

Linear & Logistic Regression

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Agenda

- Recap
- Regularization
- Linear Regression
- Logistic Regression
- Code
- Summary

Recap



Gradient Descent

The idea of gradient descent is:

Take iterative steps to update parameters in the direction of the gradient

$$w^t \leftarrow w^{t-1} - \eta \cdot \frac{\partial \ell}{\partial w^{t-1}}$$

Diagram illustrating the gradient descent update formula:

- w^t is labeled "New weights" with an upward arrow.
- w^{t-1} is labeled "Previous weights" with a downward arrow.
- η is labeled "Step size / learning rate" with an upward arrow.
- $\frac{\partial \ell}{\partial w^{t-1}}$ is labeled "gradient" with a downward arrow.



Gradient Descent Training



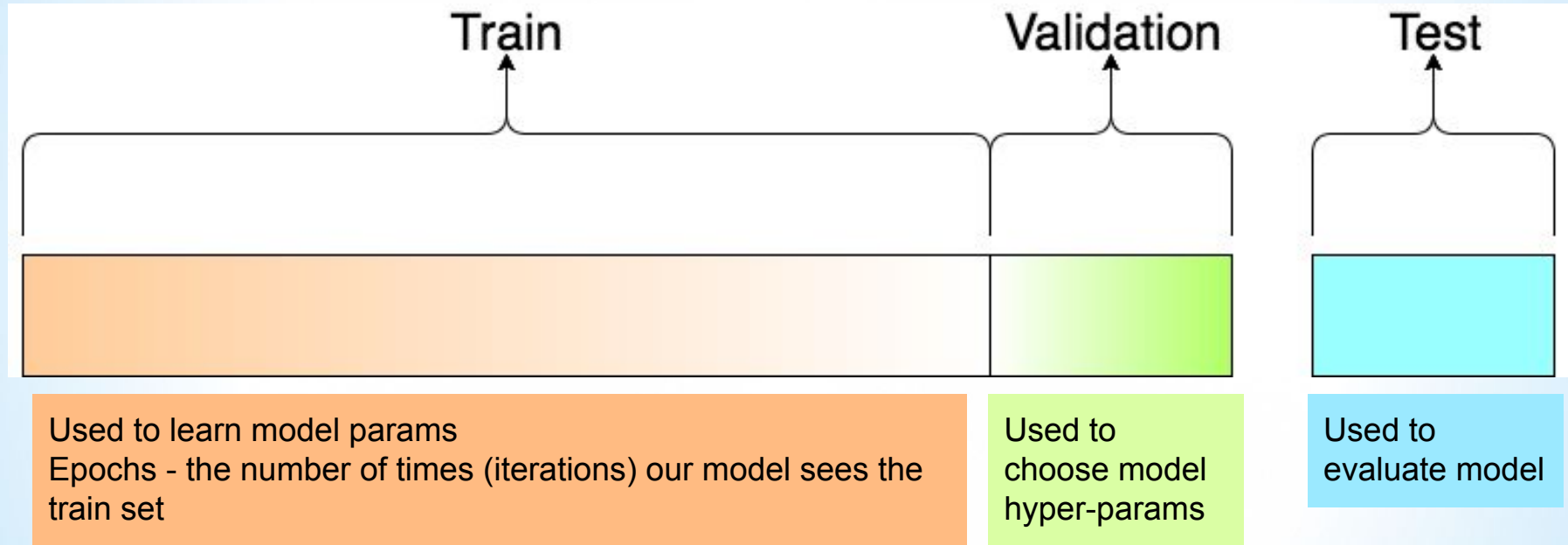
Algorithm 1 Gradient Descent Training

Input:

- Function $f(\mathbf{x}; \Theta)$ parameterized with parameters Θ .
- Training set of inputs $\mathbf{x}_1, \dots, \mathbf{x}_n$ and desired outputs $\mathbf{y}_1, \dots, \mathbf{y}_n$.
- Loss function L .

```
1: while stopping criteria not met do
2:   Compute the loss  $\mathcal{L}(\Theta) = \sum_i L(f(\mathbf{x}_i; \Theta), \mathbf{y}_i)$   <-- slow! goes over all data.
3:    $\hat{\mathbf{g}} \leftarrow$  gradients of  $\mathcal{L}(\Theta)$  w.r.t  $\Theta$ 
4:    $\Theta \leftarrow \Theta - \eta_t \hat{\mathbf{g}}$ 
5: return  $\Theta$ 
```

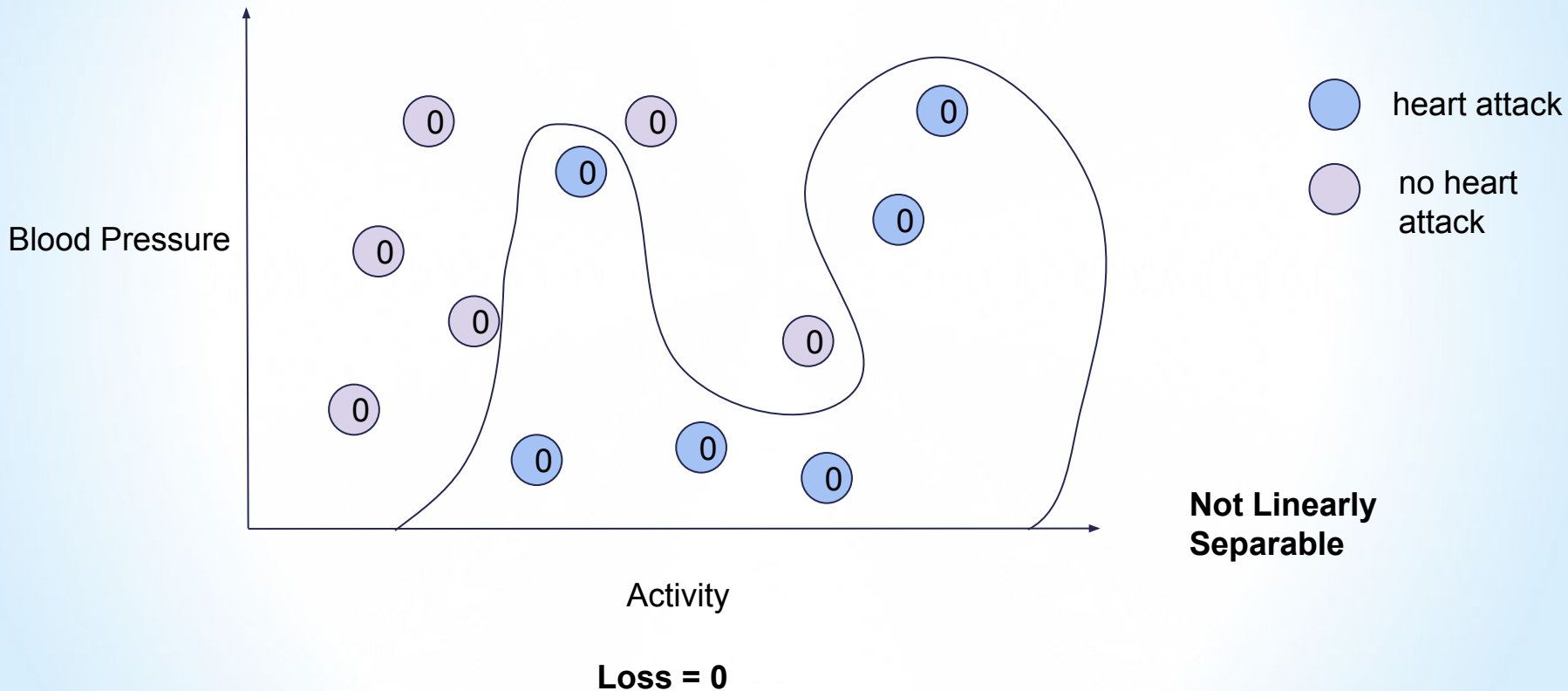
Reminder:



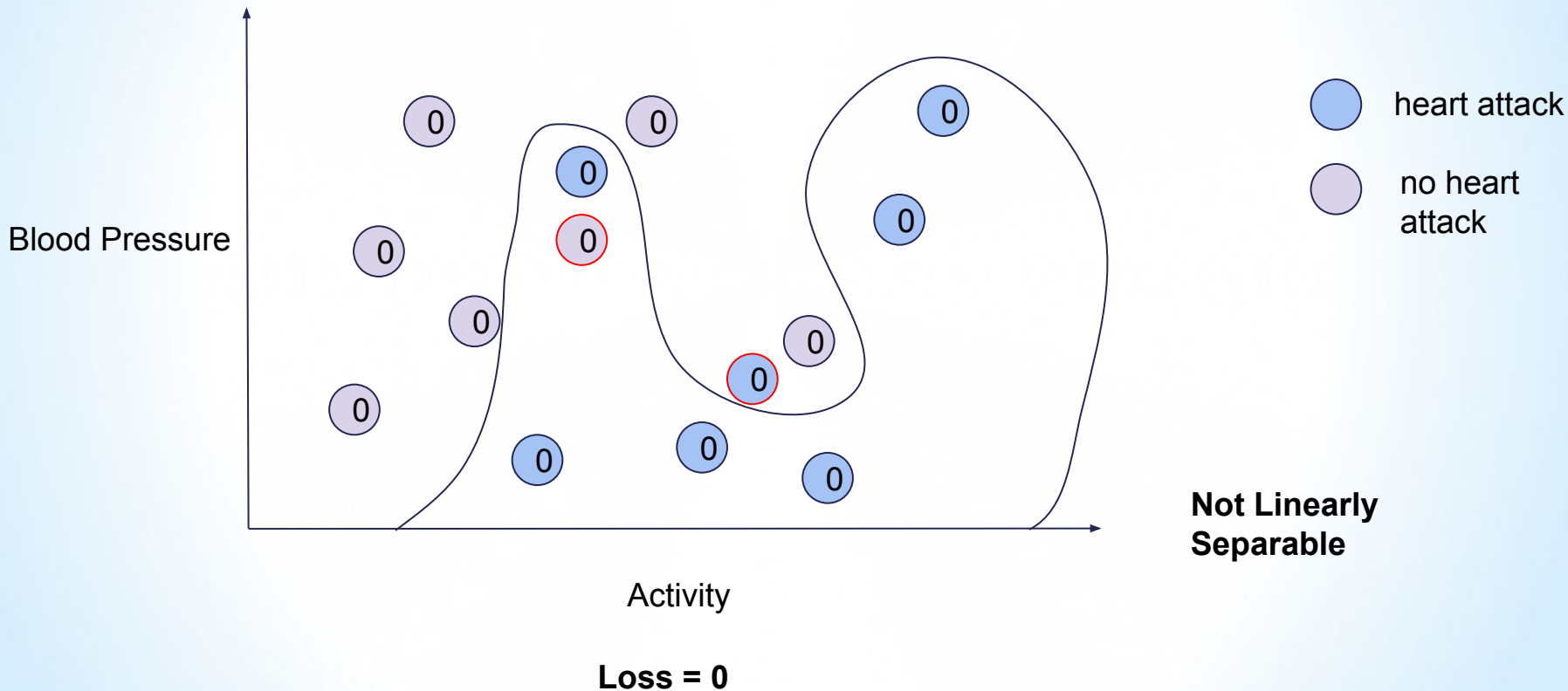
Regularization



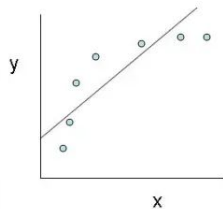
Overfitting



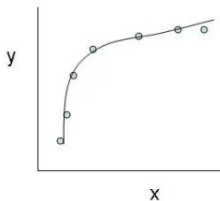
Overfitting - How Does it look on test set?



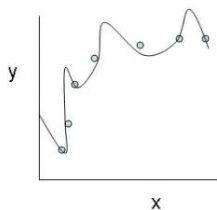
Reminder: Underfitting & Overfitting



Underfit



Just right



Overfit

How to Handle Overfitting?

1. Reduce the number of features
 - a. Manually select which features to keep
 - b. Automatic feature selection
2. Regularization
 - a. Keep all the features, but reduce the total weight of parameters θ_j
 - b. Regularization works well when we have a lot of slightly useful features
3. Normalization

Minimum Description Length Principle

This principle says we'll prefer a concise hypothesis over getting the entire training set right

$$w^* = \arg \min_w \frac{1}{m} \sum L(x_i, y_i; w) + \lambda \Omega(w)$$



l_p norms can be used as regularizers



High lambda - underfitting

Low lambda - overfitting

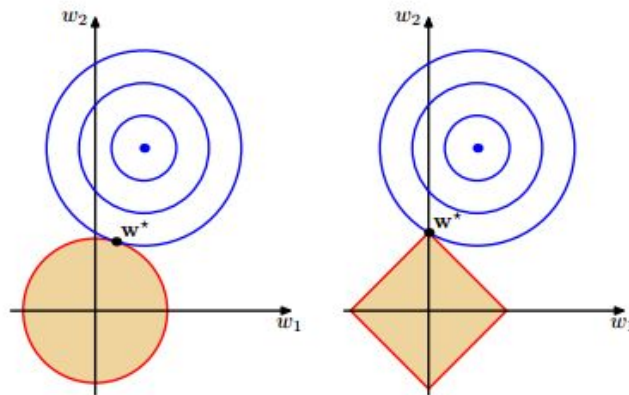
$$\|\mathbf{w}\|_2^2 = \sum_{d=1}^D w_d^2$$

$$\|\mathbf{w}\|_1 = \sum_{d=1}^D |w_d|$$

$$\|\mathbf{w}\|_p = \left(\sum_{d=1}^D w_d^p \right)^{1/p}$$

l_p norms

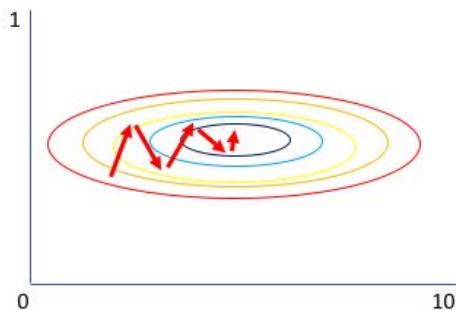
- Can be used as regularizers
- l_2 norm: convex, smooth, easy to optimize
- l_1 norm: encourages sparse w , convex, but not smooth at axis points
- $p < 1$: norm becomes non convex and hard to optimize



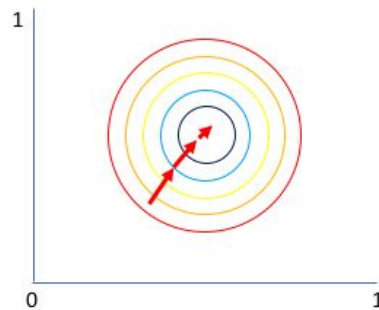
Normalization

Why is it important?

1. Helps gradient descent to converge
2. We don't necessarily want large features to have larger impact
3. Some sort of regularization - lower hypothesis search space



Gradient of larger parameter
dominates the update

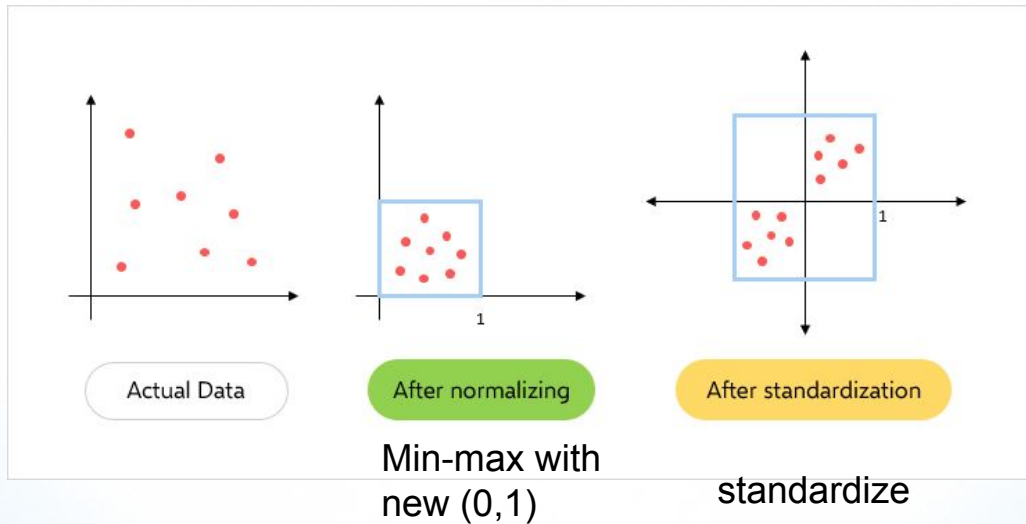


Both parameters can be
updated in equal proportions

Normalization methods

$$\text{Min-Max: } v' = \frac{v - \min_A}{\max_A - \min_A} (\text{new_max}_A - \text{new_min}_A) + \text{new_min}_A$$

$$\text{Z-Score: } v' = \frac{v - \text{mean}_A}{\text{stand_dev}_A}$$



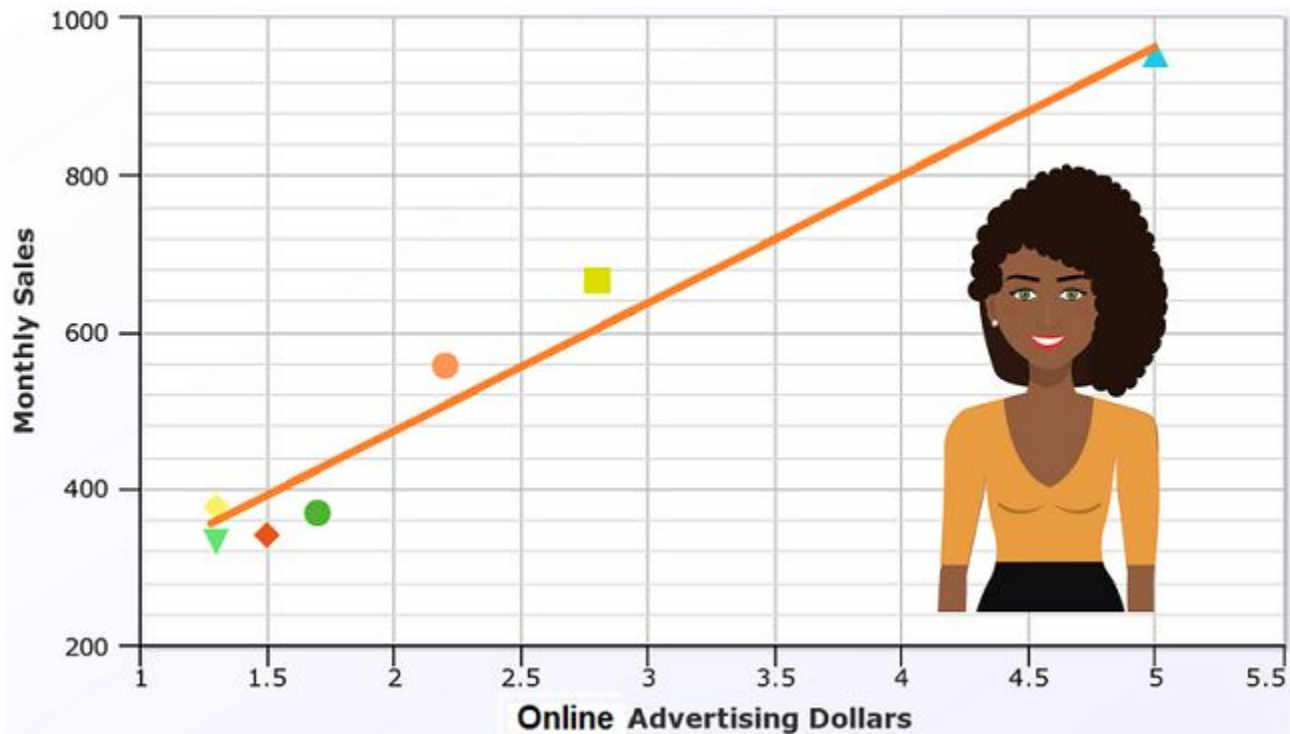
Linear Regression

$$L = \frac{1}{2} \| (Xw - y) \|^2$$

$$\text{Monthly sales} = w * x + b$$

slope

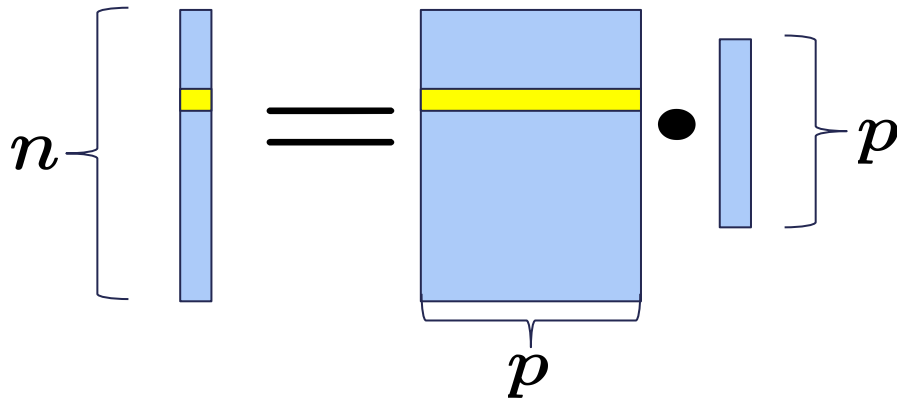
intersect



Matrix Form

1-padding for x

$$\hat{y}_{n \times 1} = X_{n \times p} w_{p \times 1}$$



Residual Sum of Squares Loss

$$L = \frac{1}{2} \|(Xw - y)\|^2 = \frac{1}{2} \underbrace{(Xw - y)^T (Xw - y)}_{\epsilon^T \epsilon}$$

Objective is
to min Loss
Function

is a scalar

Residual Sum of Squares Loss

$$L = \frac{1}{2} \| (Xw - y) \|^2 = \frac{1}{2} \underbrace{(Xw - y)^T (Xw - y)}_{\epsilon^T \epsilon}$$

Objective is
to min Loss
Function

is a scalar

$$L = \frac{1}{2} (w^T X^T X w - w^T X^T y + y^T y - y^T X w)$$

Common Matrix Derivatives

● *Notations*

○ x is a scalar

○ \mathbf{x} is a vector

Rule	Comments
$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$	order is reversed, everything is transposed
$(\mathbf{a}^T \mathbf{B} \mathbf{c})^T = \mathbf{c}^T \mathbf{B}^T \mathbf{a}$	as above
$\mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a}$	(the result is a scalar, and the transpose of a scalar is itself)
$(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$	multiplication is distributive
$(\mathbf{a} + \mathbf{b})^T \mathbf{C} = \mathbf{a}^T \mathbf{C} + \mathbf{b}^T \mathbf{C}$	as above, with vectors
$\mathbf{AB} \neq \mathbf{BA}$	multiplication is not commutative

Scalar derivative	Vector derivative
$f(x) \rightarrow \frac{df}{dx}$	$f(\mathbf{x}) \rightarrow \frac{df}{d\mathbf{x}}$
$bx \rightarrow b$	$\mathbf{x}^T \mathbf{B} \rightarrow \mathbf{B}$
$bx \rightarrow b$	$\mathbf{x}^T \mathbf{b} \rightarrow \mathbf{b}$
$x^2 \rightarrow 2x$	$\mathbf{x}^T \mathbf{x} \rightarrow 2\mathbf{x}$
$bx^2 \rightarrow 2bx$	$\mathbf{x}^T \mathbf{B} \mathbf{x} \rightarrow 2\mathbf{B}\mathbf{x}$

Residual Sum of Squares Loss

$$L = \frac{1}{2} (w^T X^T X w - w^T X^T y + y^T y - y^T X w)$$

Scalar derivative	Vector derivative
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$bx^2 \rightarrow 2bx$	$\mathbf{x}^T \mathbf{B} \mathbf{x} \rightarrow 2\mathbf{B} \mathbf{x}$



Closed Solution

$$\begin{aligned}\frac{\partial L}{\partial w} &= \frac{1}{2} \frac{\partial}{\partial w} (w^T X^T X w - w^T X^T y + y^T y - y^T X w) \\ &= X^T (X w - y)\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial w} &= X^T (X w - y) = 0 \\ w_{OLS} &= \underbrace{(X^T X)^{-1}}_{\text{pseudo-inverse of } X} X^T y\end{aligned}$$

Covariance
matrix

Correlation
between
data and
labels



Why can't we always use a closed form solution?

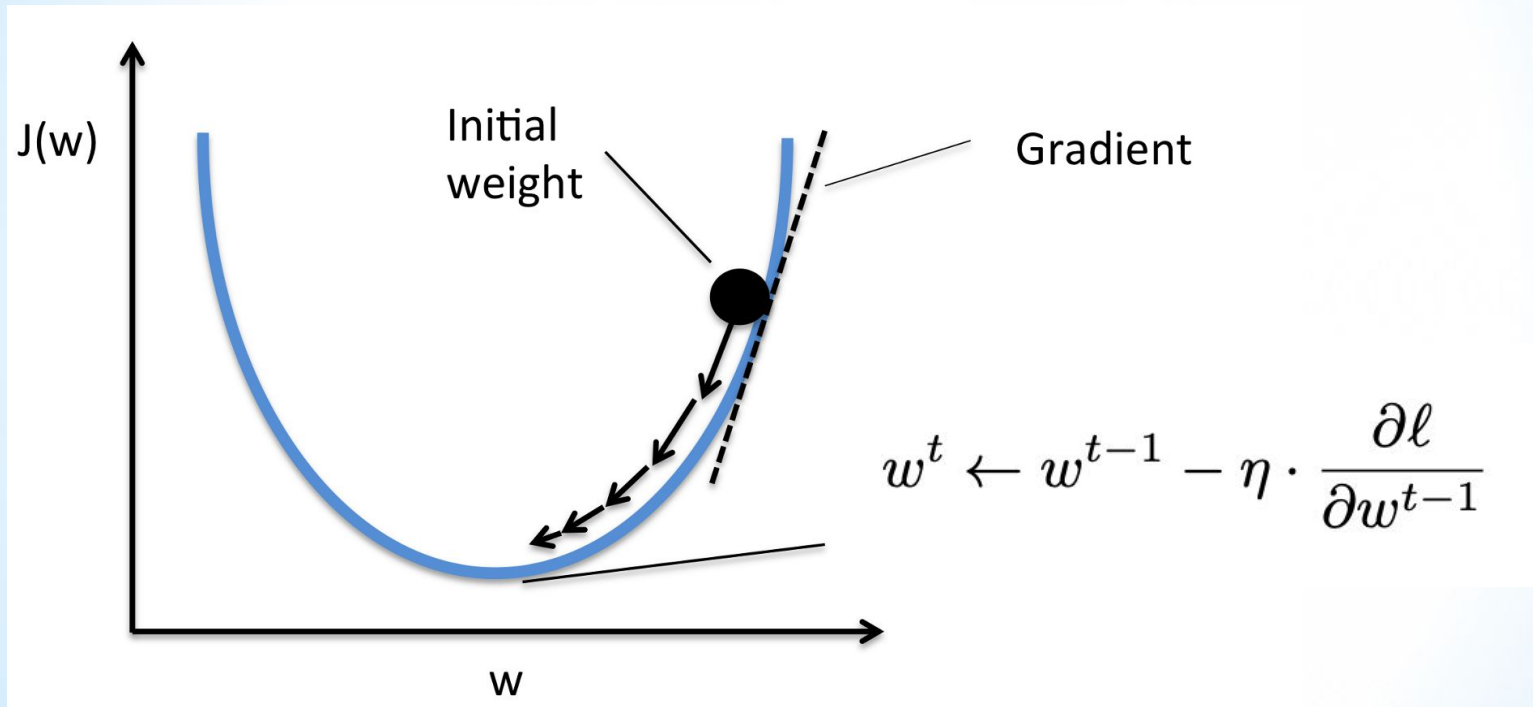
Assumes linear independence

Doesn't necessarily exist - might be impossible to invert $(X^TX)^{-1}$

Solving inverse $(X^TX)^{-1}$ is computational expensive $O(n^3)$

Scale is an issue

Reminder: Gradient Descent for the Rescue



Gradient Descent for OLS (Ordinary Least Squares)

Let's plug the gradient into gradient descent formula

$$\frac{\partial L}{\partial w} = X^T (Xw - y)$$

$$\underbrace{w'}_{\text{new}} = \underbrace{w}_{\text{old}} - \underbrace{\eta}_{\text{learning rate}} \underbrace{\frac{1}{n}}_{\text{normalization}} X^T (Xw - y)$$

Weighted Least Squares (WLS)

- This can be used when some samples are more valuable than others (imbalance, noise)
- W is a diagonal matrix

$$L = \frac{1}{2} \left\| W^{\frac{1}{2}} (Xw - y) \right\|^2$$

$$\hat{w}_{WLS} = (X^T W X)^{-1} X^T W y$$

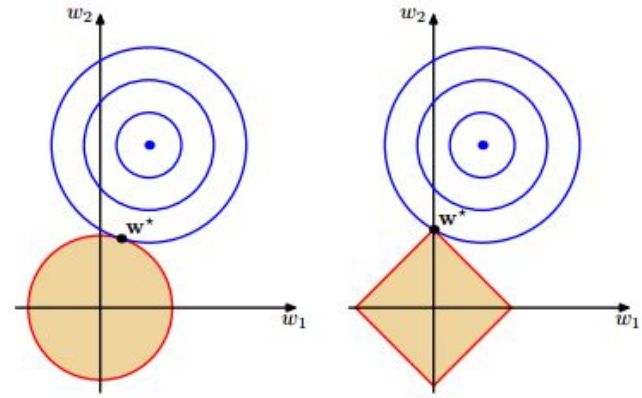
Reminder: Regularization

l_p norms can be used as regularizers

$$\|\mathbf{w}\|_2^2 = \sum_{d=1}^D w_d^2$$

$$\|\mathbf{w}\|_1 = \sum_{d=1}^D |w_d|$$

$$\|\mathbf{w}\|_p = (\sum_{d=1}^D w_d^p)^{1/p}$$



Ridge: L_2 Loss Regularization for OLS

- Minimize Objective function :

$$L = \underbrace{\frac{1}{2} ||(Xw - y)||^2}_{\text{OLS Model fitting term}} + \underbrace{\lambda ||w||^2}_{\text{Regularization term}}$$

- Where λ is the regularization parameter
- We can easily modify both our gradient descent function and our closed-form solution to fit the new loss function.

Ridge: L_2 Loss Regularization

$$L = \frac{1}{2} \|Xw - y\|^2 + \lambda \|w\|^2$$

$$\frac{\partial L}{\partial w} = X^T (Xw - y) + \lambda w = 0$$

$$w_{ridge} = \underbrace{(X^T X + \lambda I)^{-1} X^T}_{\text{pseudo-inverse of } X \text{ with diagonal loading}} y$$

$$I = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{bmatrix}$$

Lasso: L_1 Loss Regularization

- OLS with L_1 penalty

$$w_{lasso} = \underset{w}{\operatorname{argmin}} L = \underbrace{\frac{1}{2} \|(Xw - y)\|^2}_{\text{OLS Loss}} + \lambda \underbrace{\|w\|}_{\text{L1 Regularization}}$$

$$\|w\| = \sum_k^p \|w_k\|$$

- Causes sparse weights
- Can be treated as “automatic feature selection”
- Harder to solve (solved using **Coordinate Descent**)

Elastic Net: Ridge and Lasso Combined

$$\begin{aligned} w_{elastic} &= \underset{w}{\operatorname{argmin}} L \\ &= \frac{1}{2} \|(Xw - y)\|^2 + \lambda_1 \|w\| + \lambda_2 \|w\|^2 \end{aligned}$$

- Was found to be equivalent to SVM (will be discussed in SVM lecture)

Summary of Linear Regression

OLS	OLS + GD	Lasso	Ridge
Closed Solution	$\frac{1}{2} \ Xw - y\ ^2$	$\frac{1}{2} \ Xw - y\ ^2 + \lambda \ w\ $	$\frac{1}{2} \ Xw - y\ ^2 + \lambda \ w\ ^2$

Logistic Regression

$$\ell_{\text{cross-ent}} = - \sum_k \mathbf{y}_{[k]} \log \hat{\mathbf{y}}_{[k]}$$

$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{x}\mathbf{W} + \mathbf{b})$$

Classification vs Regression



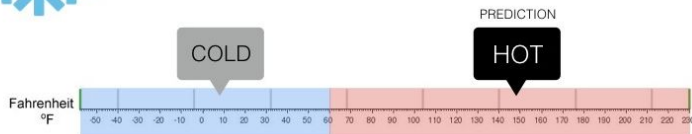
Regression

What is the temperature going to be tomorrow?

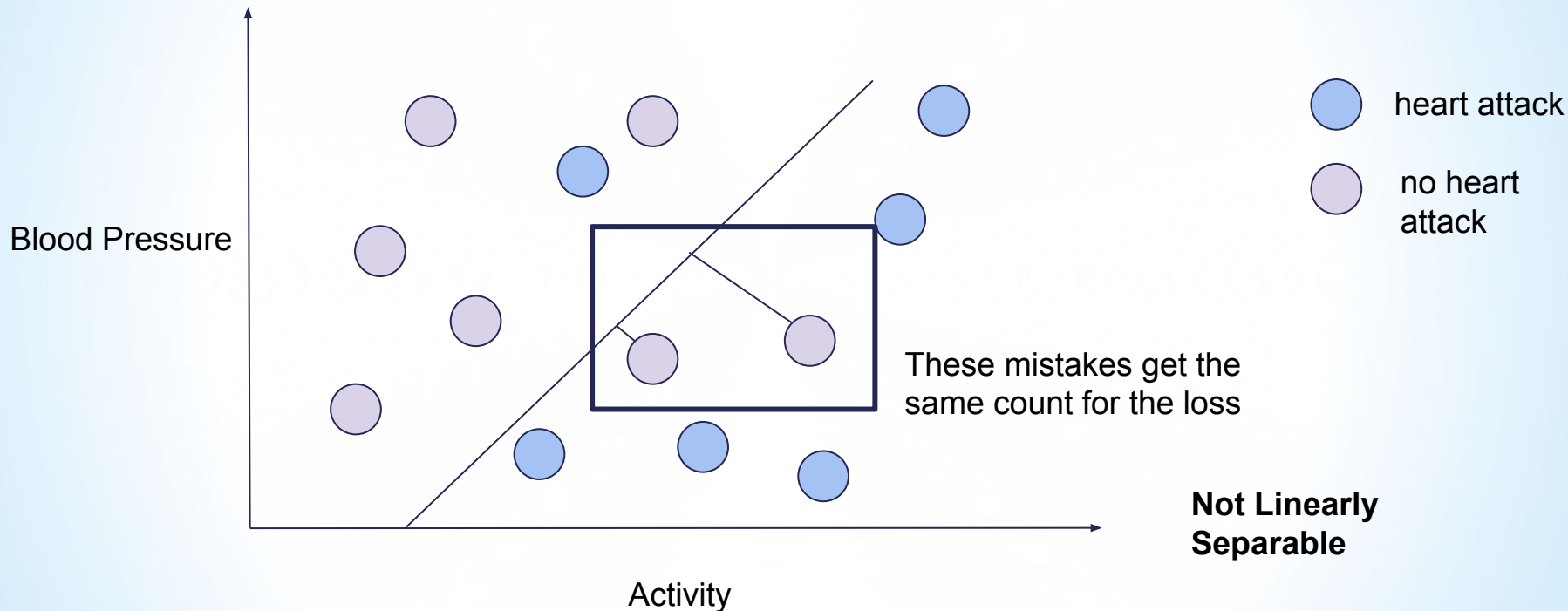


Classification

Will it be Cold or Hot tomorrow?

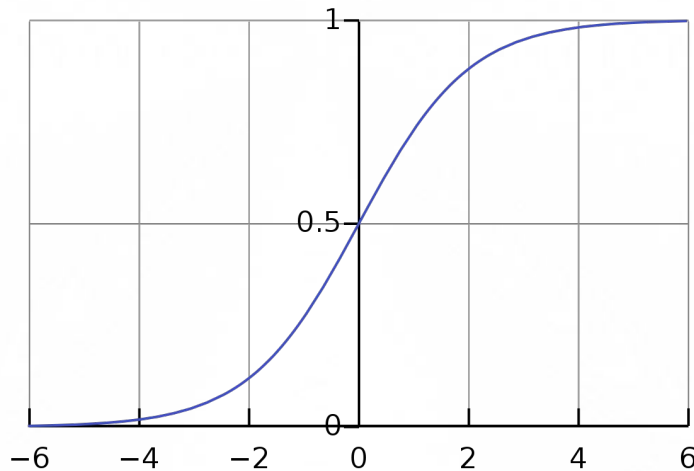


Why not use the 1-0 loss from before?



Can we do better? Sigmoid

$$\sigma(z_i) = \frac{1}{1+e^{-z_i}}$$



Derivative of Sigmoid

$$\sigma(z_i) = \frac{1}{1+e^{-z_i}}$$

What is the derivative of a sigmoid?

- *Where $z=wx$*

Derivative of Sigmoid

$$\sigma(z_i) = \frac{1}{1+e^{-z_i}}$$

$$\frac{\partial \sigma(z)}{\partial z} = \sigma(z)(1 - \sigma(z))$$

● Where $z=wx$

Sigmoid:

$$\begin{aligned} \left[\frac{1}{1+e^{-x}} \right]' &= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1+e^{-x}-1}{(1+e^{-x})^2} = \frac{1+e^{-x}}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2} \\ &= \frac{1}{1+e^{-x}} \left(1 - \frac{1}{1+e^{-x}} \right) = \text{sigmoid} * (1 - \text{sigmoid}) \end{aligned}$$

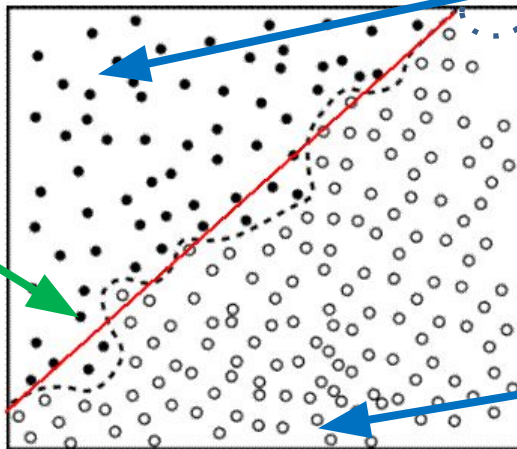
Logistic Regression with Sigmoid Intuition

- Different/Better Hypothesis & Objective

If X is near the linear separator w then

- wX is small
- $e^{-wX} \approx 1$
- $h_w(X) \approx 0.5$ (i.e. 50% probability)

Linear separator



If X is positive and far from the linear separator w then

- wX is big positive
- $e^{-wX} \approx 0$
- $h_w(X) \approx 1$

$h_w(x)$

x

If X is negative and far from the linear separator w then

- wX is big negative
- $e^{-wX} \gg 10000$
- $h_w(X) \approx 0$

Another Intuition for Logistic Regression

The logistic regression model tries to predict the odds of an event:

$$\frac{p(X)}{1 - p(X)}$$

$$\frac{p(X)}{1 - p(X)} = e^{\beta^T X} = e^{\beta_1 X_1 + \dots + \beta_p X_p}$$

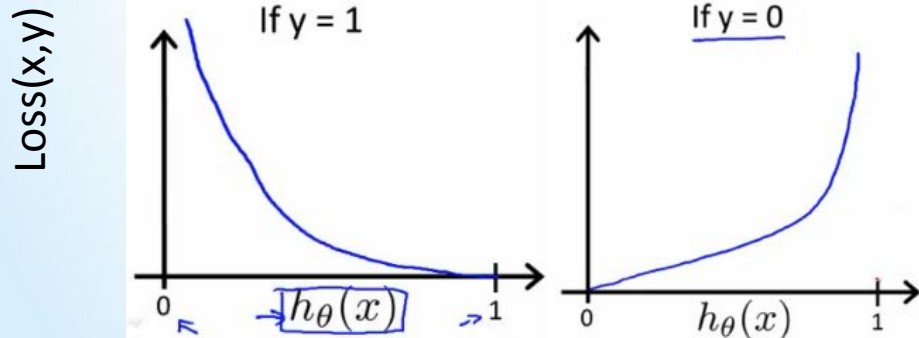
$$\log \left(\frac{p(X)}{1 - p(X)} \right) = \beta^T X$$

$$p(X) = \frac{\exp(\beta^T X)}{1 + \exp(\beta^T X)}$$

Cross Entropy Loss/Negative Log Likelihood

- Let's define a loss for each observation based on a **fix separator**

$$\text{Loss}(x,y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

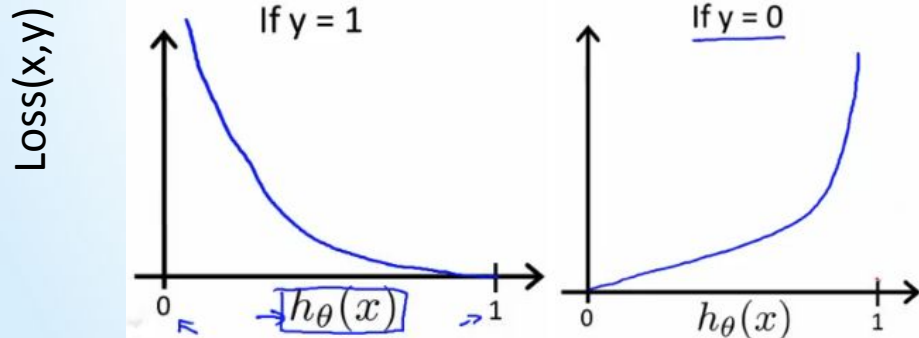


Cross Entropy Loss/Negative Log Likelihood

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$$\text{Loss}(x,y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

$$-y\log(H_{\theta}(x)) - (1 - y)\log(1 - H_{\theta}(x))$$



LR Derivation Simplified

- Optimize

$$G = y \cdot \log(h) + (1 - y) \cdot \log(1 - h)$$

- Where

$$h = 1 / (1 + e^{-z}) \quad z(\theta) = x\theta:$$

- Derivation

$$\frac{dG}{d\theta} = \frac{dG}{dh} \frac{dh}{dz} \frac{dz}{d\theta}$$

$$\frac{dG}{dh} = \frac{y}{h} - \frac{1-y}{1-h} = \frac{y-h}{h(1-h)}$$

$$\frac{dh}{dz} = h(1-h)$$

$$\frac{dz}{d\theta} = x$$

Derivative of sigmoid

$$\frac{dG}{d\theta} = (y - h)x$$

- Result














Reminder: Multi-Class & Multi-Label

Multiclass classification

Assigns a single class out of m possible classes
(y output an integer between 1 and m)

Multilabel classification

Assign a 0 or 1 labels for each of the m possible classes
(y output a binary vector of size m)

	Multi-Class	Multi-Label
$C = 3$		
	Samples	Samples
	   	  
	Labels (t)	Labels (t)
	  	  
	$[0 \ 0 \ 1]$ $[1 \ 0 \ 0]$ $[0 \ 1 \ 0]$	$[1 \ 0 \ 1]$ $[0 \ 1 \ 0]$ $[1 \ 1 \ 1]$

Heuristic Solutions to Multiclass Problems

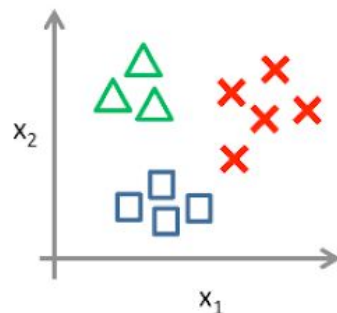
One vs All (One vs Rest)

Training a single classifier per class, with the samples of that class as positive samples and all other samples as negatives - then choose the class with maximal confidence

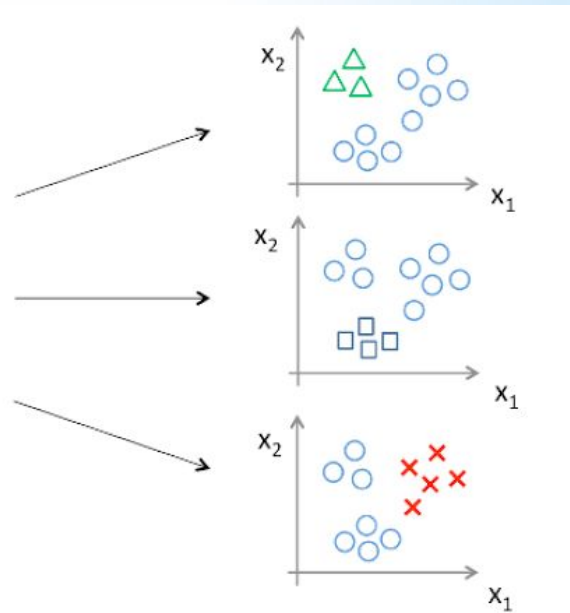
One vs One

Train $K(K-1)/2$ binary classifiers, each receives the samples of a pair of classes from the original training set, and learn to distinguish these two classes. During prediction time, a voting scheme is applied

One-vs-all (one-vs-rest):




Class 1: **Green**
 Class 2: **Blue**
 Class 3: **Red**



Cross Entropy Multiclass

$$\ell_{\text{cross-ent}} = -\log \hat{\mathbf{y}}_{[t]}$$



$$\ell_{\text{cross-ent}} = -\sum_k \mathbf{y}_{[k]} \log \hat{\mathbf{y}}_{[k]}$$

Sigmoid for Multiclass (Softmax)

$$\sigma(z_i) = \frac{1}{1+e^{-z_i}}$$



$$\text{softmax}(\mathbf{v})_{[i]} = \frac{e^{\mathbf{v}_{[i]}}}{\sum_{i'} e^{\mathbf{v}_{[i']}}}$$

Log-linear model
(aka "logistic regression")

$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{x}\mathbf{W} + \mathbf{b})$$

$$\hat{y}_{[i]} = \frac{e^{(\mathbf{x}\mathbf{W} + \mathbf{b})_{[i]}}}{\sum_i e^{(\mathbf{x}\mathbf{W} + \mathbf{b})_{[i]}}}$$

Cross Entropy Multiclass Derivative - Try this @ home

$$\begin{aligned}
 \frac{\partial J(\theta)}{\partial \theta_j} &= \frac{\partial}{\partial \theta_j} \frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))] \\
 &\stackrel{\text{linearity}}{=} \frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \frac{\partial}{\partial \theta_j} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \frac{\partial}{\partial \theta_j} \log(1 - h_{\theta}(x^{(i)})) \right] \\
 &\stackrel{\text{chain rule}}{=} \frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \frac{\frac{\partial}{\partial \theta_j} h_{\theta}(x^{(i)})}{h_{\theta}(x^{(i)})} + (1 - y^{(i)}) \frac{\frac{\partial}{\partial \theta_j} (1 - h_{\theta}(x^{(i)}))}{1 - h_{\theta}(x^{(i)})} \right] \\
 &\stackrel{h_{\theta}(x) = \sigma(\theta^T x)}{=} \frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \frac{\frac{\partial}{\partial \theta_j} \sigma(\theta^T x^{(i)})}{h_{\theta}(x^{(i)})} + (1 - y^{(i)}) \frac{\frac{\partial}{\partial \theta_j} (1 - \sigma(\theta^T x^{(i)}))}{1 - h_{\theta}(x^{(i)})} \right] \\
 &\stackrel{\sigma'}{=} \frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \frac{\sigma(\theta^T x^{(i)}) (1 - \sigma(\theta^T x^{(i)})) \frac{\partial}{\partial \theta_j} (\theta^T x^{(i)})}{h_{\theta}(x^{(i)})} - (1 - y^{(i)}) \frac{\sigma(\theta^T x^{(i)}) (1 - \sigma(\theta^T x^{(i)})) \frac{\partial}{\partial \theta_j} (\theta^T x^{(i)})}{1 - h_{\theta}(x^{(i)})} \right] \\
 &\stackrel{\sigma(\theta^T x) = h_{\theta}(x)}{=} \frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \frac{h_{\theta}(x^{(i)}) (1 - h_{\theta}(x^{(i)})) \frac{\partial}{\partial \theta_j} (\theta^T x^{(i)})}{h_{\theta}(x^{(i)})} - (1 - y^{(i)}) \frac{h_{\theta}(x^{(i)}) (1 - h_{\theta}(x^{(i)})) \frac{\partial}{\partial \theta_j} (\theta^T x^{(i)})}{1 - h_{\theta}(x^{(i)})} \right] \\
 &\stackrel{\frac{\partial}{\partial \theta_j} (\theta^T x^{(i)}) = x_j^{(i)}}{=} \frac{1}{m} \sum_{i=1}^m [y^{(i)} (1 - h_{\theta}(x^{(i)})) x_j^{(i)} - (1 - y^{(i)}) h_{\theta}(x^{(i)}) x_j^{(i)}] \\
 &\stackrel{\text{distribute}}{=} \frac{1}{m} \sum_{i=1}^m [y^{(i)} - y^{(i)} h_{\theta}(x^{(i)}) - h_{\theta}(x^{(i)}) + y^{(i)} h_{\theta}(x^{(i)})] x_j^{(i)} \\
 &\stackrel{\text{cancel}}{=} \frac{1}{m} \sum_{i=1}^m [y^{(i)} - h_{\theta}(x^{(i)})] x_j^{(i)} \\
 &= \frac{1}{m} \sum_{i=1}^m [h_{\theta}(x^{(i)}) - y^{(i)}] x_j^{(i)}
 \end{aligned}$$

Other Loss Functions

- Hinge: $\max(0, 1 - ywx)$ → You'll discuss this at SVM lecture
- Exponential loss
- C-loss
- ...

Explainability

ELI5 - Global explanation is the model W

y=1 top features

Weight?	Feature
+1.170	month__mar
+1.117	month__dec
+0.968	education__illiterate
+0.920	month__oct
+0.711	contact__cellular
+0.619	month__sep
+0.615	job__retired
+0.580	job__student
+0.564	default__no
+0.528	<BIAS>
+0.424	poutcome__success
+0.372	marital__unknown
+0.208	job__unknown
+0.201	housing__no
+0.193	day_of_week__wed
+0.188	housing__unknown
... 17 more positive ...	
... 21 more negative ...	
-0.682	month__jul
-0.761	month__may
-0.798	month__aug
-0.886	month__nov

Bank Marketing Data Set — [LINK](#)

Explainability

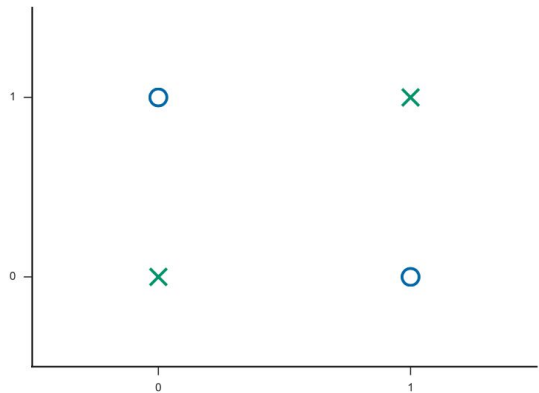
Local explanation is wx

y=1 (probability **0.961**, score **3.203**) top features

Contribution?	Feature	Value
+0.711	contact__cellular	1.000
+0.564	default__no	1.000
+0.528	<BIAS>	1.000
+0.424	poutcome__success	1.000
+0.363	previous	2.000
+0.208	job__unknown	1.000
+0.193	day_of_week__wed	1.000
+0.188	marital__single	1.000
+0.156	loan__no	1.000
+0.139	housing__yes	1.000
+0.129	age	27.000
+0.024	education__university.degree	1.000
-0.005	pdays	3.000
-0.146	month__jun	1.000
-0.271	campaign	4.000

Can we build a linear model for XOR?

XOR



$$(0,0) \cdot \mathbf{w} + b < 0$$

$$(0,1) \cdot \mathbf{w} + b \geq 0$$

$$(1,0) \cdot \mathbf{w} + b \geq 0$$

$$(1,1) \cdot \mathbf{w} + b < 0$$

$$\mathbf{w} = ?$$

Can we build a linear model for XOR?

Linear Models will underfit xor, we need non-linearity which can be achieved:

- * data pre-processing
- * kernels
- * adding non-linearity to the model

Code



Let's Think About This Together

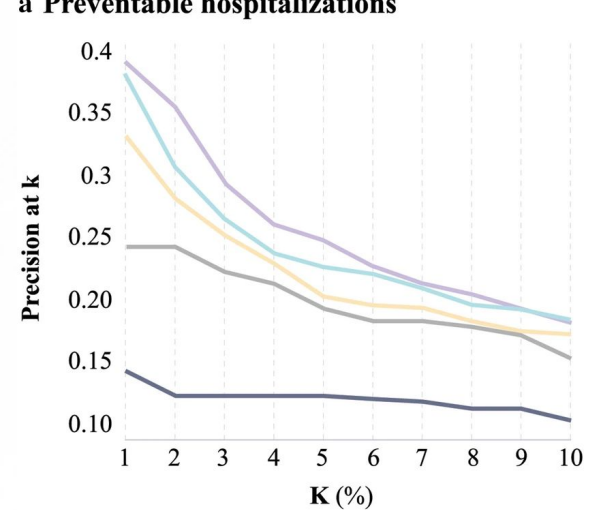
1. What are the main hyper-parameters?
2. Can it work for multi-class data (relevant only for logistic)?
3. How does it handle categorical data?
4. How does it handle missing data?
5. Is it sensitive to outliers?
6. What if some features are correlated?
7. Is it prone to overfitting?
8. Is it Interpretable?
9. Can it be parallelized?
10. Speed of training
11. Speed of prediction

Let's Think About This Together

1. What are the main hyper-parameters? Optimizer: LR, stopping criteria, initial weights, epochs, regularization lambda
2. Can it work for multi-class data (relevant only for logistic)? Yes
3. How does it handle categorical data? We need to make it numeric
4. How does it handle missing data? No
5. Is it sensitive to outliers? Yes
6. What if some features are correlated? Doesn't handle well
7. Is it prone to overfitting? Regularization
8. Is it Interpretable? Yes
9. Can it be parallelized? Not off the shelf
10. Speed of training - depends on optimizer and hyper-params
11. Speed of prediction - linear

Congestive Heart Failure

Comparison of deep learning
with traditional models to predict
preventable acute care use
and spending among heart failure
patients



Legend

- Traditional model 1 - LR
- Traditional model 2 - LR
- Enhanced model - LR
- Non-sequential machine learning models - GBM, FNN
- Sequential deep learning models - CNN, LSTM

Summary



Summary

	Linear Regression	Logistic Regression
Target Type	Regression	Classification
Loss	$\frac{1}{2} \ (Xw - y) \ ^2$	$\ell_{\text{cross-ent}} = - \sum_k \mathbf{y}_{[k]} \log \hat{\mathbf{y}}_{[k]}$ $\hat{\mathbf{y}} = \text{softmax}(\mathbf{xW} + \mathbf{b})$

Pros and Cons

Pros	Cons
<ol style="list-style-type: none">1. Fast2. Simple3. Explainable	<ol style="list-style-type: none">1. Can't handle missing data - needs imputation2. Categorical data needs translation to numeric3. Assumption of linear relations4. Sensitivity to correlated features