UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Final exam for AST4220 — Cosmology I

Day of exam: Friday December 10th 2010

Time for exam: 14.30 – 18.30

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Problem 1

Consider the conformal time defined as $dt = a\, d\eta$.

a) Show that $\eta \propto a^{1/2}$ in a matter dominated universe and $\eta \propto a$ in a radiation dominated universe.

b) Consider a universe with only matter and radiation, with matter-radiation equality at a_{eq}. Show that

$$\eta = \frac{2}{\sqrt{\Omega_m H_0^2}}[\sqrt{a + a_{eq}} - \sqrt{a_{eq}}]$$

c) Consider a de Sitter universe, with $a(t) = e^{Ht}$. What is the conformal time, as a function of the scale factor a in this universe?

Problem 2

a) Describe in a paragraph what is the Big Bang Nucleosynthesis (BBN)?

b) Discuss how the expansion rate of the universe and the neutron lifetime influence the process of BBN.

c) The dissociation energy of deuterium is 2.23 MeV. Such temperature is reached when the universe is about 1 second old. However, in the real universe deuterium did not survive until about the universe was 100 seconds old. How do you explain the discrepancy?

d) Describe the deuterium bottleneck.

e) Does BBN produces all the atomic elements we find nowadays in the periodic table? Justify.

Problem 3

Consider a massless scalar field; i.e. a scalar field $\phi(t)$ whose potential energy density is $V(\phi) = 0$. Now suppose that this scalar field is initially rolling, so $\dot{\phi} \neq 0$, and that the kinetic energy density associated with this rolling dominates the energy density of the universe.

a) Show that this implies $\rho_\phi \propto a^{-6}$, where a is the scale factor, in two ways:

   i. By recalling how the energy density of matter with an equations of state $p = \omega\rho$ scales with a.

   ii. by solving the equations of motion for ϕ in an expanding Universe.

b) Does such a Universe become curvature dominated more easily or less easily than a matter-dominated Universe? That is, everything else being equal, will such a universe become curvature dominated later or earlier than one in which the scalar field is replaced with matter

c) Could this scalar field drive an inflationary epoch at early times? Justify your response.

Problem 4

Show that

$$\begin{aligned} \rho &= \rho(t) \qquad , \qquad p = p(t) &(1)\\ \vec{u} &= H(t)\vec{r} \qquad , \qquad \Phi = \frac{2\pi G}{3}\rho r^2 &(2) \end{aligned}$$

is the homogeneous and isotropic solution of the Newtonian equations

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) &= 0 &(3)\\ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} + \frac{1}{\rho}\vec{\nabla} p + \vec{\nabla}\Phi &= 0 &(4)\\ \nabla^2 \Phi &= 4\pi G \rho &(5) \end{aligned}$$

if $\rho(t)$ and $H(t)$ satisfy the Friedman and continuity equations for pressureless matter.

Problem 5

a) Describe the physical meaning of the Jeans length ($\lambda_J \sim 1/k_J$) and how it has evolved along the different epochs our Universe went through.

b) For short wavelength modes with $k \gg k_J$, density perturbations in the matter dominated universe satisfy

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} + \frac{c_s^2 a_o^2 k^2}{a^2}\delta_{\mathbf{k}} = 0$$

i) Solve $\delta_{\mathbf{k}}(t)$, assuming that matter is the only component present and that $\Omega_m = 1$.

ii) How does the amplitude and frequency of the oscillations change with time and scale factor? (Note: conformal time, $d\eta = dt/a$, may prove helpful).

c) Why are we confident that there is a dark matter component besides baryonic matter? Which observations support this assumption?
(Note: this question is not directly related to the previous ones.)