SCHRIFTELIJK TENTAMEN COSMOLOGY
3^{rd} term 2003/2004

Question 1.

The Friedman equations correspond to an adiabatically expanding medium. On the basis of this observation one can work out the temperature history of the Universe.

a) Start by giving the full Friedmann equations (i.e. including cosmological constant, and including pressure). Describe and explain the various quantities in the equations.

b) A Newtonian equivalent of these equations can be derived too. Describe how. Subsequently specify the fundamental differences between the Newtonian equations and the genuine relativistic one (there are 3 terms that should be discussed).

c) From the Friedmann equations one can derive the equation for the the evolution of $\rho(t)$, the change of the energy density of the expanding Universe. Derive this equation.

d) Show that the derived equation in (c) implies an adiabatically expanding medium.

e) For an adiabatic expanding medium we know that $TV^{\gamma-1} = cst$, with T the temperature of the medium and V the volume. Derive the temperature change of a uniform radiation field in an expanding Universe as a function of expansion factor a_{exp}.

f) Given the fact that in an expanding Universe the frequency ν of radiation is redshifted, so that the frequency of a photon of current frequency ν_o has a frequency $\nu(z) = \nu_o(1+z)$ at redshift z, show that when the radiation field is blackbody at a particular cosmic epoch, it will remain blackbody ! (hint: combine temperature and frequency evolution of photons).

Question 2.

To reconstruct the thermal history of the Early Universe, one needs to combine the knowledge of the temperature evolution of the Universe, that of the interactionrate Γ of the various relevant physical processes and the dynamical timescale of the Universe (...). Physical transitions occur when physical processes get out of equilibrium.

a) What is the dynamical timescale of the Universe ? Write down the criterion for a physical process being in equilibrium and thus for when it runs out of equilibrium.

b) What makes the Universe such a very special physical system ? In this, take into account that the temperature is continuously decreasing (affecting the reaction rates and timescales !) and about the number of photons available (a fundamental number of cosmology !!!!).

c) Subsequently, describe in some detail the thermal history of the Early Universe. Mention at least five crucial transitions in the pre-recombination Universe, describe what happened at that transition, and approximately at what temperature/energy and redshift. Amongst those five, elaborate specifically on the epoch of primordial nucleosynthesis and that of the recombination/decoupling epoch.

to be continued ...

Question 3.

a) Einstein's general theory of relativity is a metric theory of gravity. Give the expression of the Einstein equation, and provide a physical explanation for the significance of this equation. Emphasize the essential difference with Newtonian gravity wrt. behaviour of spacetime in the general relativistic world !!!

b) What does the Cosmological Principle say ? Needed is an enumeration of the (3) ingredients of the cosmological principle and a short description of what they mean. In addition, also discuss the meaning of Weyl's principle.

c) What is the crucial significance of the cosmological principle in the context of relativistic cosmology. Describe the possible geometries allowed for a medium obeying the cosmological principle.

d) Translate the above into a metric expression for the spacetime of an evolving universe obeying the cosmological principle and Weyl's principle:
   - give the name of this metric
   - write out the metric form, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, for this spacetime
   - explain the meaning of each of the characteristic quantities. Focus in particular to those concerning the radius of curvature of spacetime, the related definition of the "scale factor" a_{exp} of the medium, and the factors expressing its curvature.

e) In the metric we usually work in terms of the comoving distance r, while an observational cosmologist usually thinks in terms of the redshift z. How is the redshift z related to the expansion factor a_{exp} ? How can you translate comoving distance r in terms of the "redshift" z of an object (hint: I am asking for an integral expression, in which one also finds the Hubble expansion rate $H(z) \equiv \dot{a}/a$).

Question 4.

Standard FRW cosmology has three important problems or "fine-tuning" issues. One of them is the monopole problem, created approximately one within the horizon at GUT transition and which should therefore crowd the present-day Universe. They do not ! Inflationary cosmology, stating that the Universe underwent a rapid de Sitter expansion at a very early phase, tries to explain why not.

The two additional problems of standard cosmology are the "flatness" problem and the "horizon" problem. Here we will elaborate on these:

a) For a FRW Universe:
   - define the meaning of the critical density ρ_{crit}.
   - give the expression for the critical density.
   - give an estimate of the value of the present-day critical density of the universe, in $[g/cm^3]$ and in $[M_\odot/Mpc^3]$.
   - give the definition of $\Omega(t)$.

to be continued ...

What is a typical value for this radius of curvature (both for a curved space $k \neq 0$ as well as for a flat space $k = 0$).

c) For the specific case of a curved matter-dominated Universe, work out the evolution of Ω as a function of expansion factor a (or redshift z) for various possible Universes. What will be the value of Ω at $a(t) = 0.001$ (recombination epoch) if the present $\Omega_0 = 0.3$. And at $a(t) \approx 0.0001$ (around matter-radiation equivalence). In general, what do you expect Ω to be for $a(t) \rightarrow 0$. Include a sketch of the Ω evolution. Explain now what the flatness problem is.

e) What is the definition for the (particle) horizon in a FRW Universe. Give the expression for the horizon distance $d_{Hor}(t)$. How does the horizon distance evolve with time t in a radiation-dominated Universe (assume $\Omega_r = 1$), and in a matter-dominated Universe (assume $\Omega_m = 1$). How can it be that the horizon seems to grow faster than the velocity of light ? Subsequently, express these horizon distances in terms of the Hubble parameter H(t).

f) Work out the typical angular diameter θ of the horizon at recombination ($z_{dec} \approx 1000$).

- You may use the following expression for the angular diameter distance D_a in a matter-dominated Universe for an object at redshift z:

$$d_A = \frac{2c}{H_0 \Omega_0^2} \frac{1}{(1+z)^2} \left\{ \Omega_0 z + (\Omega_0 - 2)\sqrt{1 + \Omega_0 z} - 1) \right\} \approx \frac{2c}{H_0 \Omega_0} \frac{1}{z} \quad (\text{for } z \gg 1)$$

- if you are left with an equation including the Hubble parameter H(t), use the expression of the evolution of H(z) as function of redshift z (from FRW equation), approximating $(1+z) \approx z$ for $z \gg 1$ and using

$$\Omega_0 H_0^2 = \frac{8\pi G}{3} \rho_0$$

.

g) Given the impressive isotropy of the Cosmic Microwave Background, how does the derived value of $d_{Hor}(z_{dec})$ imply the "horizon" problem ?

SUCCES !!!!

BEDANKT VOOR JULLIE AANDACHT EN INTERESSE DIT TRIMESTER !!!!

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