UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST3220 — Cosmology I

Day of exam: Friday 28th of March 2014

Time for exam: 14.30 – 17.30

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

The Friedmann equations

The first and second Friedmann equations and the adiabatic expansion equation are not independent equations. In this problem we will combine the second Friedmann equation and the adiabatic expansion equation to obtain the first Friedmann equation.

a) Write down the second Friedmann equation and the adiabatic expansion equation and combine them so as to eliminate the pressure term.

b) Prove that the expression you obtained in a) is equivalent to the following total derivative expression:

$$\frac{d}{dt}\left(\rho a^2 - \frac{3}{8\pi G}\dot{a}^2\right) = 0 \tag{1}$$

where ρ is the energy density and a is the scale factor.

c) Explain why the expression in b) is equivalent (or nearly equivalent) to the first Friedmann equation. What extra information is needed to make the equivalence complete.

Problem 2

The single component Universe

The Hubble factor can be defined as $H_0 = 100\text{km/s/Mpc}\ h$ where h is a parameter set by observations.

The deceleration parameter q_0 is defined by:

$$q_0 = -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2} \tag{2}$$

a) Today we measure $h = 0.68$ and $q_0 = -0.53$. If this was due to our Universe being a flat Universe dominated by a single component, what would the equation of state of that component be?

b) We observe a supernova at redshift $z = 2.5$. Calculate its luminosity distance d_L in a Universe of the type you found in a).

c) Can observations of supernovae help us to rule out the single component Universe? Are close or distant supernovae more important in this search?

Problem 3

A two-dimensional universe

In a two-dimensional universe, the adiabatic expansion equation would take the form:

$$\dot{\rho} = -2H(\rho + P) \tag{3}$$

where ρ is the energy per area, and the light speed has been set to one.

a) Find ρ_m as a function of scale factor a for pressureless matter in this universe. Explain whether this value makes sense or not.

b) What is the equation of state parameter w for a cosmological constant (that is a fluid with constant energy density (per area)) in two dimensions?

c) The first Friedmann equation is basically unchanged from the familiar three-dimensional one. Use this and the adiabatic expansion equation stated above to find the two-dimensional equivalent of the second Friedmann equation.

d) What is the condition on the equation of state parameter w for accelerated expansion in two dimensions?