UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Final exam in AST3220 — Cosmology I

Day of exam: Thursday 14th of June 2012

Exam Hours: 09.00 – 13.00

This examination paper consists of 4 pages.

Appendices: None

Permitted Materials: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

When asked for numerical values you can use that

$$H_0 = h \times \left(9.778 \times 10^9 yr\right)^{-1} \tag{1}$$

with $h = \frac{2}{3}$

Problem 1

Several cosmological observations strongly indicate that our universe is spatially flat.

a) Show that for $k = +1$,

$$\Omega(t) - 1 = \frac{1}{\dot{a}^2}$$

in units of $c = 1$.

b) How does inflation in the very early universe bring Ω closer to the value 1

c) Describe how inflation is related to structure formation in the universe.

Problem 2

In the first part of this problem we will consider a ΛCDM universe with $\Omega_{m0} = 0.3$ and $\Omega_{r0} = 8.6 \times 10^{-5}$.

a) Describe in short the physical processes at recombination. Why is recombination important in Cosmology?

b) Find the redshift of matter-radiation equality. When is this as compared to the redshift of recombination of roughly 1100?

c) Show that, for a $k = 0$ matter dominated universe, the expansion parameter satisfies

$$a(t) = a_0 \left(\frac{3H_0 t}{2} \right)^{2/3}$$

d) Find $a(z)$, $t(z)$ and $r(z)$ for this universe.

e) Using the results in d) find the proper distance to the last scattering surface in light years if you assume the Universe has been matter dominated since recombination.

Problem 3

Consider a Universe dominated by a scalar field with no potential. It is convenient to work in units of $c = \hbar = 1$.

a) Use the equation for adiabatic expansion to find an expression $\rho(a)$ for the energy density of the scalar field as a function of the scale factor.

b) Use the Friedmann equation for this Universe to find $a(t)$ and $H(t)$. You may assume that this universe started in a Big Bang.

c) Describe briefly why the time evolution of a Fourier mode of the density perturbations $\Delta_k(t)$ on scales much larger than the Jeans length λ_J in this model can be described by

$$\frac{d^2\Delta_k}{dt^2} + \frac{2}{3t}\frac{d\Delta_k}{dt} = 0 \qquad (2)$$

i.e. describe the substitutions and simplifications that where used to get to this from equation (4.26) in the notes.

d) Solve equation (2) for Δ_k.

e) What does the above solution tell you about structure formation in such a Universe?