

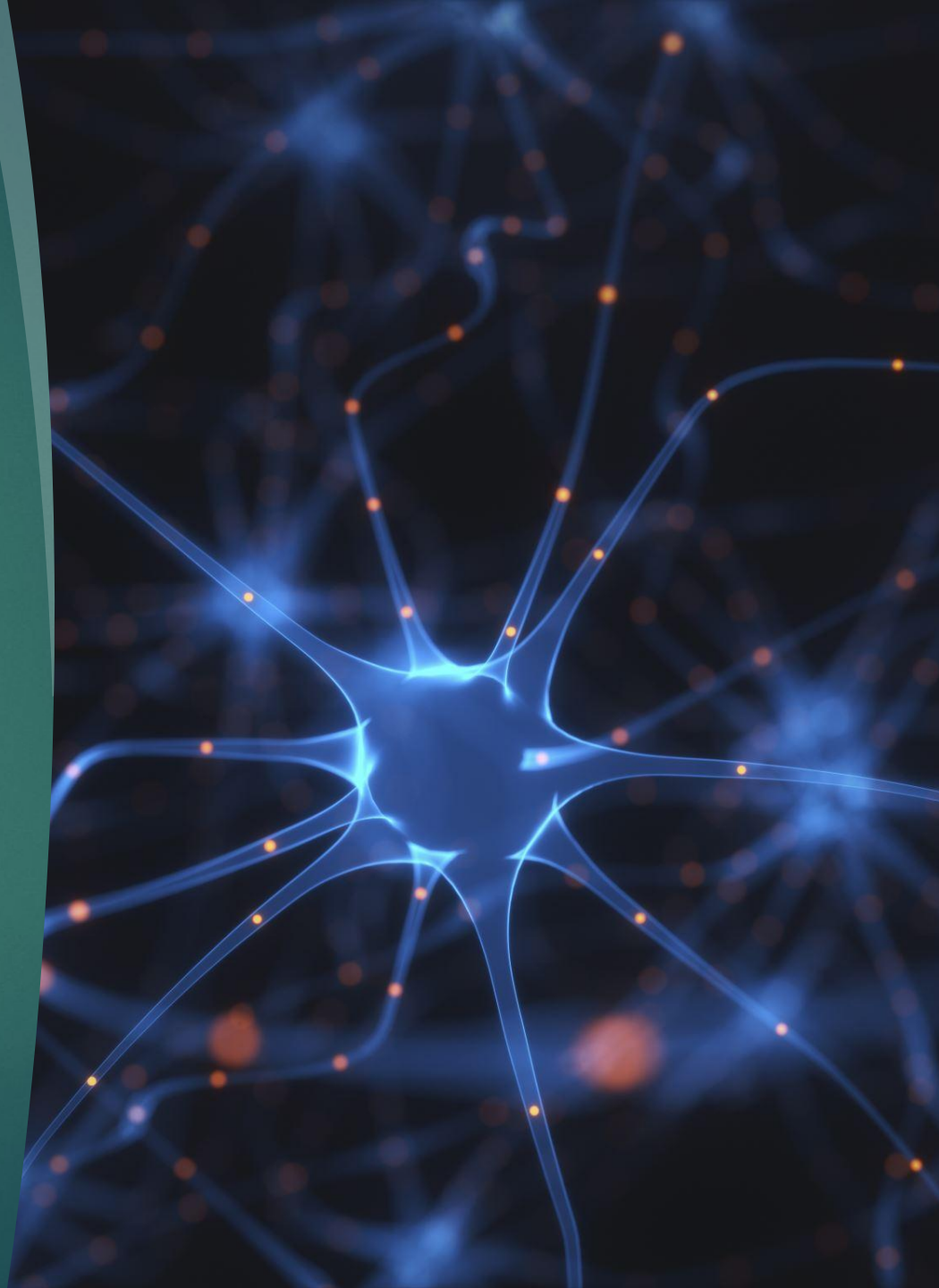


AI in Math

WHENCE, WHERE, WHITHER?

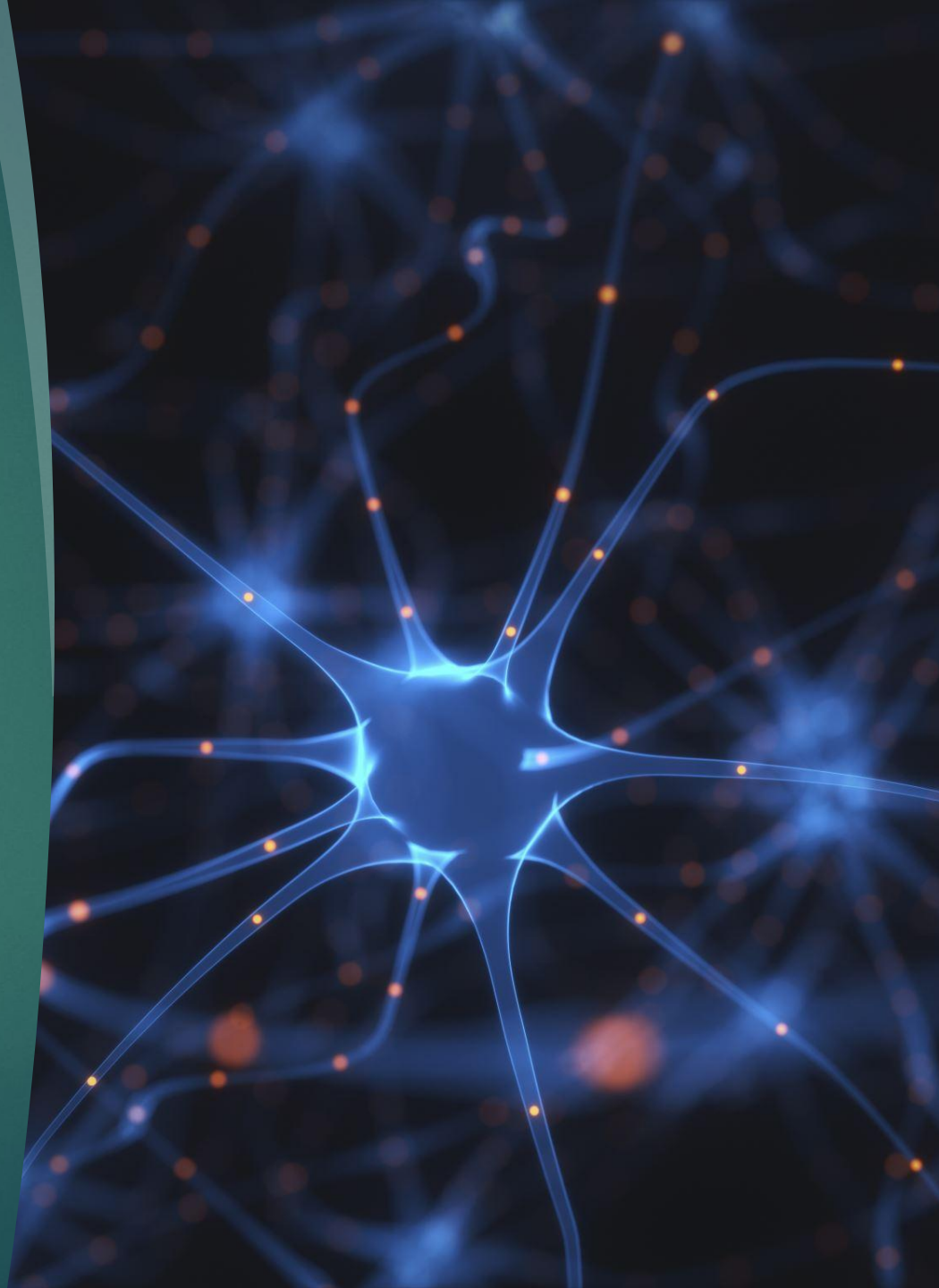
Are we there yet?

- ▶ “Machines should work; people should think!”
– Tom Watson, IBM
- ▶ From Richard V. Andree, “Selections from Modern Abstract Algebra”.

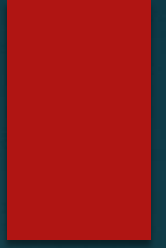


Are we there yet?

- ▶ Does it pass the Turing test?
 - ▶ (Alan Turing - born June 23, 1912)



That's so last decade!



- ▶ Fast, accurate, and dumb!
 - ▶ Calculation
 - ▶ Dedicated algorithms
- ▶ Symbolic manipulation
 - ▶ Computer Algebra Systems (CAS, Maple, Mathematica)
 - ▶ Formal Logic, Proof Checking (Prolog, QA, LEAN4)
- ▶ Visualization – Graphing, animation

Example: Solving the cubic

$$\sqrt[3]{28 + 45i\sqrt{3}} + \sqrt[3]{28 - 45i\sqrt{3}}$$

► Maple:

```
> expl := surd(28 + 45*I*sqrt(3), 3) + surd(28 - 45*I*sqrt(3), 3);  
expl := (28 + 45 I sqrt(3))^(1/3) + (28 - 45 I sqrt(3))^(1/3)  
=> simplify(%);  
8  
=>
```

What's new?

► Gemini:

Can you simplify the cube root of $(28+45\sqrt{-3})$?



Let's tackle this! We want to simplify $\sqrt[3]{28 + 45\sqrt{-3}}$.

First, let's rewrite $\sqrt{-3}$ as $i\sqrt{3}$, where i is the imaginary unit ($\sqrt{-1}$). So we have:

$$\sqrt[3]{28 + 45i\sqrt{3}}$$

We're looking for a complex number of the form $a + bi\sqrt{3}$ such that its cube equals $28 + 45i\sqrt{3}$. Let's expand $(a + bi\sqrt{3})^3$:

$$\begin{aligned}(a + bi\sqrt{3})^3 &= a^3 + 3a^2(bi\sqrt{3}) + 3a(bi\sqrt{3})^2 + (bi\sqrt{3})^3 \\ &= a^3 + 3a^2bi\sqrt{3} + 3a(b^2i^2 \cdot 3) + b^3i^3(\sqrt{3})^3 \\ &= a^3 + 3\sqrt{3}a^2bi - 9ab^2 - 3\sqrt{3}b^3i\end{aligned}$$

Example: Solving the cubic

► Gemini:

So, the simplified form of $\sqrt[3]{28 + 45\sqrt{-3}}$ is $\boxed{4 + i\sqrt{3}}$.

The Four-Colour Theorem (1976)

Appel & Haken

- ▶ Produce a list of 1,834 unavoidable configurations.
- ▶ Create an algorithm to “color” each configuration.
- ▶ Grab spare time on the university’s admin computer.
- ▶ Declare victory!

The Four-Colour Theorem (2025)

What could AI do?

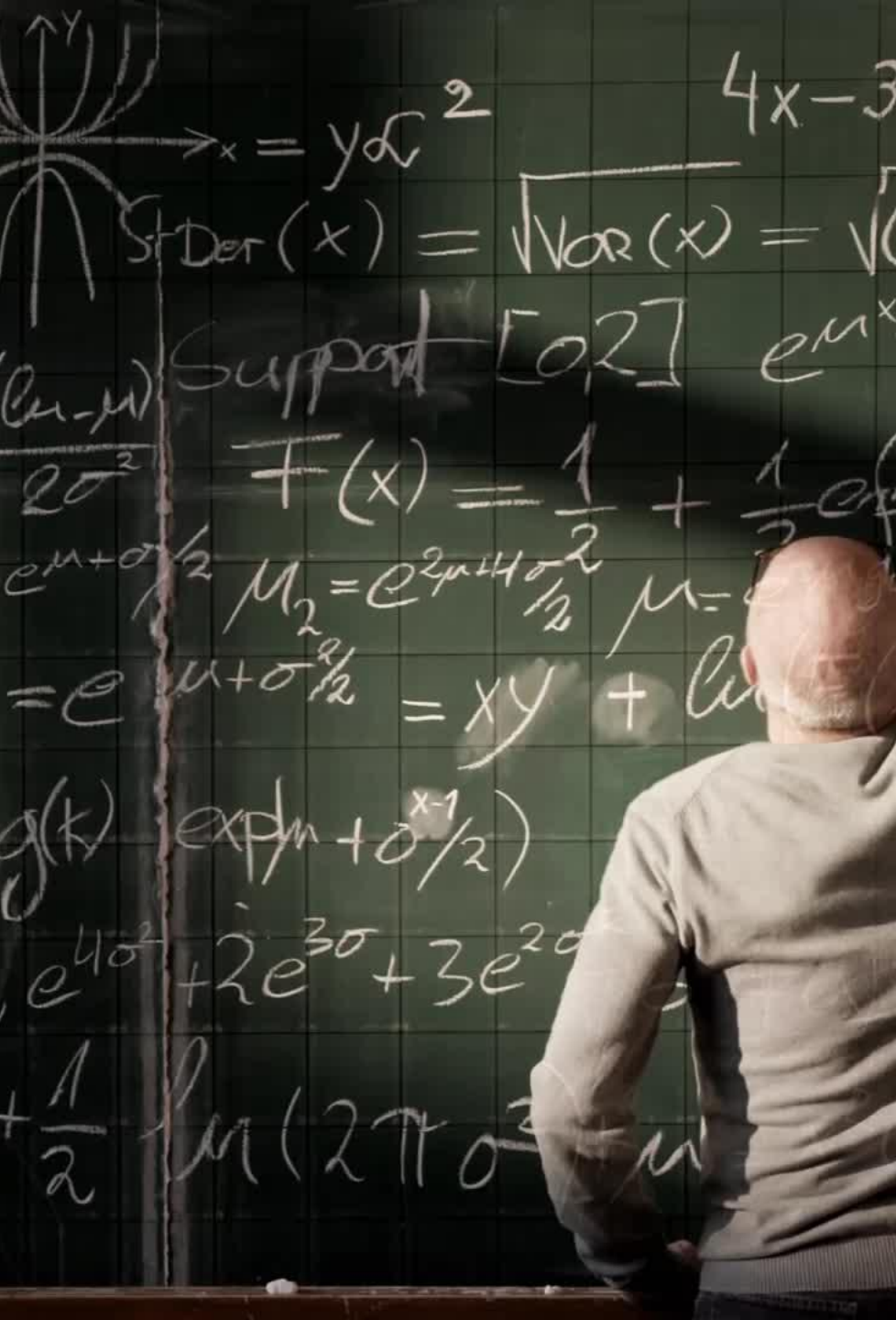
- ▶ Produce a list of 1,834 unavoidable configurations?
- ▶ Reduce / minimize the list of configurations?
- ▶ Create an algorithm to “color” each configuration?
- ▶ Color each configuration?
- ▶ Declare victory!
- ▶ Look for another way to prove the theorem?

Possible questions today...

- ▶ “Can you suggest a thesis topic for a graduate student in mathematics?”
- ▶ “What is the cutting edge of research on the Riemann hypothesis?”
- ▶ “Can you suggest any new ideas for tackling the Riemann hypothesis?”
- ▶ “How can AI be used in education, and what more is required for AI to be a good teacher/tutor?”
- ▶ “What can't AI do?”
- ▶ “Can you cite your sources, please?”

Model and Platform

- ▶ CAS?
- ▶ Gemini?
- ▶ Liaison?



What me worry?


- ▶ What are the problems with reliance on AI in mathematics?
- ▶ Are results verifiable?
- ▶ Are they understandable?
- ▶ Are they properly attributed? (Humans and AI)
- ▶ Beyond math:
 - ▶ “HAL”, “Terminator”, and “anything a hacker can do”.

“What if they gave a war and nobody came?”

- ▶ Evariste Galois (1811-1832) develops what will become known as “Galois theory”
 - ▶ Draft to Cauchy – not interested, lost it.
 - ▶ Draft to Fourier, who was on his deathbed. Draft lost.
 - ▶ Draft to Lacroix – lost.
 - ▶ Draft to Poisson – couldn’t understand it; filed away.
 - ▶ 1846, Liouville published and clarified.

The four-colour theorem

- ▶ 1852 - proposed by Guthrie.
- ▶ 1879 - “Proved” by Kempe.
 - ▶ 1890 - error found by Heawood.
- ▶ 1880 - “Proved” by Tait.
 - ▶ 1891 - error found by Petersen in 1891.
- ▶ 1976 - Proved by Appel and Haken, using a computer to solve 1,834 unavoidable configurations.
 - ▶ Proof questioned as unverifiable.



“What if they gave a war and nobody came?”

- ▶ 2029 - Riemann hypothesis proved by AI; 2,000-page proof confounds humans.

This just in...

- ▶ ChatGPT loses at chess to 70s Atari console game.
 - ▶ <https://www.pcmag.com/news/chatgpt-gets-absolutely-wrecked-in-chess-match-with-1978-atari>