Problem 10.1

Griffiths, Chapter 6, problem 2.

Problem 10.2

A spin-1/2 degree of freedom is influenced by a magnetic field that has a large z-component and a small x-component such that the Hamiltonian is

$$H = -\frac{B}{\hbar}S^z - \frac{g}{\hbar}S^x$$

Treat the x-component of the field (g) as a perturbation and
a) Compute the first order corrections to the unperturbed energy eigenvalues. Identify the dimensionless quantity that characterizes the perturbation expansion.

b) Compute the second order correction to the unperturbed energy eigenvalues.

c) Compute the first order correction to the unperturbed energy eigenstates.

Problem 10.3

This is a problem illustrating both first-order non-degenerate and degenerate perturbation theory. Consider the two-dimensional harmonic oscillator with an extra bilinear term gxy where g is a real constant.

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 y^2 + gxy$$

For $g = 0$ the exact energy eigenstates are tensor products of one-dimensional harmonic oscillator states: $|n_x, n_y\rangle = |n_x\rangle \otimes |n_y\rangle$, where $n_x, n_y \in \{0, 1, \ldots\}$. Their energies are $E_{n_x,n_y} = \hbar\omega(n_x + n_y + 1)$.

a) For $g = 0$ write down the energy eigenstates and their corresponding energies for the two lowest energy levels. What are their degeneracies?

b) Use first-order non-degenerate perturbation theory to compute how the ground state energy changes from the $g = 0$ value in a) when g is finite.

c) Use first-order degenerate perturbation theory to find how the first excited energy level splits up when g is finite.

Consider the reflection operator that interchanges $x \leftrightarrow y$

$$R|n_1\rangle \otimes |n_2\rangle = |n_2\rangle \otimes |n_1\rangle$$

d) For $g = 0$ find the eigenstates of R that are also eigenstates of H with energy $2\hbar\omega$, i.e. they belong to the first excited energy level.

e) Use non-degenerate first-order perturbation theory with the “good” states found in d) to compute how the first energy level splits up when g is finite. Compare your answer to what you got in c).