Assignment Lecture 3: Constraint Satisfaction Problems

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A problem is solved when each variable has a value that satisfies the constraints on that variable.

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- The set of values that can be assigned to a variable. e.g. $\{1, 2, 3, 4\}$.
- A set of restrictions on one, or more, of the variables. e.g. $\{A \neq B\}$.
- All constraints are met and all variables have been assigned a value.

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- Why?
  - Some efficient algorithms expect binary constraints.

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- $\mathcal{C} : \{\langle C1, C1 \neq blue\rangle, \langle C2, C2 \neq blue\rangle, \langle C3, C3 \neq blue\rangle, \langle (C1, C2, C3), Alldiff(C1, C2, C3)\rangle \langle W1, W1 \neq red\rangle, \langle W1, W1 \neq red\rangle, \langle (W1, W2), Diff(W1, W2)\rangle\}$

Encoding Soduku as a CSP

	A	B	C	D	E	F	G	H	I
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
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The AC-3 algorithm makes every variable arc consistent

AC-3 - Example

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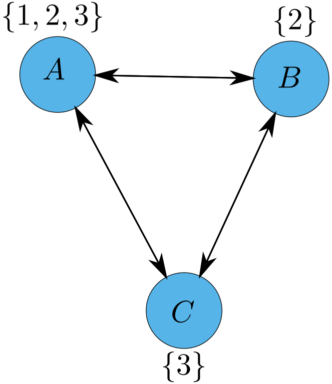

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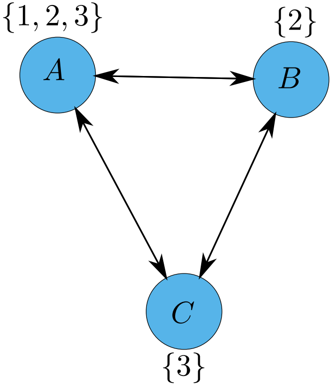

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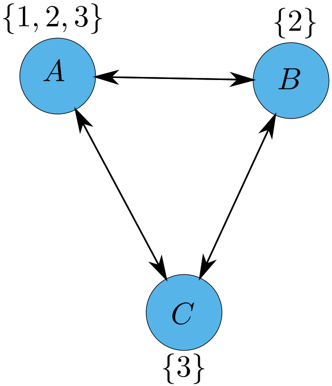

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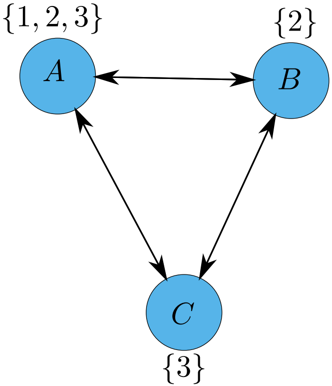

- $\mathcal{X} : \{A, B, C\}$
- $\mathcal{D} : \{A : \{1, 2, 3\}, B : \{2\}, C : \{3\}\}$
- $\mathcal{C} : \mathit{Alldiff}(A, B, C)$

AC-3 - Pseudocode

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```
function AC-3(csp)
    queue ← a queue of arcs/edges, initially all the arcs in the csp
    while queue is not empty do
        (X_i, X_j) ← POP(queue)
        if REVISE(csp, X_i, X_j) then
            if size of D_i = 0 then return false
            for each X_k in X_i.NEIGHBORS − {X_j} do
                add (X_k, X_j) to queue
    return true
function REVISE(csp, X_i, X_j)
    revised ← false
    for each x in D_i do
        if no value y in D_j allows (x, y) to satisfy the constraint between X_i and X_j then
            delete x from D_i
            revised ← true
    return revised
```

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We've now made the variables arc consistent, but what if there are still multiple values possible for variables?

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  - If it fails: restore assignment to previous state and try next value.

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- At each level in the tree we must decide which variable to deal with.

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```
function BACKTRACKING-SEARCH(csp) return BACKTRACK(csp, {})
function BACKTRACK(csp, assignment)
    if assignment is complete then return assignment
    var ← SELECT-UNASIGNED-VARIABLE(csp, assignment)
    for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
        if value is consistent with assignment then
            add {var = value} to assignment
            inferences ← INFERENCE(csp, var, assignment)
            if inferences ≠ failure then
                add inferences to csp
                result ← BACKTRACK(csp, assignment)
                if result ≠ failure then return result
                remove inferences from csp
          remove var = value from assignment
    return failure
```

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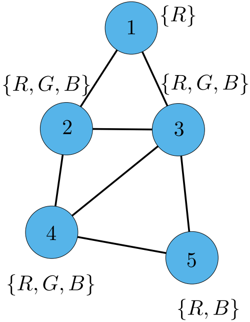

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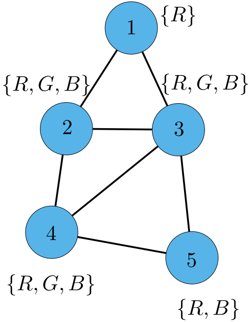

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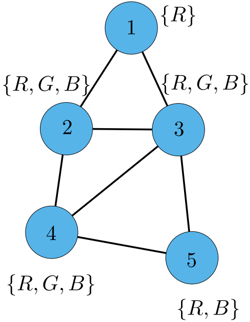

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- Show the sequence of variable assignments during a pure backtracking search (don't assume that propagation above has been done). Assume that the variables are examined in numerical order and the values are assigned in the order shown next to each node. Show assignments by writing the variable number and the letter for the value, e.g, 5R, 2G.

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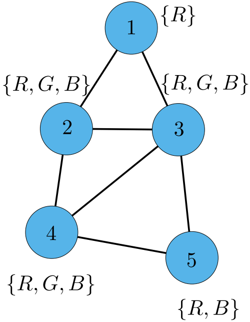

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- This time you'll apply backtracking search with forward checking. Use the same ordering convention for variables and values as above. Show the sequence of variable assignments during backward search with forward checking. Again, show assignments by writing the number of variables followed by the letter for the value.

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You will be playing sudoku!

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- $\mathcal{C}$: A hash table that associates a variable name i with another hash table, where this second hash table contains the constraints affecting variable i. Specifically, the second hash table associates the name of another variable $j (i \neq j)$ with an array of legal pairs of values for the pair of variables (i, j).

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  - What do the numbers mean?
  - How do the numbers relate to performance?
  - How do they change relative to the difficulty of the boards?
  - Can you make any improvements?

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Use provided print_sudoku_solution() function to print the board.