UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST4220 — Cosmology I

Day of exam: Wednesday December 9th 2009

Time for exam: 15.00 – 18.00

This problem set consists of 3 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

Briefly describe the sequence of reactions leading to the formation of light nucleus ^2H, ^3He, ^4He and ^7Li in the early Universe. Explain why:

a) This process cannot produce nucleus heavier than ^7Li (even though fusion processes in stars can and do produce such nucleus);

b) You expect the final yield of ^4He to be somewhat less than 30% by mass;

c) The exact yield depends on the neutron lifetime, the number of neutrino species and H_0 as well as on the baryon density.

d) Note that the mass of the proton is 1.6726×10^{-27}Kg and the neutron mass is 1.6749×10^{-27}Kg

Hints: Recall that the relevant bit of the Boltzmann distribution is the exponential, $exp(-E/kT)$; in this case the relevant energy is mc^2, so the ratio of neutrons to protons is $exp(-m_n c^2/kT)/exp(-m_p/kT)$. The temperature at which the neutron:proton ratio stabilises is around $10^{10}K$.

Problem 2

In the context of the standard (pre-inflation) Big Bang model, explain what is meant by the following phrases, and why they are described as 'problems':

a) The horizon problem; The flatness problem; The monopole problem;

b) Briefly explain the term inflation, and explain why introducing a period of inflation solves the above problems.

c) Inflation is sometimes modelled as a large positive cosmological constant. Show that in this model the expansion factor of the universe during inflation is given by $a(t) = a(t_i)exp(H(t-t_i))$, where t_i is the time that inflation starts. If we assume that inflation starts at 10^{-35}s, ends at 10^{-33}s, what is the implied value of the cosmological constant Λ?

Problem 3

The present universe has $\Omega_{m0} = 0.23$ and $\Omega_{ro} = 8.4 \times 10^{-5}$.

a) Calculate the redshift at which $\Omega_m = \Omega_r$ (the epoch of matter-radiation equality)

b) The differential equation for structure growth shows that the density perturbations should grow as $\propto a$ in a matter dominated universe. Explain why, despite this, one might expect that structures would not start to grow until the epoch of recombination at $z \sim 1100$.

c) How does the recognition that most of the matter in the universe is **non-baryonic cold dark matter** affect the growth of structure?