially have symmetry, the granular system relaxes logarithmically slowly towards new symmetric behavior, as in part (c) of the figure below.

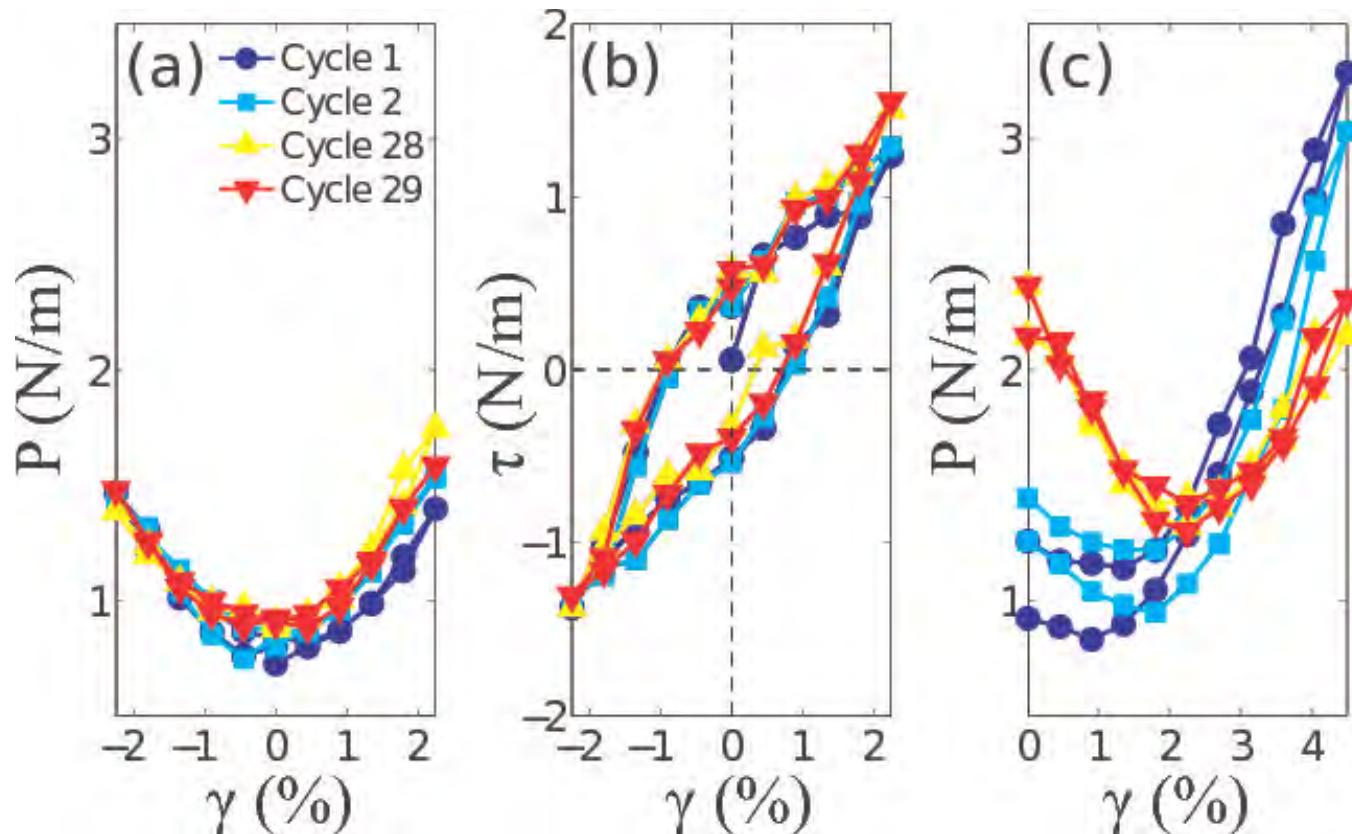


Figure 4: Example showing pressure and shear stress for symmetric cyclic shear stains (a and b) and P vs. shear strain about an initially non-symmetric point, (c). Note that in the latter case, the initial cycles (purple, blue) evolve towards symmetric cycles (red, yellow). The slow relaxation toward symmetric response in the pressure are characterized in the figure below, which shows first that this relaxation is logarithmically slow, and that the relaxation process can be represented in terms of a universal scaling function. Details are given in Ren et al. Phys. Rev. Lett. (2013) --above. Note the resemblance of (b) to the hysteresis loop of a magnetic system

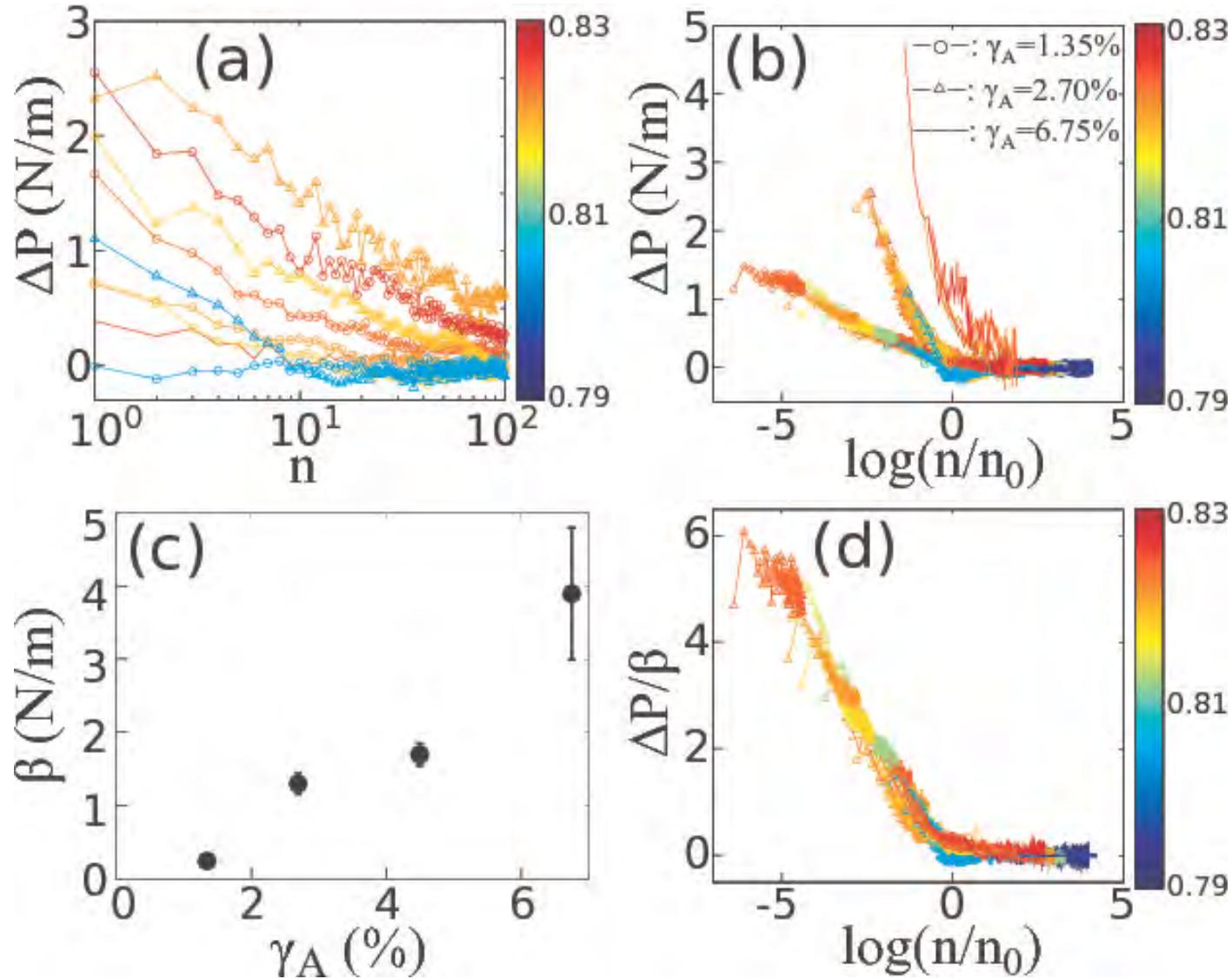


Figure 5: Relaxation for cyclic shear. The above figure shows: (a) the relaxation in ΔP , the pressure difference at the extremes of strain for each cycle, vs. cycle number, n ; (b) the fact that for a broad range of densities, the data have common scaling collapse for a given strain amplitude; and (d) that by normalizing by a strain-amplitude-dependent factor, β , all the data collapse onto a common relaxation law. The scaling factor, β , is given in part (c).

Extensive work on characterizing the network and dynamic properties of the relevant states from this cyclic shear experiment has been carried in collaboration with the group of Antoinette Tordesillas.

Mapping particle-scale shear stress to a vector order parameter: Although we proposed earlier that shear jamming is associated with an order-disorder transition, identifying the appropriate order parameter is not trivial. One approach that we have taken involves a mapping onto a space of forces, using a concept that dates to the time of Maxwell, called force tiles (Sarkar et al, Phys. Rev. Lett. **111**, 068301 (2013)). A different and particularly intriguing possibility is the idea that shear jamming can be tied in a simple way to the collective behavior of the force-scale shear stress (more properly the shear component of the force-moment tensor) that can be mapped to a vector order parameter. An example of a mapping from experimentally determined particle-scale shear stresses to the vector order parameter is shown in Figure 6, below. In Figure 6, the bars are coded by length and color to indicate their magnitude and direction. Note the similarity to a system of magnetic spins. This mapping supports a scalar product that is positive definite, such that two vectors (associated with individual grains) have an effective overlap given by this scalar product. The vector field is additive over the granular system (e.g. it is extensive), and is qualitatively analogous to the magnetization, M , of a ferromagnet near its critical (Curie) point. Shear strain (in a well controlled system) is “extensive” and is analogous to the applied magnetic field, H . For comparison, the hysteresis loop of Figure 4 (b) above is comparable to a similar loop for a magnetic system, M vs. H .

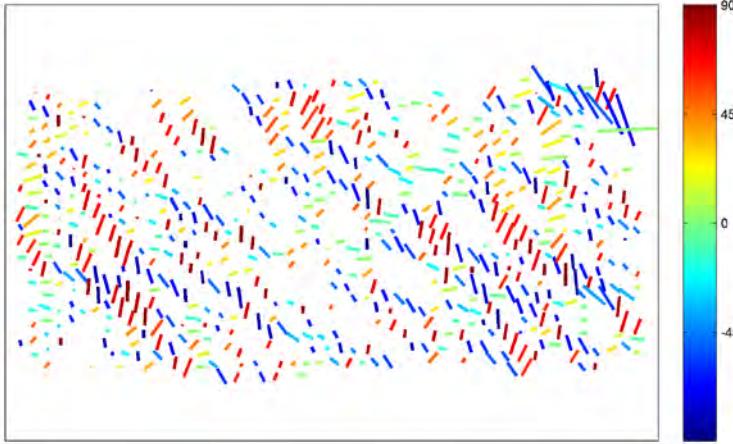


Figure 6: mapping of the shear stress states of individual particles onto vectors that resemble spins in a ferromagnet. Bars and colors indicate magnitudes and orientations. Red corresponds to a ‘spin up’ state, and blue to a spin down state. Spin up/down corresponds to local (particle-scale) shear stress with a principal stress orientation of $\pm 45^\circ$.

3D experiments: In new work, we have implemented 3D experiments, using the techniques discussed below, to probe states that are subject to shear or other strains in 3D. This project was begun in 2012. We have developed this experiment in two very substantial ways, with important contributions coming from experimental post-doc Joshua Dijksman and from theoretical/computational post-doc Nicolas Brodu. First, we have developed the laser scanning aspect of the apparatus, and physical devices for carrying out shear and axial compression experiments. Second, we have developed sophisticated computer algorithms that use the image slice data shown below to reconstruct not only the particles, but also the inter-particle contacts, and the extent of their deformation. From the deformations we infer the inter-particle forces. This means that we can characterize 3D granular states in a way that is comparable to what we have achieved with photoelasticity for 2D granular systems. The figures below illustrate important aspects of these experiments.

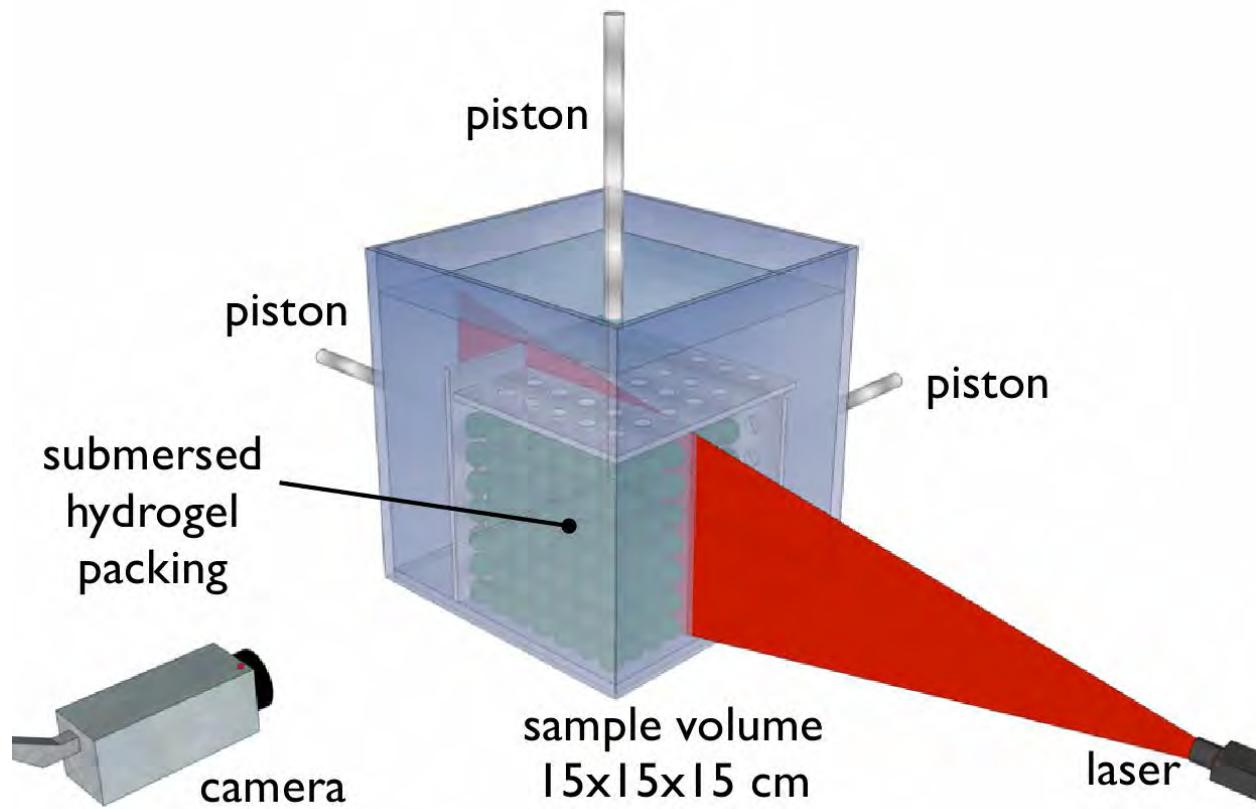


Figure 7: Schematic of 3D scanner experiment. The basic principle of operation of the 3D scanner is sketched here. Granular particles, which are transparent, are placed in an index-matched solution that also contains a small amount of fluorescent dye. Then, the system is illuminated from one side by a thin laser sheet which excites the dye in the solution. An image taken from the side then has bright regions where there is fluid in the path of the sheet, and dark regions where there are particles. It is also possible to reverse the dye scheme, and make the particles appear bright, and Figure 8 shows a typical slice image. The laser sheet is scanned laterally to produce a sequence of slices from which the 3D state can be reconstructed.

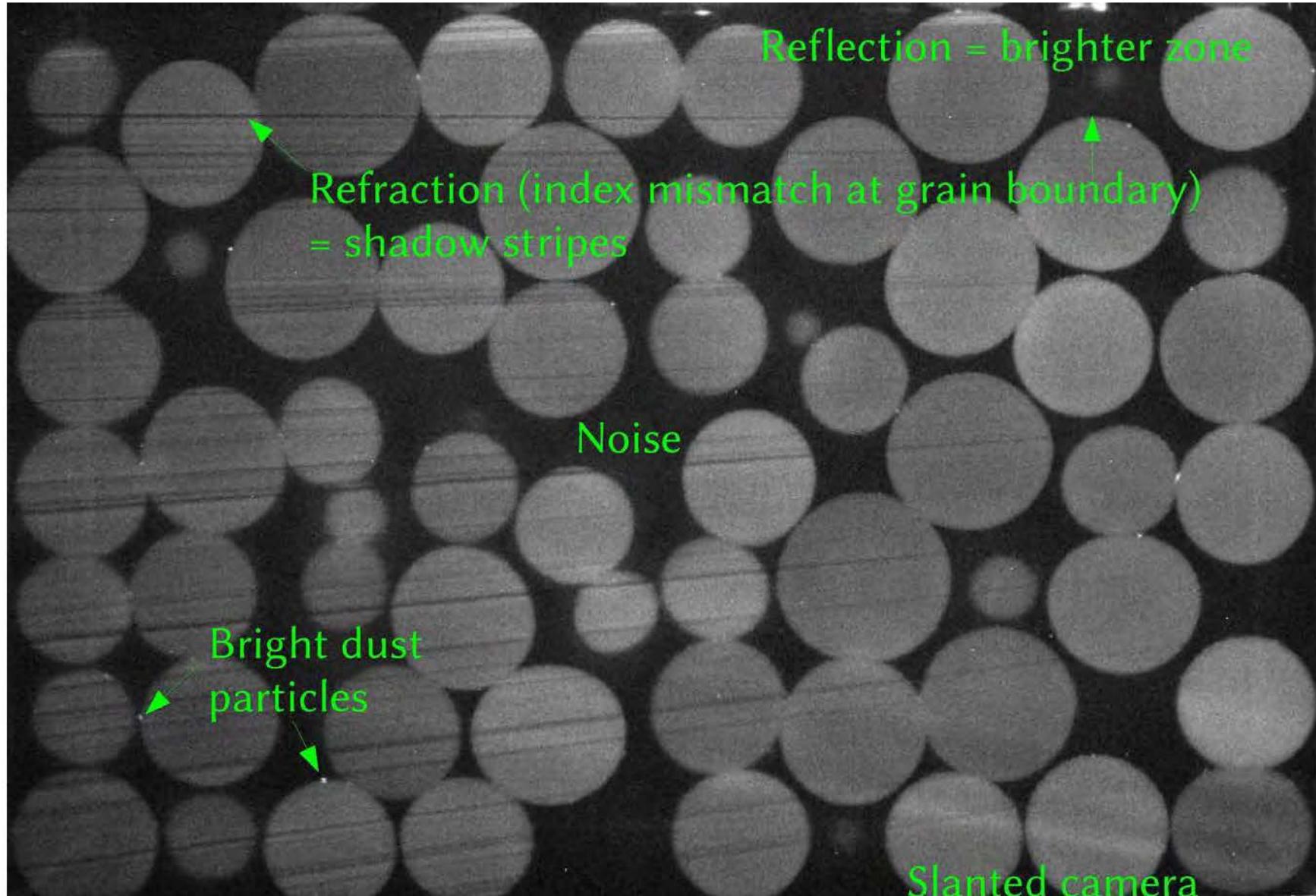


Figure 8: Example of a slice from a 3D scan. Here, the particles are hydrogel spheres, which are particularly easy to index match using water, since hydrogels are ~95% to 98% water. Some of the image artifacts that must be corrected afterward are indicated in green.

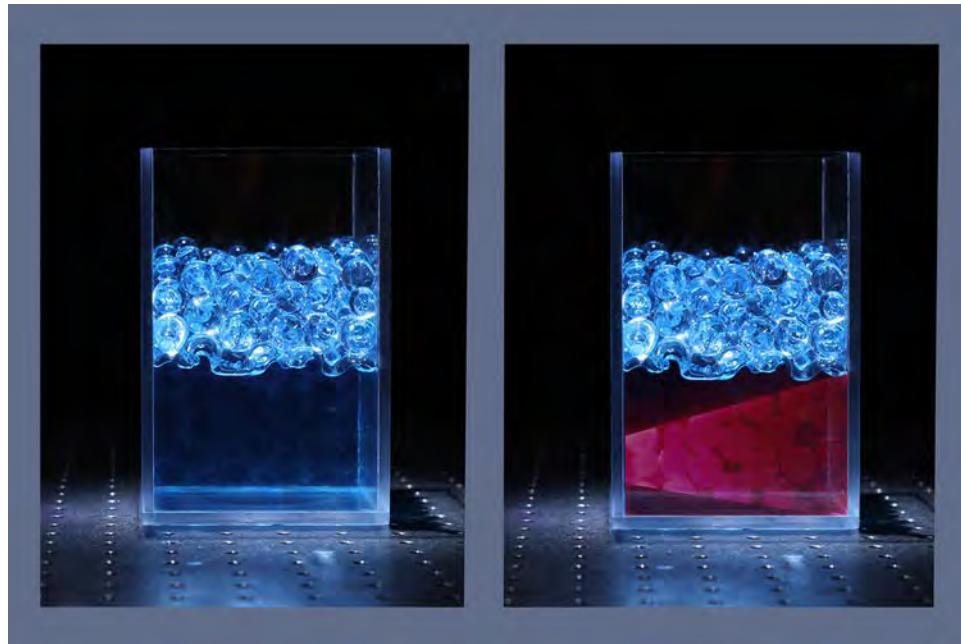


Figure 9: A more aesthetically pleasing image demonstrates the slice technique. Particles with an index of refraction that is close to the surrounding fluid ‘disappear’ when they are surrounded by fluid (left). The application of a thin laser sheet illuminates dye (in this case in the particles) but not the water (right).

The laser sheet is then swept across the whole system, and from the collection of resulting image slices, it is possible to reconstruct the location of every particle, and in principle, the amount of deformation at contacts if the particles are relatively soft. We are currently applying this technique to systems that are subject to either uniaxial compression, or to triaxial shear.

In order to reconstruct the 3D particles, it is necessary to process the slice images to account for non-uniformity of illumination, various light scattering effects, etc. Figure 10 shows the image of Figure 8, after corrections. The boundaries of the particles are shown in color. The process of sharpening the edges of the particles in one slice is aided by the fact that a particle in a given slice, which appears as a disk, corresponds to disks in neighboring slices.

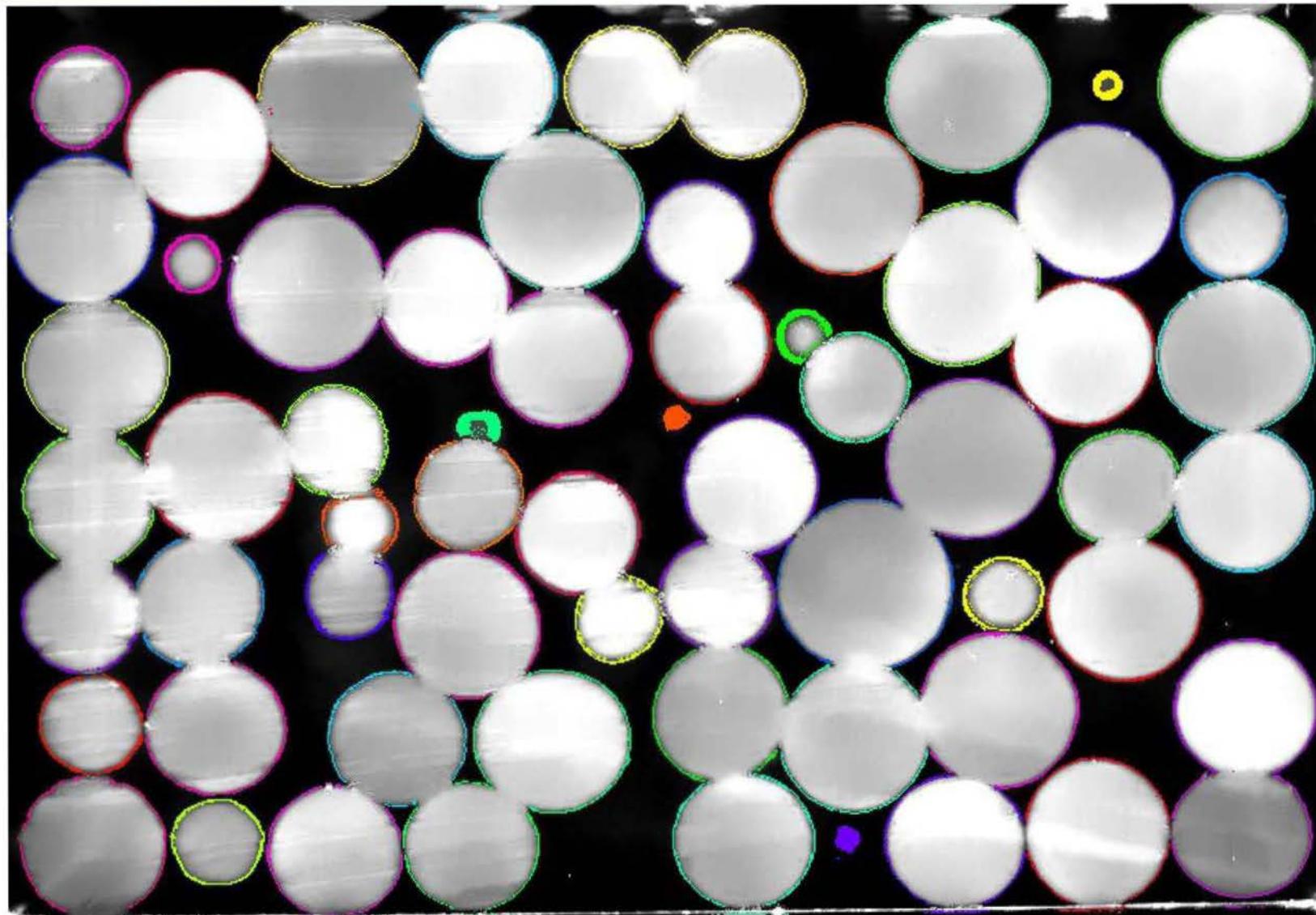
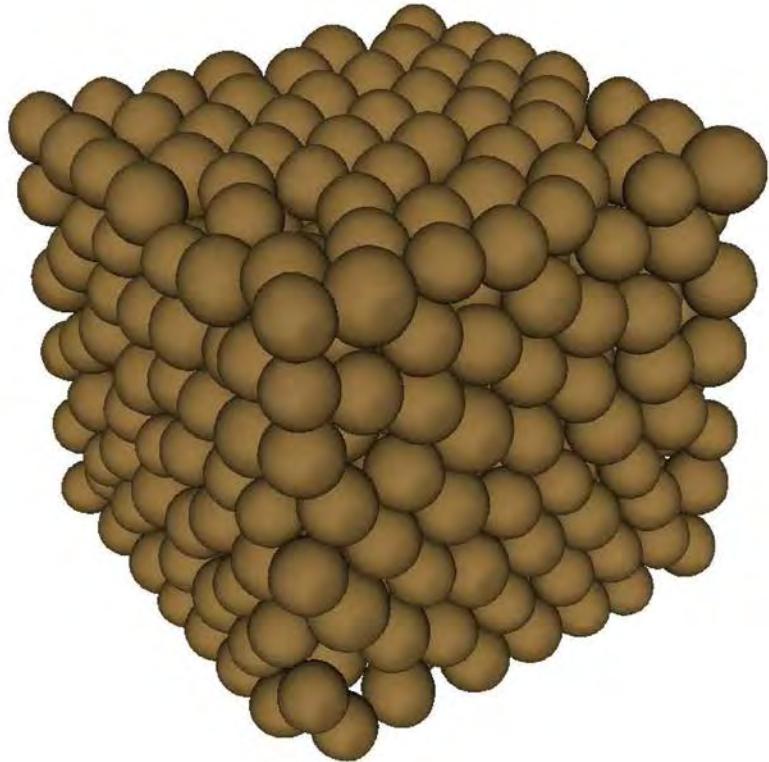
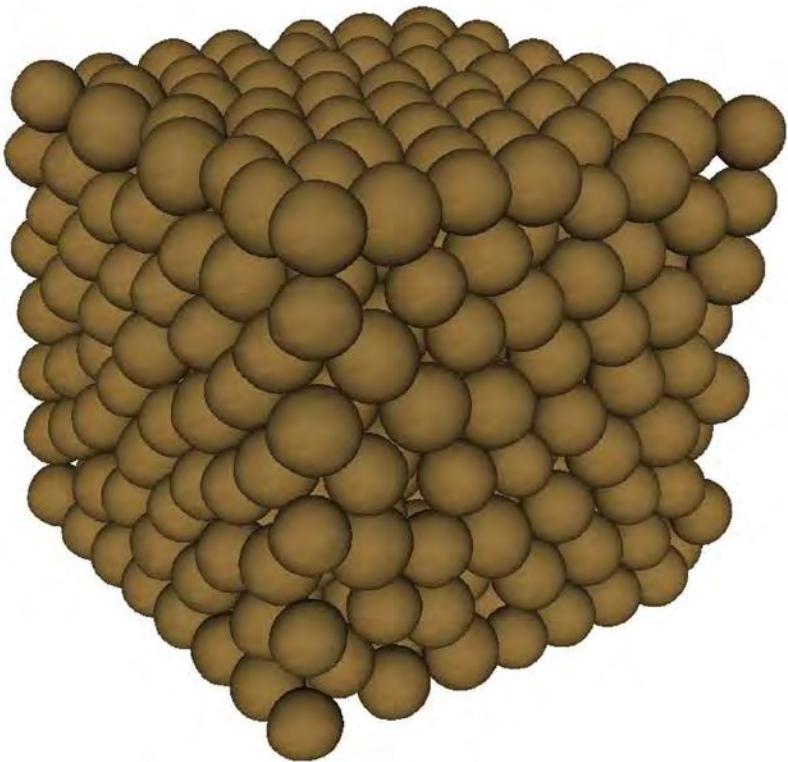


Figure 10: corrected version of the image of Figure 8, showing boundaries of particles in color. Note that some particles are just barely detectable because the laser sheet intersects only a small portion.

The next task is to use the collection of laser sheet images to reconstruct the actual particles and their contacts, as in Figure 11.



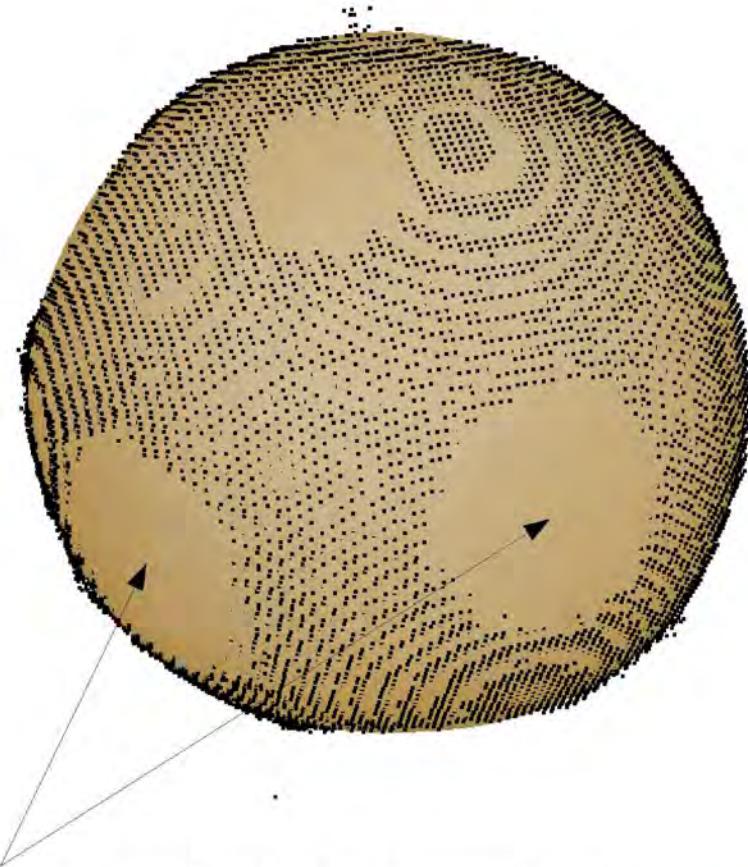
Left: toward the camera



Left: away from the camera

Figure 11: A fully reconstructed sample of particles, showing different views from the outside of the sample. These images pertain to an experiment in which uniaxial compression was applied.

An important aspect of the reconstruction technique is the fact that it allows us to resolve actual contacts. This yields the contact network, and more importantly, the deformations at contacts, which encodes the contact forces. In the figure below, we show a close-up of a particle, where the deformations due to adjoining grains are evident.



Contacts = no border between grains

Figure 12: Particle detected through slice reconstruction, where the free boundary of the particle (i.e. where there are no contacts) is indicated by points. The contacts with adjacent particles are regions without points. The radius of a contact encodes the contact force there.

Once we have the contact forces, we can construct a force network. In particular, we can construct the network of the largest forces, for various thresholds. The following figure shows a perspective view of the particles that carry the strongest 5% of the forces.

- Border/surface detection is actually performed in full 3D with home-made dedicated algorithms.

- ⇒ Better than state of art generic edge-detection routines. Very few outliers.

- The grain shape is described in a triangular spherical B-spline basis (no poles + good isotropy).

- Non-linear fit to the border data + convexity constraint
⇒ good regularization + no more outliers.

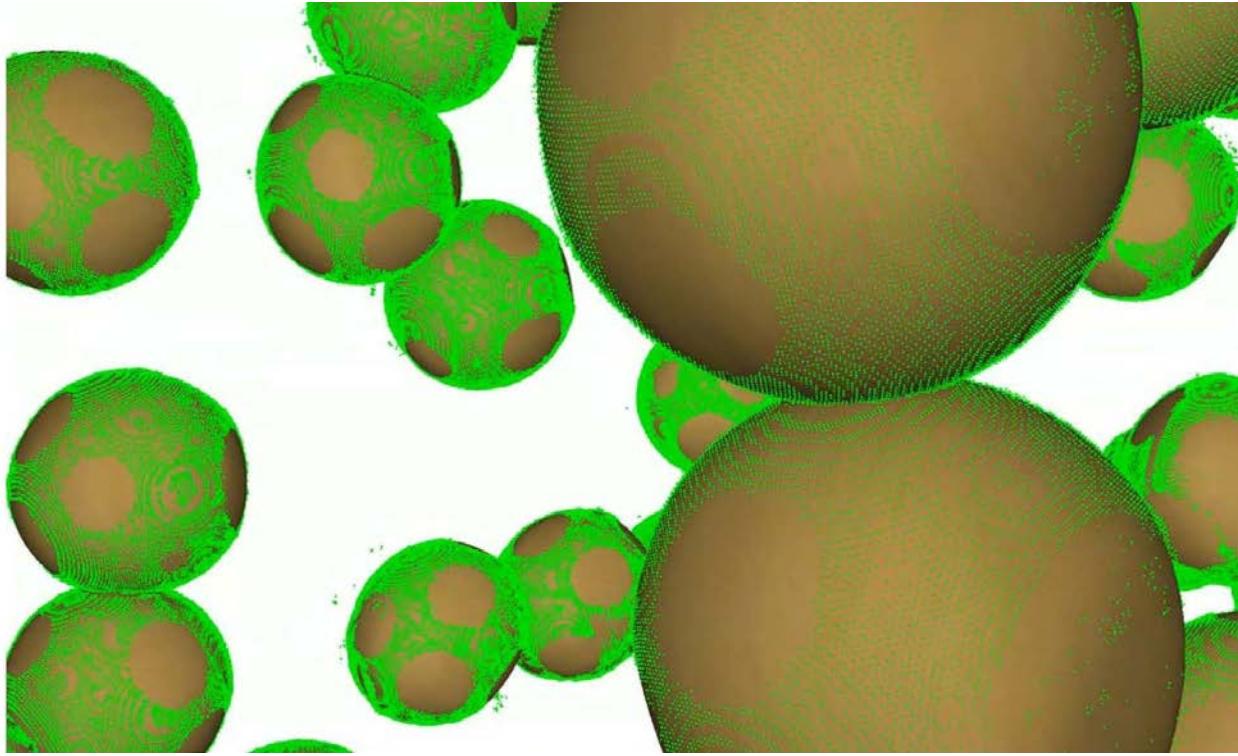


Figure 13. Internal view showing part of the system of approximately spherical hydrogel particles with the largest forces per particle (particles with the strongest 5% of force).

From the forces and contact locations, it is then possible to reconstruct the local stresses. As a test of the accuracy of this process, we carried out multiple cycles of uniaxial compression. At regular times during each cycle, we: 1) carried out a scan, 2) reconstructed the particles, 3) determined the forces at particle contacts, 4) determined the local stress, 5) Integrated this stress across the system to determine the force on the compressing boundary, 6) compared the particle-scale computation of the force to the force measured with a transducer that is attached to the piston providing the uniaxial compression. We compare the boundary forces from the particle reconstruction method and the force transducer in Figure 14. There is a single adjustable parameter for the particle-scale force computation, namely the Young's modulus of the hydrogel spheres. We adjust this single parameter to match the two force determinations. The result is a very good agreement between the two methods. This gives us considerable confidence in our image analysis/force reconstruction approach. A paper on this work, N. Brodu, J. Dijksman and R. Behringer, has been submitted recently.

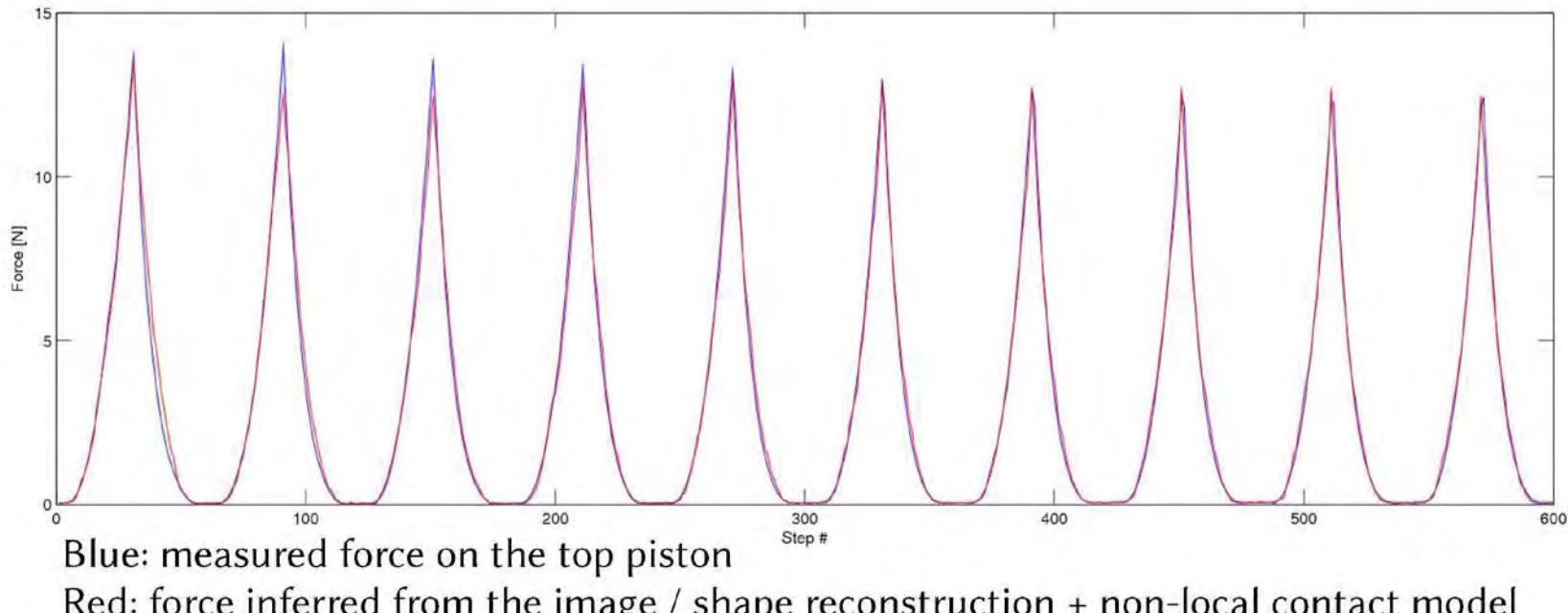


Figure 14: Forces on the upper surface of a uniaxially compressed system of hydrogel particles. Blue corresponds to the force measured by an external transducer. Red corresponds to stresses that were determined from the inter-particle forces. The agreement between the two method gives considerable validation to the particle reconstruction method.

II. Slider friction experiment

We have also developed and used a slider/friction experiment has yielded high quality information about the nature of granular stick-slip under surface traction. In particular, these results should lead to new insights into what is meant by granular friction. These data show that granular materials fail under traction by a process that dissipates energy relatively deeply in the material, by forming and breaking force chains, rather than just at the surface. In addition, the data characterize in detail the nature of granular slip and may suggest ways to maximize traction/avoid slip under a vehicle. These experiments also inform researchers about the dynamics of stick-slip in earthquake fault zones. In this regard, the energy losses for stick-slip events are power-law distributed, in analogy to the Gutenberg-Richter law for earthquake magnitudes. However, the power-law that we observe for the probability distribution function (PDF) for energy losses has an exponent of about 1.2, whereas the typical GR exponent is about 5/3 (when represented as a PDF for energy losses).

More specifically, the experiments consist of a vertically oriented layer of photoelastic disks (as in the photographs below). These disks constitute the granular material, and provide detailed information on the evolution of forces within the material at the particle scale. A solid object, the slider, is pulled across the top of this layer by a machine that moves at constant speed, V , and that connects to the slider through a spring of force constant k_p . The force on the pulling spring is measured as a function of time. In addition, coordinated video data is obtained for the region of the material that is under the slider. This is, in essence, a classic slip-friction device, with the novel feature that there is correlated information on the stick-slip dynamics of the slider/puller, and the internal structural response of the material. The experiments connect the micro-meso-scale behavior observed by photoelasticity, with the macroscale response of the slider. For the slow pulling regime that we have considered to date, classical Coulomb friction makes several simple predictions: 1) As the pulling device moves forward, the slider should stick until the pulling spring exerts a force given by $\mu_s Mg$. The slider should then slip with a lower kinetic friction coefficient, μ_k , for half a period for a harmonic oscillator of frequency $\omega = (k_p/M)^{1/2}$. The slider should then come to rest, and remain stuck until the pulling force again reaches $\mu_s Mg$. Thus, stick-slip should be periodic. In the experiments, the stick-slip behavior is in fact stochastic, with relative broad distributions for the friction coefficients, slip times, etc. The motion is not periodic. As noted, the energy losses associated with a slip event are distributed as a power-law, in analogy to the famous Gutenberg-Richter law, but the exponent differs somewhat from the classical value. The energy and time scales are nevertheless set by $(Mg)^2/k_p$, and ω^{-1} , respectively. The photoelastic video data yield several important results. Slip is typically initiated by the failure of a force chain, which can lead to the failure of other chains as the spring load falls on the remaining chains. And, creep occurs due to small structural rearrangements in the force network that precede slip.

- In collaborative work with Antoinette Tordesillas, we have characterized the dynamics and network properties of the observed stick-slip. This analysis shows in particular, that the resulting dynamics may be expressed in terms of a relatively low-dimensional attractor: *A complex systems analysis of stick-slip dynamics of a laboratory fault*, *Chaos* **24**, 013132 (2014)

With Jackie Krim, from the NC State University Department of Physics, we have modeled the transition from stick-slip to sliding dynamics, and compared the grain-scale dynamics of the present experiments to what occurs for atomic-scale stick-slip. (Prof. Krim is an expert in the latter.) We find that this transition, seen for granular materials, is remarkably similar to what is found in the atomic friction case. (Krim et al.. *Pure Appl. Geophys.* **168**, 2259-2275 (2011)). It is also of note that stick-slip phenomena have analogues in a range of different systems, including avalanches, Barkhausen noise in magnetic systems, to name a few. With Prof. Karin Dahmen of the University of Illinois, Urbana-Champaign, the PI has written a review of avalanches in slowly sheared disordered materials (to appear in an encyclopedia of granular physics, edited by Scott Franklin and Mark Shattuck).



Figure 15: Side views of granular stick-slip experiment, showing granular channel filled with photoelastic particles (left) and cart with arm to carry the high speed video camera.

Technology Transfer

- Technology transfer of an unusual kind has been achieved through collaborations with Evidence Design, and the Museum of Science and Industry in Chicago. The PI has worked extensively with museum personnel and developers to create a novel and exciting new exhibit that uses the photoelastic technology. The overall exhibit, of which our project was one part, Science Storms, was the Gold Winner in the Best Museum Environment category of the Event Design Awards. Figure 16 shows the exhibit.



Figure 16: Exhibit at the Museum of Science and Industry, Chicago, that was developed with collaborative input from the PI.