Adversarial Search

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September 23, 2021

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- We could consider them just a part of the environment that makes the environment nondeterministic (we miss out on them actually trying to defeat us).
- We can explicitly model adversarial agents using adversarial game-tree search.

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- Our agent needs to consider:
  - The possible actions of the other agents.
  - How these actions can affect the its own welfare.

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- Turn-taking - Each player takes one turn each per round.
- Two-player - Only two players involved...
- Zero-sum - The total losses and gains of both agents sums to zero.
- Perfect information - All participants have full knowledge about their own cost and utility functions and game history.

Formally Defining a Game

- The initial state:

The initial state S_0 specifies how the game is set up at the start.

Formally Defining a Game

- The initial state:
- Which players turn it is to move:

The method TO-MOVE(s) returns the player whose turn it is to move in the state s.

Formally Defining a Game

- The initial state:
- Which players turn it is to move:
- The actions available:

The method ACTIONS(s) returns set of legal moves in state s.

Formally Defining a Game

- The initial state:
- Which players turn it is to move:
- The actions available:
- The transition model:

The transition model describes the outcome of each action: $RESULT(s, a)$ returns the state that results from doing action a in state s.

Formally Defining a Game

- The initial state:
- Which players turn it is to move:
- The actions available:
- The transition model:
- A terminal test:

The method IS-TERMINAL(s) returns true if the game is over and false otherwise. States where the game has ended are called *terminal states*.

Formally Defining a Game

- The initial state:
- Which players turn it is to move:
- The actions available:
- The transition model:
- A terminal test:
- A utility function:

The method UTILITY(s, p) returns the final numeric value to player p when the game ends in terminal state s.

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- One move involves a decision from each player. Each player decision is called a ply.
- Each leaf in the search tree is assigned a utility value - usually:
  $+1 =$ win
  $-1 =$ lose
  $0 =$ draw

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- Zero-Sum.
- Perfect Information.

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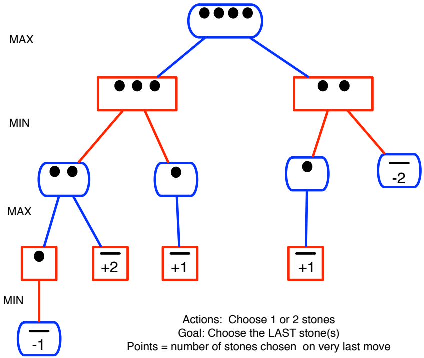
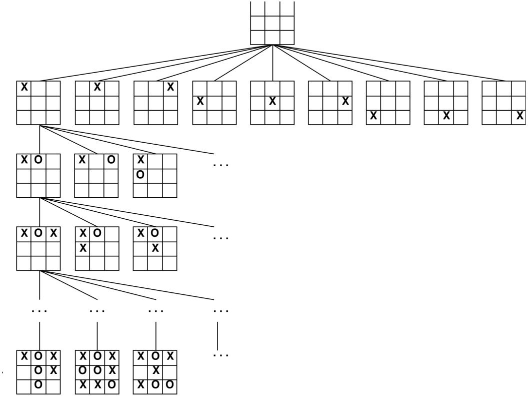


Actions: Choose 1 or 2 stones
Goal: Choose the LAST stone(s)
Points = number of stones chosen on very last move

Example: Tic-Tac-Toe



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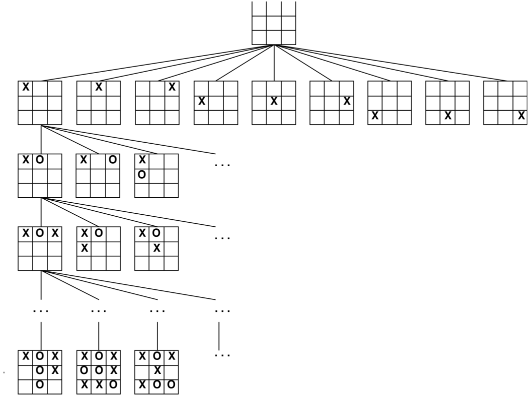

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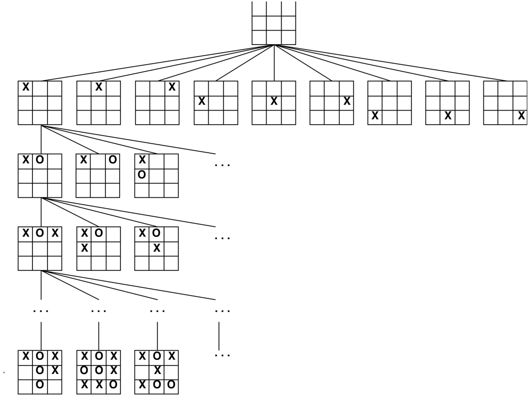

What is the best strategy?

In tic-tac-toe, when both players play optimally, neither player will win.

The Optimal Game Strategy

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- MAX must have a strategy with a response to each of MIN's possible moves.

The optimal strategy is one that leads to outcomes **at least as good** as any other strategy when playing an **infallible** opponent $\Rightarrow$ The MiniMax Value.

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- If given a choice, MAX will always move to a state with maximum value.
- MIN will always move to a state with minimum value.

$$MINIMAX(s) = \begin{cases} \text{UTILITY(s)} & \text{if TERMINAL-TEST(s)} \\ max_{a \in Actions(s)} \text{MINIMAX(RESULT(s, a))} & \text{if PLAYER(s) = MAX} \\ min_{a \in Actions(s)} \text{MINIMAX(RESULT(s, a))} & \text{if PLAYER(s) = MIN} \end{cases} \tag{1}$$

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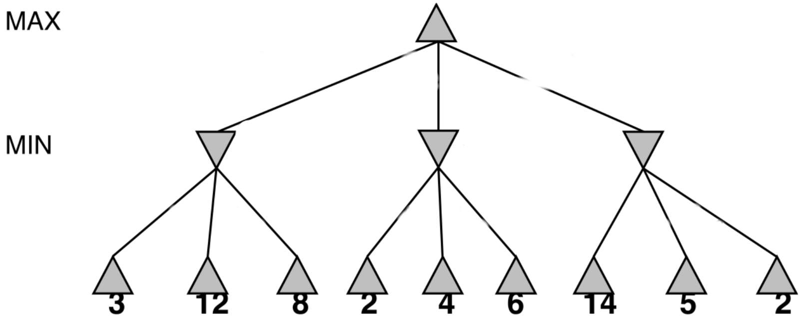
- Now, from S_{new} node choose and apply the action that leads to the highest-rated child state.

Adversarial Search - Intuitive Example



Adversarial Search - The Process

Let's formalize what we are doing in an algorithm $\Rightarrow$ The MiniMax Algorithm.

The MiniMax Algorithm

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function MINIMAX-SEARCH(*game*, *state*)
    player ← *game*.TO-MOVE(*state*)
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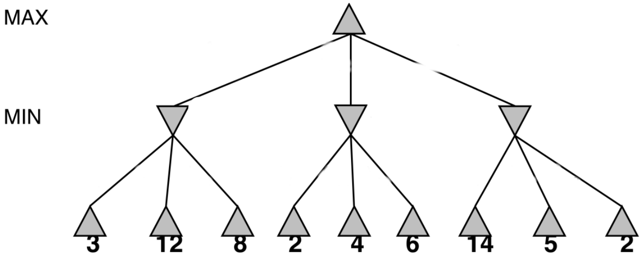

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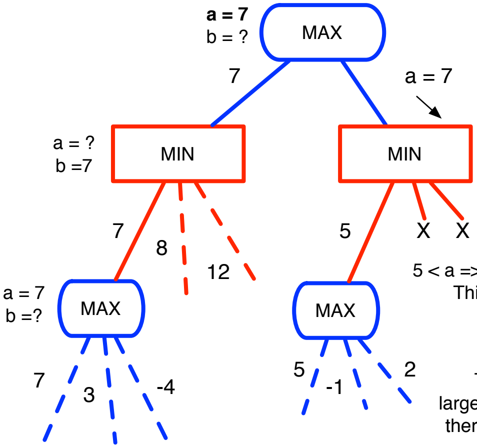
- Yes. If we keep track of the best values we have encountered, we can stop the search down a branch when we know a better value cannot be found.
- This is called pruning.

Alpha-Beta Pruning



5 < a => Stop generating children.
This node's value cannot
win at the root.
Prune it!

There could be many
large subtrees down here, but
there is no need to generate
them, since they
cannot affect the choice made
at the root.

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Both Alpha and Beta are passed between the MIN and MAX nodes.

The α-β algorithm

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function ALPHA-BETA-SEARCH(game, state)
    player ← game.TO-MOVE(state)
    value, move ← MAX-VALUE(game, state, −∞, +∞)
    return move
function MAX-VALUE(game, state, α, β)
    if game.TERMINAL-STATE(state) then
        return game.UTILITY(state, player), null
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    for each a in game.ACTIONS(state) do
        v2, a2 ← MIN-VALUE(game, game.RESULT(state, a), α, β)
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            α ← MAX(α, v)
        if v ≥ β then return v, move
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        if v ≤ α then return v, move
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α-β Pruning Example

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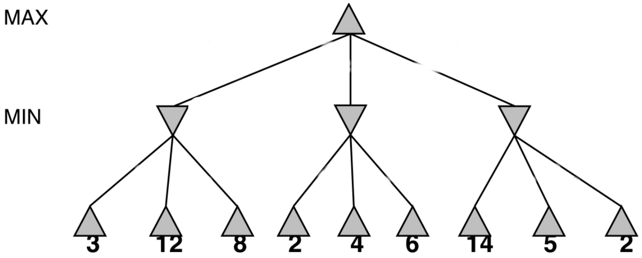

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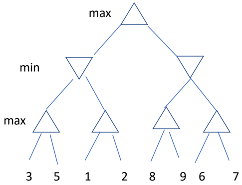

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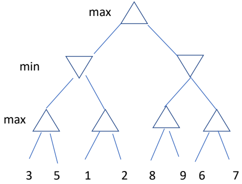

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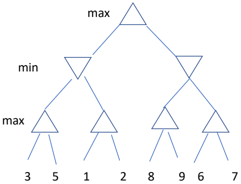

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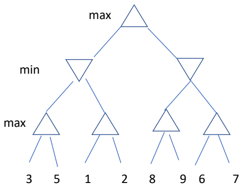

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- A simple example of the value of reasoning about which computations are relevant (a form of).
- Unfortunately, 35^{50} (e.g. chess) is still impossible!

Resource Limits

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  e.g., depth limit

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- Use EVAL instead of UTILITY
  i.e., a heuristic *evaluation function* that estimates desirability of position

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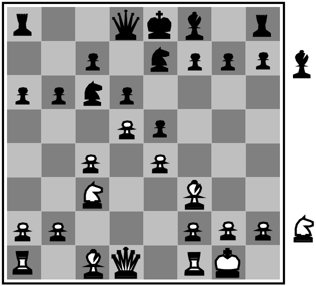
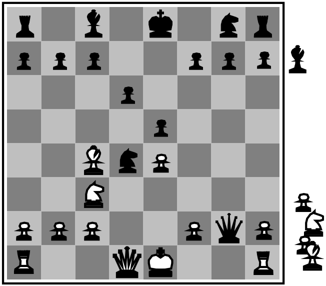
  - For non-terminal states, the evaluation function should be strongly correlated with the true chance of winning.

Evaluation Functions



Black to move

White slightly better



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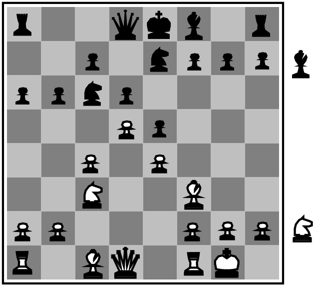
Evaluation Functions



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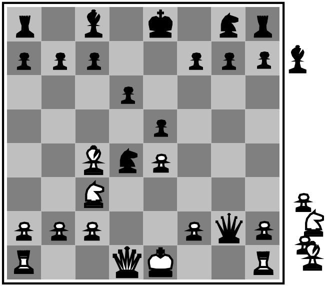

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For chess, typically a linear weighted sum of features
$\mathrm{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
e.g., $w_1 = 9$ with
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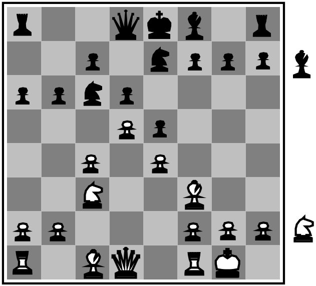

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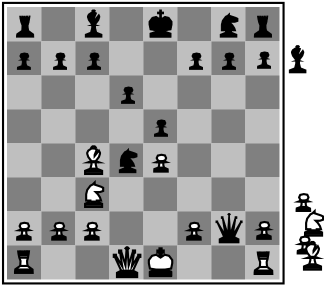

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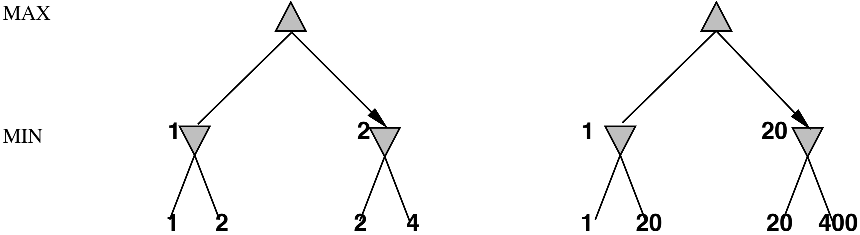
However, we can also learn the evaluation function (AlphaZero)

Cutting Off Search

```
function ALPHA-BETA-SEARCH-CUTOFF(game, state)
    player ← game.TO-MOVE(state)
    value, move ← MAX-VALUE(game, state, −∞, +∞)
    return move
function MAX-VALUE(game, state, α, β)
    if game.IS-CUTOFF(state, depth) then
        return game.EVAL(state, player), null
    v, move ← −∞
    for each a in game.ACTIONS(state) do
        v2, a2 ← MIN-VALUE(game, game.RESULT(state, a), α, β)
        if v2 > v then
            v, move ← v2, a
            α ← MAX(α, v)
        if v ≥ β then return v, move
    return v, move
function MIN-VALUE(game, state, α, β)
    if game.IS-CUTOFF(state, depth) then
        return game.EVAL(state, player), null
    v, move ← +∞
    for each a in game.ACTIONS(state) do
        v2, a2 ← MAX-VALUE(game, game.RESULT(state, a), α, β)
        if v2 > v then
            v, move ← v2, a
            β ← MIN(β, v)
        if v ≤ α then return v, move
    return v, move
```

Digression: Exact Values Don't Matter



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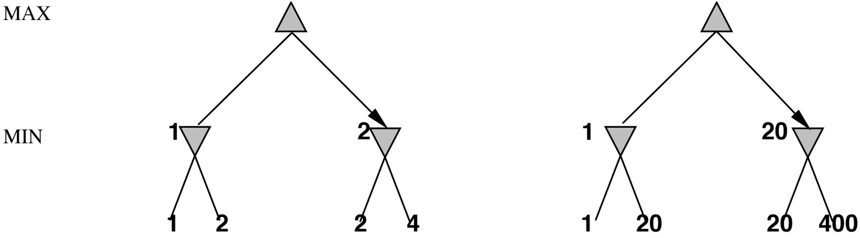

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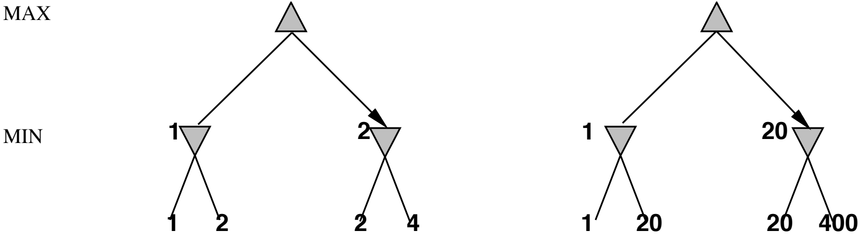

- Behaviour is preserved under any *monotonic* transformation of EVAL (the evaluation function doesn't have to be linearly correlated with the true chances of winning)
- Only the order matters:
  payoff in deterministic games acts as an ordinal utility function

Deterministic Games in Practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

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- Othello: human champions refuse to compete against computers, who are too good.
- Go ($b > 300$): human champions refused to compete against computers, who were too bad - until 2015. In 2017 Future of Go Summit, AlphaGo beat Ke Jie, the world No.1 ranked player at the time. Some players have even decided to stop playing the game...

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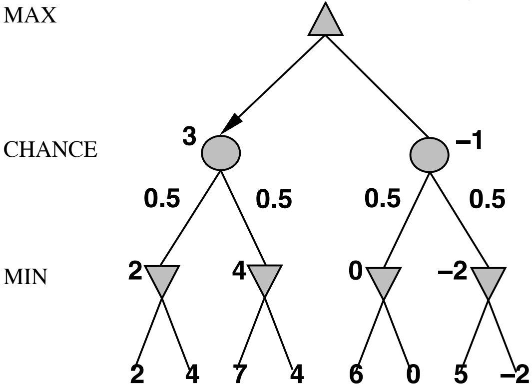
  - Branches are labelled with the associated probability (1/6 for dice).

Stochastic Games in General

Simplified example with coin-flipping:



Algorithm for Nondeterministic Games

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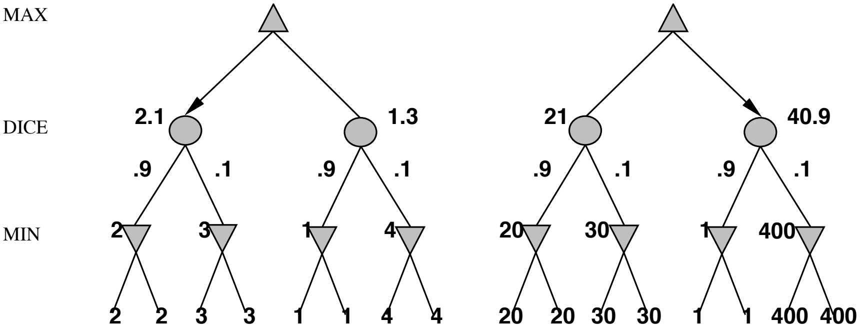
. . .

1: **if** *state* is a MAX node **then return**
   the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
2: **if** *state*is a MIN node **then return**
   the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
3: **if** *state* is a chance node **then return**
   sum of ...... EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Digression: Exact values DO Matter



EVAL should be proportional to the expected payoff

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  - good idea to think about what to think about
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- Games are to AI as grand prix racing is to automobile design