Cosmological Principle: The cosmological principle says that the universe is homogeneous and isotropic on large scales. Homogeniety means that every (large enough) chunk of the universe contains roughly the same amount of matter and energy, while isotropy means that the universe looks the same no matter which direction you are looking.

Gaussian Curvature: From the cosmological principle, the curvature of space- time has to be the same everywhere (on very large scales). This restricts the possible geometries of the universe. We can specify the shape of the universe by its Gaussian curvature, which can be positive (like on the surface of a sphere), zero (like on a flat sheet) or negative (like on the surface of a hyperboloid). **Comoving distance:** Distance that does not change because of the expansion of the universe. (The ruler also expands) $r = S_k[\int_t^{t_0} \frac{cdt'}{a(t')}]$
Robertson-Walker line element:

- The RW line element is a measure of the distance ds in spacetime of constant curvature between two events.
- In spherical coordinates: $ds^2 = c^2dt^2 - a^2(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2)$ where a is the scale factor of the universe and k specifies the type of 2 curvature.
- Can be used to measure several types of spatial distances to an object:
  - Proper distance d_p: the length we would see on a ruler going from us to an object. Setting $dt = d\theta = d\phi = 0$ and integrating ds from us to the object gives $d_P = \int_0^r \frac{a}{\sqrt{1-kr'^2}}dr'$
  - Luminosity distance d_L: the distance deduced from the relation between q luminosity L and received flux l for a static, flat universe; $d_L = \sqrt{\frac{L}{4\pi l}} = \frac{a_0^2}{a}r$ where a_0 is the present scale factor.
  - Angular diameter distance d_A : the distance deduced from the relation between angular diameter $\Delta\theta$ and proper diameter D_P for a static, flat universe; $d_A = \frac{D_P}{\Delta\theta} = ar$.

Open, Flat and Closed universes: Open, flat and closed universes: The geometry of the universe, that is, whether it's open ($k = -1$), flat ($k = 0$) or closed ($k = 1$), depends on the amount of matter and energy in the universe. The open and flat universes are infinitely large, while the closed universe has a finite volume but no boundary. The fate of the universe generally depends on the curvature: an open or flat uni- verse will expand forever, while a closed universe eventually will stop expanding and start to contract.

Friedmann Equations: The Friedmann equations: Assuming homogeniety and isotropy reduces the Einstein equation to only two equations; the Friedmann equations. They can be solved for the scale factor a given the total matter/energy density ρ and pressure p in the universe.

- $\dot{a}^2 + kc^2 = \frac{8\pi G}{3}\rho a^2$
- $\ddot{a} = -\frac{4\pi G}{3}(\rho + \frac{3\rho}{c^2})a$

Another useful equation is the equation for adiabatic expansion, which can be used to find ρ as a function of a.

- $\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + \frac{P}{c^2})$

Equations of State: In order to solve the Friedmann equations, we need a relation between the pressure p and the density ρ . This relation is the equation of state for the type of matter/energy in question. For the types of matter important to cosmology, a general equation of state on the form $p = w\rho c^2$ can be used. Inserting this into the equation for adiabatic expansion and integrating both sides gives an expression for ρ in terms of a and w: $\rho = \rho_0(\frac{a_0}{a})^{3(1+w)}$. We have $w = 0$ for non-relativistic matter, $w = \frac{1}{3}$ for radiation and $w = -1$ for vacuum energy (the cosmological constant). When the contributions from several types of matter are important, we can use the sum of the densities and pressures for the different types: $\rho = \rho_m + \rho_r + \rho_\Lambda$ and $p = (\frac{1}{3}\rho_r - \rho_\Delta)c^2$. From the second Friedmann equation we see that all the density terms as well as the radiation pressure term will contribute to a negative acceleration of the universe, while the negative Dust $P = 0$ Radiation $P = \frac{1}{3}\rho c^2$

Cosmological Models

Static Universe A static solution of the Friedmann equations is possible if the value of the cosmological constant is chosen just right to cancel the contraction caused by dust. This universe is closed and has scale factor $a = a_0 = \frac{c}{\sqrt{\Lambda}}$. A tiny perturbation of a 0 would be enough to break the equilibrium and cause the universe to expand or collapse, so this solution is unstable.

Einstein-de Sitter model In the EdS model, the universe is flat ($k = 0$)and contains only dust ($P = 0$). The scale factor is $a = a_0(\frac{t}{t_0})^{\frac{2}{3}}$, so it will expand forever, but at a smaller and smaller rate.

According to this model, the current age of the universe is 9.3 Gyrs, which is smaller than the age of some star clusters, so the model can't be completely correct. But it may still be a good description of parts of the history of the universe.

de Sitter model: The universe is flat ($k = 0$) in the de Sitter model as well, and contains nothing but vacuum energy ($\ddot{a} = \frac{\Lambda}{3}a$), (No dust, No Radiation). This leads to an exponential increase of the scale factor with time: $a = a_0 e^{H_0(t-t_0)}$. There is nothing special happening at $t = 0$, so there is no Big Bang in this model.

Like the EdS model, this model can't correspond completely to reality (there is definitely matter in the universe), but it may still be a good description of a later epoch of the universe when the vacuum energy dominates the dust and radiation.

Dust - Open and closed: Since $\Omega_{m0}+\Omega_{k0} = 1$ we can write the Friedmann eq.:

- $\frac{H^2(t)}{H_0^2} = \Omega_{m0}(\frac{a_0}{a})^3 + (1 - \Omega_{m0}(\frac{a_0}{a})^2$
- Closed $(k = 1) - \Omega m0 > 1-$ If expansion stops $\ddot{a} = \dot{a} = H = 0$. This might contract and crunch. Using this we can also find an a_{max}.
- Open $(k = -1) - \Omega m0 < 1-$ and dust. Forever expanding and no big crunch here.

Matter and Radiation

- $\frac{H^2(t)}{H_0^2} = \Omega_{m0}(\frac{a_0}{a})^3 + \Omega_{r0}(\frac{a_0}{a})^4 + \Omega_{k0}(\frac{a_0}{a})^2$

However, the spatial curvature is negligable with small a's. But stil plays a role in calculatiign geometrical quantities like distances. We can use the new expression to carry out the integrations for finding the age of the universe. This is done by saying there is an equal amount of matter and radiation at one point, leaving us with $a_{eq} = a_0 \frac{\Omega_{r0}}{\Omega_{m0}}$ from the density equations.

Flat ΛCDM Model (Cold Dark Matter): Flat, dust dominated (CDM) universe with positive cosmological constant. $\Omega_{m0} \approx 0.3$ and $\Omega_{\Lambda 0} = 1 - \Omega_{m0} \approx 0.7$. We can therefore write the Friedmann eq.:

- $\frac{H^2(t)}{H_0^2} = \Omega_{m0}(\frac{a_0}{a})^3 + (1 - \Omega_{m0})$

This is also divided into two cases:

- $\Omega_{m0} > 1, \Omega_\Lambda < 0-$ Meaning RHS becomes negative at a_{max}, resulting in contraction, then big crunch.
- $\Omega_{m0} < 1-$ RHS is always positive, forever expanding. Solving for the age of the universe, we get 13.5 Gyr, which is consistant with the oldest observed objects in the universe.

The most peculiar feature of the ΛCDM model is that the universe starts expanding at an accelerating rate. This is seen when we rewrite the Friedmann eq. and look at $\ddot{a} > 0$ $\frac{\ddot{a}}{a} = -\frac{H_0^2}{2}(\Omega_{m0}\frac{a_0^3}{a^3} - 2\Omega_{\Lambda 0})$

The crossover from deceleration to acceleration happens slightly before the cosmological constant starts to dominate the energy density of the universe. At the age of 7.3 Gyr, meaning that in this model the universe has been accelerating for the last 6.2 billion years. Looking at this in the extreme limits we see and expansion like that of the de Sitter model.

Horizons

The Event Horizon: Max distance light can travel if it is emitted now. (But turn it around, and say that the max point is where the observer is at.) Given by $d_P^{EH} = a(t)\int_t^{\inf} \frac{c}{a(t')}dt'$

The Event Horizon: The distance a light beam from the beginning of the universe could have travelled. (But turn it around, and say that the max point is where the observer is at.) Given by $d_P^{EH} = a(t) \int_t^{\inf} \frac{c}{a(t')} dt'$