

Probability and Statistics

Y-DATA School of Data Science

P&P 3

Due: 23.11.2022

PROBLEM 1. Given the joint PMF, answer the following questions.

X/Y	0	1	2
1	0.08	0.2	0.12
2	0.06	0.15	0.09
3	0.04	0.12	0.04
4	0.02	0.03	0.05

- (1) Compute the marginal PMF's of X and Y .
- (2) Compute $E(X), E(Y), E(X+Y), E(XY), Var(X), Var(Y)$.
- (3) Find the conditional distributions of $X|\{Y=0\}, Y|\{X=1\}, Y|\{X=2\}, Y|\{X=3\}$.
- (4) Calculate $P(X=1, Y=2|X+Y < 5)$.

PROBLEM 2. The joint PDF of X and Y is,

$$f_{X,Y}(x,y) = cxy^2, \quad 0 < x < y < 1$$

- (1) Find c .
- (2) Find the marginal PDF and CDF of X .
- (3) Find the marginal PDF and CDF of Y .
- (4) Compute the mean and variance of X and Y .

PROBLEM 3. Let $X \sim Ber(p)$ and it is given that $E(Y|X=0) = 1, E(Y|X=1) = 2$. Calculate $E(Y)$.

PROBLEM 4. Each morning, Hungry Harry eats some eggs. On any given morning, the number of eggs he eats is equally likely to be 1, 2, 3, 4, 5, or 6, independent of what he has done in the past. Let X be the number of eggs that Harry eats in 10 days. Find the mean and variance of X .

PROBLEM 5. Let X be a continuous RV with PDF $f_X(x)$ and Y a continuous RV with PDF $f_Y(y)$. X and Y are independent. Denote their sum by $Z = X + Y$.

- (1) Show that $f_{Z|X=x}(z) = f_Y(z-x)$.
Hint: First show that $P(Z \leq z|X=x) = P(Y \leq z-x)$.
- (2) Suppose now that X and Y are exponentially distributed with parameter λ (they are still independent). Find the conditional PDF $f_{X|Z=z}(x)$ for every $0 \leq x \leq z$.

PROBLEM 6. Let $X \sim Ber(p), Y \sim Geo(p)$ be two independent random variables and define $Z = XY$. Express in terms of p the covariance $Cov(Z, X)$ and $Cov(Z, Y)$.