UNIVERSITY OF OSLO
FACULTY OF MATHEMATICS AND NATURAL SCIENCES

Exam in: FYS3110, Quantum mechanics

Day of exam: Dec. 5. 2011

Exam hours: 14:30-18:30 (4 hours)

This examination paper consists of 3 pages.

Permitted material: Approved calculator, D.J. Griffiths: “Introduction to Quantum Mechanics”, the printed notes: “Time evolution of states in quantum mechanics”, “Symmetry and degeneracy” and “WKB connection formulae”, one handwritten A4-sheet(2 pages) with your own notes, and K. Rottmann: “Matematisk formelsamling”

Check that the problem set is complete before you start working.

Problem 1

Two spin-1/2 particles interact with each other. One of the particles is subject also to a local magnetic field. The Hamiltonian is

$$\hat{H} = \frac{J}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2 - \frac{h_1}{\hbar} S_1^z$$

where $\vec{S}_i$ is the spin-1/2 operator for spin i. J and h_1 are positive constants. J describes the interaction of the particles, while h_1 is proportional to the local magnetic field. Only consider the spin degrees of freedom in this problem. Use the notation $|\uparrow\downarrow\rangle$ to describe a state where the spin of particle 1 has z-component $+\hbar/2$ and the spin of particle 2 has z-component $-\hbar/2$.

1.1 Show that $\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right) + S_1^z S_2^z$, where $S^\pm = S^x \pm i S^y$. Use this to show that the state $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ is an eigenstate of $\hat{H}$ when $h_1 = 0$. Normalize this state and find also its energy eigenvalue when $h_1 = 0$.

1.2 Find all the other normalized eigenstates of $\hat{H}$ and their eigenvalues for $h_1 = 0$.

Set $h_1 > 0$.

1.3 The particles are in the state $|\psi(0)\rangle$ at time $t = 0$. Then time goes on and the particles evolve according to $\hat{H}$ without any other external influences. At time t the following quantitites is calculated: $G_1(t) = \langle\psi(t)|S_1^z|\psi(t)\rangle$, $G_2(t) = \langle\psi(t)|S_2^z|\psi(t)\rangle$, $G_3(t) = \langle\psi(t)|\left(S_1^z + S_2^z\right)|\psi(t)\rangle$ and $G_4(t) = \langle\psi(t)|\left(S_1^z - S_2^z\right)|\psi(t)\rangle$. Which of these quantities are <u>not</u> dependent on t? Give reasons for your answers.

1.4 Consider the local magnetic field h_1 as a perturbation and calculate the energy correction to the lowest energy level up to and including 2. order in perturbation theory.

1.5 Repeat 1.4, but find now the 1. order correction to all the other energy levels.

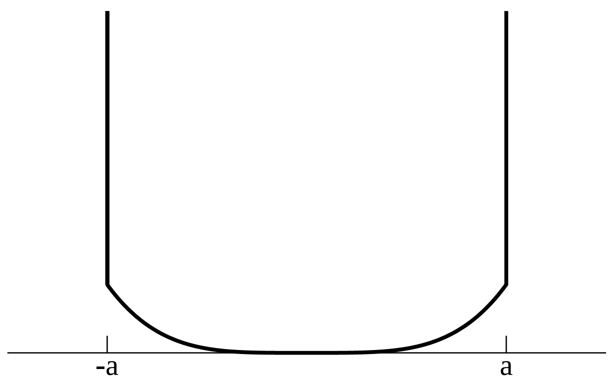
1.6 Find exact expressions for the energy eigenvalues of $\hat{H}$ and compare them with the perturbation theory results of 1.4 and 1.5.

Problem 2.

A particle with mass m (without spin) is located in a one dimensional x^4-potential with hard walls.

$$V(x) = \begin{cases} gx^4 & , -a \leq x \leq a \\ \infty & , x < -a \vee x > a \end{cases}$$

where $g \geq 0$ is a constant, see figure.



2.1 Write down all the energy eigenvalues for the particle in the potential $V(x)$ when $g = 0$.

Set $g > 0$ in the two last subproblems.

2.2 Use the WKB-method to find an expression for the energy eigenvalues with energies $E > ga^4$. Express your answer in terms of an integral. Do not try to solve the integral exactly. Instead find an approximate solution by series expansion of the integrand when $E \gg ga^4$ and find the 1. order correction (in g) to the result in 2.1.

2.3 Use the variational principle with a suitable polynomial wave function to find an upper limit on the ground state energy of the particle in the potential $V(x)$.

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