Task 2

a) I use the A_1, A_2 and A_3-matrices from the last a.lig to make the T-matrices

$$T_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^0 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & c_1c_2a_2 \\ c_2s_1 & -s_1s_2 & -c_1 & c_2a_2s_1 \\ -s_2 & c_2 & 0 & -a_2s_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1c_3s_2 & s_1 & c_1c_2c_3a_3 + c_1c_2a_2 - c_1a_3s_2s_3 \\ c_2c_3s_1 - s_2s_1s_3 & -c_2s_1s_3 - c_3s_2s_1 & -c_1 & c_2c_3a_3s_1 + c_2a_2s_1 - a_3s_2s_1s_3 \\ c_2s_3 + c_3s_2 & c_2c_3 - s_2s_3 & 0 & c_3a_3s_2 + a_2s_2 + c_2a_3s_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2(o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \qquad z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

$$o_3 - o_0 = o_3 = \begin{array}{cc} \begin{bmatrix} c_1c_2c_3a_3 + c_1c_2a_2 - c_1a_3s_2s_3 \\ c_2c_3a_3s_1 + c_2a_2s_1 - a_3s_2s_1s_3 \\ c_3a_3s_2 + a_2s_2 + c_2a_3s_3 + d_1 \end{bmatrix} & \begin{matrix} a \\ b \\ c \end{matrix} \end{array}$$

$$o_3 - o_1 = \begin{array}{cc} \begin{bmatrix} c_1c_2c_3a_3 + c_1c_2a_2 - c_1a_3s_2s_3 \\ c_2c_3a_3s_1 + c_2a_2s_1 - a_3s_2s_1s_3 \\ c_3a_3s_2 + a_2s_2 + c_2a_3s_3 \end{bmatrix} & \begin{matrix} d \\ e \\ f \end{matrix} \end{array}$$

$$o_3 - o_2 = \begin{array}{cc} \begin{bmatrix} c_1c_2c_3a_3 - c_1a_3s_2s_3 \\ c_2c_3a_3s_1 - a_3s_2s_1s_3 \\ c_3a_3s_2 + c_2a_3s_3 \end{bmatrix} & \begin{matrix} g \\ h \\ i \end{matrix} \end{array}$$

$$Z_0 \times (O_3 - O_0) = \begin{vmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ d & b & c & a & b & c \end{vmatrix} = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$$

$$Z_1 \times (O_3 - O_1) = \begin{vmatrix} s_1 & -c_1 & 0 & s_1 & -c_1 & 0 \\ d & e & f & d & e & f \end{vmatrix} = \begin{bmatrix} -c_1 f \\ -s_1 f \\ s_1 e + c_1 d \end{bmatrix}$$

$$Z_2 \times (O_3 - O_2) = \begin{vmatrix} s_1 & -c_1 & 0 & s_1 & -c_1 & 0 \\ g & h & i & g & h & i \end{vmatrix} = \begin{bmatrix} -c_1 i \\ -s_1 i \\ -s_1 h + c_1 g \end{bmatrix}$$

$$J = \begin{bmatrix} -b & -c_1 f & -c_1 i \\ a & -s_1 f & -s_1 i \\ 0 & s_1 e + c_1 d & -s_1 h + c_1 g \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) configurations where the rank of $J(q)$ is less than $\max(\text{rank}(J(q)))$ is called singularities, and occure when a robot will need an infinit amount of energy to move in a direction.

2c) I use Matlab to calculate the determinant of the J_q - matrix and get

$$\det(J_q) = -a_2 a_3 (a_2 c_2 s_3 - a_3 s_2 + a_3 c_3^2 s_2 + a_3 c_2 c_3 s_3)$$

Since a_2 and a_3 are fixed lengths, we can exclude them. We can then factorize the expression to

$$1 - c^2 = s^2$$

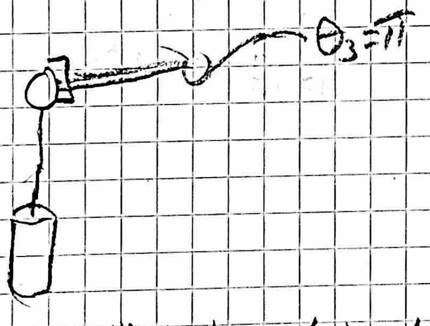
$$= a_2 c_2 s_3 - a_3 s_2 (1 - c_3^2) + a_3 c_2 c_3 s_3 = a_2 c_2 s_3 - a_3 s_2 s_3 + a_3 c_2 c_3 s_3$$

$$= s_3 (a_2 c_2 - a_3 s_2 + a_3 c_2 c_3) = s_3 (a_2 c_2 + a_3 c_{23})$$

s_2

d) This last expression telles us that if $\sin(\theta_3) = 0 \Rightarrow \theta_3 = \pi$, then we will encounter a singularity. This is then one of the options for a singularity.

e)



As the illustration tries to show, there will be a collision when $\theta_3 = \pi$

f) The consequenses of not handeling a singularity is that the robot will need infinite energy to move, and in the effort it will break the robot.