UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST4220 — Cosmology I

Day of exam: Tuesday October 11th 2010

Time for exam: 15.00 – 18.00

This problem set consists of 3 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

A low-density universe may be approximated by setting the energy density in the Friedmann equation to zero. Beside the uninteresting possibility of $\dot{a} = k = 0$ there is a non-trivial solution with curvature.

a) Find $a(t)$ for a nonzero curvature ($k \neq 0$).

b) Find the Hubble relation between the proper distance and redshift in such a model universe.

Problem 2

Our universe is spatially flat with the dominant component being matter and positive dark energy. Its fate is an unending exponential expansion. Now consider the same flat universe but with a negative dark energy $\Omega_\Lambda = 1 - \Omega_{M,0} < 0$, which provides a gravitational attraction. The universe will then start to contract.

a) Show that this will slow the expansion down to a standstill when the scale factor reaches $a_{max} = (-\Omega_\Lambda/\Omega_{M,0})^{1/3}$.

b) Show how you would calculate the age of the universe when a(t) reaches zero again. (At some point you will encounter a difficult integral. Stop there!)

Problem 3

Derive from the Friedman equation the relation between the deceleration parameter and the density parameter Ω in a flat matter dominated Friedman universe with $\Lambda = 0$.

Problem 4

Suppose the universe is filled with a fluid of density ρ and pressure p, with an equation of state $p = w\rho$, where $w \neq -1$. Ignore all other contributions to the energy density of the universe.

a) Find $\rho(a)$ in terms of w. Also, find $\rho(z)$, where z is the redshift of objects at time t.

b) Assuming that $k = 0$, find $a(t)$ in terms of w.

c) What value of w corresponds to cold matter? What is the corresponding $a(t)$?

d) What value of w corresponds to isotropic radiation? What is the corresponding $a(t)$?

e) Find the present age of the universe t_o in terms of H_o and w. Suppose h is observed to be $h = 2/3$. What range of values for w are possible if the universe is determined to be at least 15×10^9 years old?

$H_0 = h\ 100\ Km\ s^{-1}\ Mpc^{-1}$