Phase 2 Technical Report: Constraint Identification and Precedence Graph Construction for Salvagnini P4 Process Planning

1. Introduction and System Architecture

1.1 The Imperative of Constraint-Based Planning in Panel Bending

The transition from Phase 1 (Feature Recognition and Face-Adjacency Graph Construction) to Phase 2 (Constraint Identification and Precedence Graph Construction) represents the fundamental shift from geometric abstraction to physical realizability in Computer-Aided Process Planning (CAPP). In Phase 1, we successfully mapped the topology of the sheet metal part into a semantic Face-Adjacency Graph (FAG), where nodes represent planar faces and edges represent bends characterized by attributes such as angle, radius, and orientation. However, the FAG remains a static descriptor of the *final* state. It contains no information regarding the temporal feasibility of the transition states required to reach that geometry, nor does it account for the specific kinematic limitations of the manufacturing hardware.

This report details the algorithmic framework developed for Phase 2, specifically tailored to the **Salvagnini P4 Panel Bender**. Unlike press brakes, which rely on a back-gauge and manual (or robotic) following of the flange, the P4 utilizes a semi-automatic manipulator and a universal bending toolset with Automatic Blankholder Adjustment (ABA).[1] This machine topology fundamentally alters the constraint landscape. The primary limiting factor in P4 processing is not merely tool collision in the traditional sense, but **manipulator graspability**. If the machine cannot securely hold, rotate, and feed the workpiece due to geometric occlusion or lack of surface area—defined herein as the "Dead Zone"—the operation is infeasible.

Therefore, the objective of this phase is to transform the undirected, static FAG into a **Directed Acyclic Graph (DAG)** of dependencies. This transformation is driven by the rigorous identification of hard constraints, principally the **Dynamic Manipulator Grasp Validation**. We aim to develop a computational engine that, for any given intermediate state of the part, defines the forbidden zones relative to the active bend line, scans available surfaces for valid grip polygons using surface scoring metrics, and deterministically triggers Repositioning (Repo) nodes when no single grasp configuration satisfies the intersection of requirements for a sequence of bends.[4]

1.2 Evolution from FAG to DAG

The Face-Adjacency Graph $G_{FAG} = (V, E)$ serves as the input. Here, $V = \{f_1, f_2, \ldots, f_n\}$ represents the set of faces, and $E = \{b_{ij}\}$ represents the bends connecting faces f_i and f_j. While G_{FAG} captures topological connectivity, it is agnostic to the sequence.

The output, the Precedence DAG $G_{DAG} = (N, A)$, elevates the bends to nodes N (operations), where directed arcs A represent strict temporal constraints. An arc $b_x \to b_y$ implies that operation b_x *must* be completed before b_y to satisfy a hard physical constraint. These constraints are derived from:

1. **Grasp Availability:** The execution of b_y destroys the surface required to grasp the part for b_x.

2. **Geometric Occlusion:** The geometry resulting from b_y creates a collision volume preventing the tool access for b_x.

3. **Kinematic Reach:** The stability of the part during b_x is compromised if b_y is performed first (e.g., Center of Mass shifts).

The construction of this DAG is the prerequisite for Phase 3 (Sequence Optimization), where we will linearize the graph to minimize cycle time. A robust Phase 2 ensures that any sequence derived in Phase 3 is physically manufacturable.

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2. Machine Topology and Kinematic Constraints

To algorithmically identify constraints, we must first formalize the machine environment. The Salvagnini P4 is a horizontal-table panel bender, distinct in its operation from vertical press brakes. The constraints it imposes are a function of its three primary subsystems: the Press/Blade Unit, the ABA Blankholder, and the Manipulator.

2.1 The Manipulator: Kinematics and Footprint

The P4 manipulator is the primary agent of material transport. It grips the sheet, rotates it in the horizontal (XY) plane, and feeds it along the Y-axis into the bending line.[3]

-   **Degrees of Freedom (DOF):**

- T_x : Lateral translation for centering the bend line.
  - T_y : Feed translation for positioning the bend line under the blades.
  - R_z : Rotation for presenting different sides of the panel.
- **Clamping Mechanism:** The manipulator employs a pincer-style clamp (or suction cup array for loading) with a specific geometric footprint $\mathcal{F}_{grip}$. The clamping force is significant, ranging from **530 kN to 1060 kN** depending on the model (e.g., P4-2520 vs P4-3125).[1] This high force necessitates a robust contact area to prevent localized deformation of the sheet surface, particularly for thinner gauges (down to 0.5mm).[6]

2.1.1 The P4 Coordinate System

We define the global machine coordinate system $\mathcal{W}$ centered at the midpoint of the active bending length on the table edge.

- **X-axis:** Parallel to the bend line (width direction).
- **Y-axis:** Perpendicular to the bend line (feed direction). Positive Y extends towards the manipulator.
- **Z-axis:** Vertical. $Z = 0$ is the table surface.

The "Dead Zone" or "Forbidden Zone" is a region defined in this coordinate system where the manipulator clamp $\mathcal{F}_{grip}$ cannot reside during a bend cycle. If the centroid of the grasp polygon falls within this zone, or if the footprint intersects it, the grasp is invalid.

2.2 ABA Tooling and Lateral Constraints

The Automatic Blankholder Adjustment (ABA) system [1] dynamically resizes the length of the hold-down tool to match the length of the bend. This is a critical advantage for setup time (Zero Setup) but introduces a dynamic lateral constraint.

- **Mechanism:** The ABA consists of segmented tool plates that expand/contract.
- **Constraint:** The manipulator must position the sheet such that the grippers do not collide with the lateral extent of the ABA segments if the manipulator needs to be close to the bend line. While the manipulator typically retracts during the bend, the *handoff* position and the *minimum grip distance* are constrained by the ABA's physical width.

2.3 Machine Specification Table for Algorithms

The following parameters, extracted from technical datasheets [1], act as global constants in our constraint identification algorithms.

Parameter	Symbol	Value (Example: P4-2520)	Description
Max Clamping Force	F_{max}	1060 kN	Limit for calculating required grip area friction.
Min Sheet Thickness	S_{min}	0.5 mm	Lower bound for stability checks.
Max Sheet Thickness	S_{max}	3.2 mm (Steel)	Upper bound for force calculation.
Dead Zone Constant	C_{base}	596 mm (ALA config)	Base distance from bend line to manipulator.
Dead Zone Offset	K_{thick}	$4 \cdot S$	Thickness-dependent safety margin.
Manipulator Reach	R_{man}	2500 mm (Diagonal)	Maximum part size swing radius.
Max Bend Height	H_{bend}	203 mm	Limit for flange collision detection.

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3. The Face-Adjacency Graph (FAG) Analysis

The input FAG is the semantic map of the part. Before we can validate grasps, we must query the FAG to extract geometric properties relevant to physics.

3.1 Node Attributes

Each node $f_i \in V$ possesses:

- **Geometry:** A Boundary Representation (B-Rep) loop of edges.

- **Area (A_i):** Surface area available for grasping.

- **Normal Vector (n_i):** Orientation in the unfolded state (initially all $n = Z$).

- **Center of Mass (COM_i):** Local centroid.

3.2 Edge Attributes (Bends)

Each edge $e_{ij} \in E$ possesses:

- **Type:** Standard Bend, Hem, Radius.

- **Parameters:** Angle θ, Radius r.

- **Direction:** Up ($+Z$) or Down ($-Z$). This is critical for P4 constraints.
  - **Up Bends:** Flange moves *away* from the table. Generally safe for table collision, but creates an obstacle for the manipulator arm.
  - **Down Bends:** Flange moves *into* the table. On a P4, positive bends are standard; negative bends often require the P4's specific "negative bend" capability which may involve different blade kinematics or table retraction. For standard analysis, a Down Bend often implies a "flip" of the part is required *before* processing, or the machine has a specific recess. We assume standard P4 positive/negative bending capability where the table allows clearance or the part overhangs.

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4. Dynamic Manipulator Grasp Validation Algorithm

The core innovation in this Phase 2 report is the **Dynamic Grasp Validator**. This algorithm takes a candidate bend operation and the current geometric state of the part, and returns a boolean feasibility flag along with a quality score.

4.1 The Dead Zone: Calculating the Forbidden Zone

The "Dead Zone" Z_{dead} is the locus of points on the sheet where the manipulator cannot safely grip. This is defined relative to the active bend line L_{bend}.[8]

4.1.1 Derivation of the Safety Formula

According to Salvagnini technical documentation [8], the minimum distance X_{min} from the bend line to the start of the manipulator clamp is a function of the sheet thickness S. The formula is given as:

$$X_{min} = C_{base} + k \cdot S$$

Where C_{base} is a machine-specific constant derived from the depth of the bending unit

throat and the kinematics of the manipulator carriage. For the P4-2512 with ALA, C_{base} is cited as 596mm (or similar values depending on specific ALA configuration 30/600/130). The coefficient k is typically 4, representing a safety multiplier for material deformation and springback behavior.

Why $4 \cdot S$?

During bending, the material flows. The bend radius consumes material (bend allowance). The "4t" rule is a standard safety buffer in sheet metal to avoid gripping within the deformation zone, which would damage the surface or cause the gripper to slip due to thinning material.

4.1.2 Geometric Construction of $\mathcal{Z}_{dead}$

The Forbidden Zone is not a simple rectangle; it is the union of several constraint geometries.

Let $\mathcal{P}_{sheet}$ be the polygon of the flat sheet in the machine coordinate system.

1. **Press Clearance Zone ($\mathcal{Z}_{press}$):** A rectangular region extending from the bend line ($Y = 0$) into the sheet by distance X_{min}.

$$\mathcal{Z}_{press} = \{(x,y) \in \mathcal{P}_{sheet} \mid 0 \leq y \leq X_{min}\}$$

    This zone prevents the manipulator from entering the press throat or colliding with the blankholder during the active cycle.

2. **ABA Lateral Interference ($\mathcal{Z}_{ABA}$):**

    If the bend length is L_{bend}, the ABA adjusts to width $W_{ABA} \approx L_{bend}$. The manipulator cannot grip laterally adjacent to the ABA if the clearance is insufficient.

$$\mathcal{Z}_{ABA} = \{(x,y) \in \mathcal{P}_{sheet} \mid |x - x_{center}| \leq \frac{W_{ABA}}{2} + \delta_{lat}\}$$

    where δ_{lat} is the lateral safety margin.

3. **Flange Collision Zone ($\mathcal{Z}_{collision}$):**
    This is dynamic. It depends on the flanges *already bent*.

    For every face f_k that has been bent in a previous step:

    o   If f_k protrudes upwards ($Z > H_{manipulator\_clearance}$), its projection onto the XY plane acts as an obstacle.

$$\mathcal{Z}_{collision} = \bigcup_{f \in \mathrm{BentFaces}} \mathrm{Project}_{XY}(\mathrm{BoundingBox}(f))$$

○

Total Dead Zone:

$$\mathcal{Z}_{dead} = \mathcal{Z}_{press} \cup \mathcal{Z}_{ABA} \cup \mathcal{Z}_{collision}$$

4.2 Surface Scoring: Finding the Valid 'Grip Polygon'

Once $\mathcal{Z}_{dead}$ is calculated, we determine the **Available Grip Region** $\mathcal{R}_{grip}$ on a candidate face f_{cand}.

$$\mathcal{R}_{grip} = \mathrm{Geometry}(f_{cand}) \setminus \mathcal{Z}_{dead}$$

If $\mathcal{R}_{grip}$ is empty, the face cannot be used. If it is non-empty, we must evaluate its quality. We employ a multi-factor scoring algorithm.

4.2.1 Scoring Criteria

1. **Flatness and Parallelism (S_{flat}):**
   The P4 manipulator requires a planar surface parallel to the table.

$$S_{flat} = n_{face} \cdot Z_{machine}$$

If $S_{flat} < 1.0 - \epsilon$, the score is 0 (Invalid).

2. **Grip Area and Shape (S_{shape}):** The region must be large enough to accommodate the manipulator's suction cups or clamps.[1]
   ○ **Suction Cup Validation:** We use the **Maximum Inscribed Circle (MIC)** algorithm.[10]

$$r_{MIC} = \mathrm{Radius}(\mathrm{MaxInscribedCircle}(\mathcal{R}_{grip}))$$

If $2 \cdot r_{MIC} < D_{cup}$, the grasp is invalid.
   ○ **Clamp Validation:** We compute the **Maximum Inscribed Rectangle** aligned with the manipulator axis.
   ○ Score: $S_{shape} = \min(1.0, \frac{\mathrm{Area}(\mathcal{R}_{grip})}{\mathrm{Area}_{ideal}})$

3. **Distance from Bend Line (S_{dist}):**
   While we must be outside the Dead Zone, being *too* far is also problematic for accuracy

(cantilever effect), but generally, further back is safer for collision. The scoring prioritizes keeping the manipulator centrally located relative to the mass but outside the danger zone.

4. **Stability/Center of Mass (S_{stable}):** A grasp is stable if the Center of Mass (COM) of the entire part is close to the grasp centroid, minimizing torque on the suction cups/clamps.[12]

$$S_{stable} = \frac{1}{1 + \lambda \cdot ||\text{Centroid}(\mathcal{R}_{grip}) - \text{COM}_{part}||}$$

4.2.2 The Grip Scoring Function

$$Score(f_{cand}) = w_1 S_{flat} \cdot (w_2 S_{shape} + w_3 S_{stable})$$

If $Score > \text{Threshold}$, the face is a **Valid Grasp Candidate**.

4.3 Algorithm Implementation: Maximizing the Inscribed Polygon

To implement the surface scanning efficiently, we utilize computational geometry libraries (e.g., Shapely [10]).

Python

```python
def ValidateGrasp(PartState, ActiveBend, MachineSpecs):
    # 1. Compute Forbidden Zone
    X_min = MachineSpecs.C_base + 4 * PartState.Material.Thickness
    DeadZone = Polygon([( -Inf, 0 ), ( +Inf, 0 ), ( +Inf, X_min ), ( -Inf, X_min )])

    # Add ABA interference
    ABA_Width = ActiveBend.Length
    DeadZone = DeadZone.Union(Rectangle(Center=ActiveBend.Center, Width=ABA_Width))

    # Add 3D Collisions from previously bent flanges
    for flange in PartState.BentFlanges:
        if flange.Height > MachineSpecs.ManipulatorHeight:
            Obstacle = ProjectXY(flange)
            DeadZone = DeadZone.Union(Obstacle)

    # 2. Scan Candidate Faces
    ValidGrasps =
```

```
    for face in PartState.FlatFaces:
        # Subtract Dead Zone
        GripRegion = face.Geometry.Difference(DeadZone)

        if GripRegion.IsEmpty: continue

        # 3. Calculate Metrics
        MIC_Radius = CalculateMIC(GripRegion) # Max Inscribed Circle
        if MIC_Radius < MachineSpecs.MinCupRadius: continue

        Dist_COM = Distance(GripRegion.Centroid, PartState.COM)

        Score = CalculateScore(MIC_Radius, Dist_COM)
        ValidGrasps.Append((face, Score))

    return Best(ValidGrasps)
```

5. Constraint Identification & DAG Construction

With the ValidateGrasp oracle defined, we proceed to construct the Precedence Graph. This graph dictates the *necessary* order of operations.

5.1 Hard Constraints: The Physics of "Cannot"

We identify three categories of hard constraints that create directed edges in the DAG.

5.1.1 Type I: Grasp Occlusion Constraints

This is the most critical constraint for the P4.

- **Definition:** Bending edge b_i transforms the part such that the grasp required for bend b_i becomes invalid (e.g., the face moves into the Dead Zone or becomes non-planar).
- **Logic:**
  1. Simulate the state of the part S_{post-j} assuming b_j is completed.
  2. Check ValidateGrasp for b_i in state S_{post-j}.
  3. If Valid is FALSE, then b_i **must precede** b_j.
  4. Add edge $b_i \rightarrow$ to the DAG.

5.1.2 Type II: Geometric Collision Constraints (Internal/External)

- **Definition:** Standard folding logic dictates that internal features (tabs, internal cutouts) are usually bent before external flanges to allow tool access.[14]
- **Logic:**

  1. Check ray-casting from the bend line of b_i to the machine frame.
  2. If flange F_j (resulting from b_j) intersects this ray, b_j creates a collision obstacle.
  3. Add edge $b_i \rightarrow$.

5.1.3 Type III: Stack Stability Constraints

- **Definition:** Performing a large bend might shift the COM so far that subsequent small bends become unstable to manipulate (high rotational inertia).
- **Logic:** While often a "soft" constraint, if the stability score drops below a critical safety threshold, it becomes a hard constraint.

5.2 Building the DAG

The algorithm initializes a graph where nodes are bends. It iterates through all pairs (b_i, b_j).

Pair Check	Condition	Action
Grasp Check	b_j invalidates grasp for b_i	Add Edge $b_i \rightarrow$
Collision Check	b_j blocks tool path for b_i	Add Edge $b_i \rightarrow$
Tolerance Check	b_i relies on b_j for gauging (rare in P4)	Add Edge $b_j \rightarrow$

If a cycle is detected (e.g., $A \rightarrow B$ and $B \rightarrow A$), this indicates a **Deadlock**. A deadlock in P4 processing implies that no single grasp sequence can manufacture the part. This necessitates a **Regrip**.

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6. Regrip (Repo) Planning and Trigger Logic

Minimizing the number of grasp changes (Repos) is vital for cycle time reduction. However, identifying *when* a Repo is unavoidable is a Phase 2 constraint task.

6.1 The Intersection of Feasible Sets

For a sequence of bends $S = \{b_1, b_2, \ldots, b_k\}$ to be performed without a regrip, there must exist a **single** grasp configuration g that is valid for *all* $b_n \in S$.

Mathematically, let G_i be the set of valid grip polygons for bend b_i.

The sequence is valid if:

$$G_{seq} = \bigcap_{i=1}^{k} G_i \neq \emptyset$$

6.2 The Regrip Trigger Algorithm

We scan the topological sort of the tentative DAG.

1. Initialize CurrentGraspSet = G_1.

2. For next bend b_{next}:

   - Calculate NextGraspSet = G_{next}.

   - Calculate Intersection = CurrentGraspSet $\cap$ NextGraspSet.
   - If Intersection is EMPTY:

     - **TRIGGER REPO:** A Repositioning node must be inserted between $b_{current}$ and b_{next}.

     - Reset CurrentGraspSet = G_{next}.
   - Else:
     - CurrentGraspSet = Intersection.
     - Continue.

6.3 Placing the Repo Station

When a Repo is triggered, we must ensure the transition is possible.

- **Constraint:** The part must be stable on the table *during* the release and regrasp.
- The manipulator releases the part. The part sits on the table. The manipulator moves to the new location.
- **Validation:** The COM of the part must lie within the convex hull of the contact points with the table (flat faces). If the part is unbalanced (e.g., mostly bent flanges), it might tip over. This is an advanced stability constraint evaluated via the projection of the COM onto

the table plane.

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7. Detailed Analysis of P4 Specifics: The ABA Factor

The Automatic Blankholder Adjustment (ABA) is a defining feature of the Salvagnini P4. It significantly influences the "Lateral Dead Zone."

7.1 ABA Dynamics

In traditional setups, tool width is fixed. In P4, tool width W_{tool} matches bend length L.

- **Narrow Bends:** If L is small ($<$ 100mm), the ABA contracts. The manipulator must grip centrally.
- **Wide Bends:** If L is large ($>$ 2000mm), the ABA expands. The manipulator might need to grip eccentrically if the center is obstructed, or the manipulator width itself might interfere with the side-guides of the ABA mechanism.

7.2 The "ABA Shadow"

We model the ABA as a dynamic obstacle box $\mathcal{B}_{ABA}(L)$ centered on the bend.

$$\text{Width}(\mathcal{B}_{ABA}) = L + 2 \cdot \delta_{clearance}$$ The manipulator clamp, with width W_{man}, must be placed such that:$$ \cap = \emptyset$$

Unless the manipulator Z-height is strictly lower than the ABA Z-height (undercutting). However, standard P4 operation assumes the manipulator retracts to avoid the "pinch point" of the bending blades. Therefore, the **Retraction Logic** implies that the grasp must be valid at the *start* of the bend (feeding) and the *end* of the bend (retraction).

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8. Simulation and Validation of Grasp Quality

It is insufficient to merely find a geometric fit. The P4 exerts high dynamic forces.

8.1 Finite Element Validation (FEM) Integration

As suggested by research [15], simple geometric overlap is a proxy for grasp stability. For high-fidelity planning, we integrate a simplified FEM check.

- **Load Case:** Apply moment $M = \text{Force}_{bend} \times \text{LeverArm}$.
- **Resistance:** Friction force $F_f = \mu \cdot F_{clamp}$.

- **Failure Mode:** If $M > F_f \cdot r_{effective}$, the part will slip.
- **Phase 2 Implementation:** We use a pre-computed look-up table based on material thickness and grip area.
  - If Score (from section 4.2) is marginal, we trigger the FEM check.
  - If FEM fails, the face is marked invalid, potentially forcing a Repo to a more central location.

8.2 Handling "Floppy" Parts

For thin materials ($S < 0.8$ mm), large panels act as flexible membranes.

- **Sagging:** A grasp far from the COM will cause the sheet to droop.
- **P4 Limitation:** The P4 loading device typically has brush tables or ball transfers. If the leading edge droops below the table level due to gravity, it will crash into the lower blade die.

- **Constraint:** Max distance from Grip to any Edge E is constrained by
$$D_{max} = f(E_{modulus}, S)$$
.

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9. Conclusion

The output of Phase 2 is a fully validated Precedence DAG. This graph represents the "Physics of Manufacturability" for the Salvagnini P4. By rigorously defining the **Forbidden Zones** via the $X_{min} = C + 4S$ formula [8] and implementing the **Dynamic Grip Polygon** validation [1], we have ensured that every node transition in the graph is executable. The identification of **Regrip Triggers** via set intersection logic [4] solves the critical problem of continuity, ensuring that the P4 manipulator can maintain control of the workpiece throughout the entire transformation from flat sheet to folded panel.

This DAG now serves as the mathematically sound input for Phase 3, where cost functions (cycle time) will be applied to traverse this graph and generate the optimal NC code.

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10. Appendix: Algorithm Pseudocode

10.1 Dynamic Grasp Validation

Python

```
FUNCTION ValidateGrasp(Face F, Bend B, PartState P)
  # 1. Define Forbidden Zone (Dead Zone)
  X_min = GetMachineConstant(P.MachineModel) + 4 * P.Material.Thickness
  Zone_Press = Rectangle(MinX=-INF, MaxX=INF, MinY=0, MaxY=X_min)

  # 2. Add ABA Interference
  ABA_Width = B.Length + SafetyMargin
  Zone_ABA = Rectangle(Center=B.Midpoint, Width=ABA_Width, Height=X_min)

  # 3. Add Geometric Collisions (Previous Bends)
  Zone_Collision = EmptyPolygon
  FOR EACH BentFlange IN P.BentFaces
    IF BentFlange.Z_Height > Machine.ManipulatorClearance
      Zone_Collision = Union(Zone_Collision, ProjectXY(BentFlange))
    END IF
  END FOR

  Zone_Dead = Union(Zone_Press, Zone_ABA, Zone_Collision)

  # 4. Compute Available Grip Region
  Region_Grip = Difference(F.Geometry, Zone_Dead)

  # 5. Score the Region
  IF Region_Grip.Area < Machine.MinGripArea RETURN False, 0

  Radius_MIC = CalculateMaximumInscribedCircle(Region_Grip)
  IF Radius_MIC < Machine.SuctionCupRadius RETURN False, 0

  Score = (Region_Grip.Area / IdealArea) * (1 / Distance(Region_Grip.Center, P.COM))

  RETURN True, Score
END FUNCTION
```

10.2 Precedence Graph Builder

Python

```
FUNCTION BuildDependencyDAG(FAG)
 Nodes = ExtractBends(FAG)
 Edges =

 FOR B_i IN Nodes
  FOR B_j IN Nodes (where i!= j)
    # Check if performing B_j first kills the grasp for B_i
    State_Post_Bj = SimulateBend(B_j)
    ValidGrasp_Bi = ValidateGrasp(Face_for_Bi, B_i, State_Post_Bj)

    IF NOT ValidGrasp_Bi.IsValid
      # Hard Constraint: B_i MUST happen before B_j
      AddEdge(B_i -> B_j, Type="GraspDestruction")
    END IF

    # Check Geometric Collision
    IF CheckToolCollision(B_i, State_Post_Bj)
      AddEdge(B_i -> B_j, Type="ToolBlockage")
    END IF
  END FOR
 END FOR

 RETURN DAG(Nodes, Edges)
END FUNCTION
```

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