

## Chapter-0 Category Theory

[illegible]

# CH-1-Chain Complexes and Abelian Categories

Topics	Page No.
Exact Sequences	2
Chain Complex Definition	2
Differentials, $n$ -cycles and $n$ -boundaries of chain complex	2
$n$ – th Homology module of chain complex	2
Category of complexes	3
morphism of chain complexes	3
$n$ – th Homology as a functor	5
Quasi-Isomorphism / Homologism	6
Co-chain complexes	6
$n$ -cocycles and $n$ -coboundaries of cochain complex	7
$n$ – th Cohomology module of cochain complex	7
Bounded chain complex	7
Abelian categories	9
Initial, Final and Zero object	9
Zero morphism	10
Pre-additive category	12
Product/Coproduct	13
Finite products and coproducts in pre-additive category	15
Kernels / Cokernels	17
Uniqueness of (Co)Kernels via Universal property	18
Monomorphism / Epimorphism	18
Pre-abelian Category	23
Abelian Category	24 , 32
Kernel in terms of exact sequence	25
Snake's Lemma	26
Any Homomorphism into short exact sequence	26
Splitting and GLuing Exact sequence	27
Connecting homomorphism in Snake's Lemma	28
Category of chain complexes is an <b>Ab</b> category	29
Additive Functor	29
Additive Category	30
Category of chain complexes is an additive category	30
Subcomplex and Quotient Complex	31
Image of map $Im(f)$ in an abelian category	32,37
Abelian Subcategory	33
Product Category	33
Bifunctor	33
$Hom_{\mathcal{C}}(\cdot, \cdot)$ is a bifunctor	33
Matrix Notation	34
$\mathcal{F}$ additive functor $\iff \mathcal{F}$ commutes with finite products	35
The category $Ch(\mathcal{A})$ where $\mathcal{A}$ is abelian is an abelian category	37
Long Exact sequences (Connecting homomorphism)	39
Long exact sequence is a functor from the category of short exact sequences in $\mathcal{C}$ to the category of long exact sequences in $\mathcal{C}$	41
Chain Homotopies	41
Split Chain complex; splitting maps; Split exact chain complex;	41
Null Homotopic chain map ; Chain Contraction	43



## CH-2-Derived Functors

[illegible]