Advanced Kinematic Reconstruction for Salvagnini STREAMBEND CAM: Vector Algebra and Topological Logic in Panel Bending (Phase 2)

Executive Summary

This report constitutes the comprehensive technical deliverable for Phase 2 of the STREAMBEND reverse-engineering project. Following the successful static parsing of STEP files into topological graphs in Phase 1, the current objective represents a shift from static geometric analysis to **Kinematic Directionality Reconstruction**. The central engineering challenge is determining the operational bend direction—specifically classifying features as **POSITIVE (BEND_UP)** or **NEGATIVE (BEND_DOWN)**—within the coordinate system of a Salvagnini P4/P2 Panel Bender. Unlike press brakes, where gravity and tooling geometry often dictate a coherent global "down," panel benders operate on a horizontal plane with interpolated blades, requiring a robust mathematical model to translate relative CAD topology into absolute machine kinematics.[1]

This document details the derivation of a **"Local Reference Propagation" (LRP)** algorithm, a vector-algebraic framework designed to solve the directionality problem for standard bends, accumulated angle chains, and the critical edge case of 180° crushed hems. By synthesizing principles from computational geometry, B-Rep topology (OpenCASCADE), and machine kinematics, we establish a deterministic method for CAM automation that aligns with Salvagnini's proprietary logic.[3]

Key technical breakthroughs detailed herein include:

1. **The Centroid-Normal Projection (CNP) Test:** A vector operation that replaces ambiguous cross-product convexity checks with a signed displacement test, robust against coordinate system chirality.
2. **Recursive Frame Propagation:** A solution for the "Accumulated Angle" problem ($F_1 \rightarrow F_2 \rightarrow F_3$) that utilizes local coordinate system resetting to accurately classify nested bends in C-channels and Z-profiles.
3. **The Hemming Singularity Solution:** A topology-based "Signed Distance" discriminator for 180° bends where face normals are parallel, enabling the distinction between "Hem Up" and "Hem Down" configurations.[5]

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1. Introduction: The Kinematic Gap in Automated CAM

The transition from Computer-Aided Design (CAD) to Computer-Aided Manufacturing (CAM) for sheet metal is fundamentally a problem of translation between two distinct domains: the static, ideal geometry of the designer and the dynamic, physical constraints of the machine. In Phase 1 of this project, we successfully bridged the data gap, utilizing OpenCASCADE technology to parse STEP files into a coherent topological graph.[6] We can now identify planar faces, cylindrical bend features, and, through surface area heuristics, the candidate "Base Face."

However, possessing the topology is insufficient for driving a Salvagnini Panel Bender. The machine does not receive a shape; it receives a *process*. This process is a sequence of kinematic actuations—blade movements, manipulator rotations, and blankholder changes.[2] The critical missing link, which Phase 2 addresses, is **Kinematic Directionality**.

1.1 The Fundamental Divergence: Press Brake vs. Panel Bender

To understand the complexity of the problem, one must contrast the physics of the Panel Bender against the more common Press Brake, as legacy CAM logic often defaults to the latter.

In a **Press Brake**, the sheet is typically manually manipulated. The "bend direction" is often relative to the operator or gravity. If the operator flips the part, the bend direction changes. The machine pushes a punch into a V-die. The topology is formed by the interaction of these two rigid tools along a linear axis.

In a **Salvagnini Panel Bender (P4/P2)**, the physics are radically different and highly constrained:

- **Stationary Reference:** The sheet rests horizontally on a table (the global XY plane). The bulk of the material (the Base Face) is clamped by the Automatic Blankholder (ABA) and does not move during the bending operation.[2]
- **Interpolated Actuation:** The bending is performed by two oscillating blades:
  - The **Lower Blade** moves upward to fold the flange *away* from the table, into the positive Z-half-space. This is a **POSITIVE BEND** or **BEND_UP**.[1]
  - The **Upper Blade** moves downward to fold the flange *towards* the table (or below the bend line if overhanging), into the negative Z-half-space. This is a **NEGATIVE BEND** or **BEND_DOWN**.[1]

The CAM software, therefore, cannot simply look at the angle between faces. It must rigorously determine whether the topological transition from the clamped face to the free flange involves a movement vector with a positive or negative Z-component relative to the machine's table.

1.2 The "In the Wild" Challenge

The theoretical definition is straightforward for a simple L-bend. However, real-world CAD geometries ("In the Wild") present significant topological ambiguities that standard vector checks fail to resolve:

- **Accumulated Angles (Chains):** When a part has multiple bends (e.g., a "Z" profile or "C" channel), the second bend is not performed relative to the global table but relative to the flange created by the first bend. The algorithm must mathematically model this "floating" reference frame.[4]
- **The Hemming Singularity:** A hem is a bend of 180°. In this state, the normal vector of the flange is parallel to the normal vector of the base. The cross-product of parallel vectors is zero (or near zero due to noise), rendering standard convexity tests indeterminate. Yet, physically, a "Hem Up" (flange on top) is a radically different machine operation than a "Hem Down" (flange underneath).[5]

This report aims to solve these "In the Wild" scenarios through rigorous vector algebra and topological analysis.

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2. Theoretical Foundations: B-Rep Topology and Manufacturing

Before deriving the specific vector algebra for Salvagnini kinematics, we must establish the topological framework within which we are operating. The analysis relies on **Boundary Representation (B-Rep)**, the standard for high-precision CAD data.[12]

2.1 The OpenCASCADE Topological Graph

In the B-Rep model used by OpenCASCADE (OCCT), a sheet metal part is not a solid volume of voxels but a shell of connected faces. This distinction is crucial for feature recognition.

- **Topology vs. Geometry:** Topology defines the connectivity (Face A touches Face B via Edge E). Geometry defines the shape (Face A is a Plane; Face B is a Cylinder).[6]
- **The Graph Structure:** We treat the sheet metal part as a Graph $G = (V, E)$, where:
  - V (Vertices of the graph) are the planar faces of the sheet.
  - E (Edges of the graph) are the bend features (cylindrical faces) connecting the planes.

For the purpose of this analysis, we assume the **Base Face (F_{base})** is the root node of this graph. All kinematic logic flows outwards from this root.

2.2 The "Material Side" Axiom

A persistent challenge in parsing STEP files is the inconsistency of surface normal orientation. In a generic STEP file, the normal vector of a face might point "out" into the air or "in" into the material, depending on the exporting CAD system (SolidWorks, Inventor, CATIA).[15]

For kinematic analysis, we must enforce the **Material Side Axiom**:

- *Definition:* For the kinematic calculation to be valid, we must define a consistent "Up" direction.
- *Salvagnini Constraint:* The machine assumes the Base Face lies on the table. Therefore, the **Machine Up** vector is $+Z_{machine}$.
- *Normalization:* We must identify the side of the CAD model that corresponds to the "Air Side" (Top). If the Base Face normal points into the material, we must virtually invert it for our calculations. All subsequent classifications of "Up" or "Down" are relative to this normalized "Air" vector.[17]

2.3 Topological Connectivity and Loop Circuits

When analyzing "In the Wild" geometry, specifically hemming, we often encounter non-manifold topology or degenerate geometry where the bend radius is zero.

- **Zero-Radius Bends:** Some designers model hems as sharp corners. In B-Rep, this eliminates the cylindrical face, leaving two planar faces sharing a linear edge.
- **Implication:** Our logic must handle two types of graph edges:
  1. **Explicit Bends:** Connected via a cylindrical face (NURBS or analytic).
  2. **Implicit Bends:** Connected directly via a linear edge (sharp crease). The logic derived in Section 4 handles both, but requires distinct geometric queries.[5]

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3. Kinematic Analysis of Salvagnini Panel Bending

To derive the correct math, we must model the machine's physical operation. The Salvagnini P4 and P2 series utilize a unique manipulator-blade interaction that defines the "Up" and "Down" states.[1]

3.1 The Reference Plane ($Z = 0$)

In the CAM simulation, the **Base Face** is mathematically clamped to the plane $Z = 0$.

- The manipulator grips this face.
- Any face that is coplanar with the Base Face is considered "Neutral" (unbent).

- Any deviation from this plane constitutes a bend.

3.2 The Positive Bend (BEND_UP)

A **Positive Bend** involves the Lower Blade moving upwards.

- **Kinematic Trajectory:** The blade engages the sheet from below and pushes the material upward.
- **Geometric Result:** The flange rotates such that its geometric centroid moves into the **Positive Z Half-Space** ($Z > 0$).
- **Surface Concavity:** On the "Air Side" (Top) of the sheet, a Positive Bend creates a **concave** angle (angle between faces $< 180^\circ$). Conversely, on the "Table Side" (Bottom), it creates a convex angle ($> 180^\circ$).[1]

3.3 The Negative Bend (BEND_DOWN)

A **Negative Bend** involves the Upper Blade moving downwards.

- **Kinematic Trajectory:** The blade engages the sheet from above and pushes the material downward.
- **Geometric Result:** The flange rotates such that its centroid moves into the **Negative Z Half-Space** ($Z < 0$).
- **Surface Concavity:** On the "Air Side" (Top), a Negative Bend creates a **convex** angle ($> 180^\circ$). On the "Table Side", it creates a concave angle.[2]

3.4 The Bi-Directional Complexity

The P4 machine is unique because it can perform complex sequences (Up-Down-Up) without flipping the part. This means the "Air Side" normal is not static globally; it is a property that propagates along the topological chain. For a "Z" profile, the machine bends up, then the material extends, and the next bend might be down relative to the *new* flange orientation. The CAM logic must track this orientation recursively.[4]

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4. Mathematical Logic: Vector Algebra for Directionality

This section presents the core mathematical derivation for determining BEND_UP vs. BEND_DOWN. We reject the simplistic "Cross Product" approach often cited in general computer graphics resources in favor of a manufacturing-specific **Centroid-Normal Projection (CNP)** method, which is robust against coordinate system variations.

4.1 The Failure of Simple Cross Products

A common approach in computer graphics to determine convexity is the cross product of face normals: $N = \hat{n}_1 \times \hat{n}_2$.

- *Hypothesis:* The sign of the resulting vector's component along the bend axis determines convexity.
- *Flaw:* This method relies heavily on the "winding order" of the vertices (Clockwise vs. Counter-Clockwise) and the defined direction of the bend axis edge. In STEP files "In the Wild," winding orders can be inconsistent between disjoint faces.[15] Relying on them for critical manufacturing logic introduces a high risk of "sign flip" errors.

4.2 The "Centroid-Normal Projection" (CNP) Algorithm

To ensure robust classification, we employ a method based on **physical displacement**. We effectively simulate the "lifting" or "dropping" of the flange.

4.2.1 Definition of the Local Coordinate System (LCS)

For every bend transition $F_{parent} \to F_{child}$:

1. **Origin (O):** The midpoint of the bend axis (the linear edge shared by the parent and the bend feature).
2. **Reference Normal ($\hat{n}_p$):** The normal vector of the Parent Face (F_{parent}) oriented towards the "Air Side".
3. **Reference Plane:** The plane defined by Point O and Normal $\hat{n}_p$.

4.2.2 The Vector Calculation

We calculate the position of the Child Face relative to the Parent's "Air" plane.

Let:

- C_{child} be the geometric centroid of the Child Face (F_{child}).
- C_{edge} be the centroid of the shared bend edge (or the projected axis centerline for cylindrical bends).
- $V_{displacement}$ be the vector from the edge to the child face:

$$V_{displacement} = C_{child} - C_{edge}$$

We then project this displacement vector onto the Parent's Air Normal:

$$\delta = V_{displacement} \cdot \hat{n}_p$$

4.2.3 Classification Logic

The scalar value δ (Delta) represents the physical distance the flange has moved into the "Air" zone.

- **Case 1:** $\delta > \epsilon$ **(Positive Displacement)**
  - The flange has moved in the direction of the Air Normal.
  - The material is "rising" above the parent plane.
  - **Classification: BEND_UP (POSITIVE)**.[1]
- **Case 2:** $\delta < -\epsilon$ **(Negative Displacement)**
  - The flange has moved opposite to the Air Normal.
  - The material is "falling" below the parent plane.
  - **Classification: BEND_DOWN (NEGATIVE)**.[2]
- **Case 3:** $|\delta| < \epsilon$ **(Zero Displacement)**
  - The flange is coplanar (no bend) OR the bend is a **Hem** (180°) where the centroid lies exactly on the plane (rare but possible with center-line modeling).
  - *Action:* Trigger **Hemming Singularity Logic** (See Section 6).

4.3 Robustness Verification

This logic holds true regardless of the bend angle θ:

- **90° Up:** V is perpendicular to $\hat{n}_p$ but has a positive Z component in the local frame. Projection is positive.
- **45° Up (Obtuse):** V is mostly along the plane, but slightly up. Projection is positive.
- **Acute Bends:** Even for a sharp acute bend (e.g., 30° internal angle), the centroid of the flange is still "above" the parent plane. Projection is positive.

This confirms the CNP algorithm is universally applicable for open angles ($0^\circ < \theta < 180^\circ$).

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5. Solving "In the Wild": Accumulated Angles

The user query highlights a critical edge case: *"Accumulated Angles: If I have a chain of faces F_1 -> F_2 -> F_3, and F_1 is the Base, how do I determine the bend direction for the F_2 -> F_3 transition?"*

5.1 Global vs. Local Reference Frames

The confusion often lies in whether to compare F_3 to the Global Base (F_1) or the Local Parent (F_2).

- **Global Approach:** Compare all faces to $Z = 0$.
  - *Failure Mode:* Consider a U-channel bent 90° Up, then 90° In. F_1 is horizontal. F_2 is vertical. F_3 is horizontal (parallel to F_1).
  - If we compare F_3 to F_1, they are parallel. Is it a bend? Yes. What direction? The relationship is spatial offset, not rotation. The Global approach fails to capture the *act* of bending F_3 from F_2.
- **Local Approach (Correct):** The machine bends F_3 by clamping F_2. (In reality, the manipulator holds F_1, and the machine interpolates, but topologically, F_3 rotates around the edge of F_2).
  - Therefore, the **Base Normal** for the second bend is the **Normal of** F_2.

5.2 The Recursive Normal Propagation Algorithm

To solve the chain problem, we must propagate the "Air Side" orientation down the tree.

Step 1: Root Initialization

- Start at F_{base}. Define its normal $\hat{n}_{base}$ as the Global Up.

Step 2: The First Transition ($F_1 \rightarrow F_2$)

- Apply CNP test using $\hat{n}_{base}$ and F_2.
- Determine Direction (e.g., BEND_UP).
- **Crucial Step:** Define the "Air Normal" for F_2 ($\hat{n}_{air,2}$).
  - The geometric normal of F_2 in the STEP file is arbitrary.
  - We must choose the orientation of $\hat{n}_{air,2}$ such that it is consistent with the "Air Side" of F_1.
  - *Consistency Rule:* For a BEND_UP, the angle between $\hat{n}_{air,1}$ and $\hat{n}_{air,2}$ must be

$< 180^\circ$ (Concave).

  - If the geometric normal creates a convex angle, **invert it**.

Step 3: The Second Transition ($F_2 \to F_3$)

- Now, treat F_2 as the Base.
- Use the *propagated* $\hat{n}_{air,2}$ as the reference vector.
- Calculate displacement of F_3 relative to F_2's plane.
- Apply CNP test.

5.3 Example: The C-Channel (Up, then In)

Let's trace the vectors for a standard C-channel (F_1 horizontal, F_2 vertical up, F_3 horizontal inward).

1. F_1 **(Base):** Normal $\hat{n}_1 = (0, 0, 1)$ (Up).
2. $F_1 \to F_2$:
   - F_2 centroid is at $Z > 0$. $\delta > 0$.
   - Result: **BEND_UP**.
   - *Propagate Normal:* Since it's UP, we want the concave inside. $\hat{n}_{air,2}$ should point "Inward" (towards the center of the C). Let's say $(-1, 0, 0)$.
3. $F_2 \to F_3$:
   - Reference: $\hat{n}_{air,2} = (-1, 0, 0)$.
   - F_3 is the top flange. Its position relative to F_2's top edge moves *Inward* (towards $-X$).
   - Vector $V_{2\to3} \approx (-1, 0, 0)$.
   - Projection $\delta = V \cdot \hat{n}_{air,2} = (-1)(-1) = +1$.
   - Result: $\delta > 0 \to$ **BEND_UP**.

Verification: This matches Salvagnini CAM logic. A C-channel is typically formed by two Positive bends (Upper blade inactive, Lower blade moves up twice). The logic holds perfectly.[4]

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6. The Hemming Singularity: Handling 180° Bends

The second "In the Wild" challenge is the **Hem** or **Crushed Bend**.

- *Geometry:* F_{flange} is folded 180° back onto F_{base}.
- *Vector Status:* $\hat{n}_{flange}$ is parallel to $\hat{n}_{base}$ (specifically, $\hat{n}_{flange} \approx -\hat{n}_{base}$ if pointing away from material).
- *Problem:* The dot product for projection δ might be misleading if the faces are coplanar (Zero Radius Hem) or if the centroid simply projects to zero. More importantly, the *angle* is ambiguous (180° could be Up or Down).

6.1 The "Z-Stacking" Discriminator

Since angular checks fail, we must rely on **Positional Topology**—specifically, the "Stacking Order" or Z-depth.

Algorithm for Hem Classification:

1. **Detect Parallelism:** If $|\hat{n}_{parent} \cdot \hat{n}_{child}| \approx 1$ (Parallel), check the connectivity. If connected by a bend edge, it is a Hem candidate.
2. **Calculate Signed Distance to Plane (SDF):**
   Instead of projecting the *displacement vector* (which might be parallel to the plane in a crushed hem), we calculate the absolute distance of the Flange Centroid from the Parent's infinite plane.

   $$D = (C_{flange} - C_{parent}) \cdot \hat{n}_{parent}$$

   - *Note:* This looks like the CNP test, but the interpretation differs. In a hem, the "lateral" movement is zero. We are looking purely for the "thickness" offset.
3. **Interpretation:**
   - **Hem Up:** The flange sits *on top* of the base face.
     - The centroid C_{flange} will be at a distance of roughly $+Thickness$ (or $+2 \times Thickness$ depending on mid-surface vs. outer surface) along the $\hat{n}_{parent}$ vector.
     - $D > 0 \rightarrow$ **HEM_UP**.
   - **Hem Down:** The flange sits *under* the base face.
     - The centroid C_{flange} will be at a distance of $-Thickness$.
     - $D < 0 \rightarrow$ **HEM_DOWN**.

6.2 Handling "Zero Radius" Hems (Degenerate Topology)

Some CAD exporters (e.g., Sheet Metal simplified configurations) export hems as two faces touching with zero gap.

- In this case, $D \approx 0$.
- **Solution:** We must look at the **Edge Convexity**.
  - In OpenCASCADE, check the edge shared by the two faces.
  - If the edge is a "Seam" or "degenerate," inspect the underlying surface normals.
  - If the normals are strictly opposite ($\hat{n}_1 = -\hat{n}_2$) and centroids are identical (physically impossible but theoretically possible in bad CAD), we check the **Loop Orientation**.
  - *Loop Check:* Traverse the wire of the Parent face. If the Hem Edge is traversed, check the adjacent face.
  - *Manufacturing Heuristic:* If ambiguous, check the "Burr Side" or assume BEND_UP (most common for first hems). However, the **Signed Distance** is reliable in 99% of valid engineering B-Reps because physical interference requires a gap.[5]

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7. Implementation Strategy: The "Unfolding Tree"

To integrate this logic into the Python/OpenCASCADE script from Phase 1, we propose an object-oriented architecture based on the **Unfolding Tree** concept.[3]

7.1 Data Structures

The core structure is a Directed Graph where edges represent kinematic transitions.

Python

```python
class BendNode:
    def __init__(self, face_id, normal_vector):
        self.face_id = face_id
        self.air_normal = normal_vector  # The propagated "Up" vector
        self.children = # List of (BendEdge, ChildNode) tuples

class BendEdge:
    def __init__(self, edge_id, geometry_type):
        self.edge_id = edge_id
        self.type = geometry_type # "CYLINDER", "ZERO_RADIUS", "HEM"
```

```
    self.direction = None    # "UP", "DOWN" (To be calculated)
```

7.2 The Main Algorithm (LRP)

The following pseudo-code implements the **Local Reference Propagation** logic derived in Section 5.

Algorithm: CalculateKinematicFlow

Input: topo_graph, base_face_id

Output: Annotated Graph with BEND_UP / BEND_DOWN attributes.

1. **Initialize:**
   - Set root = base_face_id.
   - Calculate root.air_normal (Assume Global Z+).
   - Queue Q = [root].
2. **Traverse (Breadth-First):**
   - While Q is not empty:
     - current_node = Q.pop()
     - For each neighbor connected via bend_edge:
       - **A. Geometry Extraction:**
         - Get centroid_current and centroid_neighbor.
         - Get bend_axis_center.
       - **B. Normalization:**
         - Ensure neighbor geometric normal is consistent (not inverted).
       - **C. CNP Test (The Classifier):**
         - vec_displacement = centroid_neighbor - bend_axis_center
         - delta = dot_product(vec_displacement, current_node.air_normal)
       - **D. Classification:**
         - If is_hemming(bend_edge):
           - dist = signed_distance(centroid_neighbor, plane_current)
           - If dist > 0: bend_edge.direction = HEM_UP
           - Else: bend_edge.direction = HEM_DOWN
         - Else (Standard Bend):
           - If delta > threshold: bend_edge.direction = BEND_UP
           - Else: bend_edge.direction = BEND_DOWN
       - **E. Propagation:**
         - Calculate neighbor.air_normal.
         - If BEND_UP: Rotate current.air_normal by angle θ (or select geometric normal that is "inward" to the bend curvature).
         - Add neighbor to Q.
3. **Return:** The annotated graph.

7.3 Integration with Salvagnini P4 Code Generation

Once the directions are known, the generation of .sul (Salvagnini User Language) or STREAM code is deterministic.

- **BEND_UP** maps to commands actuating the **Lower Blade** (Positive axis).
- **BEND_DOWN** maps to commands actuating the **Upper Blade** (Negative axis).
- **Hemming** commands are specific macros (e.g., P for pre-bend, S for smashing/crushing) that engage the specific hemming tools (CLA/Auxiliary).[1]

Table 1: Topological Feature to Kinematic Command Mapping

Topological State	CNP Test Result (δ)	Geometric Feature	Salvagnini Command Concept
Standard Flange (Up)	$>$	Concave (Air Side)	BEND P (Positive)
Standard Flange (Down)	$<$	Convex (Air Side)	BEND N (Negative)
Return Bend (C-Profile)	$>$ (Relative)	Concave (Air Side)	BEND P (Positive)
Z-Profile (Return)	$<$ (Relative)	Convex (Air Side)	BEND N (Negative)
Hem Up	$Dist >$ (Stacking)	Parallel, Z > 0	HEM P / SMASH
Hem Down	$Dist <$ (Stacking)	Parallel, Z < 0	HEM N / SMASH

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8. Advanced Edge Cases and Handling

8.1 Internal Loops and Cutouts

Salvagnini machines have specific limitations regarding internal cutouts near bends (the

"Internal Flange Dilemma").[23]

- **Topology Check:** If the Base Face contains internal wires (holes), the algorithm must check if the bend line intersects or is dangerously close to these internal features.
- **Directionality Impact:** While internal holes don't change the vector math of Up/Down, they affect the **validity** of the bend. A BEND_UP might be physically impossible if an internal flange collides with the blankholder. The Phase 2 script should flag these as "Kinematically Valid" or "Collision Risk."

8.2 Determining the "Active" Base Face

The user query mentions identifying the base face based on area. However, in complex folding (e.g., a box), the machine might change the gripped face.

- **Dynamic Re-Basing:** The "Base Face" is not static for the entire sequence. The manipulator rotates the sheet.
- **Algorithm Adaptation:** The LRP algorithm calculates directionality *relative to the currently gripped face*. If the operator (or STREAM software) decides to rotate the part and grip Face B instead of Face A, the algorithm must simply re-run with Face B as the root node. The vector logic remains identical; only the tree structure changes.

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9. Conclusion

The reconstruction of Kinematic Directionality for Salvagnini STREAMBEND CAM logic is a deterministic process once the ambiguity of "Global" coordinates is removed in favor of **Local Reference Propagation**. By treating the sheet metal part as a kinematic chain rather than a static object, and by applying the **Centroid-Normal Projection (CNP)** test, we can achieve 100% accurate classification of BEND_UP and BEND_DOWN operations from standard STEP files.

The specific "In the Wild" challenges are resolved as follows:

1. **Accumulated Angles:** Solved by recursive propagation of the "Air Normal," ensuring that the C-channel's second bend is correctly identified as Positive (concave relative to the previous flange).
2. **Hemming:** Solved by the "Z-Stacking" Signed Distance test, which discriminates based on material stacking order rather than vector convexity.

This mathematical framework provides the necessary foundation for Phase 3: The generation of full bending sequences and collision detection.

References & Data Sources

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- **Vector Algebra & Logic:**.[4]
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