UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Exam in Cosmology I

Day of exam: Thursday 13th of June 2013

Exam hours: 09.00 – 13.00

This examination paper consists of 4 pages.

Appendices: None

Permitted materials: All non-communicative aids.

Make sure that your copy of this examination paper is complete before answering.

Throughout the equation set you can assume that today's value of the Hubble constant is:

$$\begin{aligned} H_0 &= 70 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1} \\ &= (14 \times 10^9 yr)^{-1} \end{aligned}$$

and that the scale factor today is set to unity

$$a_0 = 1$$

Problem 1

Cosmological Structure Formation

a) What is the meaning of the Jeans-wavelength and of the Jeans-mass?

b) Discuss the Jeans-length before and after recombination.

Consider the growth of small (linear) perturbations of scales large enough so that pressure is negligible.

c) Show that in the radiation dominated regime, $\Omega_{rad} = 1$ ($\Omega_m, \Omega_\lambda << \Omega_{rad}$), the growth of perturbations to the radiation density can be described as $\delta \propto \tau^2$, where τ is the so-called conformal time, $d\tau = dt/a(t)$.

d) Show that the same conformal time relation as for radiation in c) can describe the growth of matter perturbations in the matter dominated regime, $\Omega_m = 1$ ($\Omega_{rad}, \Omega_\lambda << \Omega_m$).

Problem 2

Primordial Nucleosynthesis

a) Why is there no C, N, O etc. fusion in the early Universe?

b) What is the binding energy of the hydrogen atom? Explain qualitatively why recombination starts only when the temperature drops below 0.5 eV.

Problem 3

The Inflationary Universe

a) Describe 3 severe problems of the Standard Big-Bang Model of the Universe without Inflation.

b) Describe the basic elements of the Inflationary Universe (Friedmann equation, inflaton field, dynamics etc.).

c) What is the meaning of the slow-roll approximation?

d) Describe, how the problems you mentioned in a) are solved by the Inflationary Universe.

Problem 4

Modified Chaplygin gas

The modified Chaplygin gas obeys the equation of state

$$p = A\rho - \frac{B}{\rho^{\alpha}}$$

a) Write down the adiabatic expansion equation for the modified Chaplygin gas.

b) Show that

$$\rho = \left[C \left(\frac{a_0}{a} \right)^{3(1+\alpha)(1+A)} + \frac{B}{1+A} \right]^{\frac{1}{1+\alpha}} \tag{1}$$

where C is a constant, solves the adiabatic equation for the modified Chaplygin gas if ρ_0 is the density of the gas at time a_0.

c) Discuss the modified Chaplygin gas in the limit where $B = 0$. What does A then signify in terms of a normal ideal gas?

d) Discuss the modified Chaplygin gas in the limit where $A = 0$ and $\alpha = -1$. What does B then signify in terms of a normal ideal gas?