Probability and Statistics

Y-DATA School of Data Science

P&P 2

Due: 16.11.2022

PROBLEM 1. Let $X \sim U(1,5)$ (discrete). Define a new random variable $Y = 3^X$.

(1) What is the probability mass function of Y?
(2) Compute the expected value of Y.

PROBLEM 2. According to the British secret intelligence service, during the war, the expected number of bombs that fall per day in each quarter of London is 2. It is known that the number of bombs that fall in one day in each quarter is a Poisson random variable.

(1) What is the probability that on some day there will be no bombs at all?
(2) What is the probability that at least 4 bombs will fall on some specific quarter in one day?

PROBLEM 3. Let X be some discrete random variable. Show that for any constants $a, b \in \mathbb{R}$,

(1) $E(aX+b) = aE(X)+b$
(2) $Var(aX+b) = a^2 Var(X)$

PROBLEM 4. Give a simple example of some random variable X and some function g to show that in general $Eg(X) \neq g(EX)$.

PROBLEM 5. In a multiple-choice exam, there are 10 questions. Each has 4 possible answers (only one correct). Alice didn't prepare for the exam, so she guessed all her answers. Let X denote the number of her correct answers.

(1) What is the distribution of X? Write its PMF.
(2) To pass the test, a student should get 55%. What is the probability that Alice passed the test?

PROBLEM 6. Let X be a random variable with the following density function,

$$f_X(x) = \begin{cases} ax & \text{if } 0 < x < 1 \\ a & \text{if } 1 \leq x < 2 \\ a(3-x) & \text{if } 2 \leq x < 3 \\ 0 & \text{o.w.} \end{cases}$$

(1) Find the constant a for which f_X is a density function.
(2) Compute the expectation and variance of X.
(3) Find the cumulative distribution function of X.

PROBLEM 7. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with distribution $Bin(48, 1/4)$. Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Use the central limit theorem to calculate

$$P(\bar{X}_{144} > 12.75)$$