UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: FYS3110, Quantum mechanics

Day of exam: Dec. 1. 2014

Exam hours: 09:00-13:00 (4 hours)

This examination paper consists of 3 pages.

Permitted material: Approved calculator, D.J. Griffiths: "Introduction to Quantum Mechanics", the printed notes: "Time evolution of states in quantum mechanics", "Symmetry and degeneracy" and "WKB connection formulae", one handwritten A4-sheet(2 pages) with your own notes, and K. Rottmann: "Matematisk formelsamling".

Check that the problem set is complete before you start working. Some subproblems have more than one question.

Problem 1

1.1 Explain briefly what the variational method is.

1.2 Let $\hat{O}$ be a composite operator: $\hat{O} = \hat{A}\hat{B}$. Show that $\hat{O}^\dagger = \hat{B}^\dagger \hat{A}^\dagger$, where $\dagger$ means hermitian conjugate. Use this result to show that if $\hat{A}$,$\hat{B}$ and $\hat{O}$ are all hermitian operators, then $\left[\hat{A}, \hat{B}\right] = 0$.

Problem 2

A particle without spin with mass m and charge $q < 0$ are located in a central symmetric potential such that the potential energy of the particle is $V(r) = V_C(r) + V_K(r)$, where $V_C(r) = -|qQ|/r$ is the potential energy of the particle in a Coulomb potential from a charge $Q > 0$ where we for simplicity have set $\epsilon_0 = 1/4\pi$, and $V_K(r) = gr^2$ is a more general potential. $g \geq 0$ and has units of energy divided by length squared.

2.1 Write down the energies and the degree of degeneracy of the two lowest energy levels when $g = 0$.

Now assume that g is small so that we can consider V_K to be a perturbation of the Coulomb system which has the Hamiltonian $H_0 + V_C$, where $H_0 = \frac{\vec{p}^2}{2m}$

2.2 Use perturbation theory to calculate the ground state energy correct to first order in g.

2.3 The second lowest energy level is degenerate for $g = 0$. For $g > 0$ this energy level will split up into sublevels. What is the number of sublevels this energy level will be split into? Justify your answer either with physical arguments or calculations valid to first order in g.

Now let g be large such that it is reasonable to consider V_C as a perturbation of $H_0 + V_K$.

2.4 Calculate the ground state energy correct to first order in Q when g is large.

In the last subproblem set $Q = 0$. At time $t = 0$ the system, which is in its ground state, is suddenly exposed to a weak electric field with an amplitude E_0 polarized along the z-axis. The field is held constant for a time t_1 before it vanishes suddenly.

2.5 Use first order time-dependent perturbation theory to calculate the probability to find the system in the first excited energy level after the field has vanished.

Problem 3

This problem is about the effect of the spin-orbit-interaction

$$H^{so} = \frac{b}{\hbar^2}\vec{L}\cdot\vec{S}$$

on a degenerate energy level (a multiplet) in an atom with many electrons. All the states in the multiplet have the same values for the total orbital angular momentum L and total spin S. We have simplified the interaction such that b is a constant with units of energy.

3.1 Write down the expression for the splitting of the multiplet which is caused by the spin-orbit-interaction. Show also that the energy difference between successive energy levels is proportional to the largest of the total angular momenta (J) of the two levels.

Spectroscopy on an unknown atom has shown that the atom has the following energy levels (E_0 is the lowest energy level): $E_0 = 0$, $E_1 = 1.70 \times 10^{-2}$eV, $E_2 = 4.08 \times 10^{-2}$eV og $E_3 = 7.14 \times 10^{-2}$eV. These energy levels are separated from the next higher energy level by roughly 0.3eV. It is therefore natural to assume that the energy levels $E_0, \ldots, E_3$ stem from the same multiplet where the spin-orbit-interaction is responsible for the splitting.

3.2 Determine S for this multiplet and show that $L > S$. Determine also the sign of b, and find the value of L. Justify your answers.

- THE END -

This integral might be useful: $\int_0^\infty dx x^n e^{-x} = n!$