UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST3220 — Cosmology I

Day of exam: Thursday 26th of March 2015

Time for exam: 15.00 – 18.00

This problem set consists of 4 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Throughout the equation set we will work in units of $c = 1$

We will also assume that the value of the scale factor today is unity:

$$a_0 = 1$$

Problem 1

Redshift dependent equations

Recall that the redshift z is defined by the equation

$$\frac{1}{a} = 1 + z \tag{1}$$

This definition provides a one-to-one correspondance between redshift and scale factor so long as $a > 0$ and $z > -1$. We can use this fact to redefine equations in terms of redshift.

a) Show that the adiabatic expansion equation in terms of redshift becomes:

$$\dot{\rho} = \frac{3\dot{z}}{(1+z)}(\rho + p) \tag{2}$$

b) Use the version of the adiabatic expansion equation above to find an expression for $\rho(z)$ for dust and cosmological constant, without first calculating $\rho(a)$.

c) Use your results from b) and the definitions of the density parameters Ω_{m0} and $\Omega_{\Lambda 0}$ to write the first Friedmann equation for flat Universe with dust and cosmological constant (ΛCDM) using only Ω_{m0}, z, H_0 and H.

d) Use the definition of the comoving coordinate and make a variable change from t to a to z to prove that the comoving coordinate is given by:

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')} \tag{3}$$

where $E(z) \equiv \frac{H(z)}{H_0}$. Find $E(z)$ for the Universe described in c).

Problem 2

$R_h = t$ and power law cosmologies

The $R_h = t$ and power law cosmologies, are alternative models to ΛCDM. $R_h = t$ cosmology is defined as a universe with a linear increase in Hubble radius

$$R_h = t, \tag{4}$$

where the Hubble radius $R_h = \frac{1}{H}$. Power law cosmology is defined by assuming a power law function of a with respect to t, i.e.

$$a = \left(\frac{t}{t_0}\right)^n \tag{5}$$

They are both proposed to describe the Universe at late time, so at low redshift.

a) Find $H(t)$ for $R_h = t$ and power law cosmology.

b) $R_h = t$ is equivalent to a power law cosmology for a special value of n, which value?

c) Find the function $E(z) \equiv \frac{H(z)}{H_0}$ for power law cosmologies.

d) Show that the comoving distance $r(z)$ in power law cosmology is given by:

$$r(z) = \frac{1}{H_0} \times \begin{cases} \frac{(1+z)^{1-1/n}-1}{1-1/n}, & n \neq 1, \\ \ln(1+z), & n = 1. \end{cases}$$

Problem 3

Supernova observations

We want to use supernovae observations to compare the ΛCDM model to the alternative models described above. When we observe supernovae, we measure their magnitude m on the sky, and their redshift. The observed magnitude is given by:

$$m = M + 5\log_{10}\left(H_0 d_L(z)\right) \tag{6}$$

where M is the absolute magnitude, which is assumed to be constant for supernovae type Ia, and d_L is the luminosity distance

a) Explain why the differences in observed magnitude in universes governed by different models, can be found by specifying just $r(z)$.

b) Assume that z is very small, and find the value of $r(z)$ for the ΛCDM universe in this limit. (Hint: In ΛCDM $3\Omega_{m0} \approx 1$ so you can safely assume that $3\Omega_{m0} z$ is also a small number.)

c) Find $r(z)$ in small z limit for the power law cosmologies.

d) Can we hope to distinguish between these models by observing supernovae at very low redshifts?