UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Final exam for AST3220 — Cosmology I

Day of exam: Thursday 12th of June 2014

Time for exam: 14.30 – 18.30

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

Neutrinos

In the standard ΛCDM model neutrinos are considered massless and the standard three neutrino families are assumed. Oscillations however have shown that neutrinos have nonzero mass but have not set an absolute neutrino mass scale. If neutrinos have a total mass below about 1eV they are i) hot, have a high velocity dispersion and ii) are relativistic at CMB time. Recall that neutrinos are very, very! weakly interacting.
Assume that he fraction of dark matter in neutrinos can be expressed by f_ν in other words $f_\nu = \Omega_\nu/\Omega_m$ ($\Omega_m = \Omega_c + \Omega_\nu$)

a) Write down how the differential equation describing the linear growth of perturbations:

$$\frac{d^2\Delta}{dt^2} - 2H\frac{d\Delta}{dt} = 4\pi G\bar{\rho}\Delta - c_s^2 k^2 \Delta$$

can be written for the (cold) dark matter overdensity field Δ_c and expressed in terms of f_ν and motivate it. Consider scales that are above the Jeans scale but below the neutrino free streaming length.

b) Compute the growth rate of perturbations assuming that f_ν is small i.e., ($<< 1$). (recall that this would apply on scales below the neutrinos free streaming length) i.e. $D(t)$ where $\Delta(x,t) = D(t)\Delta_0(x)$ and $\Delta_0(x)$ is the overdensity at some initial time.

c) Name and explain at least one observational consequence of this: how would this affect the large-scale structure (statistical) properties?

d) Now instead assume neutrinos are masseless but there are many more than the standard. As such they are relativistic (behave like radiation). What would be one observational consequence of this?

Problem 2

Growth of Matter Perturbations

a) Justify that the growth of perturbations in the linear regime (and for scales below the horizon) can be factorized as $\delta(z) = D(z)\delta(z0)$

b) Now express $D(z) = g(z)a$ and plot the evolution of the growth factor in the following form D(z)/a, i.e. g(z) as function of z in:

- Einstein de sitter universe
- standard ΛCDM universe

c) Extrapolate it into the future for a LCDM universe

Problem 3

Dark Matter and Large Scale Structure

a) Draw the CMB angular power spectrum for the (flat) ΛCDM universe. How would it look for a Universe with significant curvature (all else remaining equal)?

b) Temperature fluctuations in the CMB correspond to density fluctuations.On large scales, is it a hot or a cold spot that corresponds to an overdensity? Why?

c) What are the properties that the dark matter should have and why?

Problem 4

Inflation

a) Imagine that inflation happened half an hour after the Big Bang rather than after 10^{-35} seconds. What observational consequences would that have? (You may assume that the inflation field converts completely to photons in the reheating process.)

The recent measurements from the BICEP 2 experiment and those of the Planck satellite favour very different models for

inflation. However, there are types of inflation which might fit them both. One is the *radion gauge inflation* which is a scalar field with the following potential:

$$V(\phi) = M^4 \frac{(\phi/E_{\rm PL})^2}{\alpha + (\phi/E_{\rm PL})^2} \tag{1}$$

where α is a parameter and M is the mass scale of this inflationary field.

b) Calculate the slow roll parameter ϵ for the radion gauge inflation.

c) Calculate the slow roll parameter η for the radion gauge inflation.

d) Inflation is said to end when ϵ is 1, find an expression for $\phi_{\rm end}$ when radion gauge inflation ends. (Assume that ϕ is always positive and that $\alpha < \frac{1}{4\pi}$)

e) Assuming that inflation starts at ϕ_i, find an integral expression for the number of e-folds of inflation in this model. (You do not need to compute the integral.)