Problem 11.1

Consider the 1d harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

with energy eigenstates $|\psi_n\rangle$ and corresponding energies $E_n = \hbar\omega(n + 1/2)$.

a) Use the Feynman-Hellman theorem to calculate $\langle\psi_n|x^2|\psi_n\rangle$.

b) Use the Feynman-Hellman theorem to show that $\langle\psi_n|\frac{p^2}{2m}|\psi_n\rangle = \langle\psi_n|\frac{1}{2}m\omega^2 x^2|\psi_n\rangle$

Problem 11.2

Use perturbation theory to estimate the first order correction to the ground state energy of Hydrogen due to the finite size of the proton. To do this, assume that the proton is a uniformly charged sphere of radius $b = 1 \cdot 10^{-15}$m. The electric potential is thus

$$V(r) = \begin{cases} \frac{e}{4\pi\epsilon_0 b}\left(\frac{3}{2} - \frac{r^2}{2b^2}\right) & r \leq b \\ \frac{e}{4\pi\epsilon_0 r} & r > b \end{cases}$$

As this problem only asks for an estimate you may expand your expressions to lowest non-vanishing order in the dimensionless quantity b/a_0 where a_0 is the Bohr radius. Compare the magnitude of your answer to the fine- and hyperfine corrections. How does your estimate change if the proton has all its charge on its surface?

Problem 11.3

Griffiths Chapter 6. Problem 21.