UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST3220 — Cosmology I

Day of exam: Friday 27th of March 2015

Time for exam: 12.15 – 15.15

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Throughout the equation set we will work in units of $c = 1$

We will also assume that the value of the scale factor today is unity:

$$a_0 = 1$$

Problem 1

Redshift dependent equations

Recall that the redshift z is defined by the equation

$$\frac{1}{a} = 1 + z \tag{1}$$

This definition provides a one-to-one correspondance between redshift and scale factor so long as $a > 0$ and $z > -1$. We can use this fact to redefine equations in terms of redshift.

a) Show that the adiabatic expansion equation in terms of redshift becomes:

$$\dot{\rho} = \frac{3\dot{z}}{(1+z)}(\rho + p) \tag{2}$$

b) Use the version of the adiabatic expansion equation above to find an expression for $\rho(z)$ for dust and cosmological constant, without first calculating $\rho(a)$.

c) Use your results from b) and the definitions of the density parameters Ω_{m0} and $\Omega_{\Lambda 0}$ to write the first Friedmann equation for flat Universe with dust and cosmological constant (ΛCDM) using only Ω_{m0}, z, H_0 and H.

d) Use the definition of the comoving coordinate and make a variable change from t to a to z to prove that the comoving coordinate is given by:

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')} \tag{3}$$

where $E(z) \equiv \frac{H(z)}{H_0}$. Find $E(z)$ for the Universe described in c).

Problem 2

$R_h = t$ and power law cosmologies

The $R_h = t$ and power law cosmologies, are alternative models to ΛCDM. $R_h = t$ cosmology is defined as a universe with a linear increase in Hubble radius

$$R_h = t, \tag{4}$$

where the Hubble radius $R_h = \frac{1}{H}$. Power law cosmology is defined by assuming a power law function of a with respect to t, i.e.

$$a = \left(\frac{t}{t_0}\right)^n \tag{5}$$

They are both proposed to describe the Universe at late time, so at low redshift.

a) Find $H(t)$ for $R_h = t$ and power law cosmology.

b) $R_h = t$ is equivalent to a power law cosmology for a special value of n, which value?

c) Find the function $E(z) \equiv \frac{H(z)}{H_0}$ for power law cosmologies.

d) Show that the comoving distance $r(z)$ in power law cosmology is given by:

$$r(z) = \frac{1}{H_0} \times \begin{cases} \frac{(1+z)^{1-1/n}-1}{1-1/n}, & n \neq 1, \\ \ln(1+z), & n = 1. \end{cases}$$

Problem 3

Supernova observations

We want to use supernovae observations to compare the ΛCDM model to the alternative models described above. When we observe supernovae, we measure their magnitude m on the sky, and their redshift. The observed magnitude is given by:

$$m = M + 5\log_{10}\left(H_0 d_L(z)\right) \tag{6}$$

where M is the absolute magnitude, which is assumed to be constant for supernovae type Ia, and d_L is the luminosity distance

a) Explain why the differences in observed magnitude in universes governed by different models, can be found by specifying just $r(z)$.

b) Assume that $z >> 1$, and find approximate behaviour of $r(z)$ as a function of z for the ΛCDM universe in this limit.

c) Find $r(z)$ in the large z limit for the power law cosmologies.

d) The power law cosmologies are only used to describe cosmology in the late time of the Universe history. Do your answers in b) and c) help you understand why?