Task 1

a) We use the τ_2 from task 1 in oblig 3

$$\tau_2 = \ddot{q}_2 m_2 L_2^2 + m_2 L_2^2 \dot{q}_1^2 c_2 s_2 + m_2 g c_2 L_2$$

When working with joint 2, we will eliminate all q_1's and q_3's and get

$$\tau_2 = \ddot{q}_2 m_2 L_2^2 + m_2 g c_2 L_2$$

Since we have only calculated for two joints, not three, we'll assume that joint 3 will only extend joint 2. Then $L_2 \rightarrow L_2 + L_3$ and $m_2 \rightarrow m_2 + m_3$. The distrobution of mass will not be correct, but this will still be more accurate solution.

$$\tau_2 = \ddot{q}_2 (m_2 + m_3)(L_2 + L_3)^2 + (m_2 + m_3) g c_2 (L_2 + L_3)$$

For the rest of this oblig, I'll write m_2 and L_2 for $m_2 + m_3$ and L_2 for $L_2 + L_3$

b)

$T_2 = r T_m - B \dot{\theta}_2$

$r \cdot T_m = \ddot{q}_2 m_2 L_2^2 + m g c_2 L_2 + B \dot{\theta}_2$

$r = 200$

Performing the Laplace-transformation:

$J = m_2 L_2^2$

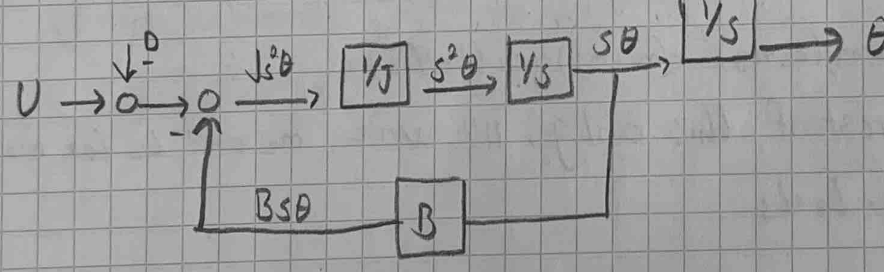
$D = g = m g c_2 L_2$

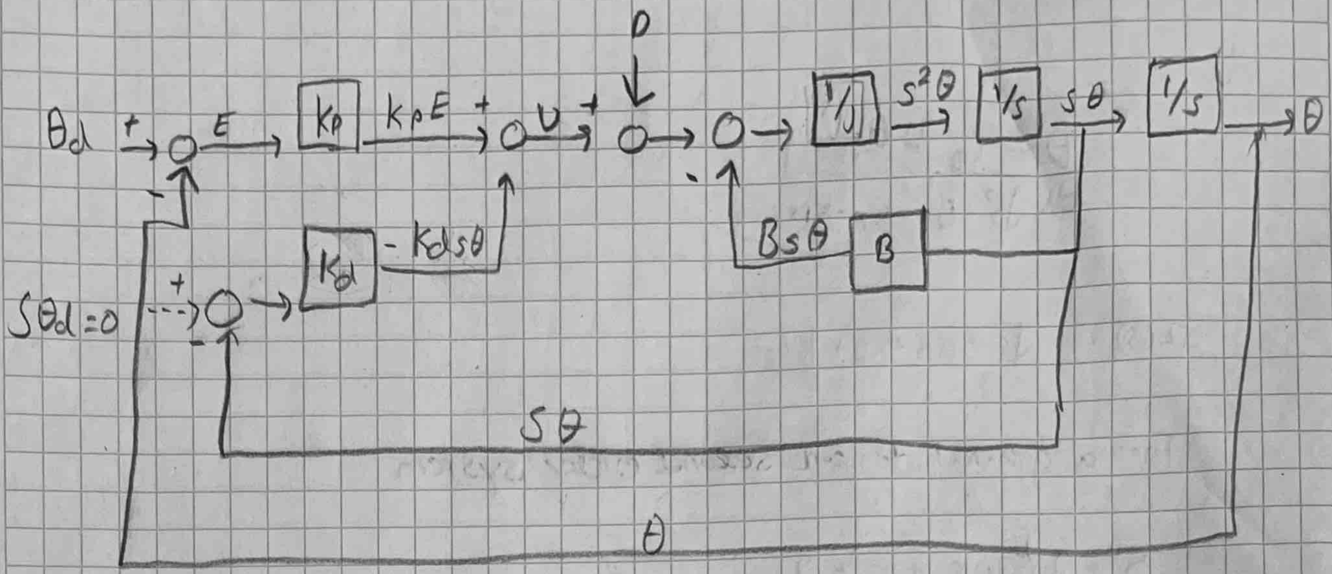
B = Internal damping

$U = J s^2 \theta + B s \theta + D$

T_m is the torque applied to the motor

c) $U = J s^2 \theta + B s \theta + D$





$U(s) = Js^2\theta + Bs\theta$ dynamic system

$u(s) = K_pE(s) - K_ds\theta$ controller

$Js^2\theta + Bs\theta = k_p\theta_d - k_p\theta - K_ds\theta$

$\theta(Js^2 + Bs + k_p + K_ds) = k_p\theta_d$

$\frac{\theta}{\theta_d} = \frac{k_p}{Js^2 + Bs + k_p + K_ds}$ = Transfer function d)

$$\frac{\Theta}{\Theta_d} = \frac{k_p}{Js^2 + Bs + k_p + k_d s}$$

$$\Omega(s) = Js^2 + Bs + k_p + k_d s$$

For a general damped second order system

$$s^2 + 2\zeta\omega s + \omega^2 = 0$$

For a critically damped system $\zeta = 1$

$\omega = 6$

$\Omega(s) = 0$ gives us

$$Js^2 + Bs + k_p + k_d s = 0$$

$$s^2 + \frac{B}{J}s + \frac{k_p}{J} + \frac{k_d}{J}s = 0$$

$$s^2 + \frac{B + k_d}{J}s + \frac{k_p}{J} = 0$$

$L_2 = L_2 + L_3 = 3,583\,dm$

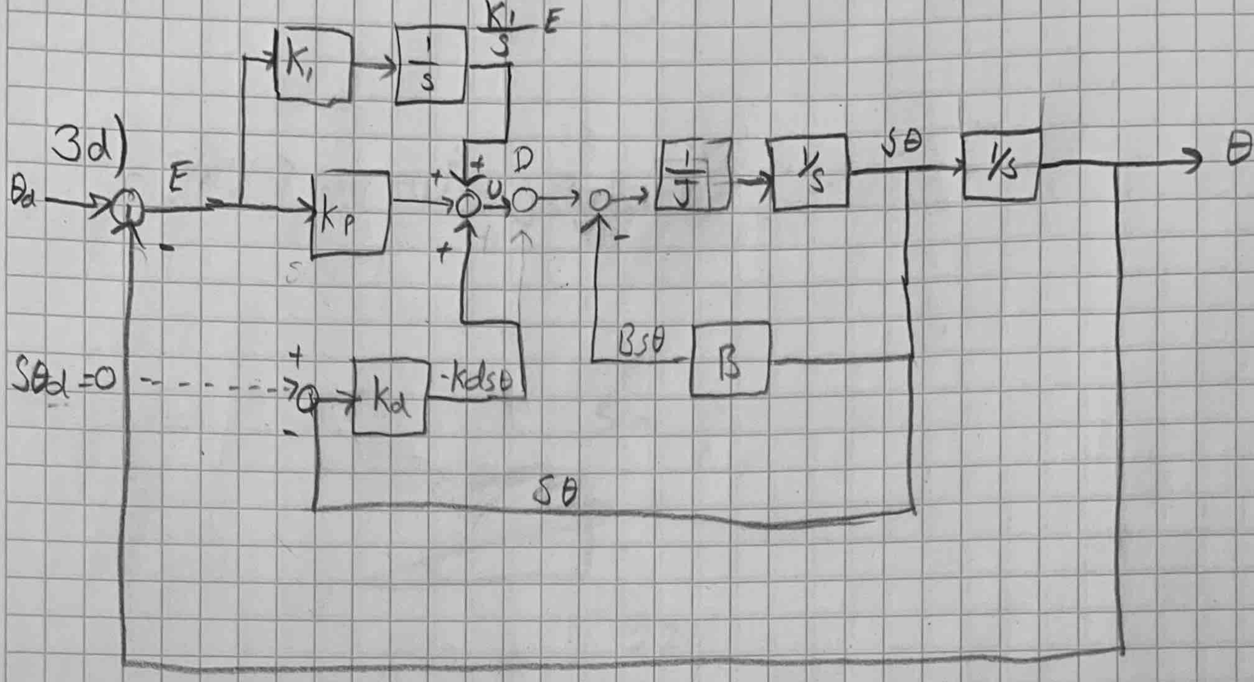
$m_2 = m_2 + m_3 = 0,413\,kg$

matched with the general system gives us

$$\frac{k_p}{J} = \omega^2 \Rightarrow k_p = J\omega^2 = m_2 L_2^2 \omega^2 = 36\, m_2 L_2^2 = 36 \cdot 3,583^2 \cdot 0,413 = 190$$

$$\frac{B + k_d s}{J} = 2\zeta\omega s \Rightarrow k_d = 2\zeta\omega J - B = 12\, m_2 L_2^2 = 12 \cdot 3,583^2 \cdot 0,413 = 63$$

To get values within a reasonable range we use the length i dm.



3d)
θ_d
E
K_I
$\frac{1}{s}$
$\frac{K_I}{s}E$
K_p
D
u
$\frac{1}{J}$
$1/s$
$s\theta$
$1/s$
θ
$Bs\theta$
B
$s\theta_d = 0$
K_d
$-K_d s\theta$
$s\theta$