Tutorial 4
CS3241 Computer Graphics (AY23/24)

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Recap

Lecture 4:

- Matrices (translation, rotation, scale)
- Matrix stacks (Current Transformation Matrix or CTM)

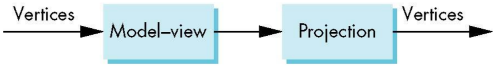
Lecture 5:

- View transformation
- Projection
- `GL_MODELVIEW` and `GL_PROJECTION` in context of CTM

Recap

CTMs in OpenGL

- OpenGL has a **model-view** and a **projection** matrix in the pipeline



- Each has a CTM and can be manipulated by first setting the correct matrix mode

v = object space

$M_{model} v$ = World space

$M_{view} M_{model} v$ = Camera space

```
void display( void ) {
  ...
  glviewport( 0, 0, 800, 600 );
  glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );

  glMatrixMode( GL_PROJECTION );
  glLoadIdentity();
  gluPerspective( viewFovY, viewAspect, nearDist, farDist );

  glMatrixMode( GL_MODELVIEW );
  glLoadIdentity();
```

$M_{view} = R_{cam} T_{cam}$

camera space

```
  gluLookAt( viewerPosX, viewerPosY, viewerPosZ,
             lookAtX, lookAtY, lookAtZ, upX, upY, upZ );
```

world space

```
  glTranslate3d( 100.0, 200.0, 300.0 )
  glScale3d( 3.0, 3.0, 3.0 );
```

$M_{model} = T S$

object space

```
  glBegin(GL_QUADS);
    glColor3d( 1, 0, 0 );  glVertex3d(0, 0, 0);
    glColor3d( 0, 1, 0 );  glVertex3d(1, 0, 0);
    glColor3d( 0, 0, 1 );  glVertex3d(1, 1, 0);
    glColor3d( 1, 1, 1 );  glVertex3d(0, 1, 0);
  glEnd();

  glutSwapBuffers();
}
```

```
void display( void ) {
  ...
  glviewport( 0, 0, 800, 600 );
  glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );

  glMatrixMode( GL_PROJECTION );
  glLoadIdentity();
  gluPerspective ( viewFovY, viewAspect, nearDist, farDist );

  glMatrixMode( GL_MODELVIEW );
  glLoadIdentity();

  gluLookAt( viewerPosX, viewerPosY, viewerPosZ,
             lookAtX, lookAtY, lookAtZ, upX, upY, upZ);

  glPushMatrix();
    glTranslate3d( 100.0, 200.0, 300.0 );
    glScale3d( 3.0, 3.0, 1.0 );
    glColor3d( PINK );
    drawUnitSquare();
  glPopMatrix();

  glPushMatrix();
    glTranslate3d( 400.0, 400.0, 500.0 );
    glScale3d( 6.0, 6.0, 1.0 );
    glColor3d( VIOLET );
    drawUnitSquare();
  glPopMatrix();

  glutSwapBuffers();
}
```

camera space

world space

object space

world space

object space

Square 1 : $M_{view} T_1 S_1 V$

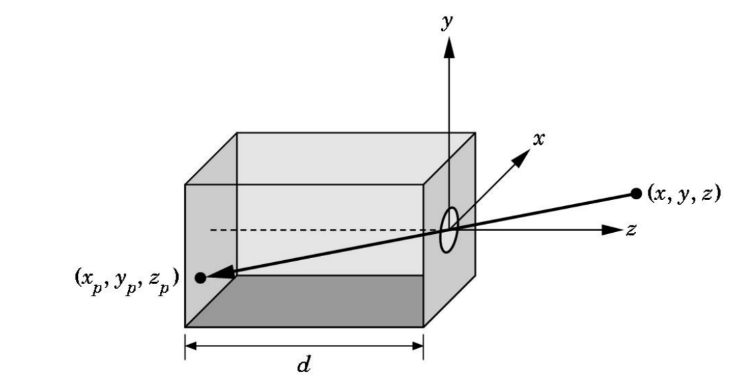
Square 2 : $M_{view} T_2 S_2 V$

Call	$\boxed{CTM}$	Rest of stack
gluLookAt	M_{view}	-
glPushMatrix / glTranslate3d	$M_{view} T_1$	M_{view}
glScale3d	$M_{view} T_1 S_1$	M_{view}
glPopMatrix	M_{view}	-
glPushMatrix	M_{view}	M_{view}
glTranslate3d	$M_{view} T_2$	M_{view}
glScale3d	$M_{view} T_2 S_2$	M_{view}

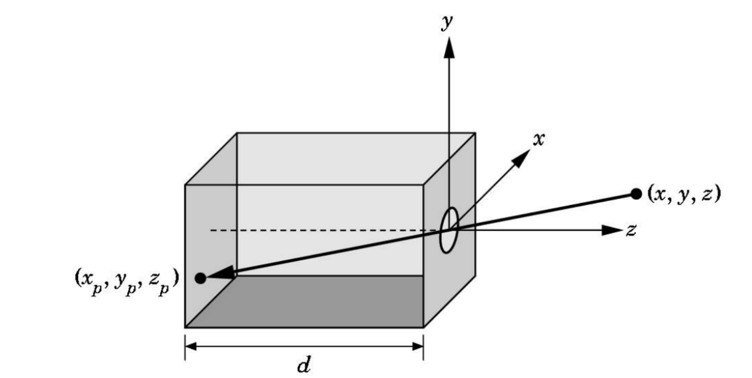
Tutorial

Question 1a

Referring to Lecture 1 Slide 31. If an imaginary image plane is *d* unit distance in front of the pinhole camera, what are the coordinates of the projection (on the imaginary image plane) of the 3D point (x, y, z)?



Question 1a



$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} \text{ and by definition } z' = d$$

$$x' = \frac{dx}{z} \quad y' = \frac{dy}{z} \quad z' = d$$

Question 1b

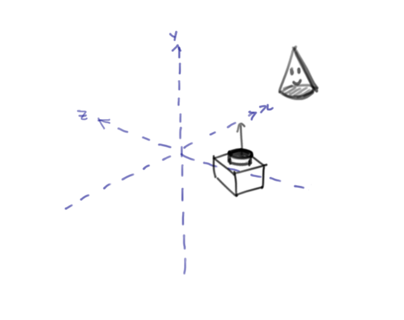
In the above setup, the camera's center of projection is conveniently located at the origin of the "world" coordinate frame, and pointed in the z direction. If the camera's center of projection is not located at the origin, and the camera is pointed in an arbitrary direction, the calculation of the projection becomes very messy. How would you make it less messy?

Question 1b

Reorient the world **with respect to the camera's rotation and translation**.

Visualization: https://imgur.com/a/sXuYgaM

Explanation

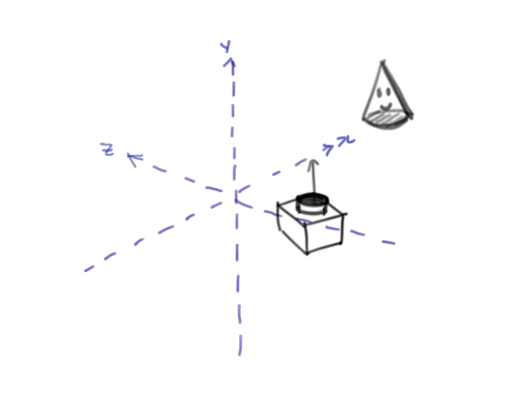


- Camera axes and world axes are not not equivalent
- The cone is represented in world axes.
- We undo the transformation by **translating everything by the camera's distance from the origin**,
- and then **rotating everything by the camera's rotation**.
- And now the camera axes and world axes are aligned.

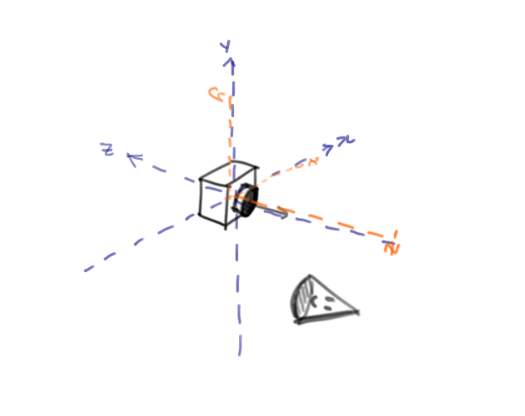
Question 2

Why do we want to perform view transformation?

See Q1b



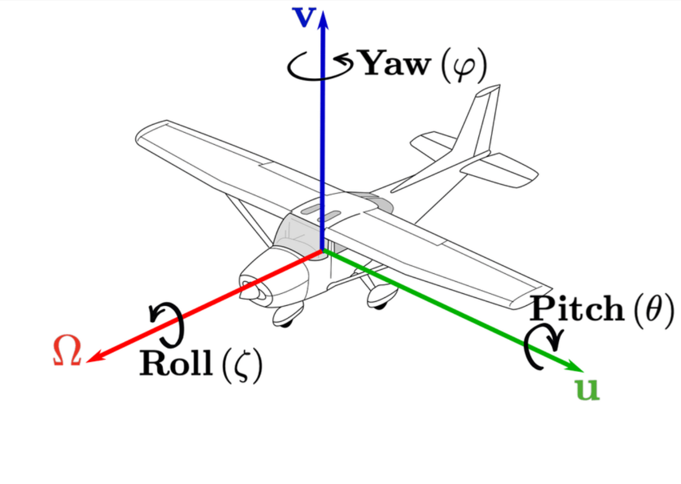
See Q1b



Question 3

Explain the purpose of the "up-vector" provided to the `gluLookAt()` function.

To prevent the camera from 'rolling'



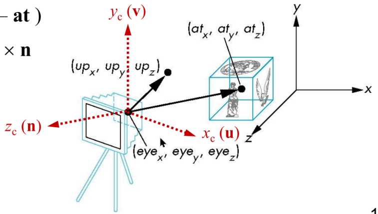
By defining the "up-vector we establish a vertical plane for the y and z axes of the camera coordinates.

Question 3b

Why does the “up-vector” not need to be perpendicular to the view direction?

We can derive our 3 axes as such:

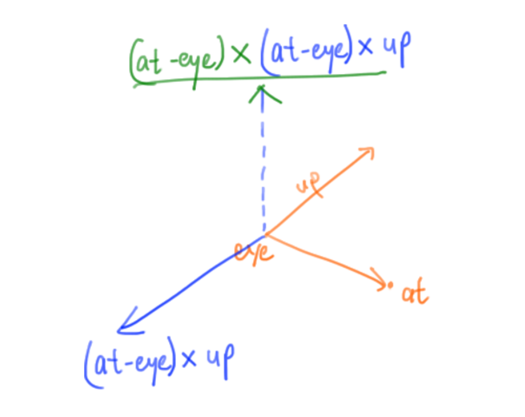
- The vectors **u**, **v**, **n** can be derived as follows
  - **n** = normalize(**eye** – **at**)
  - **u** = normalize(**up**) × **n**
  - **v** = **n** × **u**



18

As long as the up-vector is **not parallel** to the view direction and **is not zero vector**, it already uniquely defines the y-axis of the camera.

We can derive our 3 axes as such:



Question 4

Replace the following `gluLookAt()` function call with one or more calls to `glRotated()` and `glTranslated()`.
When using `glRotated()`, you are allowed to rotate about the x-axis, y-axis and z-axis only.

```
gluLookAt( ex, ey, ez, ex, ey, ez+1, 0, -1, 0 );
```

Analysis of `gluLookAt`

eye = (e_x, e_y, e_z)
at = $(e_x, e_y, e_z + 1)$
eye - at = $(0, 0, -1)$
up = $(0, -1, 0)$

z axis: n = eye - at = $(0, 0, -1)$
x axis: $u = \|up\| \times \|n\| = (1, 0, 0)$
y axis: $v = \|n\| \times \|u\| = (0, 0, -1) \times (1, 0, 0) = (0, -1, 0)$
camera position = (e_x, e_y, e_z)

- It is made up of a **translation first**, **then a rotation**
  - $\mathbf{M}_{\text{view}} = \mathbf{R}\,\mathbf{T}$
    - The translation $\mathbf{T}$ moves the camera position back to the world origin
    - The rotation $\mathbf{R}$ rotates the axes of the camera frame to coincide with the corresponding axes of the world frame

1. Translate the world towards camera: `glTranslate(-ex, -ey, -ez);`
2. Rotate the world to align with camera:
   - Notice that the camera z and y coordinates are flipped $z_c = n = -(0, 0, 1)$ and $y_c = v = -(0, 1, 0)$
   - `glRotated(180, 1, 0, 0)`
   - `glRotated(180, 0, 1, 0); glRotated(180, 0, 0, 1);`

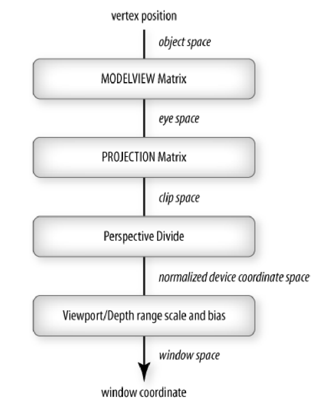
Question 5

A vertex, whose camera coordinates are (4, 6, -6), is being projected using the following OpenGL orthographic projection:

```
glOrtho( -10, 10, -10, 10, 0, 8 );
```

What will be the vertex's Normalized Device Coordinates (NDC)?

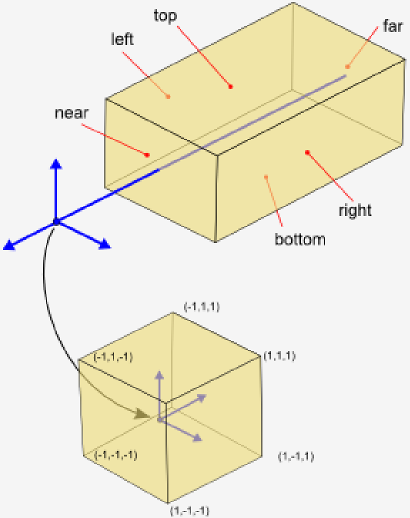
Coordinates through pipeline



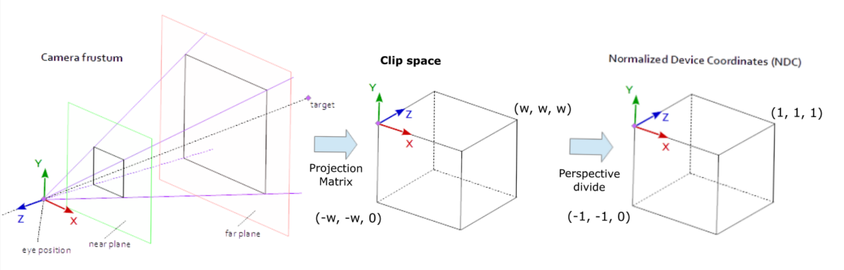
Camera coordinates to NDC space:

1. If vertex is within the clipping region, it is mapped in NDC space
2. NDC space is scaled to a $2 \times 2 \times 2$ **volume**

Coordinate spaces: Orthographic Projection



Coordinate spaces: Perspective Projection



Orthographic projection

- The mapping can be found by
  - First, **translating** the view volume to the origin
  - Then, **scaling** the view volume to the size of the canonical view volume

$$\mathbf{M}_{\text{ortho}} = \mathbf{S}\left(\frac{2}{right-left}, \frac{2}{top-bottom}, \frac{2}{near-far}\right) \cdot \mathbf{T}\left(\frac{-(right+left)}{2}, \frac{-(top+bottom)}{2}, \frac{(far+near)}{2}\right)$$

  - Note that $z = -near$ is mapped to $z = -1$, and $z = -far$ to $z = +1$

Orthographic projection

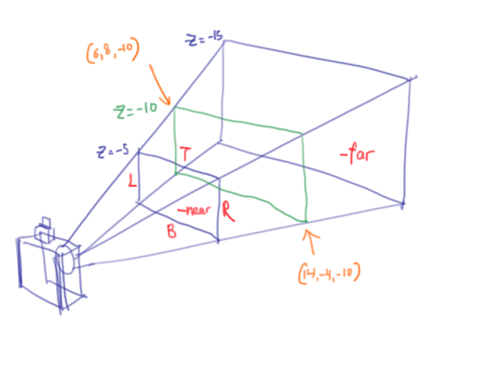
```
glOrtho( l, r, b, t, n, f );
```

$$
\begin{aligned}
\mathbf{T} &= T(\frac{-(10-10)}{2}, \frac{-(10-10)}{2}, \frac{8+0}{2}) \\
&= T(0, 0, 4) \\
\mathbf{S} &= S(\frac{2}{10-(-10)}, \frac{2}{10-(-10)}, \frac{2}{0-8}) \\
&= S(0.1, 0.1, -0.25) \\
\mathbf{M}v &= \mathbf{ST}(4, 6, -6) \\
&= \mathbf{S}(4, 6, -2) \\
&= (0.4, 0.6, 0.5)
\end{aligned}
$$

Question 6

A rectangle has vertices A: (6, -4, -10), B: (14, -4, -10), C: (14, 8, -10), D: (6, 8, -10) in the camera space.

Write a `glFrustum` function call to set up a view frustum that will maximize the image size of the rectangle, and the entire rectangle must appear in the image. The near and far plane distances should be set as 5 and 15 respectively.




```
glOrtho( 3, 7, -2, 4, 5, 15);
```

Question 7

A viewpoint at (vx, vy, vz) is looking at the center (cx, cy, cz) of a sphere of radius R. Complete the following OpenGL program to set up a view transformation and an orthographic projection so that the entire sphere appears as big as possible in a square viewport.

```
double PI = 3.141593;
double R = ...;      // radius of sphere.
double cx, cy, cz;  // center of sphere.
double vx, vy, vz;  // viewpoint position.
...
double D = Distance( cx, cy, cz, vx, vy, vz );

// Write your code below.
```

Code

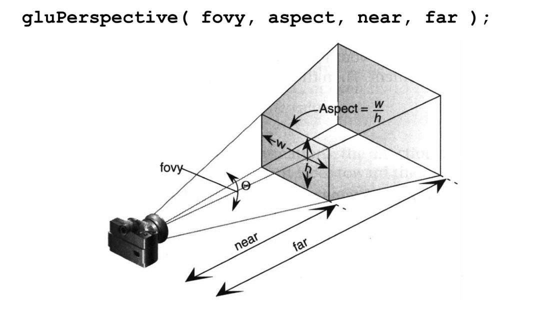
```
glMatrixMode(GL_PROJECTION); // Camera coordinates
glLoadIdentity(); // Always reset the matrix
// we are already looking at the camera's center,
// with the top/bottom/left/right points of the circle touching the clipping boundaries
// near = front most point on z-axis, far = furthest point on z-axis
glOrtho(-R, R, -R, R, D-R, D+R);

glMatrixMode(GL_MODELVIEW); // World Coordinates
glLoadIdentity(); // Always reset the matrix
// eye, at, up
gluLookAt(vx, vy, vz, cx, cy, cz, 0, 1, 0);
```

Question 8

Re-implement the `gluPerspective()` function by using the `glFrustum()` function. You can make use of the tangent function `tan()`, which takes an angle parameter (in radians).

```
void gluPerspective(
    double fovy, double aspect,
    double near, double far) {
  const double PI = 3.141592;
}
```

Question 8



$$\text{left} = -\frac{h}{2},\ \text{right} = \frac{h}{2},\ \text{bottom} = -\frac{w}{2},\ \text{top} = \frac{w}{2}.$$

Let aspect ratio be $a = \frac{w}{h}$.
Let fovy be θ.

By trigonometry, $h = 2\tan(\frac{\theta}{2}) \times \text{near}$.
By definition, $w = ah$.

```
void gluPerspective( double fovy, double aspect,
                     double near, double far )
{
    // Note that fovy is in degrees.
    const double PI = 3.141592;

    double h2 = near * tan( fovy/2.0 * PI / 180.0 );
    double w2 = aspect * h2;
    glFrustum( -w2, w2, -h2, h2, near, far );

}
```

Attendance taking

Thanks! Get the slides here after the tutorial.

https://trxe.github.io/cs3241-notes