Formelark for eksamen i TDAT3024, matematikkdelen

Appendix A. Fourierrekker

A.1. **Fourierrekker.**

$$a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \tfrac{\pi n x}{L} + b_n \sin \tfrac{\pi n x}{L} \right),$$

$$\begin{aligned} a_0 &= \frac{1}{2L} \int_{-L}^{L} f(x)\,\mathrm{d}x \\ a_n &= \frac{1}{L} \int_{-L}^{L} f(x)\, \cos \tfrac{\pi n x}{L}\,\mathrm{d}x \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(x)\, \sin \tfrac{\pi n x}{L}\,\mathrm{d}x. \end{aligned}$$

A.2. **Sinusrekker.**

$$\sum_{n=1}^{\infty} b_n \sin \tfrac{\pi n x}{L},$$

$$b_n = \frac{2}{L} \int_{0}^{L} f(x)\, \sin \tfrac{\pi n x}{L}\,\mathrm{d}x.$$

A.3. **Cosinusrekker.**

$$a_0 + \sum_{n=1}^{\infty} a_n \cos \tfrac{\pi n x}{L},$$

$$\begin{aligned} a_0 &= \frac{1}{L} \int_{0}^{L} f(x)\,\mathrm{d}x \\ a_n &= \frac{2}{L} \int_{0}^{L} f(x)\, \cos \tfrac{\pi n x}{L}\,\mathrm{d}x \end{aligned}$$

Appendix B. Vektoranalyse

Vektorfelt

$$\mathbf{F}(x,y,z) = P(x,y,z)\,\hat{\mathbf{i}} + Q(x,y,z)\,\hat{\mathbf{j}} + R(x,y,z)\,\hat{\mathbf{k}}.$$

Gradient

$$\begin{aligned} \operatorname{grad} f &= \\ \nabla f &= \tfrac{\partial f}{\partial x}\,\hat{\mathbf{i}} + \tfrac{\partial f}{\partial y}\,\hat{\mathbf{j}} + \tfrac{\partial f}{\partial z}\,\hat{\mathbf{k}} \end{aligned}$$

Curl

$$\begin{aligned} \operatorname{curl} \mathbf{F} &= \\ \nabla \times \mathbf{F} &= \left(\tfrac{\partial R}{\partial y} - \tfrac{\partial Q}{\partial z} \right) \hat{\mathbf{i}} + \left(\tfrac{\partial P}{\partial z} - \tfrac{\partial R}{\partial x} \right) \hat{\mathbf{j}} + \left(\tfrac{\partial Q}{\partial x} - \tfrac{\partial P}{\partial y} \right) \hat{\mathbf{k}} \end{aligned}$$

Divergens

$$\begin{aligned} \operatorname{div} \mathbf{F} &= \\ \nabla \cdot \mathbf{F} &= \tfrac{\partial P}{\partial x} + \tfrac{\partial Q}{\partial y} + \tfrac{\partial R}{\partial z} \end{aligned}$$

Formler

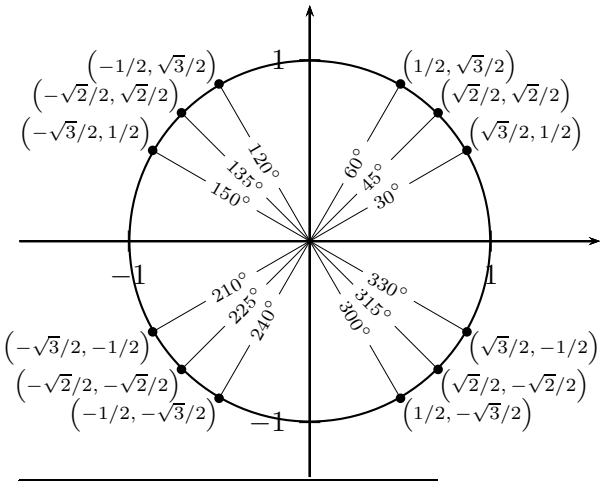
$$\begin{aligned} \operatorname{div}(\operatorname{curl} \mathbf{F}) &= 0 \\ \operatorname{curl}(\operatorname{grad} f) &= \mathbf{0} \end{aligned}$$

Appendix C. Laplace-transformasjon

	$f(t)$	$\mathscr{L}(f)(s)$
1	1	$\frac{1}{s}$
2	t	$\frac{1}{s^2}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
5	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
6	$u(t-a)$	$\frac{1}{s}e^{-as}$
7	$\delta(t-a)$	e^{-as}
8	$u(t-a)(t-a)$	$\frac{1}{s^2}e^{-as}$
9	$u(t-a)(t-a)^n$	$\frac{n!}{s^{n+1}}e^{-as}$
10	$u(t-a)\cos\omega(t-a)$	$\frac{s}{s^2+\omega^2}e^{-as}$
11	$u(t-a)\sin\omega(t-a)$	$\frac{\omega}{s^2+\omega^2}e^{-as}$
12	e^{at}	$\frac{1}{s-a}$
13	te^{at}	$\frac{1}{(s-a)^2}$
14	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
15	$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
16	$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
17	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
18	$t\sin\omega t$	$\frac{2s\omega}{(s^2+\omega^2)^2}$
19	$y'(t)$	$sY(s)-y(0)$
20	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
21	$e^{b(t-a)}u(t-a)$	$\frac{1}{s-b}e^{-as}$
22	$g(t-a)u(t-a)$	$G(s)e^{-as}$
23	$f(t)e^{at}$	$F(s-a)$

Appendix D. Trigonometriske identiteter

$$
\begin{aligned}
\cos^2 x + \sin^2 x &= 1 \\
\cos 2x &= \cos^2 x - \sin^2 x \\
\sin 2x &= 2\cos x \sin x \\
\cos^2 x &= \frac{1}{2}(1 + \cos 2x) \\
\sin^2 x &= \frac{1}{2}(1 - \cos 2x) \\
\cos(\pi/2 - x) &= \sin x \\
\sin(\pi/2 - x) &= \cos x \\
\cos(x + y) &= \cos x \cos y - \sin x \sin y \\
\sin(x + y) &= \sin x \cos y + \cos x \sin y \\
\cos x \cos y &= \frac{1}{2}\Big(\cos(x - y) + \cos(x + y)\Big) \\
\sin x \sin y &= \frac{1}{2}\Big(\cos(x - y) - \cos(x + y)\Big) \\
\sin x \cos y &= \frac{1}{2}\Big(\sin(x - y) + \sin(x + y)\Big)
\end{aligned}
$$



x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{12} = 15°$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$
$\frac{\pi}{10} = 18°$	$\sqrt{\frac{5+\sqrt{5}}{8}}$	$\frac{\sqrt{5}-1}{4}$
$\frac{\pi}{6} = 30°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{5} = 36°$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{\frac{5-\sqrt{5}}{8}}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{3\pi}{10} = 54°$	$\sqrt{\frac{5-\sqrt{5}}{8}}$	$\frac{\sqrt{5}+1}{4}$
$\frac{\pi}{3} = 60°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{2\pi}{5} = 72°$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{\frac{5+\sqrt{5}}{8}}$
$\frac{5\pi}{12} = 75°$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$
$\frac{\pi}{2} = 90°$	0	1

Appendix E. Nummeriske skjema

Eulers metode

$$
\begin{aligned}
w_0 &= y(t_0) \\
w_{n+1} &= w_n + h\, f(t_n, w_n)
\end{aligned}
$$

Trapesmetoden

$$
\begin{aligned}
w_0 &= y(t_0) \\
s_1 &= f(t_n, w_n) \\
s_2 &= f(t_n + h, w_n + h s_1) \\
w_{n+1} &= w_n + \frac{h}{2}(s_1 + s_2)
\end{aligned}
$$

Midtpunktsmetoden

$$
\begin{aligned}
w_0 &= y(t_0) \\
s_1 &= f(t_n, w_n) \\
s_2 &= f(t_n + h/2, w_n + (h/2)s_1) \\
w_{n+1} &= w_n + h s_2
\end{aligned}
$$

Runge Kutta RK4

$$
\begin{aligned}
w_0 &= y(t_0) \\
s_1 &= f(t_n, w_n) \\
s_2 &= f(t_n + h/2, w_n + (h/2)s_1) \\
s_3 &= f(t_n + h/2, w_n + (h/2)s_2) \\
s_4 &= f(t_n + h, w_n + h\, s_3) \\
w_{n+1} &= w_n + \frac{h}{6}(s_1 + 2s_2 + 2s_3 + s_4)
\end{aligned}
$$

Foroverdifferanse for frstederivert

$$u_t(x,t) \approx \frac{1}{k}\big(u(x,t+k) - u(x,t)\big)$$

Bakoverdifferanse for frstederivert

$$u_t(x,t) \approx \frac{1}{k}\big(u(x,t) - u(x,t-k)\big)$$

Sentrert differanse for andrederivert

$$u_{xx}(x,t) \approx \frac{1}{h^2}\big(u(x+h,t) - 2u(x,t) + u(x-h,t)\big)$$