UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Final exam for AST4220 — Cosmology I

Day of exam: Friday December 9th 2011

Time for exam: 14.00 – 18.00

This problem set consists of 5 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

In this exam we are assuming the units where $c = 1$.

Problem 1

The Big Bang model is extremely successful in accounting for many of the basic features of our Universe. However, there are some properties within this model one cannot answer.

a) Describe two main puzzles of the Big Bang model?

b) Describe how Inflation could be a solution to these puzzles.

c) Besides solving the above problems, Inflation also gives us an extra positive feature. Describe it.

d) Explain the need of reheating at the end of inflation.

Problem 2

Describe in detail what is meant by the

a) Deuterium bottleneck.

b) Freeze-out of a particle species.

Problem 3

Suppose the Universe was radiation dominated for times t before decoupling time a_d, with $p = \rho/3$.

a) Show that
$$\rho(a) = \rho_d(\frac{a_d}{a})^4$$
where $a_d = a(t_d)$.

b) Solve the Friedman equations for $k = 0$ to obtain $a(t)$.

c) Show that
$$\rho(t) = \frac{3}{32\pi G}t^{-2}$$
.

d) Suppose $\rho = N\alpha T^4$, where N is the effective number of radiative species and α is the Stefan-Boltzmann constant. Determine at what cosmic time the Universe reaches a temperature T, i.e. find $t(T)$.

e) Suppose that nucleosynthesis ends when $T \approx 10^9$K. Does the corresponding cosmic time $t(10^9)$ increase with N or decreases with N?

f) Briefly describe how $t(10^9)$ affects the ratio of neutron density to proton density ρ_n/ρ_p.

g) How does this ratio affect the present Helium abundance?

Problem 4

Supposed the Universe is static ($da/dt = 0$) and filled with a gas of mass density ρ, pressure p, velocity $\mathbf{v}$, and gravitational acceleration $\mathbf{F}$. The gas obeys te mass equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \tag{1}$$

and the momentum equation

$$\rho(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) = -\nabla p + \rho \mathbf{F} \tag{2}$$

a) Let

$$\rho = \bar{\rho} + \delta\rho, \qquad p = \bar{p} + \delta p, \qquad \mathbf{v} = \delta \mathbf{v}$$
$$\mathbf{F} = \delta \mathbf{F}, \qquad \nabla \cdot \delta \mathbf{F} = -4\pi G \delta\rho$$

The overbared quantities are those that follow the equations of the static universe (1,2), and the deltas are linear perturbations in these quantities. Also let $c_s^2 \equiv dp/d\rho$. Derive the first order linear approximation equation

$$\frac{\partial^2 \delta\rho}{\partial t^2} - c_s^2 \nabla^2 \delta\rho - 4\pi G \bar{\rho} \delta\rho = 0$$

.

b) Show that this equation has solutions of the form

$$\delta\rho = A(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x} - w(k)t)}$$

.

c) Determine $w(k)$, in terms of $k_J^2 \equiv 4\pi G \bar{\rho}/c_s^2$.

d) What is the physical meaning of c_s and of k_J?

e) Let the wavelength of a density perturbation be $\lambda = 2\pi/k$. For what range of λ will a perturbation grow? What happens to the perturbation if λ is not in this range?

f) Suppose a standing wave perturbation at some wavelength λ_1 starts at $t = 0$ with maximum amplitude, and reaches its next maximum amplitude at decoupling time $t = t_d$. Find an expression for λ_1 in terms of t_d, k_J and c_s.