Chapter 1: Probability

1. Sample space: all possible events for random experiment
2. Event: Subset of S
3. Probability Set Function (p.s.f)
4. Properties of p.s.f
5. Boole's Inequality
6. Conditional Probability
7. Law of Total Probability
8. Bayes' Theorem
9. Indepentdent Events
10. Mutually Independent Events

1. Sample Space

(1) Finite
(2) Countable
(3) Uncountable

2. Event

3. Probability Set Function

Let $B = \{A_1, A_2, \cdots\}$
(1) $P(A) \geq 0$ for all $A \in B$
(2) $P(S) = 1$
(3) If $A_1, A_2, \cdots, \in B$ (i.e. $A_i \cap A_j = \emptyset \quad \forall i \neq j$) are pairwise mutually exclusive, then $P(\cup_{i=1}^{\infty} A_i) = \sum(A_i)$

4. Properties of p.s.f.

(1) $P(S) = 1$
(2) $P(\emptyset) = 0$
(3) $P(A) \leq 1$
(4) $P(A \cap B^c) = P(A) - P(A \cup B)$
(5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(6) If $A \leq B$, then, $P(A) \leq P(B)$

5. Boole's Inequality

If $A_1, A_2, \cdots$ is a sequence of events, then

$$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i)$$

6. Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

7. Law of Total Probability

Let $B_1, B_2, \cdots B_n$ is a collection of mutually exclusive and exhaustive events (i.e $\sum_{i=1}^{n} B_i = S$)

Then $P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A|B_i)$

8. Bayes Theorem

Let $B_1, B_2, \cdots B_n$ is a collection of mutually exclusive and exhaustive events (i.e $\sum_{i=1}^{n} B_i = S$)

Then

$$P(B_i|A) = \frac{P(B_i) \cdot P(A|B_i)}{\sum_{j=1}^{n} P(B_j) \cdot P(A|B_j)}$$

9. Independent Events

A and $B$are independent, then

$$P(A|B) = P(A)$$
$$P(A \cup B) = P(A) \cdot P(B)$$

10. Mutually Independent Events

Suppose $A_1, A_2, \cdots, A_n$ are events defined on S,
for any $i_1, \cdots i_k$ from 1 to n, we have

$$P(A_{i_1} \cup A_{i_2} \cup \cdots A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_k})$$