Recreational Math Meetup
March 25, 2025

Creating Opportunities to Think Like a Mathematician: Exploring Classroom Possibilities

K-12 Mathematics

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Our Agenda

* Reflections on our experiences learning mathematics in K-12 classrooms

* A visit to a Grade 6 mathematics classroom where students explore division situations involving fractions

* Reflections on how the teacher structures student engagement to support student thinking and reasoning

* Reflections on mathematical ideas that emerge as students work together on the problems posed

* Thoughts on creating these kinds of opportunities for students to learn mathematics

What was learning mathematics like for you . . .

in elementary school?
in middle school?
n high school?

1. Take a few moments of private think time.
2. Find a partner.
3. Partner A shares thoughts for 1 minute while Partner B listens.
4. Partner B shares thoughts for 1 minute while Partner A listens.
5. For 1 minute, partners A and B reflect together, making sure each has time to talk.

What do you notice? What do you wonder?

Max Ray, a mathematics educator, is presenting a math problem that addresses how to make sense of a division situation that involves fractions.

Imagine we are stepping into a Grade 6 classroom with Max and we know something about what it means to add, subtract, multiply, and divide with whole numbers and how to add, subtract, and multiply fractions.

Imagine you have a new puppy!!!

You have 7 cups of dog food.

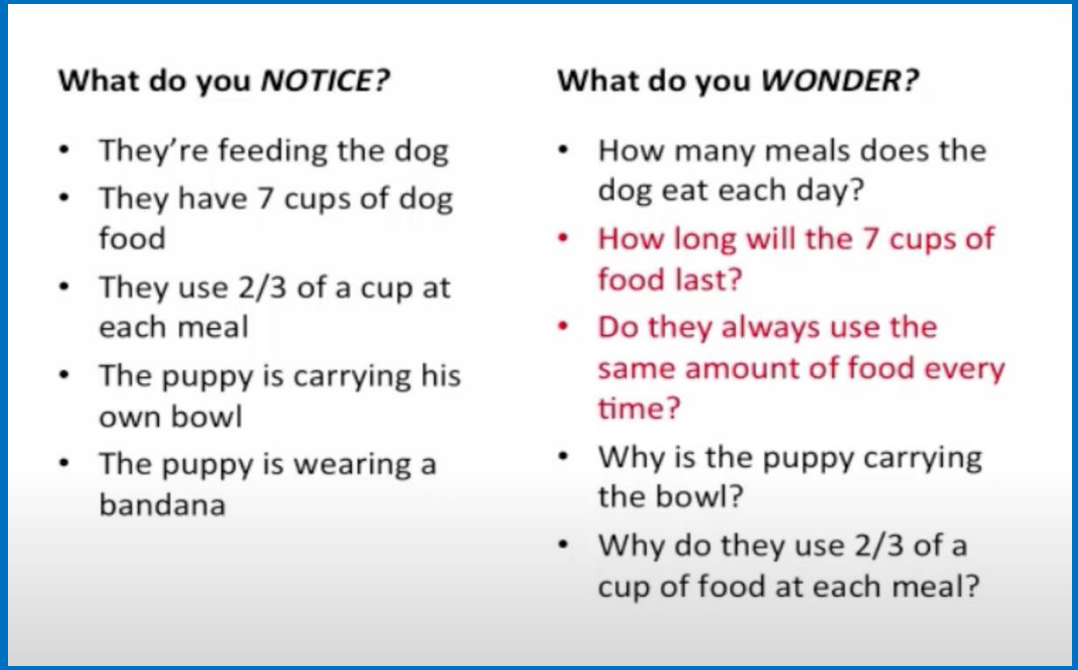
You use two-thirds of a cup of food at each meal.

What do you notice?

What do you wonder?

What students noticed and wondered . . .

We tell students this puppy gets one meal a day and it is always the same amount per meal.



We focus on what helps us think about this situation if we want to know how many meals we can feed our puppy before we run out of food.

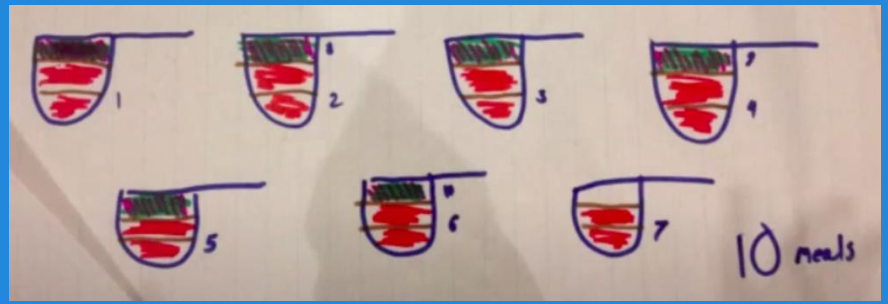
We want to get a sense of what students understand before they even begin to work toward an answer to our question . . .

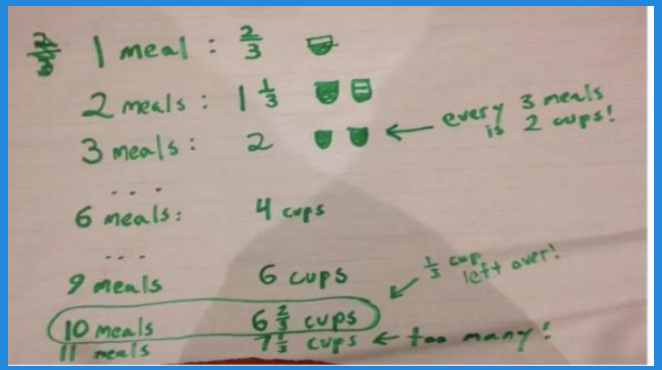
About how many meals will the dog food last?

- Definitely more than 7 meals
- Maybe about 10 meals
- Seven and two-thirds
- Definitely less than 14 meals
- Definitely less than 20 meals
- More than 5 meals
- Maybe about 15 meals
- Definitely less than 21 meals

The teacher records what students are thinking . . . without judgement . . .

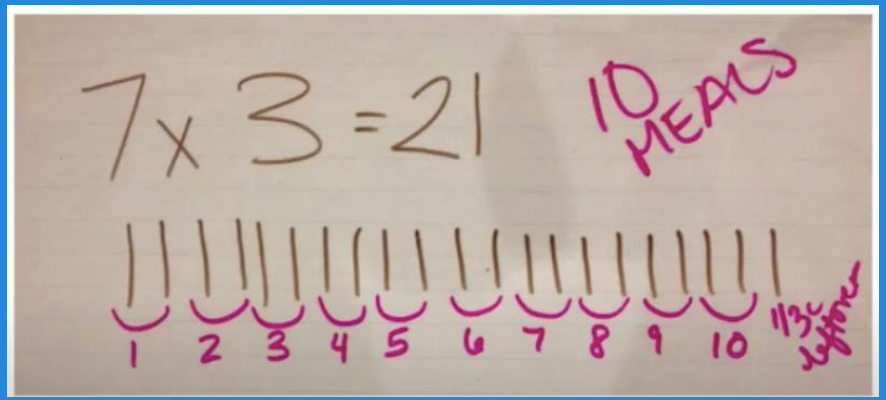
Students get into small groups to work together to find out how long the dog food lasts.





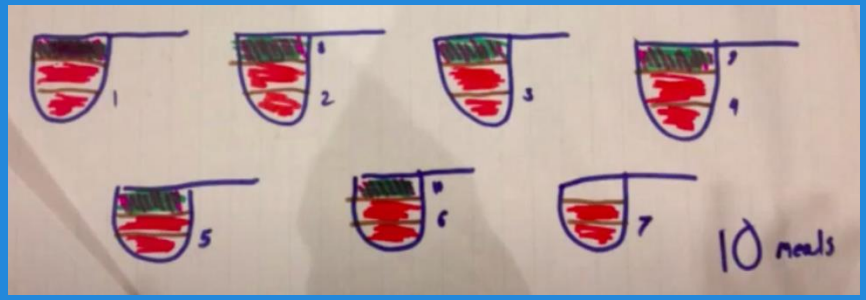
Focusing vs funneling question

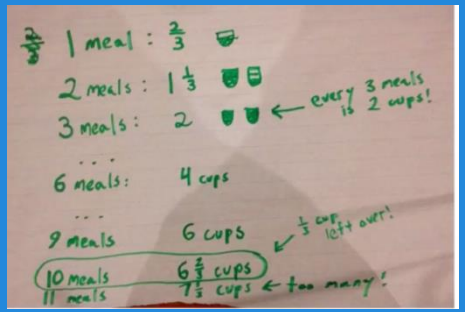
Assessing and advancing questions

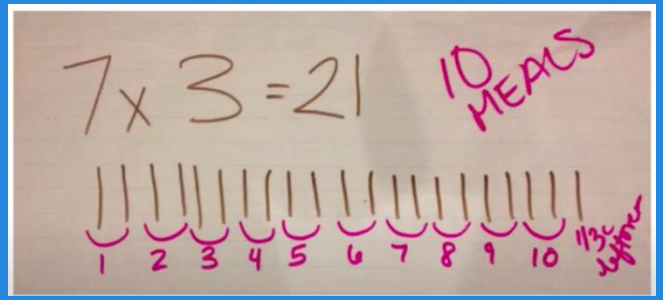


While they work, the teacher circulates, asks the kinds of questions that help surface their thinking, and thinks about in what order groups will present their work to the whole class.

Time for groups to share their work.

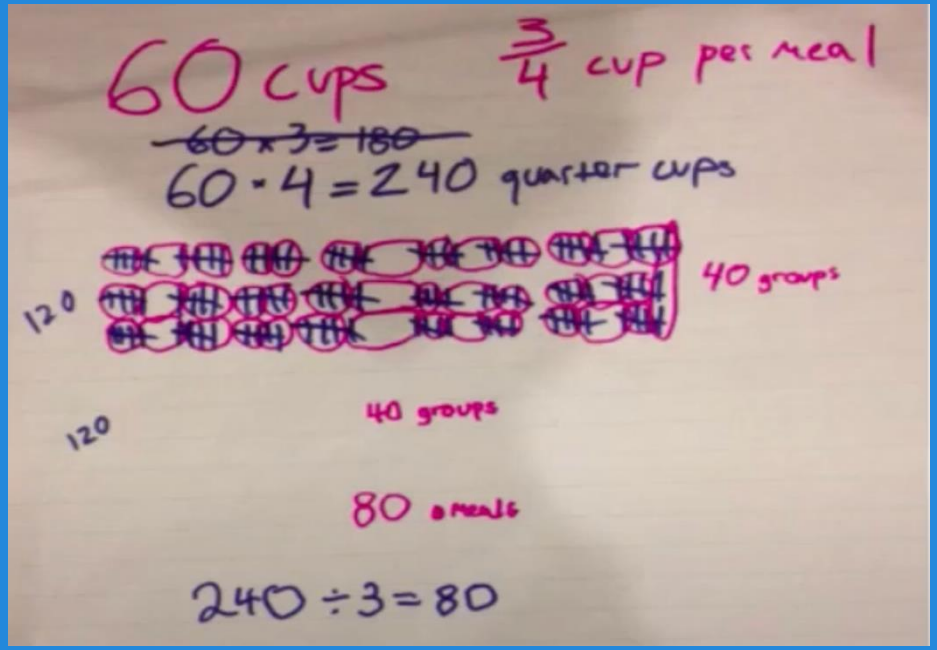






Groups present their work and answer questions from other students. The teacher asks what is the same about these solution strategies? What is different about these solution strategies? Will these strategies always work??

Try out their or someone else's strategy on a more complicated but similar situation . . .



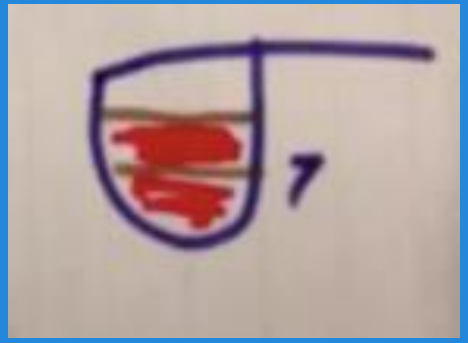
What if you had 60 cups of dog food and you used ¾ cup of dog food for each meal?

$$(60 * 4) \div 3 = 80$$

$$\frac{60}{1} \div \frac{3}{4} = \frac{60}{1} * \frac{4}{3} = \frac{80}{1}$$

As students work on these kinds of problem situations they become more efficient in their solution strategies.

There are also related ideas that could be important to explore . . .



This shows 2/3 of a serving of dog food in each cup.

This also shows that each cup contains 3/2 of a serving.

So how much of a serving remains after you feed your dog 10 servings?

See https://files.eric.ed.gov/fulltext/ED412122.pdf for more on what this kind of work this looks like in classrooms . . .

What's the Context for the Shift to Thinking and Reasoning?

2011 Common Core State Standards for Mathematics

2011 Common Core State Standards for Mathematical Practice

NCTM Mathematics Teaching Practices*

* NCTM (2014). Principles to Actions: Ensuring Mathematical Success for All

2011 CCSS for Mathematics Content: A Grade 6 Fraction Division Example

The Number System 6.NS

A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. 6.NS.A.1

   Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

2011 CCSS Mathematical Practice K-12

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

2011 CCSS Mathematical Practice K-12 Example

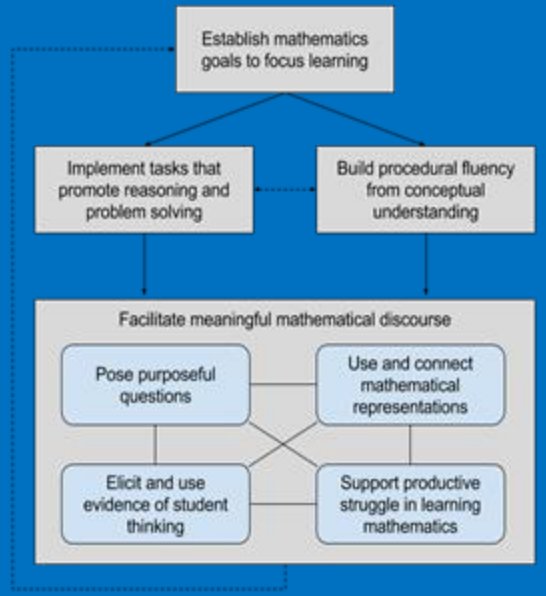
1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematics Teaching Practices

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

From Principles to Actions: Ensuring Mathematical Success for ALL by NCTM (2014)



From *Taking Action: Implementing Effective Mathematics Teaching Practices* by Margaret S. Smith (2017)

Equity-Based Mathematics Teaching Practices

"...we relate the eight effective teaching practices to specific equity-based practices shown to strengthen mathematical learning and cultivate positive student mathematical identities. Equitable mathematics classrooms provide every student with access to meaningful mathematics by leveraging students' strengths (mathematical competencies), drawing on students as resources of knowledge, and challenging spaces of marginality. These classroom communities of collaboration and coherent discourse position each and every student to make sense of mathematics and develop positive mathematics identities." (*Taking Action*, p. 6)

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple sources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematical learning.

From *Taking Action: Implementing Effective Mathematics Teaching Practices* by Margaret S. Smith
Adapted from *The Impact of Identity in K–8 Mathematics: Rethinking Equity-Based Practices*

Many Thanks
for Your Thoughts
and Questions