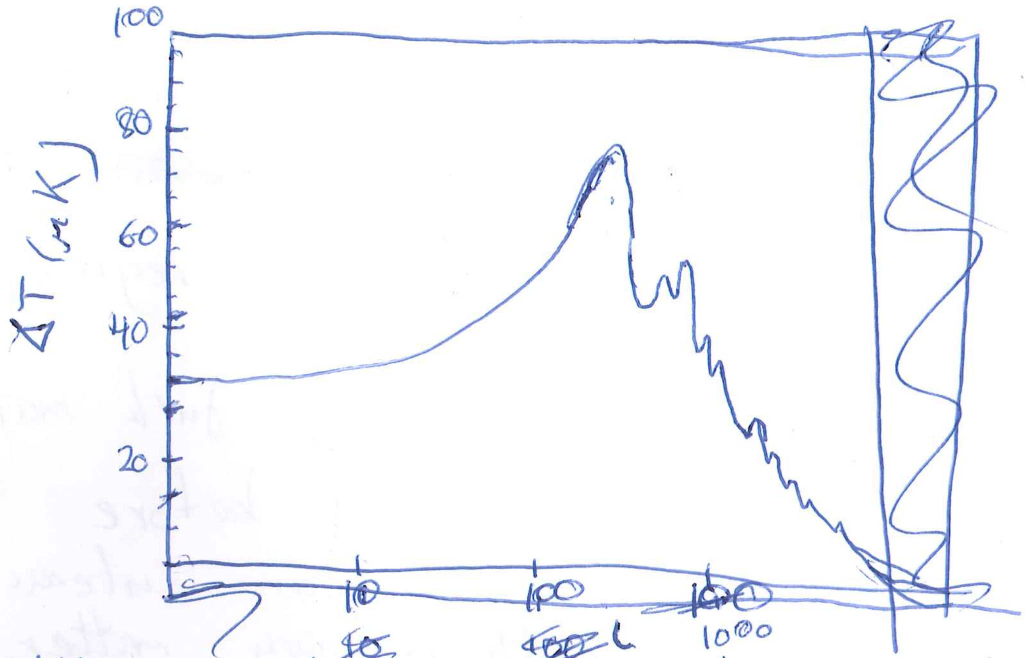
Suggestion for solution of final exam AST3220 spring 2014

Problem 1a) The CMB angular power spectrum:



With curvature the spectrum would shift. To the right (the peaks move to higher l) if the curvature is negative (hyperbolic universe) and to the left (the peaks move to lower l) if the curvature is positive (... universe)

b) Cold spots correspond to overdensities on large scales. This is because the CMB photons that come from an overdense area have had to climb out of the gravitational potential, thereby being redshifted / loosing energy / getting colder. In cases where the photon did not start from the overdense region at recombination, the photon first gains energy entering the potential. However, due to the continued structure formation during the photons travel time through the overdense region, more matter has accreted in the region making the gravitational well steeper to climb out of than into. This leads to a net loss of energy for the photon on the journey ~~passed pst~~ passing through the overdense region

c) For structure formation to work, dark matter needs ~~to~~ to start clumping long before ~~the~~ recombination, and hence can interact only very weakly with ordinary matter. It of course needs to be a massive particle as it can not have behaved as a relativistic particle for a long time. It also needs to be stable otherwise its mass would dictate that it would have had to decay a long time ago. Also its particular dispersion indicates that it must be particle-like (not a large composite object like a rock, or brown dwarf).

Problem 2 Radion gauge inflation

$$V(\varphi) = M^4 \frac{(\varphi/E_{PL})^2}{\alpha + (\varphi/E_{PL})^2}$$

a) The slow roll parameter ε is given in formula (3.5) in the notes:

$$\varepsilon = \frac{E_{PL}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

$$V' = M^4\left[\frac{2(\varphi/E_{PL}^2)}{\alpha + (\varphi/E_{PL})^2} - \frac{2\frac{\varphi}{E_{PL}^2}\left(\frac{\varphi}{E_{PL}}\right)^2}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^2}\right]$$

$$= 2M^4 \frac{\frac{\varphi}{E_{PL}^2}}{\alpha + (\varphi/E_{PL})^2}\left(\frac{\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2 - \left(\frac{\varphi}{E_{PL}}\right)^2}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)}\right)$$

$$= 2M^4 \frac{\alpha\frac{\varphi}{E_{PL}^2}}{\left[\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right]^2}$$

$$\varepsilon = \frac{E_{PL}^2}{16\pi}\left[\frac{\frac{2\cancel{M^4}\,\alpha\frac{\varphi}{E_{PL}^2}}{\left[\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right]^2}}{\frac{\cancel{M^4}\left(\frac{\varphi}{E_{PL}}\right)^2}{\left[\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right]}}\right]^2 = \frac{E_{PL}^2}{16\pi}\left[\frac{2\alpha}{\varphi\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)}\right]^2$$

$$= \frac{E_{PL}^2}{4\pi}\,\frac{\alpha^2}{\varphi^2\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^2}$$

b) η is given in equation (3.6) in the notes as:

$$\eta = \frac{E_{PL}^2}{8\pi} \frac{V''}{V}$$

$$V'' = \frac{2M^4\alpha}{E_{PL}^2}\left[\frac{1}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^2} - \frac{4\varphi\left(\frac{\varphi}{E_{PL}^2}\right)}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^3}\right]$$

$$= \frac{2M^4\alpha}{E_{PL}^2}\left[\frac{\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2 - 4\left(\frac{\varphi}{E_{PL}}\right)^2}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^3}\right]$$

$$= \underline{\frac{2M^4\alpha}{E_{PL}^2}\left[\frac{\alpha - 3\left(\frac{\varphi}{E_{PL}}\right)^2}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^3}\right]}$$

$$\eta = \frac{E_{PL}^2}{8\pi}\left[\frac{\frac{2\cancel{M^4}\alpha}{\cancel{E_{PL}^2}}\,\frac{\alpha - 3\left(\frac{\varphi}{E_{PL}}\right)^2}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^3}}{\frac{\cancel{M^4}\left(\frac{\varphi}{\cancel{E_{PL}}}\right)^2}{\cancel{\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2}}}\right]$$

$$= \underline{\underline{\frac{E_{PL}^2\alpha}{4\pi}\,\frac{\left(\alpha - 3\left(\frac{\varphi}{E_{PL}}\right)^2\right)}{\varphi^2\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^2}}} = \cancel{[illegible]}$$

$$\left(= \underline{\varepsilon\left(1 - \frac{3\left(\frac{\varphi}{E_{PL}}\right)^2}{\alpha}\right)} \right)$$

(Note that $\eta \leq \varepsilon$ so slow roll conditions are satisfied when $\varepsilon \ll 1$)

c) ~~&~~ Inflation ends when $\varepsilon = 1$. This means that

$$1 = \frac{E_{PL}^2}{4\pi} \frac{\alpha^2}{\varphi_{end}^2\left(\alpha + \left(\frac{\varphi_{end}}{E_{PL}}\right)^2\right)^2}$$

$$\varphi_{end}^2\left(\alpha + \left(\frac{\varphi_{end}}{E_{PL}}\right)^2\right)^2 = \frac{E_{PL}^2}{4\pi}\alpha^2$$

Assuming φ to be positive we can take the positive root of this as the negative root is of no interest to us.

$$\varphi_{end}\left(\alpha + \left(\frac{\varphi_{end}^2}{E_{PL}}\right)^2\right) = \frac{E_{PL}}{\sqrt{4\pi}}\alpha$$

We rename x such that $x = \frac{\varphi_{end}}{E_{PL}}$

$$x(\alpha + x^2) = \frac{\alpha}{\sqrt{4\pi}}$$

$$\cancel{\alpha}x^3 + \alpha x - \frac{\alpha}{\sqrt{4\pi}} = 0$$

φ_{end} is then ~~the~~ E_{PL} times the x that solves this cubic equation whilst still being a real and positive solution.

d) From equation (3.10) in the notes we know that the number of e-folds of inflation is give by:

$$N = \frac{8\pi}{E_{PL}^2} \int_{\varphi_{end}}^{\varphi_i} \frac{V}{V'} d\varphi$$

which becomes in the case of radion gauge inflation:

$$N = \frac{8\pi}{E_{PL}^2} \int_{\varphi_{end}}^{\varphi_i} \frac{\frac{M^4 (\varphi/E_{PL})^2}{\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2}}{\frac{2M^4 \alpha (\varphi/E_{PL}^2)}{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)^2}} d\varphi$$

$$= \frac{8\pi}{E_{PL}^2} \int_{\varphi_{end}}^{\varphi_i} \frac{\left(\alpha + \left(\frac{\varphi}{E_{PL}}\right)^2\right)\varphi}{2\alpha} d\varphi$$

$$= \frac{4\pi}{E_{PL}^2} \int_{\varphi_{end}}^{\varphi_i} \left(\varphi + \frac{1}{\alpha}\frac{\varphi^3}{E_{PL}^2}\right) d\varphi$$

$$= \frac{4\pi}{E_{PL}^2} \left[\frac{\varphi^2}{2} + \frac{1}{4\alpha}\frac{\varphi^4}{E_{PL}^2}\right]_{\varphi_{end}}^{\varphi_i}$$

$$= \pi \left[2\left(\frac{\varphi}{E_{PL}}\right)^2 + \frac{1}{\alpha}\left(\frac{\varphi}{E_{PL}}\right)^4\right]_{\varphi_{end}}^{\varphi_i}$$

$$= \pi \left[2\left(\frac{\varphi_i}{E_{PL}}\right)^2 + \frac{1}{\alpha}\left(\frac{\varphi_i}{E_{PL}}\right)^4 - 2\left(\frac{\varphi_{end}}{E_{PL}}\right)^2 - \frac{1}{\alpha}\left(\frac{\varphi_i}{E_{PL}}\right)^4\right]$$

Problem 3

a) Before recombination the nuclei were ionized so the universe was filled with free electrons and ionized nuclei. This made Compton scattering between photons and charged particles (especially electrons) a very common occurrance and the mean free path of the photons very short.

~~At recomb~~ Recombination designates the process of forming ~~stabl~~ neutral atoms from the charged particles when the universe had reached a temperature low enough for these to be stable. After this the universe was basically transparent to the photons. The last scattering surface defines the ~~sur~~ area of sky mapped out by the points where each of the CMB photons scattered off an electron for the last time before the universe became fully neutralized. If recombination had occured instantaneously this would have been a ~~near~~ ~~2 dimensional~~ ~~surfac~~ very nearly a perfect 2D sphere, ~~at~~ as it is this is not entirely so, though compared to the scales in the universe still very close to that.

b) Primordial nucleosynthesis is the process of formation of heavier elements in the early universe.
All heavier element must be formed through an initial formation of deuterium. Hence, the primordial nucleosynthesis can not take place before deuterium can be formed stably (deuterium bottleneck).
From there on most of the yield will go to the formation of ^4He, though trace amounts of deuterium, tritium, ^3He, litium and beryllium are also formed.
The net reaction is that of

$$2H + 2n \longrightarrow {}^4He$$

and as neutrons decay into protons, the final yield of ^4He is limited by the amount of neutron left in the universe when the deuterium bottleneck is reached.
Detailed calculations involving the Boltzman equation yields a mass fraction of helium of $X_{He} = 0.22$ (or with a slightly simpler approach 0.24). which agrees with observation.

c) The four fundamental forces:

The strong force is the strongest force and is mediated by the ~~g~~ gluons (of 8 types)

The electromagnetic force is the second strongest force and is mediated by the photon.

The weak (nuclear) force is the third strongest of the forces an is mediated by the neutral Z boson and the two charged $W^{\pm}$ particles. These force carriers are very heavy, making the weak force short distanced.

The gravitational force is the weakest of the fundamental forces and is believed to be mediated by the undiscovered gravitons (though their nature is not entirely clear as we have no quantum theory for gravity).

The ~~*~~ gravitational and the weak force act on all the particles in the standard model. The electromagnetic force acts on all charged particles, so all but the neutrinos of the standard model. The strong force acts on the quarks (and correspondingly on the particles composed of quarks).

d) Hubble's law:

$$v_r(t) = H(t)\, d_p(t)$$

At a given time t, ~~the~~ ~~universe~~ points in the universe are moving apart with a velocity $v_r(t)$ given by the Hubble factor $H(t) = \frac{\dot{a}}{a}$, where a is the scale factor, multiplied by the proper distance $d_p(t)$ between the points.

e) Galaxy with $d_p = 18.0\,\mathrm{MpC}$ with redshift speed $v_r = 1300\,\frac{km}{s}$

From this we estimate Hubble's constant to be: $H = \frac{v}{d_p} = \frac{1300\,\frac{km}{s}}{18.0\,\mathrm{MpC}}$

$$= 72\,\frac{km}{s\,\mathrm{MpC}}$$

This happens to be quite close to our current good estimates for the Hubble constant. The ~~Virgo~~ galaxy for the measurement is also far enough away not to be completely dominated by peculiar galaxy motion. However, it is still close enough for a single measurment to be influenced by peculiar motion. In addition the Virgo cluster and our own local group is (very weakly) gravitationally bound in the Virgo supercluster which may also bias the measurement.

e) If the Universe had been completely matterdominated, the age of the universe would be given as

$$t = \frac{2}{3 H_0}$$

As it is, a roughly ok estimate is just $t = \frac{1}{H_0}$ as the cosmological constant ~~ma~~ accelerates the expansion and means slower expansion, and longer timespans in the past. In this case

$$t = \frac{1 s\, Mpc}{72\, Km} = \frac{1 \cdot s \cdot 3{,}0857 \cdot 10^{19} km}{72\, km}$$

$$= \frac{3{,}0857 \cdot 10^{19} s}{72 \cdot 3600 \frac{s}{h} \cdot 24 \frac{h}{d} \cdot 365 \frac{d}{yr}} \approx \cancel{13}\, 14 \cdot 10^{9} yr$$

which is ~~quite~~ equal to our current estimate up to the level of accuracy of the numbers used in this calculation.