TMA4220 - Numerical Solution of Partial Differential Equations Using Finite Element Methods

Programming Project

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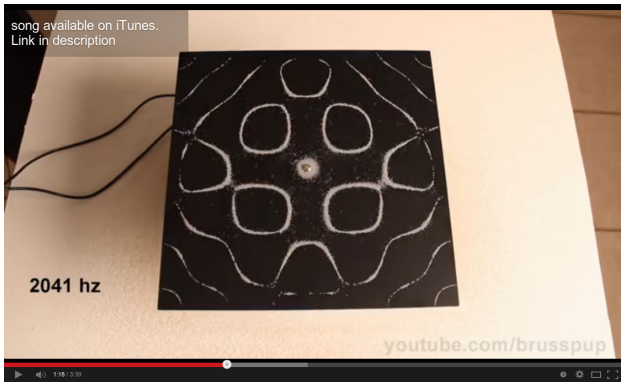
Finite Element Methods on Free Vibrations

Introduction

The Free Vibration Equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \sigma(u) \tag{1}$$

- Is it possible to model



with Finite Element Methods?

Introduction

- We want a variational formulation of (1)
- Usual procedure: Multiply (1) with a test function v and integrate over the domain to obtain

$$\rho \int_{\Omega} \ddot{u} \cdot v \mathrm{d}\Omega = -\int_{\Omega} \epsilon(v) \cdot C \cdot \epsilon(u) \, \mathrm{d}\Omega \tag{2}$$

- Semi-discretization on (2) yields

$$M\ddot{\boldsymbol{u}} = -A\boldsymbol{u} \tag{3}$$

$$A = [A_{ij}] = \int_{\Omega} \overline{\epsilon}\left(\varphi_i\right)^T C\overline{\epsilon}\left(\varphi_j\right) \mathrm{d}\Omega$$

$$M = [M_{ij}] = \int_{\Omega} \rho\varphi_i^T \varphi_j \mathrm{d}\Omega$$

- Assuming $\boldsymbol{u} = \boldsymbol{u}e^{i\omega t}$ on (3) yields the generalized eigenvalue problem

$$\omega^2 M\boldsymbol{u} = A\boldsymbol{u} \tag{4}$$