UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST3220 — Cosmology I

Day of exam: Friday 30th of March 2012

Time for exam: 15.00 – 18.00

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

In this exercise you can assume units of $c = 1$

a) Show that $\Omega > 1$ for a $k = +1$ universe, $\Omega = 1$ for a $k = 0$ universe and $\Omega > 1$ for a $k = -1$ universe,

b) Show that $\rho(t) = \rho_0 a_0^3 / a^3(t)$ for a matter dominated universe.

c) The deceleration parameter q_0 is defined by

$$q_0 = -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2}.$$

Suppose the universe is matter dominated, so that $\Omega_0 = \Omega_{m0}$. Express q_0 in terms of Ω_{m0}.

d) Now suppose both matter and vacuum energy (cosmological constant) are important. Show that the first evolution equation (first Friedmann equation) can be written as

$$\dot{a}^2 + k = \frac{C}{a} + Da^2. \tag{1}$$

e) What are the constants C and D?

Problem 2

Consider a flat universe filled with dust and a dark energy with equation of state $p_{\mathrm{DE}} = wc^2\rho_{\mathrm{DE}}$. For convenience you can choose $a_0 = 1$.

a) Use the adiabatic expansion equation to find the evolution of the energy density of the dark energy ρ_{DE} as a function of redshift z.

b) Use the above solution to find an equation for the redshift of equality z_{eq} when the dark energy density is equal to the energy density of matter in terms of Ω_{m0}.

Imagine you are trying to extract cosmological information from observations of supernovae. The observations provide you with a redshift z and a magnitude μ. For constant magnitude objects like supernovae the magnitude is given by the luminosity distance according to the following formula:

$$\mu = 5\log_{10}(\frac{d_L}{10\text{pc}}) - \mu_0 \tag{2}$$

where d_L is the luminosity distance, μ_0 is a constant and pc is the astronomical distance unit parsec.

c) Describe a method for extracting the comoving coordinate distances r of the supernovae. In particular show that r can be written as

$$r = \frac{1}{1+z} 10^{\left(1+\frac{\mu+\mu_0}{5}\right)}\text{pc} \tag{3}$$

Imagine you have plenty of measurements so that you can find a good approximation for $r(z)$ and it's derivatives with respect to z from the observations.

d) Use the integral defining the comoving coordinate to find a formula for H expressed through $r(z)$ and its derivatives with respect to z.

e) Further prove that the deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ can be expressed as:

$$q(z) = -\left(1 + (1+z)\frac{d}{dz}\ln\left(\frac{dr}{dz}\right)\right) \tag{4}$$

f) Use the Friedmann equations to show that the deceleration parameter q can be expressed in the following form:

$$q(z) = \frac{1}{2}\left(1 + \frac{3w_{\mathrm{DE}}\left(\frac{1+z}{1+z_{\mathrm{eq}}}\right)^{3w_{\mathrm{DE}}}}{1 + \left(\frac{1+z}{1+z_{\mathrm{eq}}}\right)^{3w_{\mathrm{DE}}}}\right) \tag{5}$$

g) Use the results from i) and h) to obtain a formula for the equation of state parameter w_{DE} at the time $z = z_{\mathrm{eq}}$ in terms of z_{eq} and the derivatives of the comoving coordinate with respect to redshift at the redshift of equality.