TMA4220 - PDEs /w FEM

Programming Project

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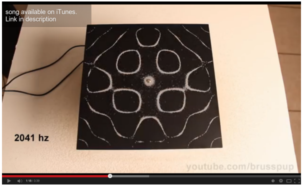
Finite Element Methods on Free Vibrations

Introduction

The Free Vibration Equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \sigma(u) \qquad (1)$$

- Is it possible to model



with Finite Element Methods?

Introduction

- We want a variational formulation of (1)
- Usual procedure: Multiply (1) with a test function v and integrate over the domain to obtain

$$\rho \int_{\Omega} \ddot{u} \cdot v \mathrm{d}\Omega = -\int_{\Omega} \epsilon(v) \cdot C\epsilon(u) \mathrm{d}\Omega \tag{2}$$

Introduction

- Semi-discretization on (2) yields

$$M\ddot{\boldsymbol{u}} = -A\boldsymbol{u} \tag{3}$$

$$A = [A_{ij}] = \int_\Omega \varepsilon(\varphi_i)^T C\varepsilon(\varphi_j)\,\mathrm{d}\Omega$$

$$M = [M_{ij}] = \int_\Omega \rho\varphi_i^T\varphi_j\mathrm{d}\Omega$$

- Assuming $\boldsymbol{u} = \boldsymbol{u}e^{i\omega t}$ on (3) yields the generalized eigenvalue problem

$$\omega^2 M\boldsymbol{u} = A\boldsymbol{u} \tag{4}$$

Building A and M

$$\varphi_{\hat{i},1} = \begin{bmatrix} \varphi_{\hat{i}} \\ 0 \\ 0 \end{bmatrix}, \quad \varphi_{\hat{i},2} = \begin{bmatrix} 0 \\ \varphi_{\hat{i}} \\ 0 \end{bmatrix}, \quad \varphi_{\hat{i},3} = \begin{bmatrix} 0 \\ 0 \\ \varphi_{\hat{i}} \end{bmatrix}.$$

$$\varphi_{\hat{i}}\Big|_{T_k} = c_0 + c_1 x + c_2 y + c_3 z$$

$$\varepsilon\left(\varphi_{\hat{i},d}\right) = \begin{bmatrix} c_1 \\ 0 \\ 0 \\ c_2 \\ c_3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ c_2 \\ 0 \\ c_1 \\ 0 \\ c_3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ c_3 \\ 0 \\ c_1 \\ c_2 \end{bmatrix}$$

Building A and M

- Stiffness matrix A:

$$\varepsilon(\varphi_i)^T C\varepsilon(\varphi_j)\,\mathrm{area}(T_k)$$

- Mass matrix M:

$$\rho \cdot \det(B) M^P_{\alpha,\beta}$$

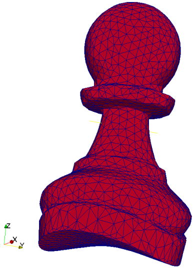
Implementation

- Solve $\omega^2 M\boldsymbol{u} = A\boldsymbol{u}$ for the k lowest eigenvalues using MATLAB's `eigs`
- Use the different eivenvectors $\boldsymbol{u_i}$ to make animations
- $\boldsymbol{x} = \boldsymbol{x_0} + \alpha \boldsymbol{u_i} \sin(t)$
- Collection of MATLAB scripts/functions
- Open to different geometries (chess pawn, gun, plate) and materials
- ParaView for the post processing

Boundary condition

Can we have boundary conditions?

- Physical interpretation
- Free vibration
- Can use Dirichlet



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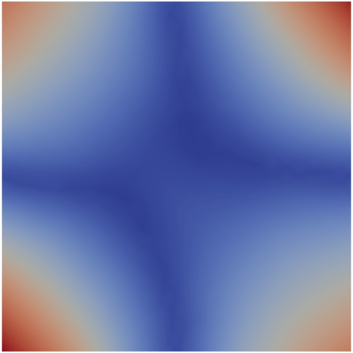
Back to the plate

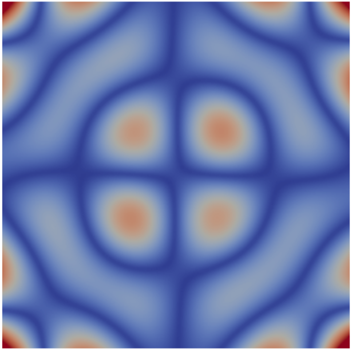
- An example of our results (3443 Hz):



- Needs finer mesh

Back to the plate (cont.)

- Finer mesh gives (3031.6 Hz):



- Picks up frequencies closer together.