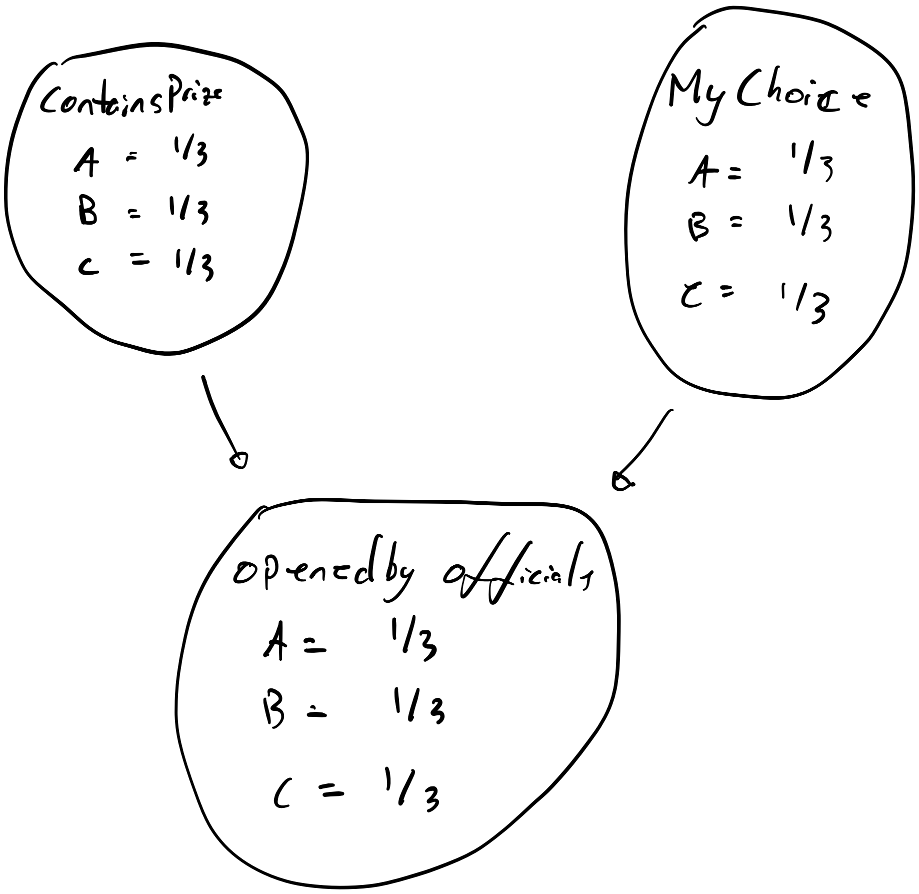
Monty hall problem:

Bayesian network:

A, B, C = Doors



probability table opened by Officials:

Contain price	Door A			Door B			Door C		
My Choise	A	B	C	A	B	C	A	B	C
open Door A	0	0	0	0	1/2	1	0	1	1/2
open Door B	1/2	0	1	0	0	0	1	0	1/2
open Door C	1/2	1	0	1	1/2	0	0	0	0

The problem is symmetric, so i do the calculations for one scenario

My choice = A

P(A contains price) = 1/3
P(B contains price) = 1/3
P(C contains price) = 1/3

P(open B | A prize) = 1/2
P(open B | B prize) = 0
P(open B | C prize) = 1

Should i switch my choice?

$p(A \text{ prize} \mid \text{open } B)$

$p(C \text{ prize} \mid \text{open } B)$

Bayes rule:

$$P(B_r \mid A) = \frac{P(B_r \cap A)}{P(A)} = \frac{P(A \mid B_r)\, P(B_r)}{\sum P(A \mid B_i)\, P(B_i)}$$

$P(C \text{ prize} \mid \text{open } B) =$

$$\frac{P(\text{open } B \mid C \text{ prize})\; P(C \text{ prize})}{P(\text{open } B \mid A \text{ prize})\, p(A \text{ prize}) + P(\text{open } B \mid B \text{ prize})\, p(B \text{ prize}) + P(\text{open } B \mid C \text{ prize})}$$

$$= \frac{1 \cdot 1/3}{1/2 \cdot 1/3 + 0 + 1 \cdot 1/3} = \frac{2}{3}$$

$$P(A\text{ prize} \mid \text{open } B) =$$

$$\frac{P(\text{open } B \mid A\text{ prize})\, P(A\text{ prize})}{P(\text{open } B \mid A\text{ prize})\, P(A\text{ prize}) + P(\text{open } B \mid B\text{ prize})\, P(B\text{ prize}) + P(\text{open } B \mid C\text{ prize})\, P(C\text{ prize})}$$

$$= \frac{1/2 \cdot 1/3}{1/2 \cdot 1/3 + 0 + 1 \cdot 1/3} = \frac{1}{3}$$

if i choose A and the host open B, the chance that the prize is in $A = \frac{1}{3}$ and in $C = \frac{2}{3}$. So i should switch doors