Universitetet i Oslo

AST4310

Stellar Spectra B (SSB)
LTE Line formation

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November 9, 2016

Abstract

In this project we investigate how radiative transfer can help us understand the solar and terrestrial atmosphere through studying their continua. We will also study the formation of spectral lines assuming LTE. All source code available on GitHub.

Contents

1 Stratification of the solar atmosphere

1.1 FALC temperature stratification

In this exercise we will study the stratification of the solar atmosphere using the FALC model by Fontela et al. First, a routine was created to fetch the FALC data:

```
# reading falc.dat
h, tau5, colm, temp, vturb, nhyd, nprot, nel,
    ↪ ptot, pgasptot, dens = np.loadtxt("falc
    ↪ .dat",usecols=(0,1,2,3,4,5,6,7,8,9,10),
    ↪  unpack=True)
```

This was used to in the next few exercises were we study its properties.

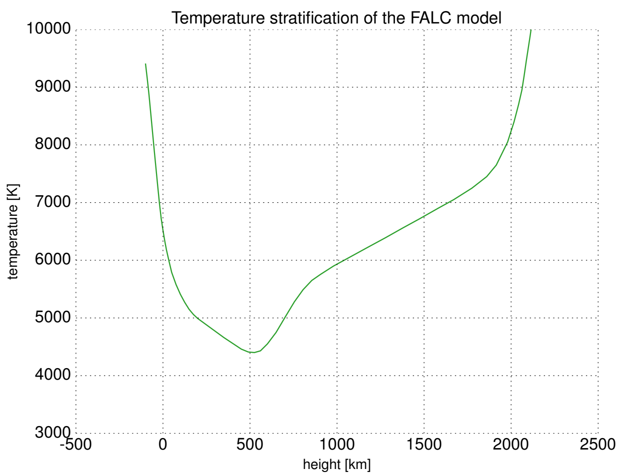

Figure 1: Recreated the temperature stratification in the FALC data.

1.2 Falc Density stratification

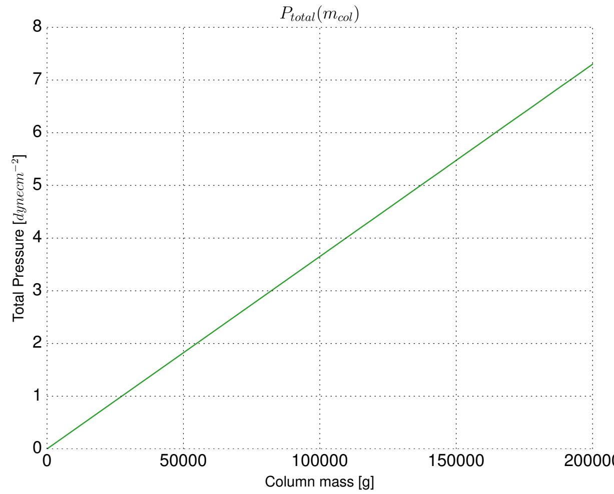

Figure 2: $p_{total}(m)$

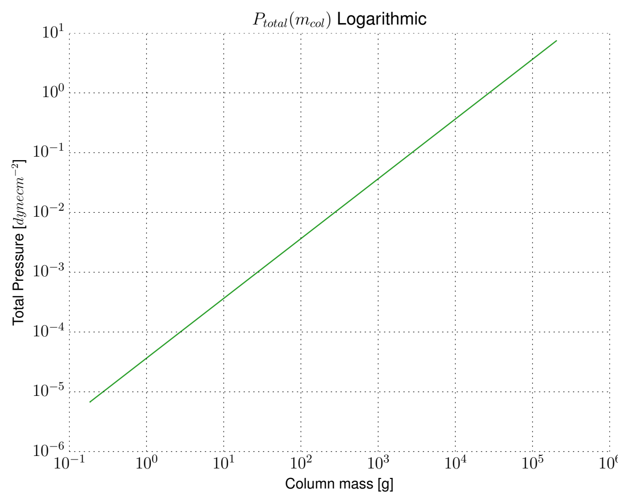

Figure 3: $p_{total}(m)$ logarithmic

We observe that by plotting the total pressure scales linearly with mass, meaning we an write $P_{tot} = Cm$. By calculating the average over all pressures and column masses we get $C = g_{surface} = 2.7 \cdot 10^4 cm/s^2$. In the FALC producing code, complete mixing is assumed. This can be shown by checking the element density fractions in figure 4. Here it is easy to see how the Helium and Hydrogen abundance is stable with height and that the left over density corresponds to metals.

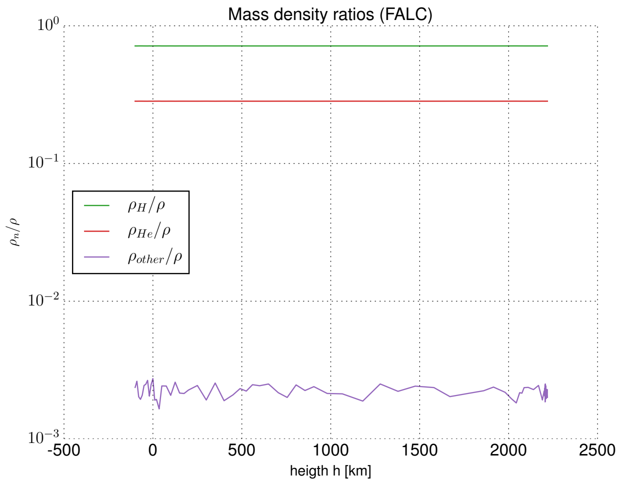

Figure 4: Abundance ratios. The mixing is more or less independent of height and the total number of elements always sum up to 1, as expected.

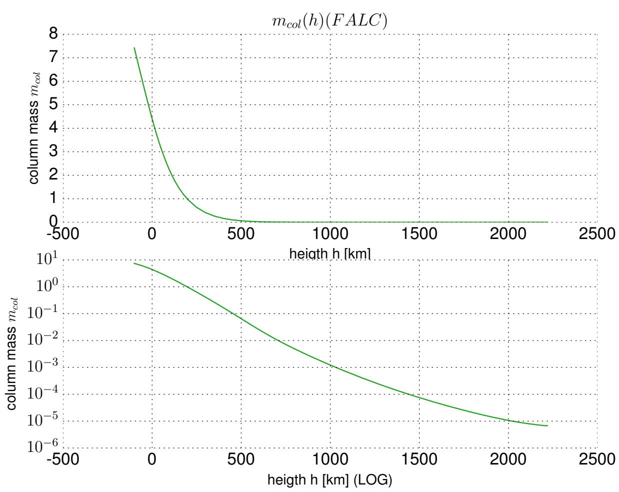

Figure 5: Column mass against height. An almost linear curve indicates that the change in column mass is almost constant with some deviations due to pressure dependence.

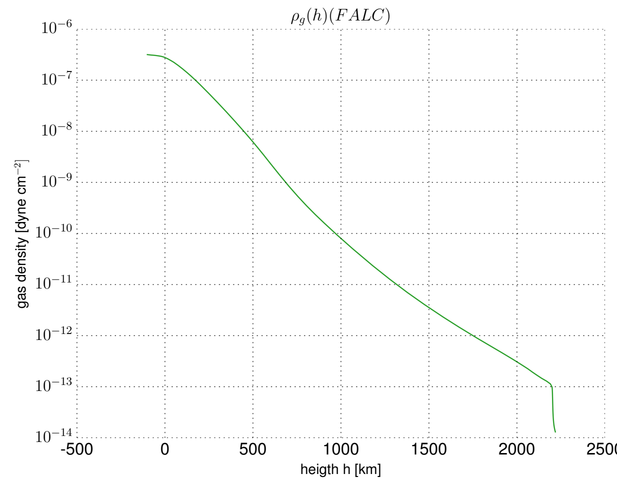

Figure 6: Gas density against height.

In figure 6 we look at the gas density as a funciton of height. We can find the pressure scale height H_ρ by $\rho = \rho(0)exp(-h/H_\rho)$ with $H_\rho = \frac{kT}{Mg}$ where k is the Boltzmann constant, T is temperature in Kelvin, M is the mean molecular weight (m_H in our case) and g is the surface gravity. This gives us $H_\rho = 196.3km$ for $h = -100$ which is not realistic considering the assumption of $M = m_H$. Another estimate can be done by looking at how fast the figure decreases, this leaves us with $H_\rho = 150km$.

By looking at 7, a plot of the gas pressure against height, along with the $(n_H + n_e)kT$ we see slight deviations. Because the sun does not consist of only hydrogen, we must add helium to the ideal gas law. This is shown in the next figure and corrects for these deviations.

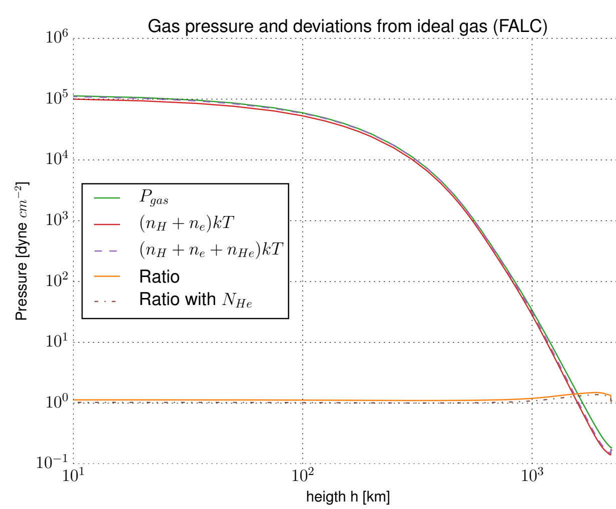

Figure 7: Gas pressure and deviations from ideal gas.

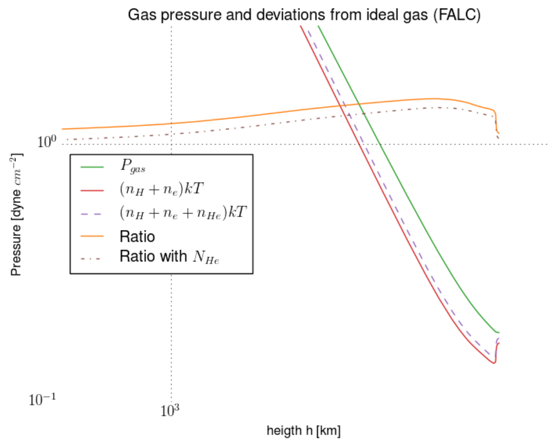

Figure 8: Enlarged deviation in the hight atmosphere

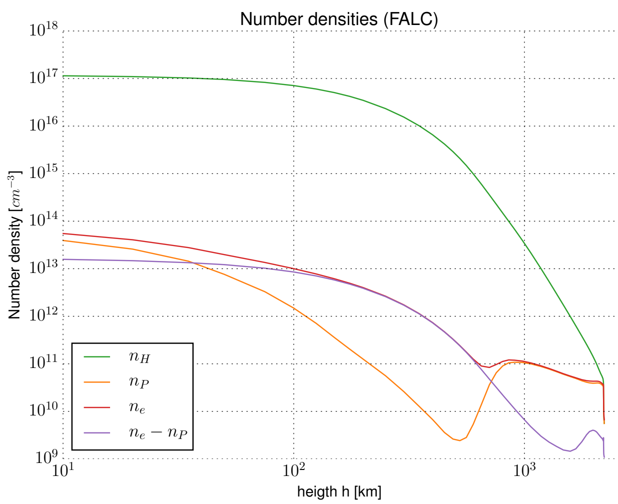

Figure 9: Total hydrogen density against height with electron density, proton density and density of electrons that do not result from hydrogen ionization. These are found by $n = (n_H - n_p)$ We also notice that the electrons that do not result from hydrogen ionization are paralell to the hydrogen density at in the lower atmosphere, this is the result of ionization. As we move farther away, the number densities decrease, because the atmosphere thins out.

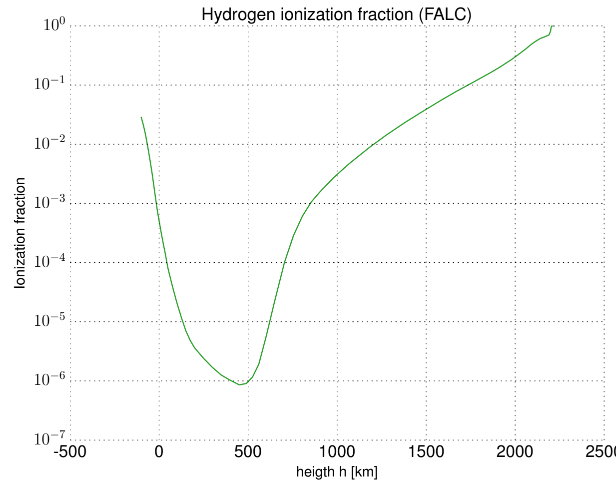

Figure 10: Ionization fraction of hydrogen logarithmically against height. This resembles the temperature plot because ionization is heavily dependent on temperature.

By assuming TE the photon density can be estimated to $N_{phot} \simeq 20T^3$, which corresponds to a medium dominated by collisions. This corresponds to a low photon density. For $h = -100$ this gives $N_{phot} = 1.6 \cdot 10^{13} cm^3$, where hydrogen has a magnitude of 10^{17}, meaning TE is reasonable. However, higher up in the atmosphere, we this is no longer viable and we replace it with $N_{phot} = 20T_{eff}^3/2\pi$ where $T_{eff} = 5770K$ leaving $N_{phot} \simeq 6.1 \cdot 10^{11} cm^3$ at h = 2500 km where we have more photons than hydrogen atoms and the medium is transparent.

```
[trygvels@mothallah SSB]$ python readfalc.py
Photon density at deepest model location:  1.66117e+13
Hydrogen density at deepest model location:  1.351e+17
Photon density at highest model location:  6.11473e+11
Hydrogen density at highest model location:  5.575e+09
```

1.3 Comparison with the earth's atmosphere

In this section we will repeat some of the analysis for similar data on the earths atmosphere. The data used here are from Allen 1976. First, a routine was created to fetch the earth data:

```
h, logP, temp, logdens, logN= np.loadtxt("
    ↪ earth.dat",usecols=(0,1,2,3,4), unpack=
    ↪ True)
```

With this, we can plot the the behaviour of the temperature, pressure, particle density and gas density against height. We notice how the Particle density, pressure and gas density look quite similar meaning they all depend on the same quantity.

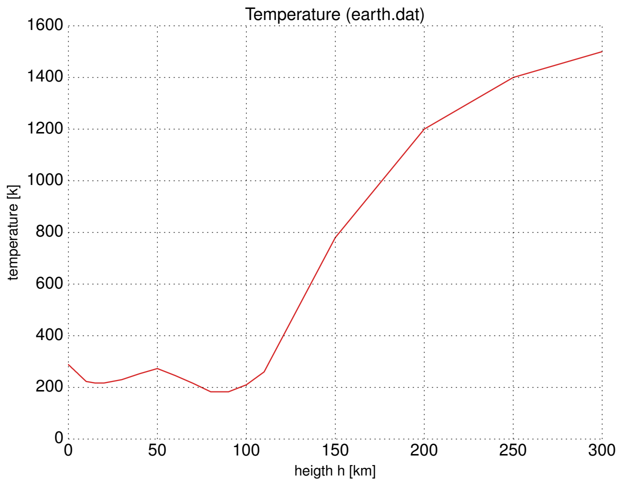

Figure 11: Temperature as a function of height for earth atmosphere

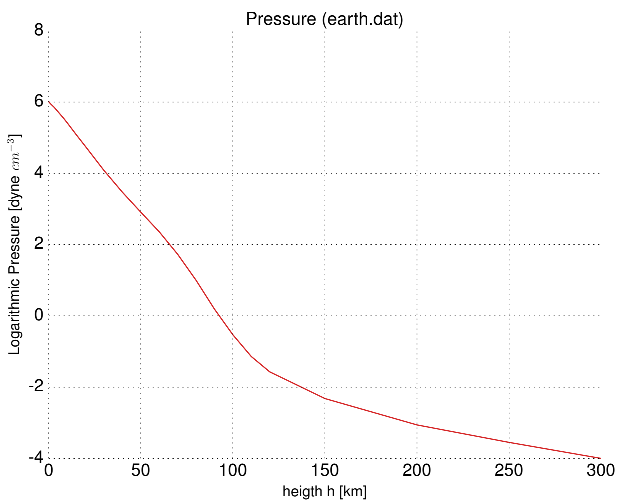

Figure 12: Pressure as a function of height for earth atmosphere

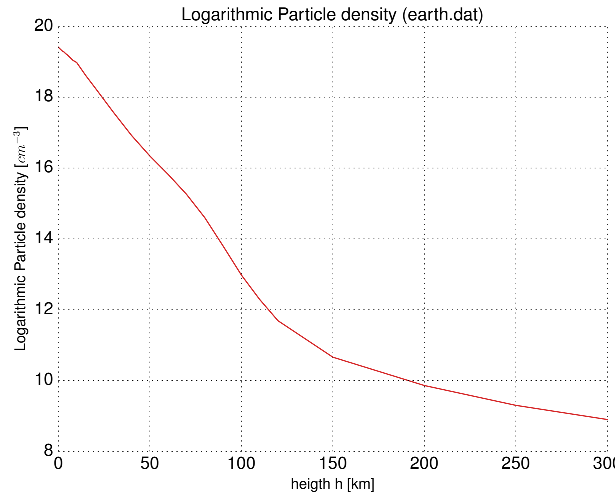

Figure 13: Particle density as a function of height for earth atmosphere

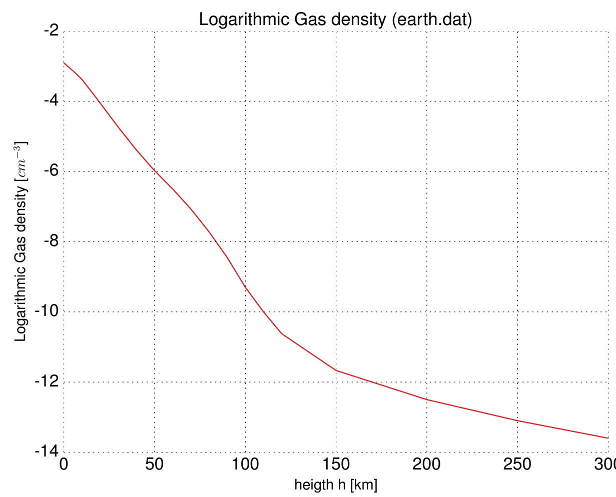

Figure 14: Gas density as a function of height for earth atmosphere

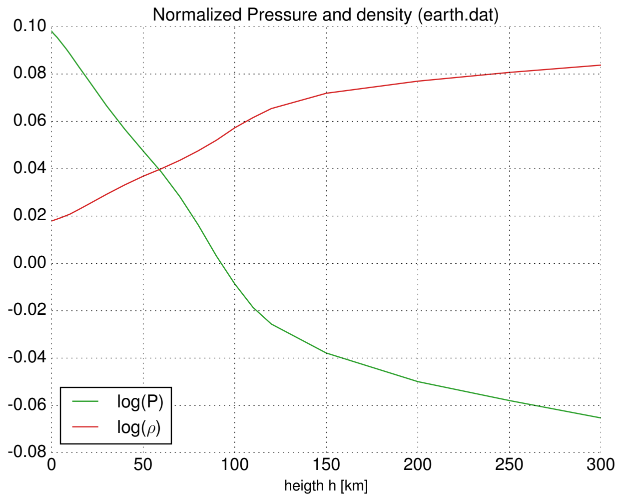

Figure 15: Pressure and density stratifications together in normalized units.

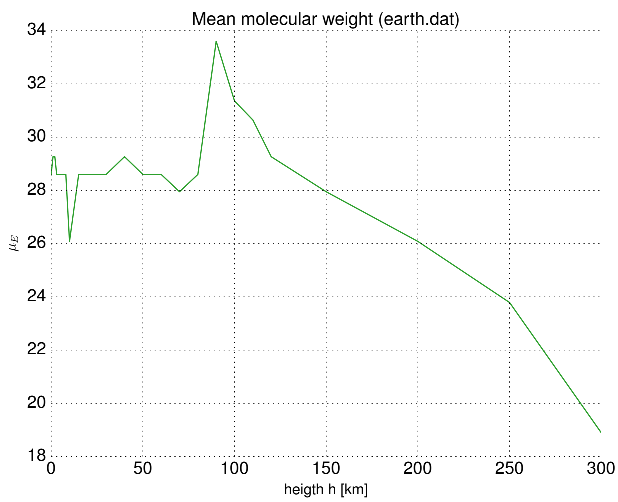

Figure 16: Mean molecular weight against height. This quantity decreases with height because lighter atoms crowd the upper atmosphere.

We can now find the density scale height by using the formula described in the previous section with the mean molecular weight given. This yields $H_\rho = 8.54km$. The difference here is due to the surface temperature and gravity. Because mount everest is 8848 meters high, the air is almost 1000 times less dense with this scale hight.

The ratio of the particle densities at $h = 0$ for solar and terrestial atmospheres is about 200. By calculating the column mass for the earth we get 1044gcm^{-2}. For the sun we get 4.4gcm^{-2}. We can also calculate $N_{phot} = \pi \frac{R^2}{D^2} N^{top} = 4.16 \cdot 10^7$ where N^{top} corresponds to the density on top of the earth atmosphere. If we calculate it from the earth, we get $N^E_{phot} = 4.78 \cdot 10^8 cm^{-3}$. These numbers tell us that there are 10 times more photons coming from the earth atmosphere than we receive from the sun. The photon density is smaller by 12 orders of magnitude. However, they correspond to different wavelengths.

2 Continuous spectrum from the solar atmosphere

In this section we will look at the formation of the solar continuum.

2.1 Observed solar continua

We will be using data provided by Allen once again. We will specifically be looking at radiation in the bandwidth $\lambda = 0.2 - 5\mu m$. We will be looking at are radially emergent intensity nad astrophysical flux with and without smoothed lines. First, a routine was created to fetch the solar continua data:

```
# reading solspect.dat
wav, F,Fcont,I,Icont= np.loadtxt("solspect.dat
    ↪ ",usecols=(0,1,2,3,4), unpack=True)
```

Then they were all potted in figure 17.

In order to check that the continuum intensity reaches $I^c_\lambda = 4.6 \cdot 10^{10}$ erg cm^{-2} s^{-1} ster^{-1} μm^{-1} at $\lambda = 0.41\mu$m, we use the statement:

```
print 'max(Ic)␣=␣',np.max(Icont*factor),'at',
    ↪ wav[np.where(Icont*factor == np.max(
    ↪ Icont*factor))]
```

```
[trygvels@mothallah SSB]$ python readsolspect.py
max(Ic) = 4.6 at [ 0.41]
```

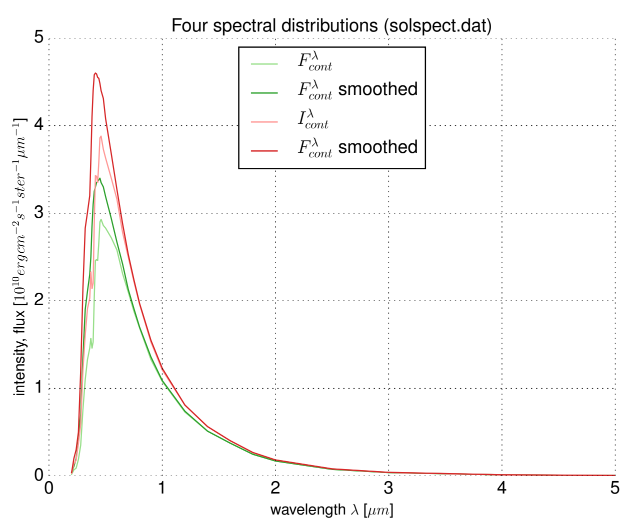

Figure 17: Spectral distributions, the flux has a slight difference in angle dependence.

These units can be converted into values per frequency bandwidth instead of wavelength as seen in figure 18

```
[trygvels@mothallah SSB]$ python readsolspect.py
max(Ic) = 4.20266666667e-05 at [ 0.8]
```

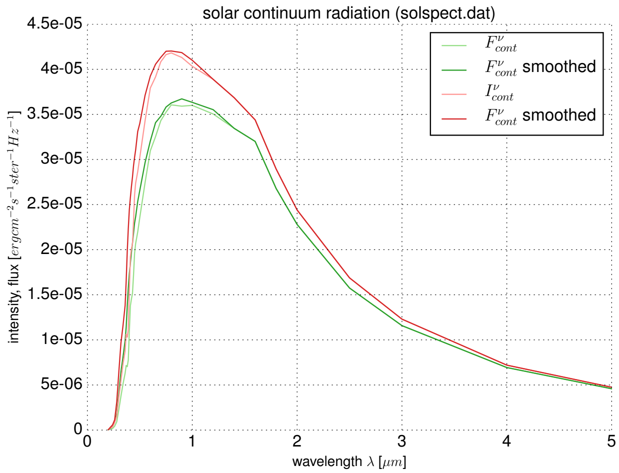

Figure 18: Spectral distributions converted into values per frequency bandwidth

The planck function can be used to estimate a temperature which gives a good fit to the observed temperature. I found that $T = 6250K$ is a good fit, but the plot was unfortunately lost.

Next we solve the planck function for temperature and set $B(T_b) = I$. This will give us a plot of the brightness temperature, which peaks at $\lambda \simeq 1.6\mu m$. This corresponds to the infrared radiation meaning most of the transported radiation comes from this.

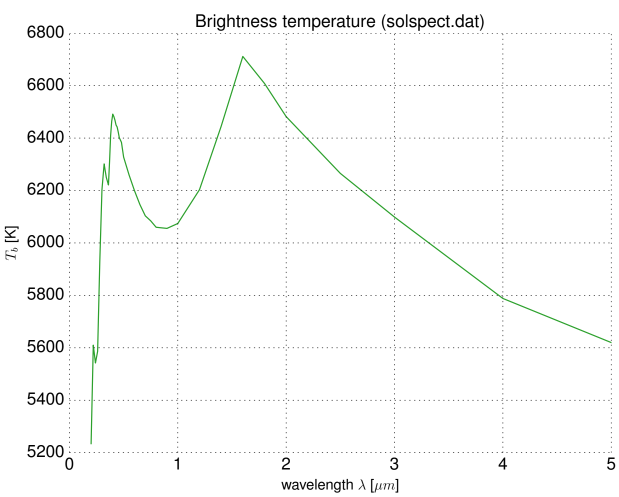

Figure 19: Brightness temperature

2.2 Continuous extinction

In this part of the project we will have a look at the extinction profiles. We pull the code exthmin.py which lets us work with the H^- extinction provided by Gray 1992. This code lets us look at the total extinction in H^- in units of cm^2 per neutral hydrogen atom. The extinction at the surface is given in figure 20.

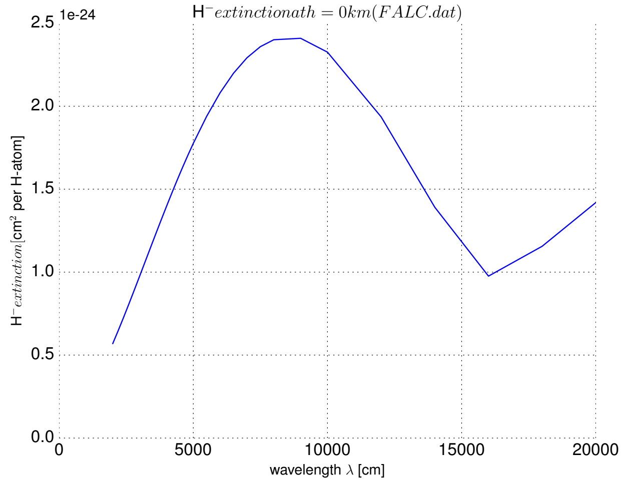

Figure 20: Variation of the H^- extinction with wavelength at surface

This is not hydrogenic because it has two bound electrons. We now look at how this extinction behaves with heigth. This is done by importing the FALC data once again and looking at $\lambda = 0.5\mu m$.

We now include the extinction due to free electrons as well, multiplying the Thomson cross-section with the number density of electrons and adding it to the hydrogen extinction gives us a a reasonable value.

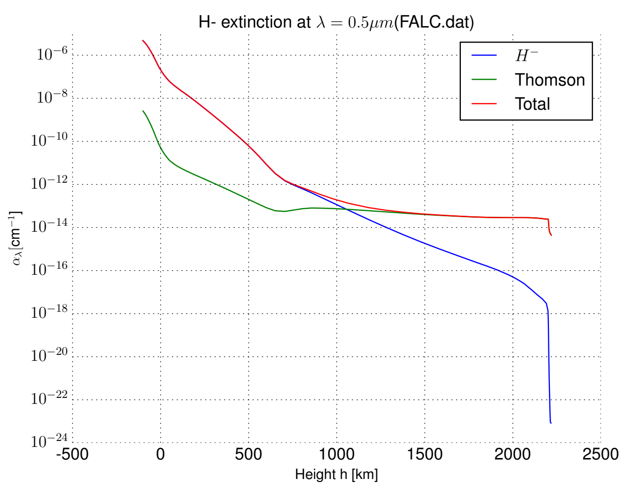

Figure 21: Separate and total extinctions

By looking at the figure, we see that neutral hydrogen dominates at low altitudes. Electron density also decrease but stabilize at around 850 μm. Hydrogen keeps decreasing but the but ionization does as well at high altitutes.

2.3 Optical Depth

We now want to study the optical depth from the data in the FALC model and the continuous extinction. It is given by $\tau(h_0) = -\int_{\infty}^{h_0} \alpha_\lambda^c dh$ which can be calculated numerically with trapezoidal integration yielding 22

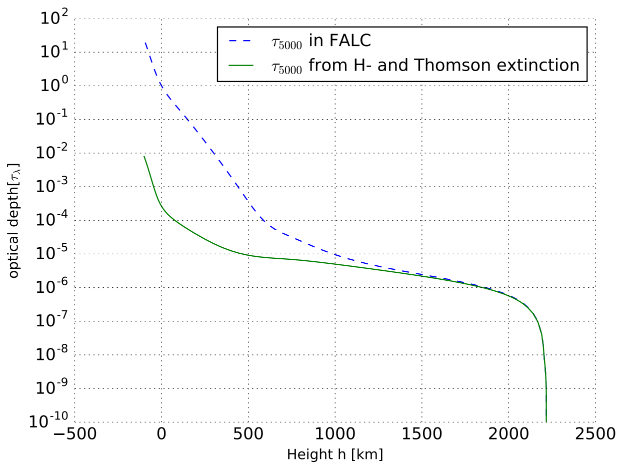

Figure 22: Optical depth at $\lambda = 500nm$ calculated and observed (Some unexpected values at h<500 ie. bad match, probably wrong units in program)

2.4 Emergent intensity and height of formation

The calculation of the emergent intensity from the solar disk is a small adjustment to the trapezoidal integration loop. We now want to calculate $I_\lambda = \int_0^\infty S_\lambda e^{-\tau_\lambda} d\tau_\lambda$. The intensity contribution function $\frac{dI}{dh} = S_\lambda e^{-\tau}\alpha$ gives us the contribution of each layer at a height h to the emergent intensity. The mean of this gives us the "Mean height of formation. These values can be calculated in our script:

```
[trygvels@mothallah SSB]$ python emergentI.py
Mean height of formation:  -3.13971570857 computed
continuum intensity wl = 0.5 :  4.28874e+14 observed
continuum intensity wav=0.5 :  4.08e+14 Mean height
of formation:  24.1092633752 computed continuum
intensity wl = 1 :  1.2541e+14 observed continuum
intensity wav=1 :  1.23e+14 Mean height of formation:
-17.9438162565 computed continuum intensity wl
= 1.6 :  4.27783e+13 observed continuum intensity
wav=1.6 :  4.03e+13 Mean height of formation:
75.9259958352 computed continuum intensity wl
= 5 :  5.74415e+11 observed continuum intensity
wav=5 :  5.7e+11
```

We can with these numbers for mean heights of formation conclude that light with short wavelengths form higher up in the atmosphere that longer wavelengths.

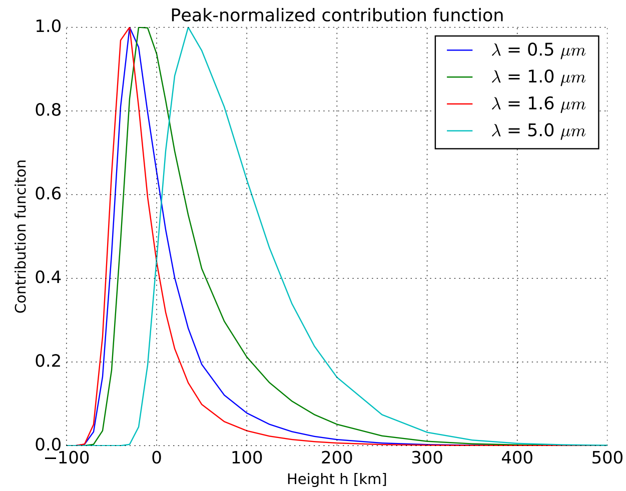

Figure 23: Peak-normalized contribution function against height.

By comparing the mean height of formation with the locations where $\tau = 1$ and $Tb = T(h)$ we can check the validity of the LTE eddington-Barbier approximation.

Table 1:

λ	h($\tau = 1$)	h	<h>
500nm	0.0km	10km	-3.1km
1000nm	10km	1065km	24.1km
1600nm	-30km	1476km	-17.9km

Table 2: We see that the difference between the values are too great to conclude that the LTE E-B approximation holds.

2.5 Disc-center intensity

By looping the integration over all wavelengths we can get a complete intensity for the solar continuum. This is plotted against the observed solar continuum in figure 24

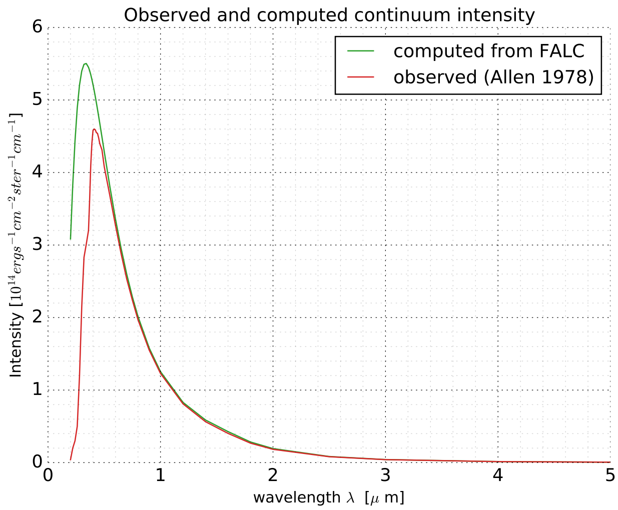

Figure 24: Emergent continuum compared with observed solar continuum.

2.6 Limb darkening

The code from the previous exercise is modified to include a loop over wavelength and the μ-term as seen in the source code `limbdark.py`. This term is used to find the intensity that emerged under the angle $\cos\theta = \mu$ given by:

$I_\lambda = \int_0^\infty S_\lambda e^{-\tau_\lambda/\mu} d\tau_\lambda/\mu$.

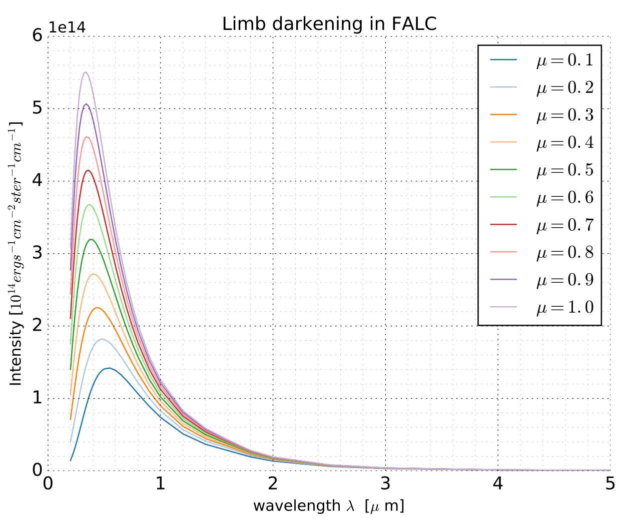

Figure 25: Intensity evaluated for different $\mu = cos\theta$ as a function of wavelength.

Due to a bug in my code, i was unable to reproduce a sufficient plot of the ratio of the intensity as a function of radius or μ. But from what was posted on the course web page, i can conclude that because $\tau = 1$ at different heights for different wavelengths. For long wavelengths the temperature where $\tau = 1$ is higher, giving a higher intensity.

2.7 Flux integration

The astrophysical flux can be found by integrating over the emergent intensity found in the previous exercise. $F = 2\int_0^1 I(0,\mu)\mu d\mu$ this was done using `flux.py` which was provided on the course web page.

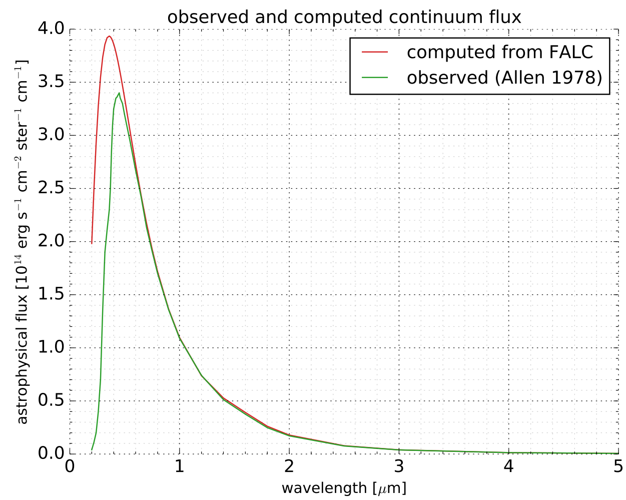

Figure 26: Emergent solar flux compared to observed flux

3 Spectral lines from the solar atmosphere

In this last part of the project, we will look at the formation of the Na I D_1 line at 5890Å.

3.1 Observed Na D line profiles

First we pull the data from the provided data file and pick out the first and third column.

3.2 Na D wavelengths

Once the data is fetched, we plot it against vacuum wavelength.

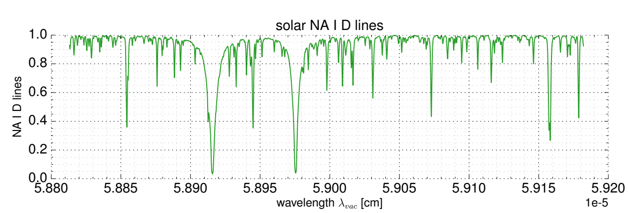

Figure 27: Solar Na D lines against vacuum wavelength

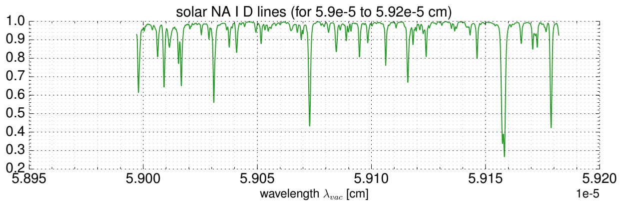

Figure 28: Second half

We check the positions of the NaI D lines using pythons `min()`-functions leaving the yielding results:

`[trygvels@raak SSB]$ python read31.py Vacuum wavelength of minima: 5.892e-05 ) [trygvels@raak SSB]$ python read31.py Vacuum wavelength of minima: 5.916e-05` Which correspond to that of Moore.

Next, we want convert the vacuum wavelength to air wavelength: $\lambda_a ir = 0.99972683\lambda_{vac} + 0.0107 - 196.25/\lambda_{vac}$ And plot the data against this new wavelength.

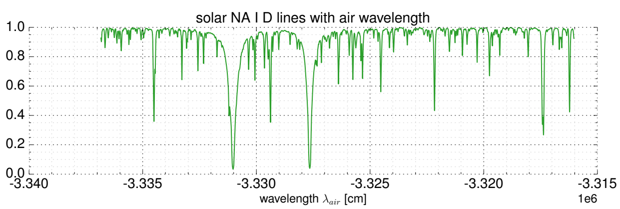

Figure 29: Solar Na D lines against air wavelength

3.3 Computing NaD line profile

Finally we want to compute the solar Na I D_1 line. We now assume LTE and evalueate line extinction as a function of height and wavelength which we add to the extinction in the integration loop from before.

The line extinction is given by:

$$\alpha_\lambda^l = \frac{\sqrt{\pi}e^2}{m_e c}\frac{\lambda^2}{c} b_l \frac{n_l^{LTE}}{N_E} N_H A_E f_{lu} \frac{H(a,v)}{\Delta\lambda_D}[1 - \frac{b_u}{b_l}e^{-hc/\lambda kT} \quad (1)$$

Where $\frac{n_l^{LTE}}{N_E}$ is the Saha-Boltzmann distribution for a given element, H is the voigt function and $\Delta\lambda_D$ is the doppler width. In order to make sure the Saha and Boltzmann distribution functions were working as expected they were plotted as well.

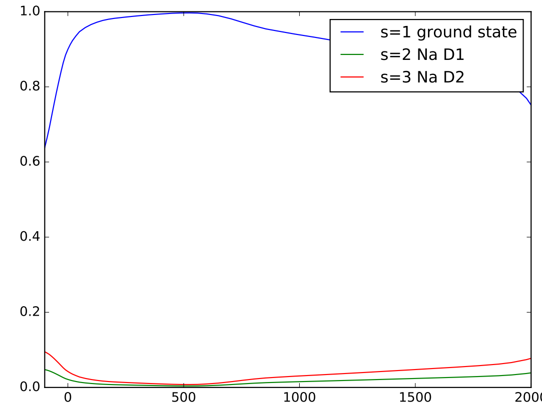

Figure 30: The Boltzmann distribution

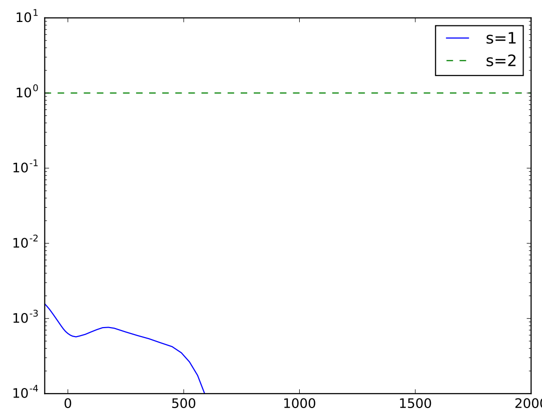

Figure 31: The Saha distribution

The rest of the functions were implemented and looped over for wavelength and height for calculating the sodium line extinction, which was added to the previously calculated extinction. This produced the final results.

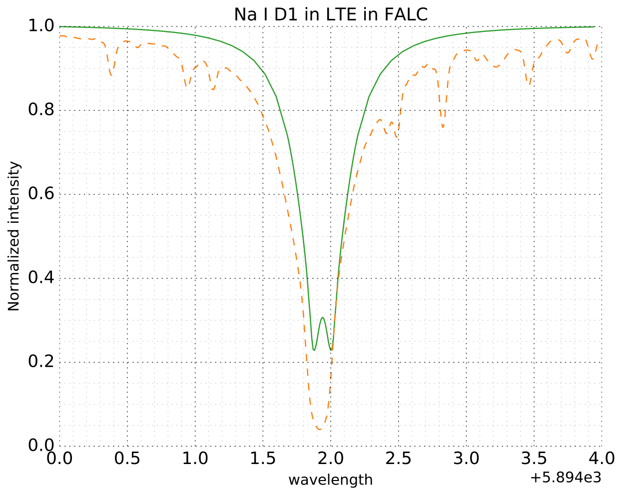

Figure 32: The computed intensity (Solid) and the observed intensity (Dashed). A slight shift of the observed intensity might be a result of the different resolution of the wavelength used for that plot. The other lines in the observed line correspond to other elements. The minimum of the computed line shallower than the observed, this corresponds to a breakdown of the LTE assumption.

References

[1] Rutten, R.J. (2010), *STELLAR SPECTRA A. Basic line formation.* Sterrekundig Instituut, Utrech University. Institutt for Teoretisk Astrofysikk, Oslo University.