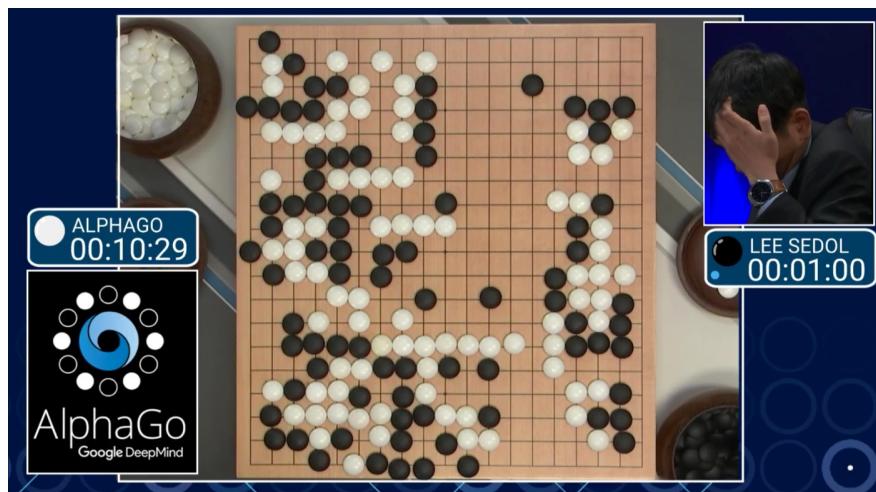
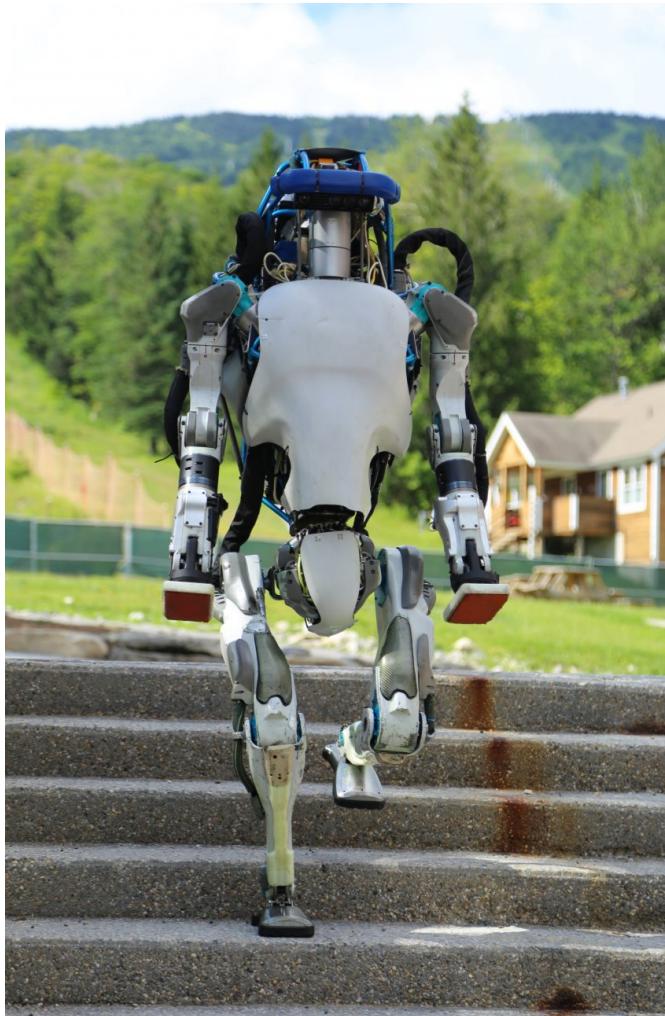
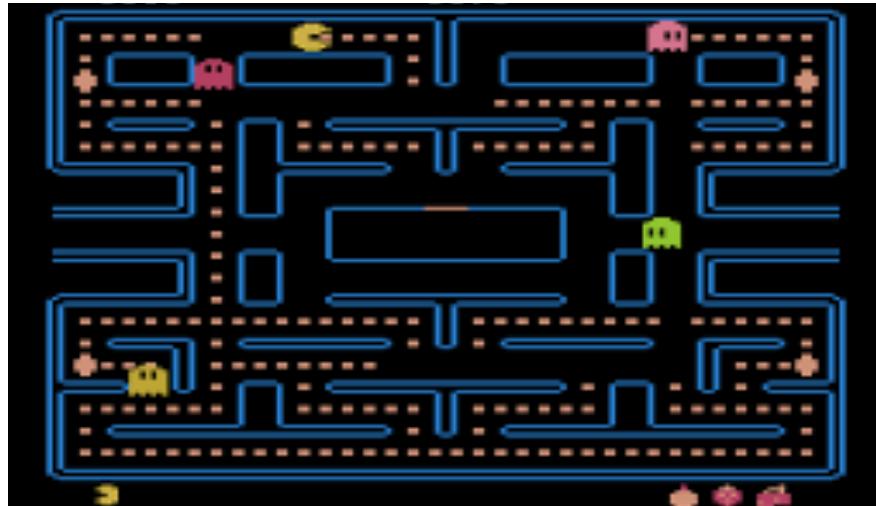


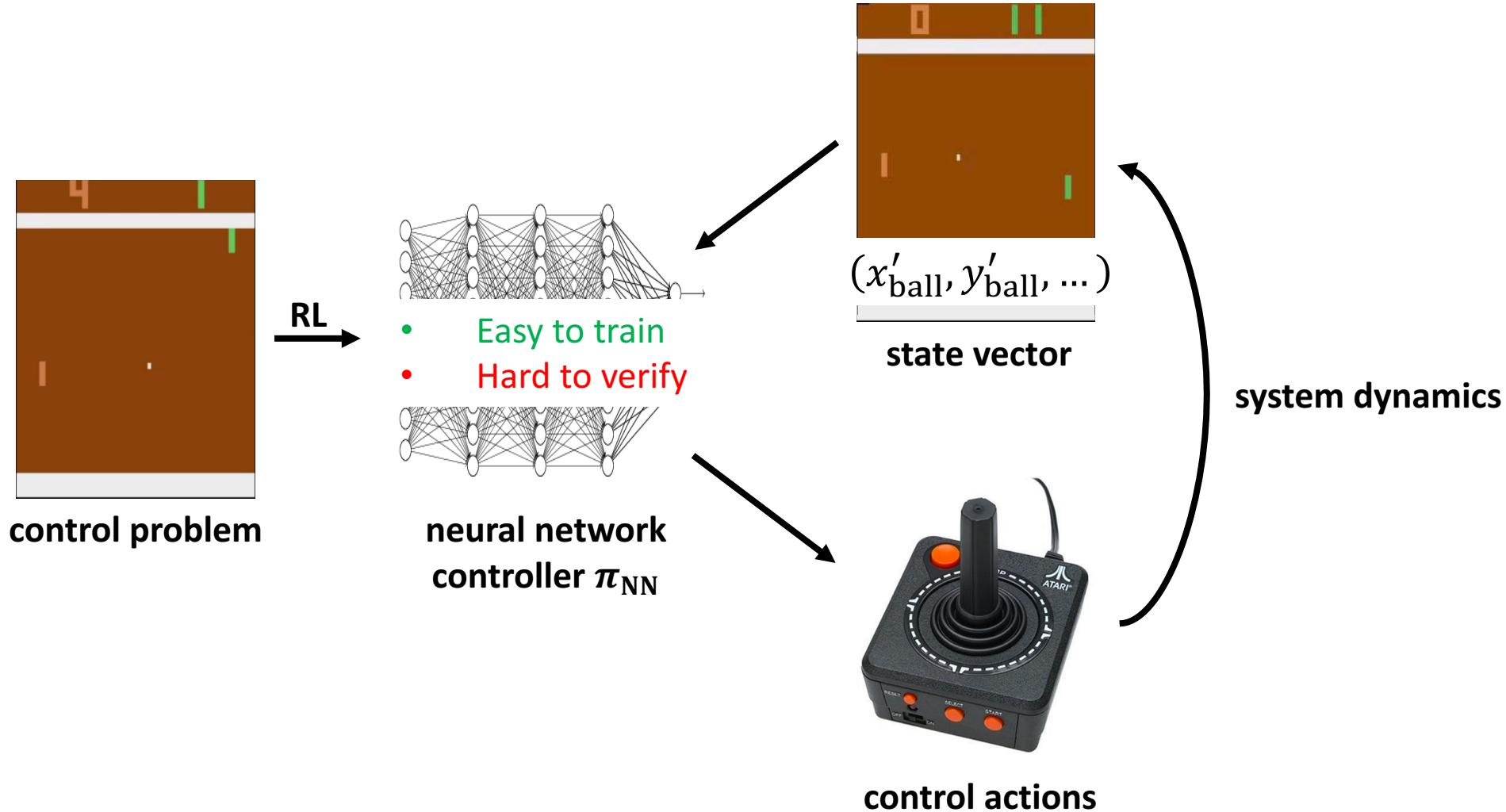
Verifiable Reinforcement Learning via Policy Extraction

Osbert Bastani, Yewen Pu, Armando Solar-Lezama

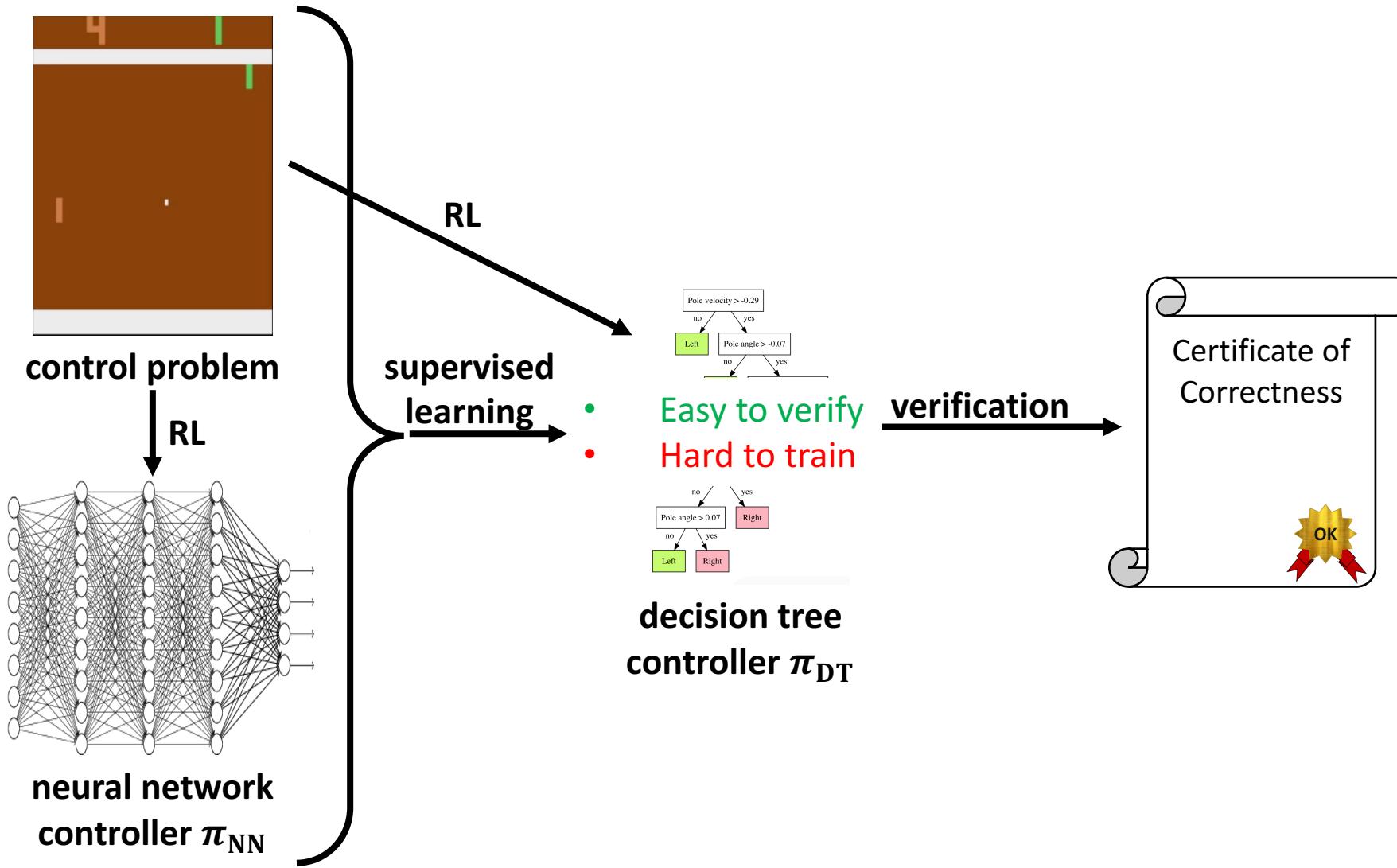
Deep Reinforcement Learning



Deep Reinforcement Learning



Our Approach



Background

Imitation Learning

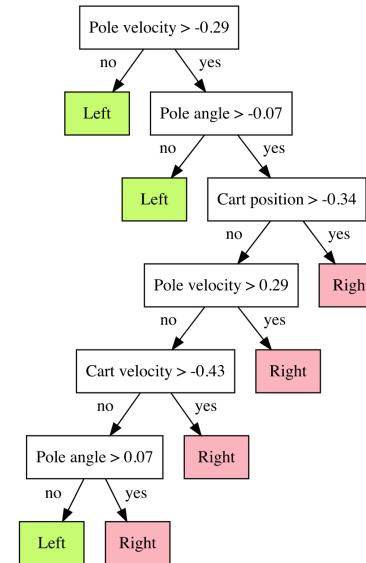
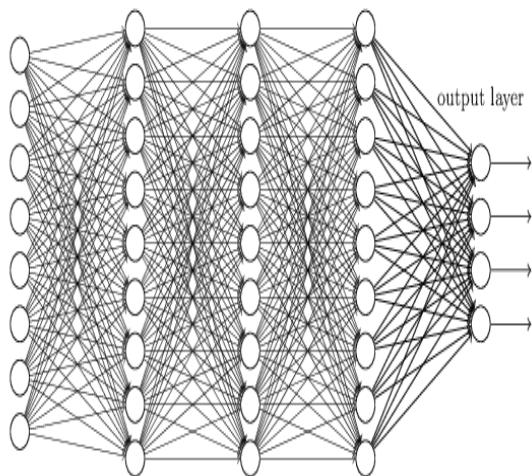


Demonstrations from Human Expert

Controller

Abbeel & Ng 2004

Imitation Learning

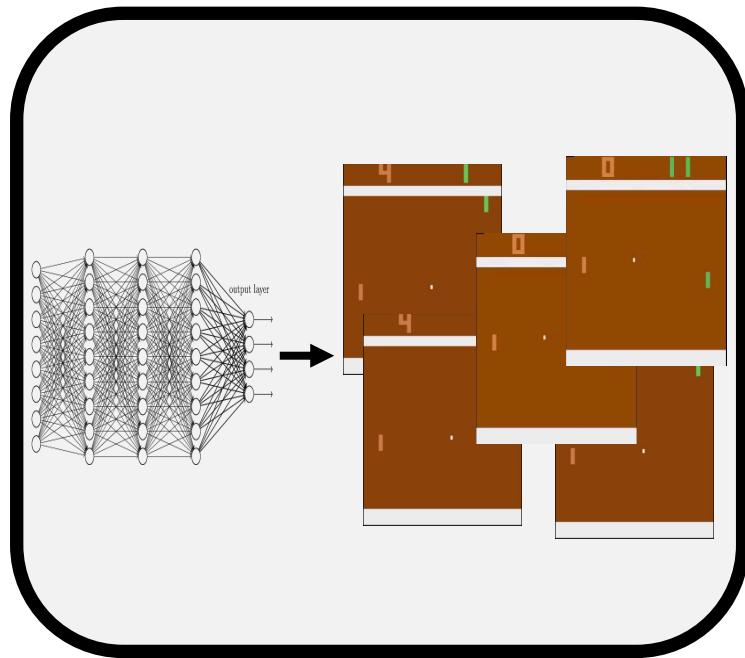


Demonstrations from Neural Network

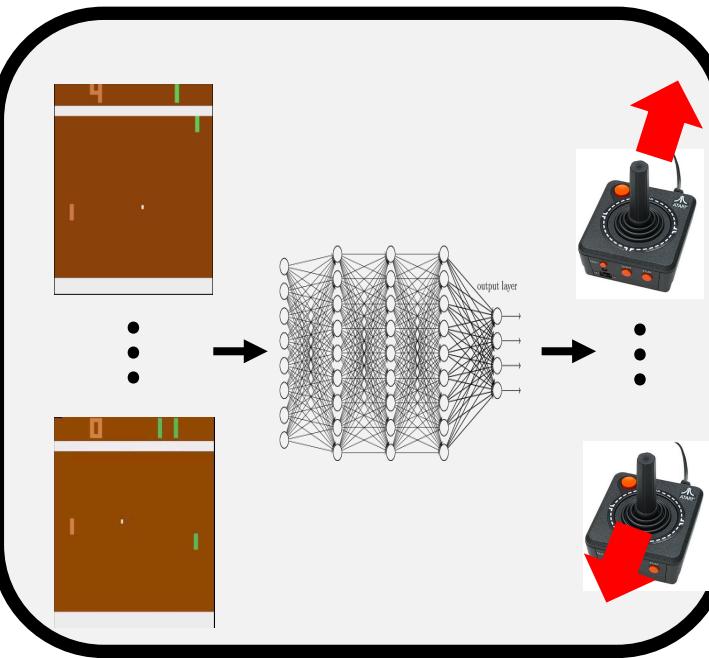
Decision Tree Controller

Abbeel & Ng 2004

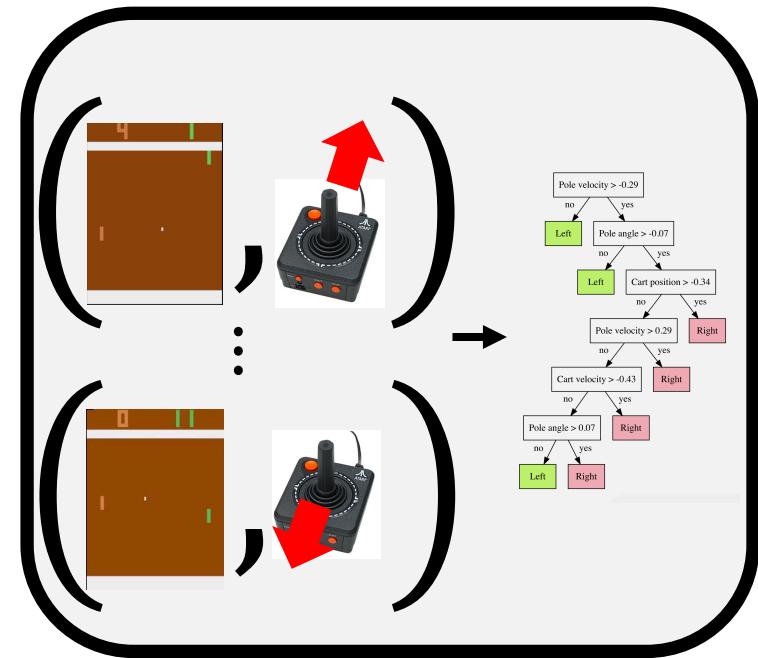
Imitation Learning



Step 1: Use NN to generate states



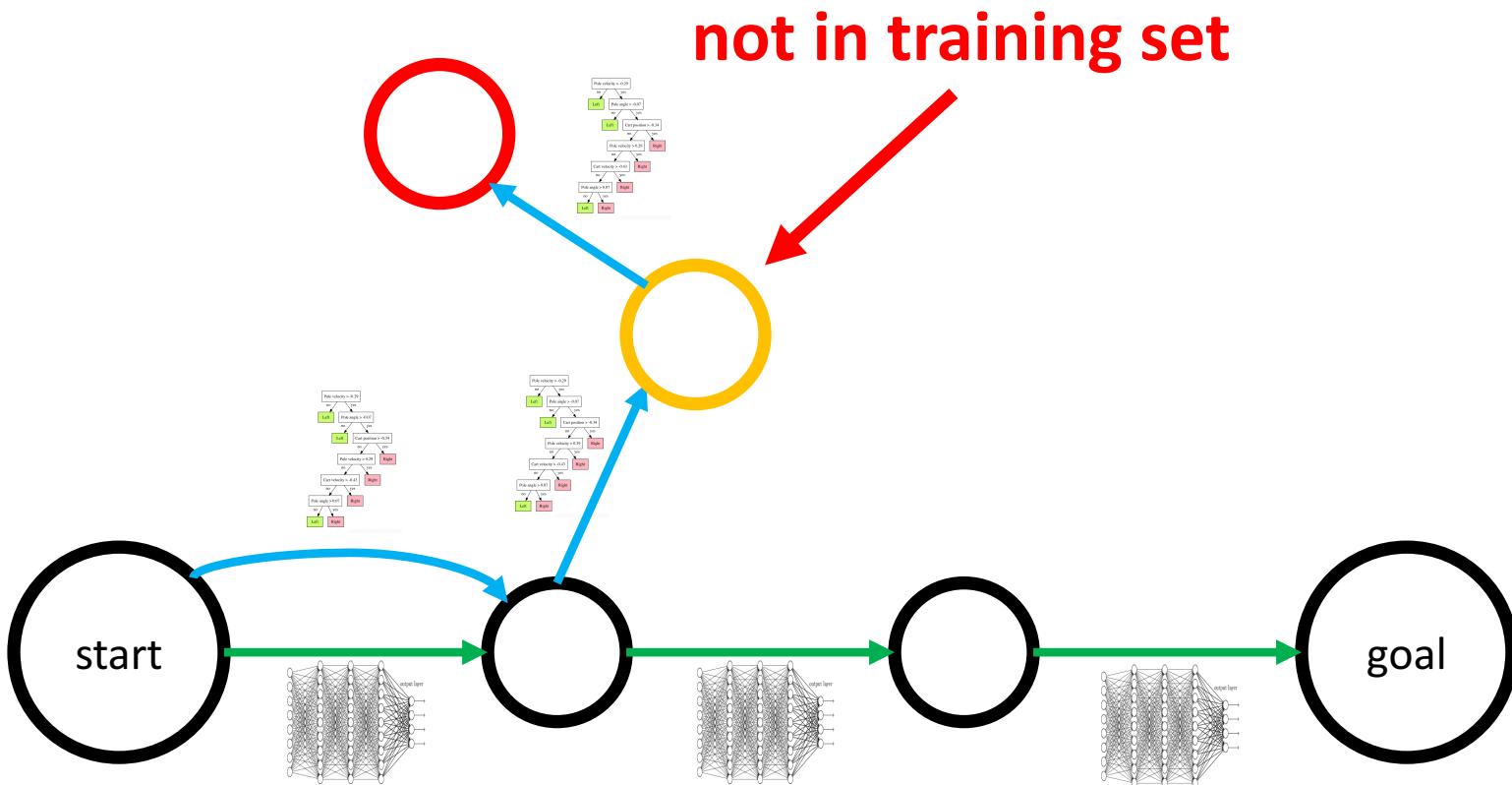
Step 2: Use NN to obtain actions



Step 3: Use supervised learning
to train a decision tree

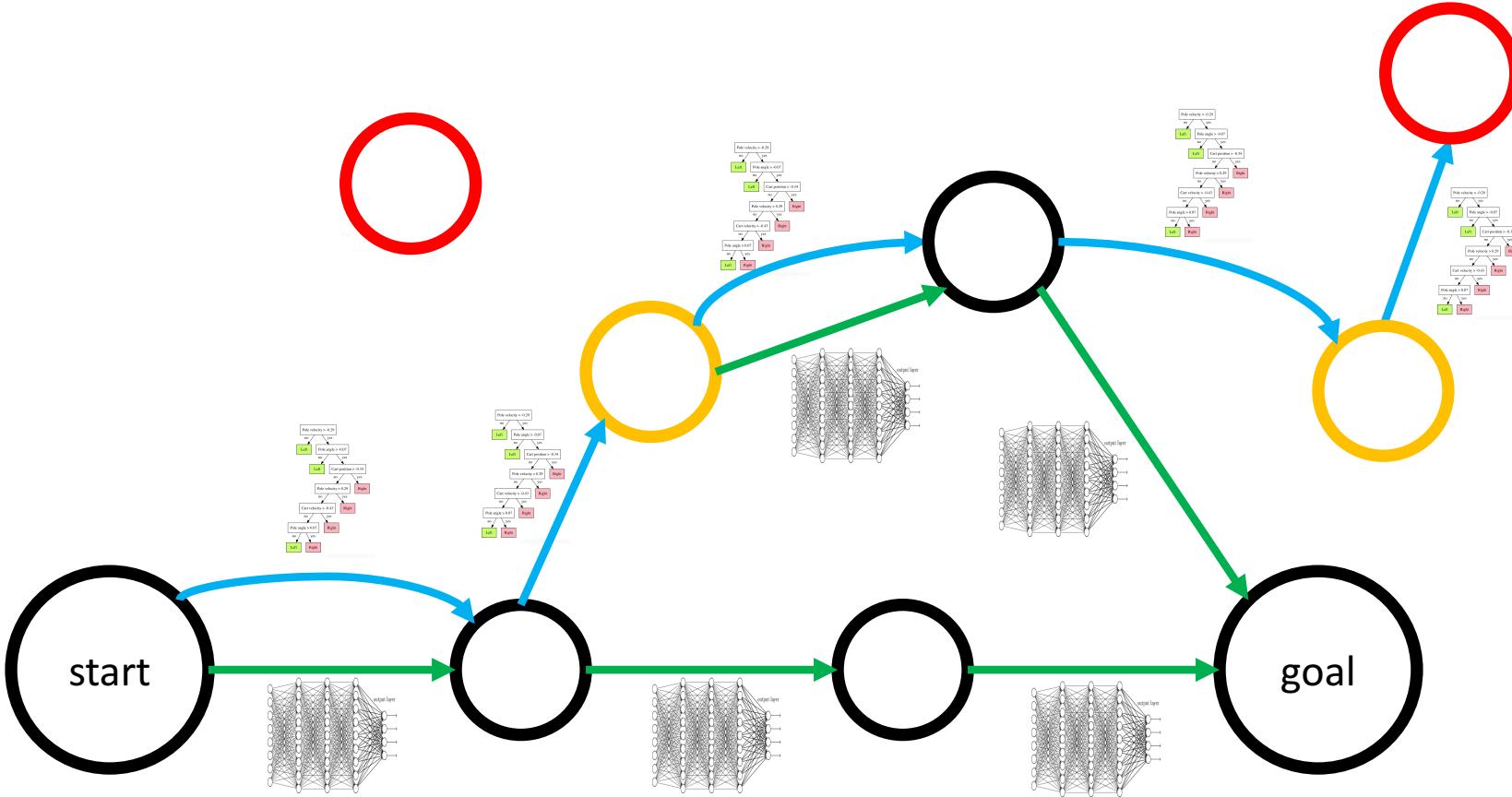
Ross & Bagnell 2011

Imitation Learning



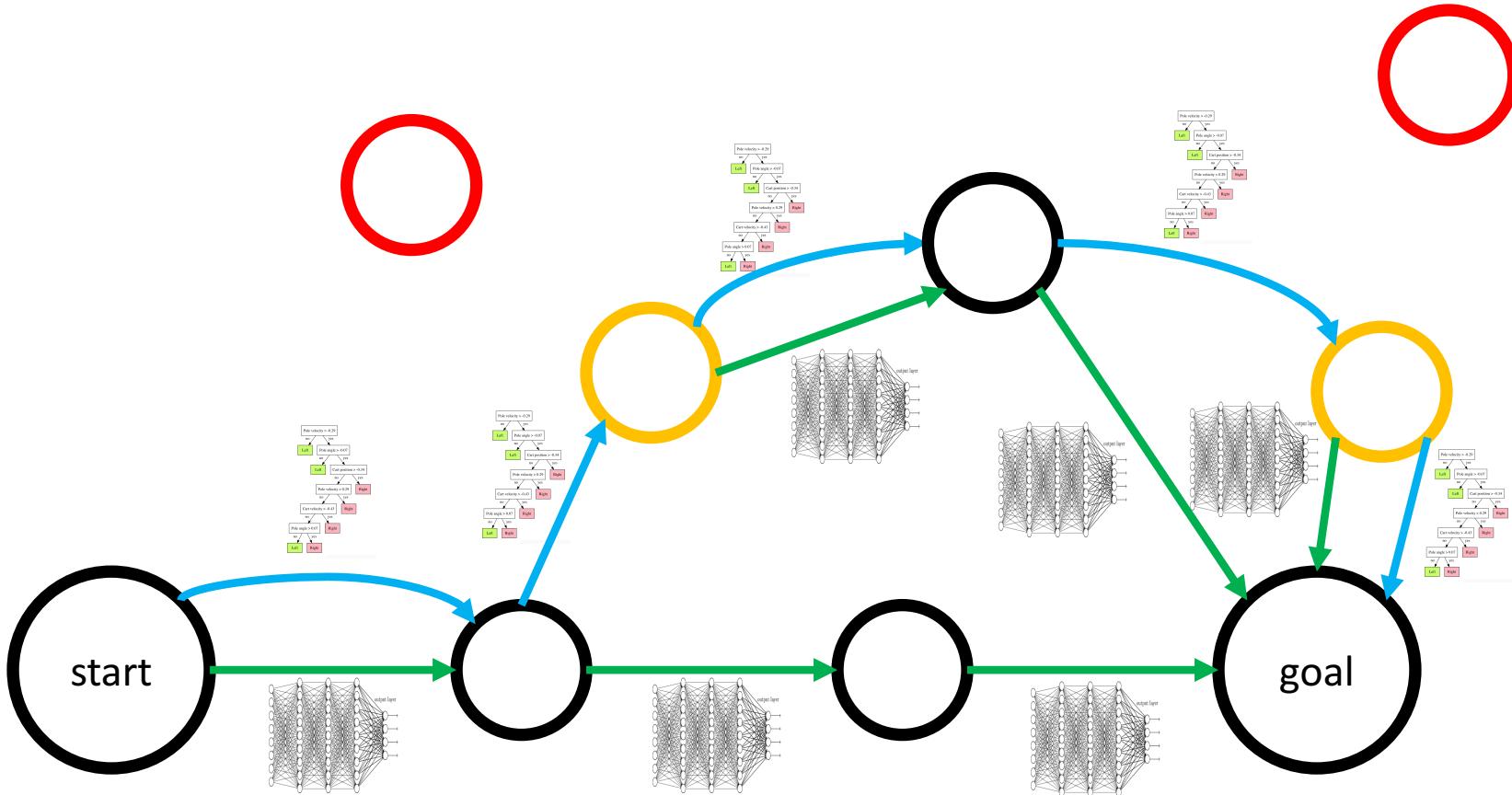
Ross & Bagnell 2011

Dataset Aggregation (DAgger)



Ross & Bagnell 2011

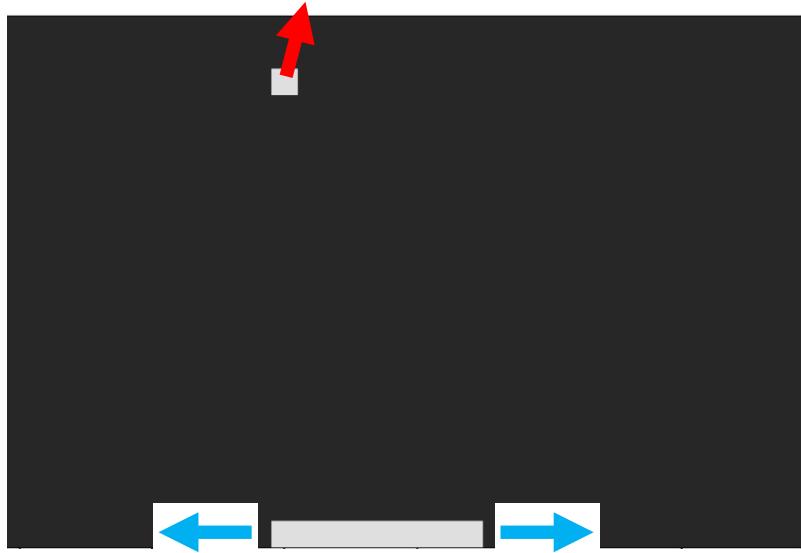
Dataset Aggregation (DAgger)



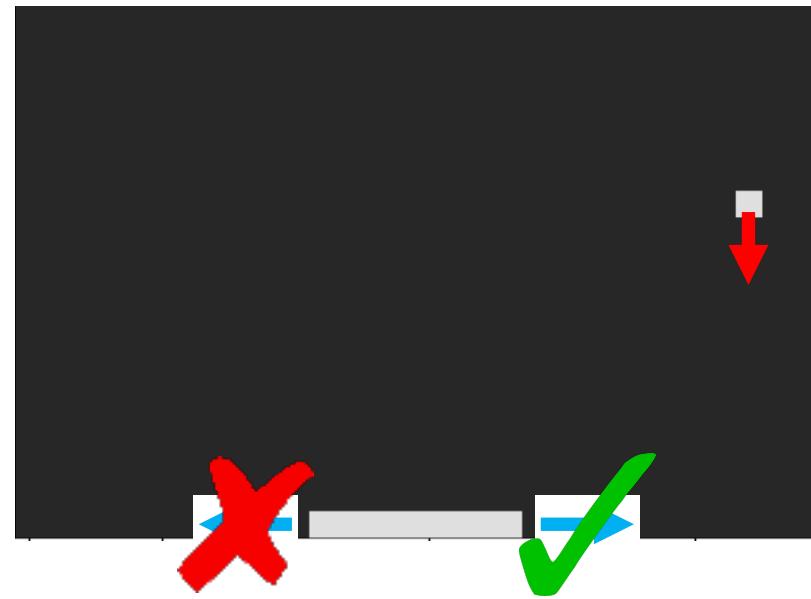
Ross & Bagnell 2011

Viper Algorithm

Insight: Critical States



**actions are similar
(non-critical state)**

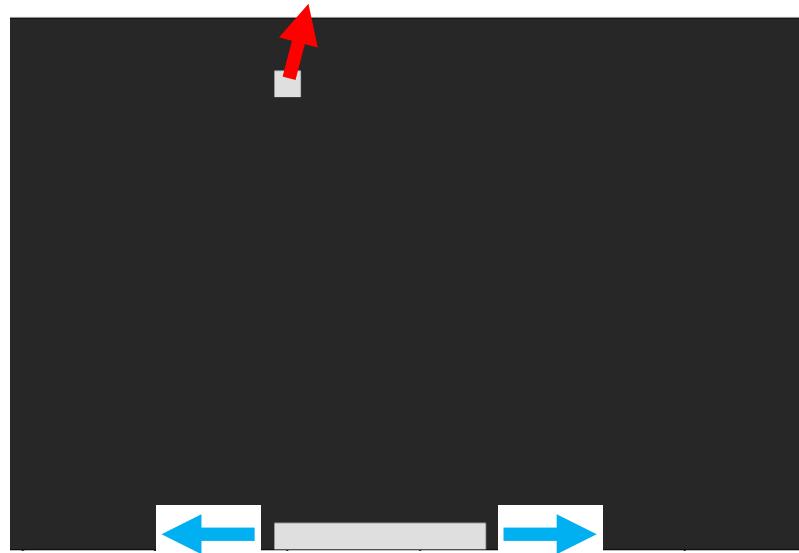


**must move right!
(critical state)**

Our Approach: Leverage the Q -Function

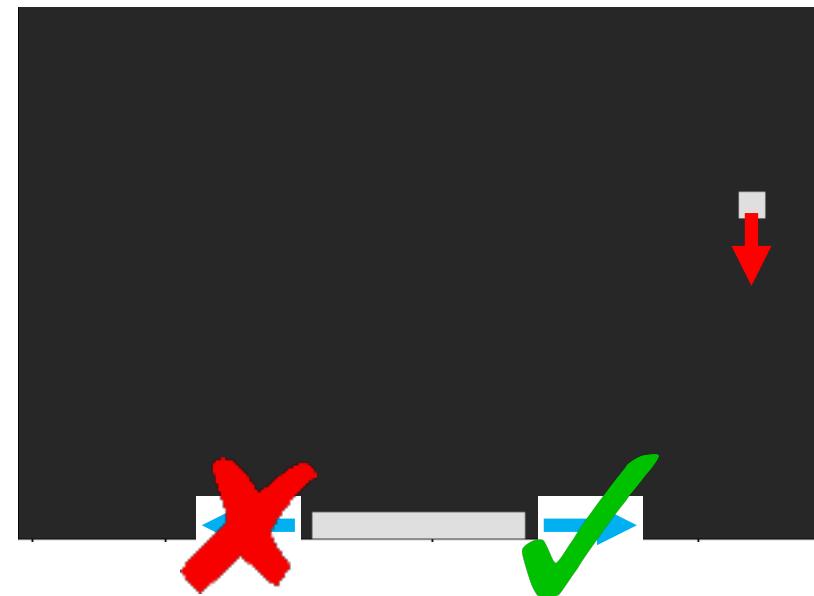
$Q(s, a) = \text{"how good is action } a \text{ in state } s?" \in \mathbb{R}$

Our Approach: Leverage the Q -Function



non-critical state (low priority)

$$Q(s, \pi_{\text{NN}}(s)) \approx \underbrace{\min_{a \in A} Q(s, a)}_{\text{optimal } Q \text{ value}} \quad \underbrace{\min_{a \in A} Q(s, a)}_{\text{worst-case } Q \text{ value}}$$



critical state (high priority)

$$Q(s, \pi_{\text{NN}}(s)) \gg \underbrace{\min_{a \in A} Q(s, a)}_{\text{optimal } Q \text{ value}} \quad \underbrace{\min_{a \in A} Q(s, a)}_{\text{worst-case } Q \text{ value}}$$

Viper Algorithm

- DAgger treats all state-action pairs equally:

$$\pi_{\text{DT}} = \arg \min_{\pi} \sum_{s \in D} \mathbb{I}[\pi(s) = \pi_{\text{NN}}(s)]$$

- Viper weights state-action pairs by the Q -function:

$$\pi_{\text{DT}} = \arg \min_{\pi} \sum_{s \in D} \left(Q(s, \pi_{\text{NN}}(s)) - \min_{a' \in A} Q(s, a') \right) \mathbb{I}[\pi(s) = \pi_{\text{NN}}(s)]$$

optimal Q value worst-case Q value

Theoretical Guarantees

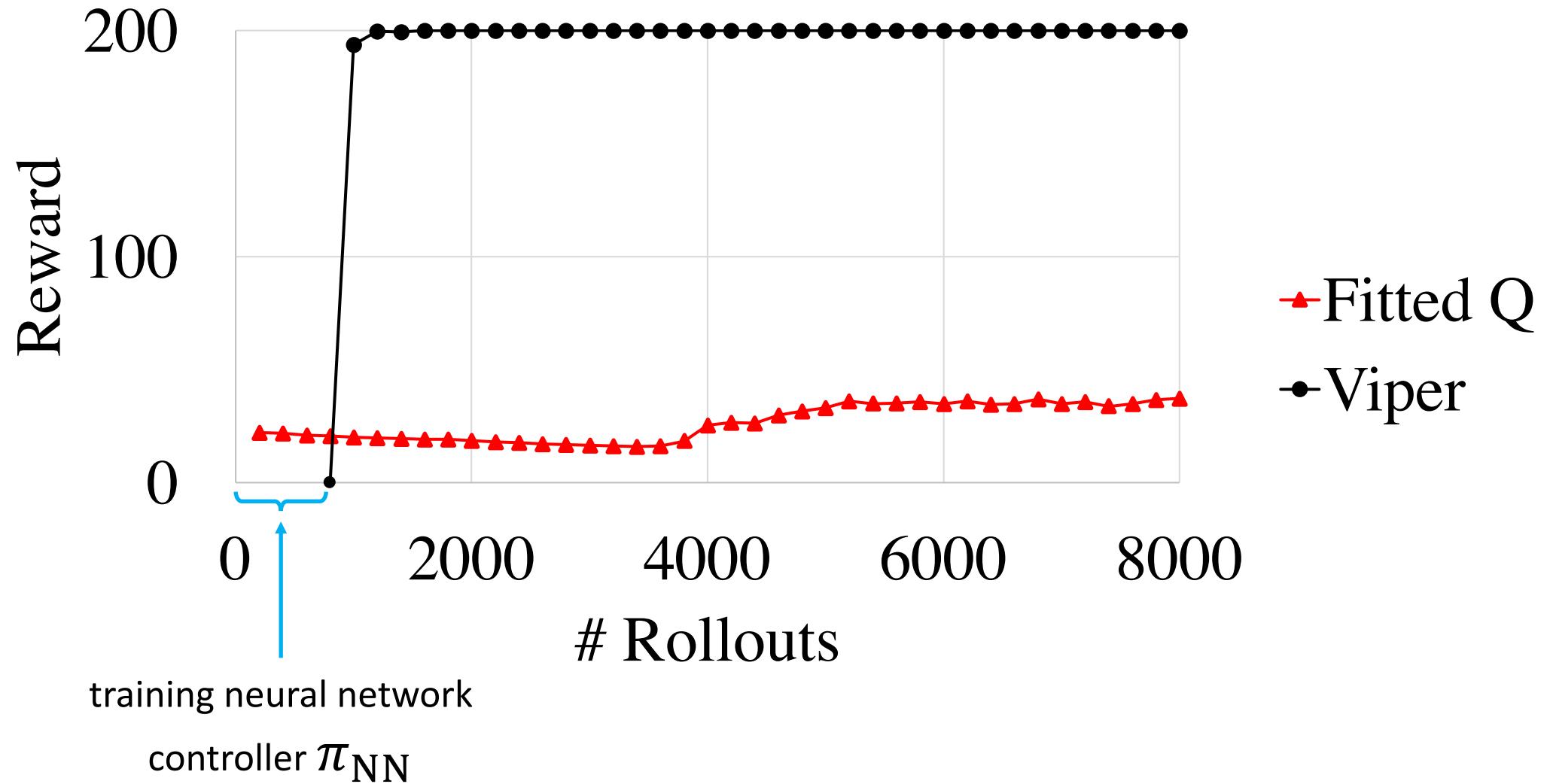
Theorem. *For any $\delta > 0$, there exists a policy $\hat{\pi} \in \{\hat{\pi}_1, \dots, \hat{\pi}_N\}$ such that*

$$J(\hat{\pi}) \leq J(\pi^*) + T\epsilon_N + \tilde{O}(1)$$

with probability at least $1 - \delta$, as long as $N = \tilde{\Theta}(\ell_{max}^2 T^2 \log(1/\delta))$.

Evaluation

vs. Decision Trees via RL (on Cart-Pole)

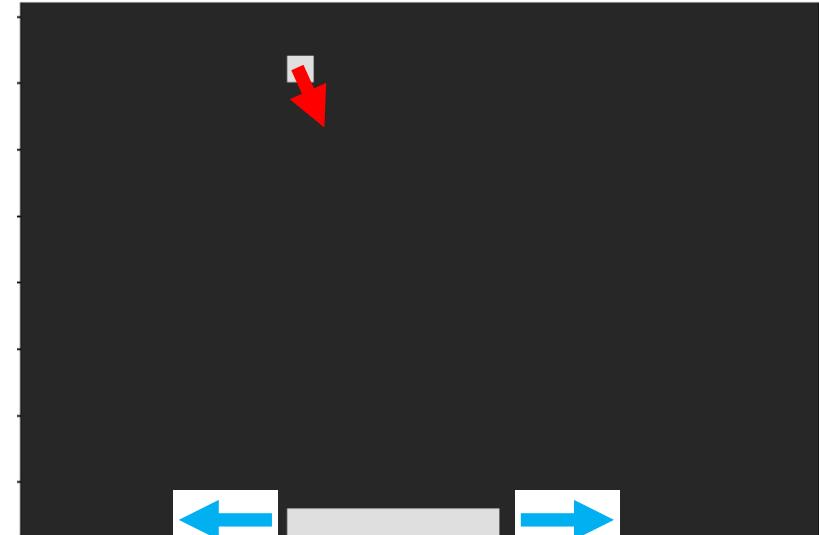


vs. to DAgger (on Atari Pong)



Verifying Correctness of a Toy Pong Controller

- **Toy Pong**
 - states = \mathbb{R}^5
 - actions = {left, right, stay}
- **Neural network:**
 - trained using policy gradients
 - 600 neurons
- **Decision tree:**
 - extracted using Viper
 - 31 nodes



Verifying Correctness of a Toy Pong Controller

- **Inductive invariant:**

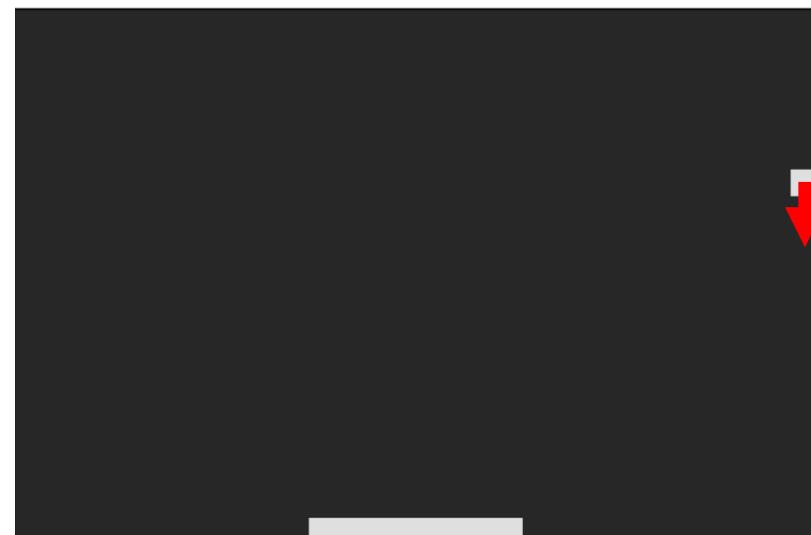
$$s(0) \in \text{blue} \Rightarrow s(t) \in \text{blue}$$

- **Verification algorithm**

- dynamics are piecewise linear
- SMT formula over linear arithmetic
- solved by Z3 in < 5 seconds

- **Results:**

- error when ball starts on the right
- fixed when paddle is slightly longer!



Conclusion

Verifiability is critical to enabling application of deep reinforcement learning to safe-critical systems