TMA4220 Numerical Solution of Differential Equations by element methods

Part 2: Vibration analysis

Candidate numbers

October 24, 2013

Abstract

Introduction

With the displacement of spatial points in x_1- and x_2-direction represented as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

the strain on each point is

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} \\ \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \end{bmatrix}$$

as many notes on linear elasticity will let you know.[1]

The connection between the strain ϵ and the stress σ is

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{bmatrix}$$

$$\boldsymbol{\sigma} = C\boldsymbol{\epsilon}$$

where E is the Young's modulus and ν is the Poisson's ratio. Young's modulus characterises the solids stiffness, i.e. large E means that you need a large force to deform

[1] Note however that this is the form usually called engineer strain.

the solid. Poisson's ratio is the ratio between the strains in x_1- and x_2-direction when submitting the solid to a stress in only one of the directions. To clarify: Consider a stress in only the x_1-direction direction[2], then $\nu > 0$ will say that the solid contracts in the x_2-direction and elongates in the x_1-direction. Worth noting is that $\nu < 0$ is possible, and some materials actually have this property. Weird.

During the course of this analysis we will let E and ν completely describe a solid's properties. No stress or strain due to difference in temperature.

[2] σ_{11} being the only stress different from zero.