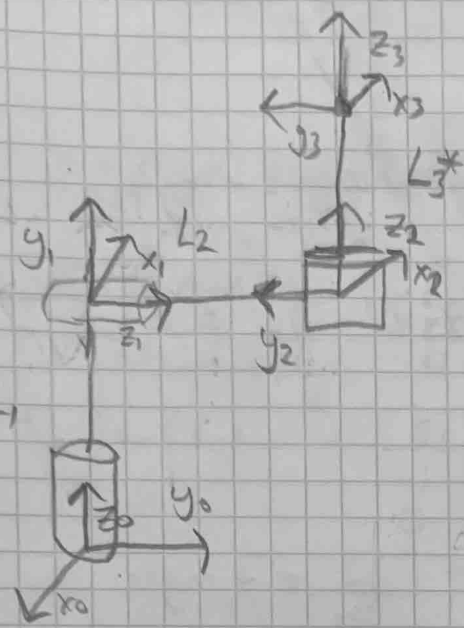
Oppg. 4

4.1)



link	θ_i	d_i	a_i	α_i
1	$\theta_1^* + 180$	L_1	0	90
2	θ_2^*	L_2	0	90
3	0	L_3^*	0	0

$\cos(\theta_1^*+180) = c_1 \cdot -1 - s_1 \cdot 0 = -c_1$

$\cos(\theta+180) = -c_i$

4.2) $A_i^{i-1} = \begin{bmatrix} c_i & -s_i c\alpha_i & s_i s\alpha_i & a_i c_i \\ s_i & c_i c\alpha_i & -c_i s\alpha_i & a_i s_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\sin(\theta_1^*+180) = s_1 \cdot -1 + c_1 \cdot 0 = -s_1$

$\sin(\theta+180) = -s_1$

$A_1^0 = \begin{bmatrix} -c_1 & 0 & -s_1 & 0 \\ -s_1 & 0 & c_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A_2^1 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Videre bruker jeg standard matrise-multiplikasjon slik vi har lært i lin-alg og som e beskrevet i tidligere eksameners hjelpeark

$A_1^0 = A_1^0$

$A_2^0 = A_1^0 A_2^1$

$A_3^0 = A_1^0 A_2^1 A_3^2$

For ordens skyld opplyser jeg om at $L_3^* = L_3 + d_3$

oppg 4

4.2) $$A_1^0 = \begin{bmatrix} -c_1 & 0 & -s_1 & 0 \\ -s_1 & 0 & c_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^0 = \begin{bmatrix} -c_1c_2 & s_1 & c_1s_2 & -L_2s_1 \\ -c_2s_1 & -c_1 & s_1s_2 & L_2c_1 \\ s_2 & 0 & c_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = \begin{bmatrix} -c_1c_2 & s_1 & c_1s_2 & L_3^*c_1s_2 - L_2s_1 \\ -c_2s_1 & -c_1 & s_1s_2 & L_2c_1 + L_3^*s_1s_2 \\ s_2 & 0 & c_2 & L_1 + L_3^*c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Her bruker jeg origo fra 3. ledd for å finne p_x, p_y og p_z

4.3) $$p_z = L_1 + L_3^* c_2 \Rightarrow L_3^* = \frac{p_z - L_1}{c_2}$$

$$p_x = L_3^* c_1 s_2 - L_2 s_1$$

$$p_y = L_2 c_1 + L_3^* s_1 s_2$$

$$p_x c_1 = L_3^* c_1^2 s_2 - L_2 s_1 c_1$$

$$p_y s_1 = L_2 c_1 s_1 + L_3 s_1^2 s_2$$

$$p_x c_1 + p_y s_1 = \underline{L_3^* c_1^2 s_2} - \underline{L_2 s_1 c_1} + \underline{L_2 c_1 s_1} + \underline{L_3^* s_1^2 s_2}$$

$$p_x c_1 + p_y s_1 = L_3^* s_2 \Rightarrow \frac{p_x c_1 + p_y s_1}{L_3^*} = s_2$$

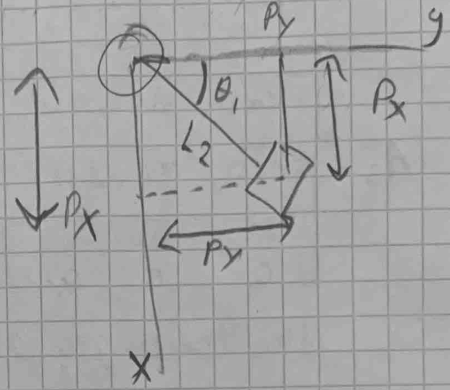
$$c_2 = \sqrt{1 - s_2^2} = \sqrt{1 - \left(\frac{p_x c_1 + p_y s_1}{L_3^*}\right)^2}$$

$$\theta_2 = \operatorname{atan}\left(\frac{\frac{p_x c_1 + p_y s_1}{L_3^*}}{\sqrt{1 - \left(\frac{p_x c_1 + p_y s_1}{L_3^*}\right)^2}}\right)$$

oppg 4.3

c) $\sin\theta_1 = \frac{P_x}{L_2}$ $\cos\theta = \frac{P_y}{L_2}$

$$\theta_1 = \operatorname{atan}\left(\frac{\frac{P_x}{L_2}}{\frac{P_y}{L_2}}\right) = \operatorname{atan}\left(\frac{P_x}{P_y}\right)$$



Her bruker jeg A-matrisene fra 4.2

4.4

$$A_1^0 = \begin{bmatrix} -c_1 & 0 & -s_1 & 0 \\ -s_1 & 0 & c_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^0 = \begin{bmatrix} -c_1c_2 & s_1 & c_1s_2 & -L_2s_1 \\ -c_2s_1 & -c_1 & s_1s_2 & L_2c_1 \\ s_2 & 0 & c_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = \begin{bmatrix} -c_1c_2 & s_1 & c_1s_2 & L_3^*c_1s_2 - L_2s_1 \\ -c_2s_1 & -c_1 & s_1s_2 & L_2c_1 + L_3^*s_1s_2 \\ s_2 & 0 & c_2 & L_1 + L_3^*c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Z finner man fra 3. kolonne i A-matrisene

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c_1s_2 \\ s_1s_2 \\ c_2 \end{bmatrix}$$

O finner man fra siste kolonne i A-matrisene

$$O_3 - O_0 = \begin{array}{c} \begin{bmatrix} L_3^*c_1s_2 - L_2s_1 \\ L_2c_1 + L_3^*s_1s_2 \\ L_1 + L_3^*c_2 \end{bmatrix} \end{array} \begin{array}{c} a \\ b \\ c \end{array} \quad O_3 - O_1 = \begin{bmatrix} L_3^*c_1s_2 - L_2s_1 \\ L_2c_1 + L_3^*s_1s_2 \\ L_3^*c_2 \end{bmatrix} \begin{array}{c} d \\ e \\ f \end{array}$$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

Rot Rot Prism

$$z_0 \times (O_3 - O_0) = \begin{matrix} 0 & 0 & 1 & 0 & 0 & 1 \\ a & b & c & a & b & c \end{matrix} = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} = \begin{bmatrix} -L_2c_1 - L_3^*s_1s_2 \\ L_3^*c_1s_2 - L_2s_1 \\ 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_0) = \begin{matrix} -s_1 & c_1 & 0 & -s_1 & c_1 & 0 \\ d & e & f & d & e & f \end{matrix} = \begin{bmatrix} c_1 f \\ s_1 f \\ -s_1e + c_1d \end{bmatrix} = \begin{bmatrix} L_3^*c_1c_2 \\ L_3^*s_1c_2 \\ L_2s_1c_1 - L_3^*c_1^2s_2 - L_2s_1c_1 - L_3s_1^2s_2 \end{bmatrix}$$

$$= \begin{bmatrix} L_3^*c_1c_2 \\ L_3^*s_1c_2 \\ -L_3^*s_2 \end{bmatrix}$$

4.4 Siste delen er nå bare å sette sammen Jacobian

$$J_v = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \end{bmatrix}$$

Siden vi bare skal ha J_v tar vi bare 3 øverste rader

$$J_v = \begin{bmatrix} -L_2 c_1 - L_3^* s_1 s_2 & L_3^* c_1 c_2 & c_1 s_2 \\ L_3^* c_1 s_2 - L_2 s_1 & L_3^* c_2 s_1 & s_1 s_2 \\ 0 & -L_3^* s_2 & c_2 \end{bmatrix}$$

4.5 For å finne singularitetr, altså når en robot mister en frihetsgrad, må vi finne ut når determinanten til $J_v = 0$.

$$\det \begin{bmatrix} -L_2c_1 - L_3^* s_1 s_2 & L_3^* c_1 c_2 & c_1 s_2 \\ L_3^* c_1 s_2 - L_2 s_1 & L_3^* c_2 s_1 & s_1 s_2 \\ 0 & -L_3^* s_2 & c_2 \end{bmatrix}$$

her bruker jeg matlab

$$= -L_3^{*2} s_2 \underbrace{(c_1^2 + s_1^2)}_{1} \underbrace{(c_2^2 + s_2^2)}_{1}$$

$$= -L_3^{*2} s_2$$

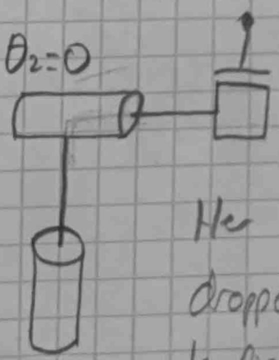
Altså trenger vi nå å sette $-L_3^{*2} s_2 = 0$

siden $L_3^* = L_3 + d_3$, og L_3 ikke kan være lik 0, og d_3 ikke kan være negativ, kan ikke L_3^* settes lik 0.

Dermed må $s_2 = 0$ for å komme i en singularitet.

For at $s_2 = 0$ må $\theta_2 = 0°$ V $\theta_2 = 180°$

$\theta_2 = 0$



Dette er altså en singularitet.

Her har roboten mistet en frihetsgrad. Man kunne altså droppet det andre roterende leddet og hatt samme konfigurasjon, men nå som en 2-DOF. Altså blir vår 3-DOF til en 2-DOF. Da har den mistet en frihetsgrad.

```
syms s1 s2 s3 c1 c2 c3 L2  L3

Jv = [-L2*c1-L3*s1*s2 L3*c1*c2 c1*s2
      L3*c1*s2-L2*s1 L3*c2*s1 s1*s2
      0 -L3*s2 c2]

simplify(det(Jv))
```

Command Window

```
ans =

-L3^2*s2*(c1^2 + s1^2)*(c2^2 + s2^2)
```