TDT4305 2021 - Assignment 4 solution

Adwords problem

1. Given the following table of advertisers and their bids on queries, compute the advertiser-query pairs using the three algorithms. For all algorithms, tie-break on the index, smallest index first.

Advertiser	Query	Bid
a_1	q_1	0.5
a_1	q_3	1
a_2	q_2	0.5
a_3	q_2	0.5
a_3	q_4	1
a_4	q_1	0.75

a) Assume for the Greedy algorithm that all bids are 1 or 0 and the budget of each advertiser a_i is B_i=2. Fill in the table for the Greedy algorithm.

Time	Query	Candidates	Budget left	Accu. revenue	Notes
1	q_1	**a_1**, a_4	B_1=1	1	Tie-break
2	q_2	**a_2**, a_3	B_2=1	2	Tie-break
3	q_3	**a_1**	B_1=0	3	
4	q_4	**a_3**	B_3=1	4	
5	q_3	a_1	x	4	
6	q_3	a_1	x	4	
7	q_2	**a_2**, a_3	B_2=0	5	Tie-break
8	q_4	**a_3**	B_3=0	6	

Assume the following budgets B_i for advertisers a_i in the next two algorithms:

Advertiser	Budget
a_1	3
a_2	1
a_3	1
a_4	2

b) Fill in the table for the Balance algorithm.

Time	Query	Candidates & bids	Budget left	Accu. revenue	Notes
1	q_1	**(a_1, 0.5)**, (a_4, 0.75)	B_1=2.5	0.5	Largest remaining budget
2	q_2	**(a_2, 0.5)**, (a_3, 0.5)	B_2=0.5	1	Tie-break
3	q_3	**(a_1, 1)**	B_1=1.5	2	
4	q_4	**(a_3, 1)**	B_3=0	3	
5	q_3	**(a_1, 1)**	B_1=0.5	4	
6	q_3	(a_1, 1)	x	4	No budget left
7	q_2	**(a_2, 0.5)**, (a_3, 0.5)	B_2=0	4.5	Only remaining budget
8	q_4	(a_3, 1)	x	4.5	No budget left

c) Fill in the table for the Generalized Balance algorithm.

Time	Query	Candidates & bids	Scores	Budget left	Accu. revenue	Notes
1	q_1	(a_1, 0.5), **(a_4, 0.75)**	$0.5(1-e^{-1})\approx0.31$ $0.75(1-e^{-1})\approx\underline{0.47}$	B_4=1.25	0.75	Highest score
2	q_2	**(a_2, 0.5)**, (a_3, 0.5)	$0.5(1-e^{-1})\approx\underline{0.31}$ $0.5(1-e^{-1})\approx0.31$	B_2=0.5	1.25	Tie-break
3	q_3	**(a_1, 1)**		B_1=2	2.25	
4	q_4	**(a_3, 1)**		B_3=0	3.25	
5	q_3	**(a_1, 1)**		B_1=1	4.25	
6	q_3	**(a_1, 1)**		B_1=0	5.25	
7	q_2	**(a_2, 0.5)**, (a_3, 0.5)		B_2=0	5.75	Only remaining budget
8	q_4	(a_3, 1)		x	5.75	

2. What is the definition of the competitive ratio?
    - *See [MMDS] section 8.2.3.*
3. What is broad matching and why is it useful?
    - *See [MMDS] section 8.4.3.*
4. What is second-price auction and why is it useful?
    - *See [MMDS] section 8.4.3.*