Privacy-Preserving Natural Language Processing

Lecture 3 – Introduction to Differential Privacy

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Intro

Overview

First lecture: privacy definitions, arbitrary hard choices

Second lectures: still arbitrary choice in anonymization, but GDPR notion of possibility of deidentification

Today: towards probabilistic approach to privacy

Differential privacy basics

Precursors of differential privacy

"Let us begin with a short story. Envision a database of a hospital containing the medical history of some population. On one hand, the hospital would like to advance medical research which is based (among other things) on statistics of the information in the database. On the other hand, the hospital is obliged to keep the privacy of its patients, i.e. leak no medical information that could be related to a specific patient. The hospital needs an access mechanism to the database that allows certain 'statistical' queries to be answered, as long as they do not violate the privacy of any single patient."

I. Dinur and K. Nissim (2003). **"Revealing Information While Preserving Privacy".** In: *Proceedings of the Twenty-Second ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems.* SIGMOD/PODS03: International Conference on Management of Data and Symposium on Principles Database and Systems. San Diego California: ACM, pp. 202–210

Precursors of differential privacy

"We focus on binary databases, where the content is of n binary (0-1) entries"

I. Dinur and K. Nissim (2003). **"Revealing Information While Preserving Privacy".** In: *Proceedings of the Twenty-Second ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems*. SIGMOD/PODS03: International Conference on Management of Data and Symposium on Principles Database and Systems. San Diego California: ACM, pp. 202–210

First DP papers

"The database consists of some number n of rows. We are completely agnostic about the type of rows – they may be tuples of attributes, strings, or even pictures."

A. Blum, C. Dwork, F. McSherry, and K. Nissim (2005). **"Practical Privacy: The SuLQ Framework".** In: *Proceedings of the Twenty-Fourth ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems.* SIGMOD/PODS05: International Conference on Management of Data and Symposium on Principles Database and Systems. Baltimore Maryland: ACM, pp. 128–138

Explicit notion of independence of rows

Dependency Between Database Records

"We explicitly assume that the database records are chosen independently from each other, according to some underlying distribution D. We are not aware of any work that does not make this assumption (implicitly or explicitly)."

C. Dwork and K. Nissim (2004). **"Privacy-Preserving Datamining on Vertically Partitioned Databases".** In: *Proceedings of the 24th Annual International Cryptology Conference - CRYPTO 2004*. Ed. by M. Franklin. Vol. 3152. Santa Barbara, CA, USA: Springer Berlin Heidelberg, pp. 528–544

Explicit notion of independence of rows

"The intent of the independence assumption is to characterize what information is under the control of a given individual. Specifically, **if there is information about a row that can be learned from other rows, this information is not truly under the control of that row.** Even if the row in question were to sequester itself away in a high mountaintop cave, information about the row that can be gained from the analysis of other rows is still available to an adversary. It is for this reason that **we focus our attention on those inferences that can be made about rows without the help of others**."

A. Blum, C. Dwork, F. McSherry, and K. Nissim (2005). **"Practical Privacy: The SuLQ Framework".** In: *Proceedings of the Twenty-Fourth ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems.* SIGMOD/PODS05: International Conference on Management of Data and Symposium on Principles Database and Systems. Baltimore Maryland: ACM, pp. 128–138

Dataset

Dataset (or Database)

The essential assumption (in words)

Each person corresponds to a single row in a table

The semantics of columns is arbitrary:

- It could be an actual "table" with some sensitive attributes (e.g., age, income)
- It could be just an arbitrary unstructured object (e.g., personal photo, text)

Dataset

Another implicit assumption

Each row contains data (values or attributes) that "belong" to this row's person

Implication of the above assumption

Example: If person A is removed from the dataset, you cannot certainly tell her private attributes from some other row B, C, D.

Examples of implicit assumptions

Name	Income	Age
Bob	700	21
Alice	2,800	32
Charlie	3,500	45

Table 1: Looks legit, no clear violation of the assumptions

Name	Income	Age	Someone else's income
Bob	700	21	Alice: 2,800
Alice	2,800	32	Charlie: 3,500
Charlie	3,500	45	Alice: 2,800

Table 2: Something looks wrong with this dataset

Examples of implicit assumptions

Name	Income	Age
Bob	700	21
Alice	2,800	32
Charlie	3,500	45
Alice	2,800	32

Table 3: Something looks wrong with this dataset

Statistical queries

Real-valued query

"Trusted server that holds a database of sensitive information. Given a query function f mapping databases to reals, the so-called true answer is the result of applying f to the database. [...]"

C. Dwork, F. McSherry, K. Nissim, and A. Smith (2006). **"Calibrating Noise to Sensitivity in Private Data Analysis".** In: *Theory of Cryptography*. Ed. by S. Halevi and T. Rabin. Red. by D. Hutchison et al. Vol. 3876. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 265–284

Query

Let X be a database from a universe (a set) of all possible databases $\mathcal{X}$

Real-valued query is

$$f : \mathcal{X} \mapsto \mathbb{R}$$

Query example – ”counting” query

"A query consists of a pair (S, f) where S is a set of rows in the database and f is a function mapping database rows to $\{0, 1\}$. The true answer is $\sum_{i \in S} f(d_i)$"

- implicitly assuming d_i is the i-th entry from S
- also note inconsistencies in notations, inevitable in every new field being just formalized!! Here f maps a "row" to zero or one, previous slide f would be the whole sum

C. Dwork, F. McSherry, K. Nissim, and A. Smith (2006). **“Calibrating Noise to Sensitivity in Private Data Analysis”.** In: *Theory of Cryptography*. Ed. by S. Halevi and T. Rabin. Red. by D. Hutchison et al. Vol. 3876. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 265–284

How could a counting query leak private information?

"It's just a statistics (a sum), so I can hide in the crowd!"

What is private information

The secret "bit" $f(d_i) \mapsto \{0, 1\}$

which eventually corresponds to whether or not you are in the database

- If your are in the database, you cannot control others
- What if the database is very small?
- What if the database is queried repeatedly?
- What if you are an outlier?

Statistical queries

How to protect privacy in counting queries

Example

Name	Hospitalized in year	Age	Illegal drug use
Alice	2024	32	yes
Bob	2020	21	no
Charlie	2023	45	no
...			
Xander	2020	31	yes

Table 4: A sensitive database example from a clinic

(Important to clarify for context: If you are Alice, how did you end up in such a database?)

Query: How many persons in the database take illegal drugs?

Example: How many persons in the database take illegal drugs?

Name	Hospitalized in year	Age	Illegal drug use
Alice	2024	32	yes
Bob	2020	21	no
Charlie	2023	45	no
...			
Xander	2020	31	yes

What might be public information: the size of the dataset (say $n = 100$)

The true answer to the query: 2

If Xander reveals his drug use, and we know Alice is in the database, her privacy is revealed!

Statistical queries

How to protect privacy

Privatizing the query result

How would you go about it?

Solution: Alter the query result

Changes to the true query result must be **irreversible**!

"A natural approach, and one that has been explored by others in the 1980's, is to add random noise to the answer" (Blum, Dwork, McSherry, and Nissim, 2005)

"It is evident that without randomness there is no privacy: if everything is pre-determined, and all possible choices we make are predictable or pre-programmed by our adversaries, then there is nothing that we can build our privacy on." (Ekert and Renner, 2014)

A. Blum, C. Dwork, F. McSherry, and K. Nissim (2005). **"Practical Privacy: The SuLQ Framework".** In: *Proceedings of the Twenty-Fourth ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems.* SIGMOD/PODS05: International Conference on Management of Data and Symposium on Principles Database and Systems. Baltimore Maryland: ACM, pp. 128–138

A. Ekert and R. Renner (Mar. 2014). **"The Ultimate Physical Limits of Privacy".** In: *Nature* 507.7493, pp. 443–447

How to randomize the query result?

Recall: The counting query output is a natural number

How should we randomize it?

– Draw a random value from a probability distribution

– Which one? How parametrized?

What we want

The randomized output will be likely close to the true value,
and unlikely far away

The Laplace distribution

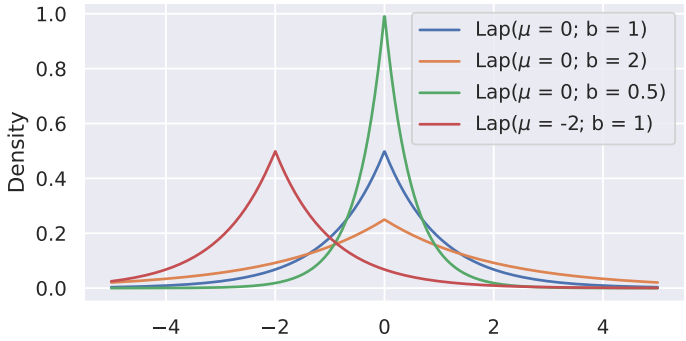


Figure 1: Laplace PDF (probability density function)

$$\mathrm{Lap}(x; \mu, b) = \frac{1}{2b} \exp\left(-\frac{|x - \mu|}{b}\right)$$

How to choose the parameters?

Location and scale

– Location: the true value

– Scale?

Let's pick $b = 1$ just for lack of better ideas :)

It's all nice and random, but what does it give us?

Query: How many persons in the database take illegal drugs?

Name	Hospitalized in year	Age	Illegal drug use
Alice	2024	32	yes
Bob	2020	21	no
Charlie	2023	45	no
...			
Xander	2020	31	yes

Table 5: Database D, including Alice

Assume that the true answer is 51

It's all nice and random, but what does it give us?

What if Alice decided **not to be part of the data**?

– She has the right to decide about her privacy, GDPR, etc.

Name	Hospitalized in year	Age	Illegal drug use
Bob	2020	21	no
Charlie	2023	45	no
...			
Xander	2020	31	yes

Table 6: Database D', **excluding** Alice

The true answer to the query would be now 50

We have two databases: With and without Alice

Privatize for D (with Alice), true answer is 51

$$Y \sim \frac{1}{2b} \exp\left(\frac{-|x-51|}{b}\right) = \frac{1}{2} \exp\left(-|x-51|\right)$$

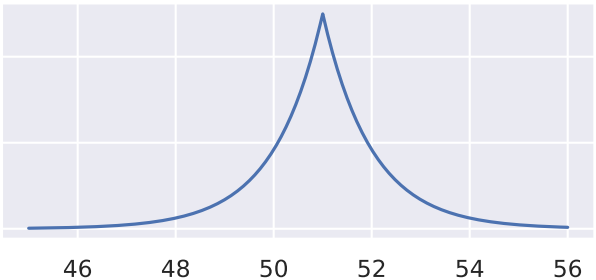


Figure 2: Laplace density, $\mu = 51, b = 1$

Now without Alice

Privatize for D' (without Alice), true answer is 50

$$Y' \sim \frac{1}{2b} \exp\left(\frac{-|x-50|}{b}\right) = \frac{1}{2} \exp\left(-|x-50|\right)$$

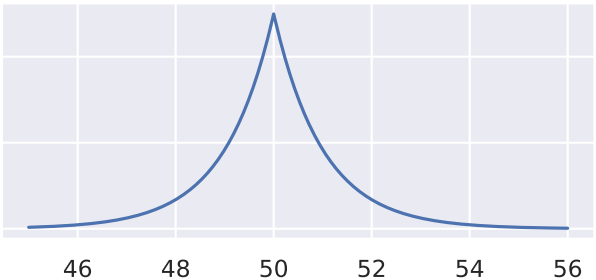


Figure 3: Laplace density, $\mu = 50, b = 1$

Both variables in one plot

$Y \sim \frac{1}{2} \exp(-|x - 51|)$ $Y' \sim \frac{1}{2} \exp(-|x - 50|)$

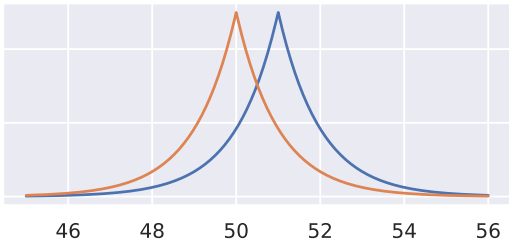


Figure 4: Blue = with Alice, red = without Alice

Now observe a particular value $y \in \mathbb{R}$, for example 52

– Was it sampled from Y or from Y'?

If we observed 52, was it sampled from Y or from Y'?

We cannot really tell!

But we can compare probabilities of this event[1]

$$\Pr[Y = 52] = \tfrac{1}{2}\exp\left(-|52 - 51|\right) = 0.5\exp(-1)$$
$$\Pr[Y' = 52] = \tfrac{1}{2}\exp\left(-|52 - 50|\right) = 0.5\exp(-2)$$

How much more likely from Y?

$$\frac{\Pr[Y=52]}{\Pr[Y'=52]} = \frac{0.5\exp(-1)}{0.5\exp(-2)} = \exp(-1)\exp(2) = \exp(1) = 2.718$$

How much more likely from Y'?

$$\frac{\Pr[Y'=52]}{\Pr[Y=52]} = \frac{0.5\exp(-2)}{0.5\exp(-1)} = \exp(-1) = 0.367$$

[1] We must compare densities, so our math will not break

Both variables in one plot

$Y \sim \frac{1}{2} \exp(-|x - 51|)$ $Y' \sim \frac{1}{2} \exp(-|x - 50|)$

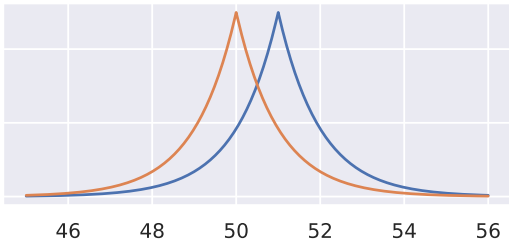


Figure 5: Blue = with Alice, red = without Alice

Now observe a particular value $y \in \mathbb{R}$, for example 50

– Was it sampled from Y or from Y'?

If we observed 50, was it sampled from Y or from Y'?

$$\Pr[Y = 50] = \tfrac{1}{2}\exp\left(-|50 - 51|\right) = 0.5\exp(-1)$$
$$\Pr[Y' = 50] = \tfrac{1}{2}\exp\left(-|50 - 50|\right) = 0.5\exp(0)$$

How much more likely from Y?

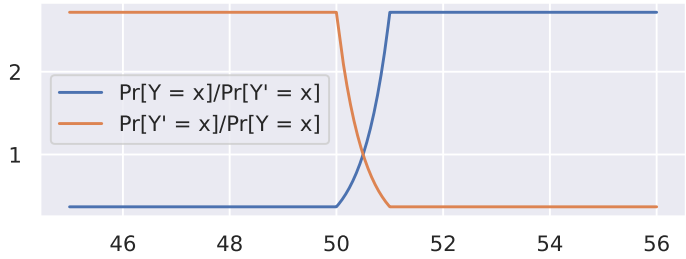
$$\frac{\Pr[Y=50]}{\Pr[Y'=50]} = \frac{0.5\exp(-1)}{0.5\exp(0)} = \exp(-1) = 0.367$$

How much more likely from Y'?

$$\frac{\Pr[Y'=50]}{\Pr[Y=50]} = \frac{0.5\exp(0)}{0.5\exp(-1)} = \exp(1) = 2.718$$

It seems like the odds ratio is bounded from above...

Can we generalize it for any observed x?



Seems like the maximum we can get is $2.718 = e = \exp(1)$

Let's try to prove it

We need triangle inequality for absolute values:[2]

$|x| - |y| \leq |x - y|$

$$\frac{\Pr[Y = x]}{\Pr[Y' = x]} = \frac{\frac{1}{2}\exp(-|x - 51|)}{\frac{1}{2}\exp(-|x - 50|)} = \exp(|x - 50| - |x - 51|)$$

$$\leq \exp(|(x - 50) - (x - 51)|)$$

$$= \exp(|x - 50 - x + 51|)$$

$$= \exp(|1|) = \exp(1)$$

[2] Homework: Prove it! $x, y \in \mathbb{R}$

Let's try to prove it (part 2)

$$\frac{\Pr[Y' = x]}{\Pr[Y = x]} = \frac{\frac{1}{2}\exp\left(-|x-50|\right)}{\frac{1}{2}\exp\left(-|x-51|\right)} = \exp(|x-51| - |x-50|)$$

$$\leq \exp(|(x-51) - (x-50)|)$$

$$= \exp(|x-51-x+50|)$$

$$= \exp(|-1|) = \exp(1)$$

What does that mean?

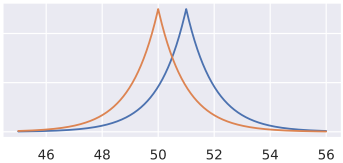


Figure 6: Blue = with Alice, red = without Alice,

$Y \sim \frac{1}{2}\exp\left(-|x-51|\right) \qquad Y' \sim \frac{1}{2}\exp\left(-|x-50|\right)$

$\frac{\Pr[Y=x]}{\Pr[Y'=x]} \leq \exp(1) \qquad \frac{\Pr[Y'=x]}{\Pr[Y=x]} \leq \exp(1)$

No matter what value we get after privatizing the counting query – we can only get some limited "information" about whether it came from Y or Y'.

What if the true answer would be the same for D and D'?

Alice did not take drugs

$$Y \sim \tfrac{1}{2}\exp\left(-|x-50|\right) \qquad Y' \sim \tfrac{1}{2}\exp\left(-|x-50|\right)$$

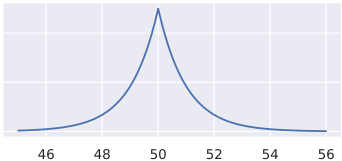


Figure 7: Both have the same distribution

$$\frac{\Pr[Y=x]}{\Pr[Y'=x]} = 1 \leq \exp(1) \qquad \frac{\Pr[Y'=x]}{\Pr[Y=x]} = 1 \leq \exp(1)$$

So the maximum "loss" only when the presence of Alice changes the query output

What if the true answer would be 22?

Alice did not take drugs

$Y \sim \frac{1}{2}\exp\left(-|x-22|\right) \qquad Y' \sim \frac{1}{2}\exp\left(-|x-22|\right)$

– bounded by 1 as in the previous slide

Alice did take drugs

$Y \sim \frac{1}{2}\exp\left(-|x-23|\right) \qquad Y' \sim \frac{1}{2}\exp\left(-|x-22|\right)$

– our general proof still holds!

$\frac{\Pr[Y=x]}{\Pr[Y'=x]} \leq \exp(1) \qquad \frac{\Pr[Y'=x]}{\Pr[Y=x]} \leq \exp(1)$

Summary

Four counting queries, the maximum difference of the query result is 1 (when a particular person is not in the dataset)

We used scale $b = 1$ for the Laplace distribution, which gives us upper bound on privacy loss

$$\frac{\Pr[Y=x]}{\Pr[Y'=x]} \leq \exp(1) \qquad \frac{\Pr[Y'=x]}{\Pr[Y=x]} \leq \exp(1)$$

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Credits

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