16.2.21) $F = (\sin x)i + (\cos y)j + (xz)k$

$C = r(t) = (t^3)i + (-t^2)j + (t)k$

$r'(t) = (3t^2)i + (-2t)j + (1)k$

$\int_0^1 (3t^2)\cdot(\sin t^3) + (-2t)(\cos(-t^2) + (1)(t^3 t)\, dt$

$= \int_0^1 3t^2 \sin t^3 - 2t\cos(-t^2) + t^4\, dt$

Ehs19-5)

Eksamen i TDAT3024

Oppgave nr:_____ Kandidat nr:_____

Antall sider:_____ Side:_____ Dato:_____

Denne kolonne er forbeholdt sensor

~~2019 3c)~~

$F(x,y,z) = (yz)i + (xz + x + z)j + (xy, y, 1)k$

2d) $\operatorname{div}((2xy + z^2)i + (x^2 + 2y)j + (2xz + 1)k$

$= 2y + 2 + 2x = 2(x + y + 1)$

3)a) $\sin^3(\pi x)$, $D_g = [0,1]$ (even)

$u_t = 4u_{xx}$, $u(x,y) = F(x) \cdot G(t)$

b) $u_t = F(x) \cdot G'(t)$, $u_{xx} = F''(x) \cdot G(t)$

$F(x) \cdot G'(t) = 4F''(x) \cdot G(t)$

$u(0,t) = 0$, $u(\pi,t) = 0$

$\frac{G'(t)}{G(t)} = 4 \cdot \frac{F''(x)}{F(x)}$

$F''(x) = kF(x)$

c)

4)a) $y'' + 2y' + 2y = f(t)$, $y(0) = y'(0) = 0$

$z_1 = y$, $z_2 = y'$, $z_2' = y''$

$z_2' + 2z_2 + 2z_1 = f(t)$, $z_1(0) = z_2(0) = 0$

b) $h = 0.1$

$y_1 = y_1' + \cos(y_2)$

$y_2 = \cos y_1 - y_2'$

$f_1(y_1, y_2) = y_1 - 2y_2$, $f_2(y_1, y_2) = 2y_1 - y_2$

$w_0 = y(t_0)$, $w_n + hf(t_n, w_n)$

w_i	t	y	$\frac{dy}{dt}$	$w_{i,j}$
0	0	$w_{1,1} = 1$, $w_{2,1} = 1.1$		
1	.1	$w_{1,2} = h \cdot f_1(1, 1) = 0.1 \cdot -1 = -.1$, $w_{2,2} = 0.95$		
2	.2	$w_{1,3} = hf_1(.1, -.1) + = 0.1$		
3	.3			
4	.4			
5	.5			

3 a) $F(x,y,z) = \overset{P}{yz}\,i + \overset{Q}{(xz+2y)}\,j + \overset{R}{(xy-2z)}\,k$

$\text{curl } F = (R'(y) - Q'(z))i + (P'(z) - R'(x))j + (Q'(x) - P'(y))k$

$= (x-x)i + (y-y)j + (z-z)k = 0i + 0j + 0k = \underline{0}$

b) Siden curl $F = 0$, er $F(x,y,z)$ konservativt.

c) $f_x = yz$, $f_y = xz+2y$, $f_z = xy-2z$

$f = \int f_x\,dx = xyz + g(y,z)$

$f_y = xz + g'(y,z)$

$xz + g'(y,z) = xz + 2y \Rightarrow g'(y,z) = 2y$

$\int 2y\,dy = y^2 + C$

$\cancel{\Rightarrow f(x,y,z) = xz + y^2}$

$f_z = xy + g'(y,z) \Rightarrow g'(y,z) = -2z$

$\int 2z\,dz = -z^2 + C$

$\underline{\underline{f(x,y,z) = xyz + y^2 - z^2 + C}}$

d) $x(t) = t^2$, $y(t) = \cos(\pi t)$, $z(t) = 1-t$, $0 \leq t \leq 1$

$\int_C F \cdot dr = f(x(1), y(1), z(1)) - f(x(0), y(0), z(0))$

$= f(1,-1,0) - f(0,1,1)$

$= (1-0) = \underline{\underline{1}}$