TDT4136 Introduction to Artificial Intelligence

Lecture 13: Examples

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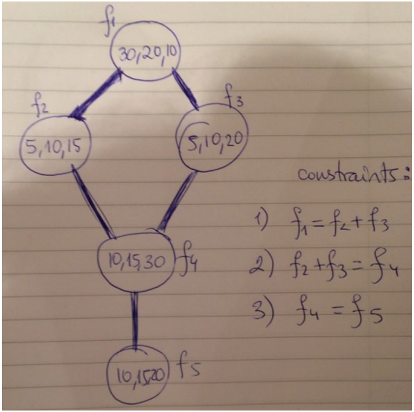
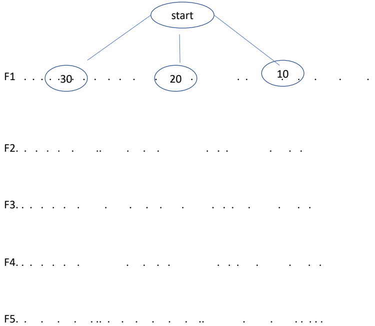
CSP -1

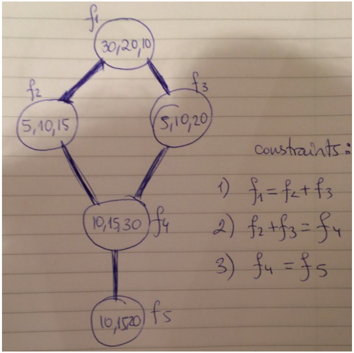
This problem is about agricultural water management.
The network consists of 5 water pipes with various capacities of water flow: f1, f2, f3, f4, f5 .
You are given the following diagram representing the constraint network. The circles represent the variables and are shown as f1, f2, f3, f4 and f5 while the integers in each circle/node show the possible values of flow for that node.



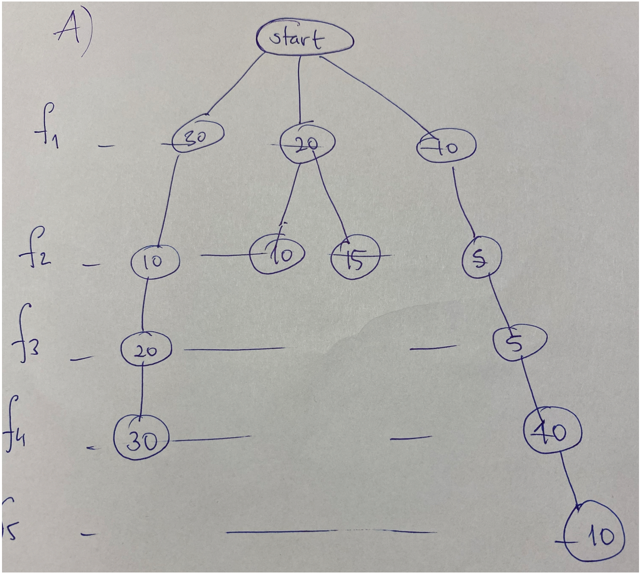
CSP - 2

A) Draw a search tree of the system using Backtracking with forward checking. For each node draw only the valid successors at that point. Select the values to try in the same order as in the nodes of the diagram. Complete the search tree on the left.





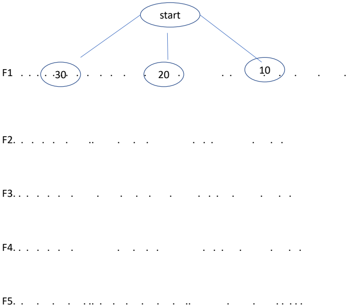
CSP - 3

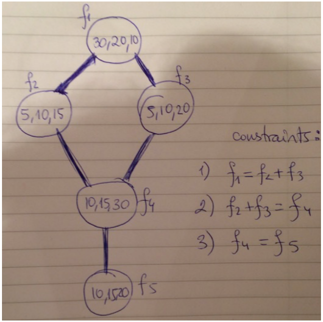


CSP - 4

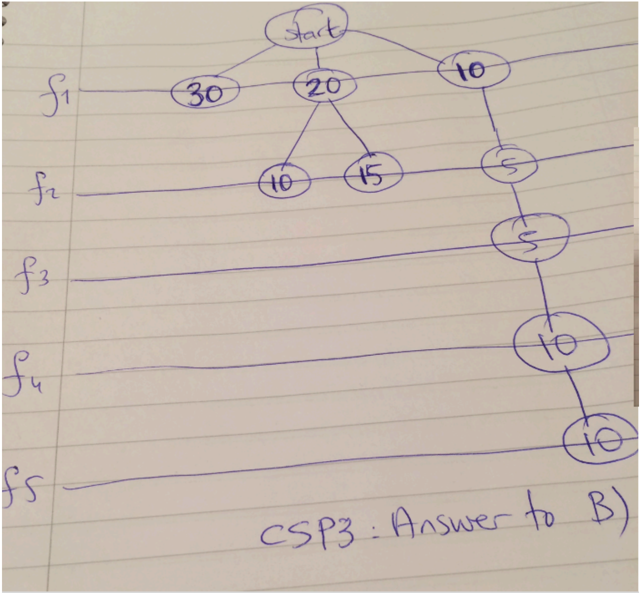
B) Draw the search tree that results from applying backtracking with forward checking and propagating through domains that are reduced to singleton (i.e., only 1 value left) domain

Use the same partial search tree in Question A and complete the search tree. Again draw only the nodes with valid successors, and try values in the same order as shown in the initial domain representations





CSP - 5



Game Theory - 1

This problem is about watering rice farms in Sumatra. Two farms are located in the riverside, upstream and downstream farms. Water is not sufficient - meaning if both farmers water their rice crops at the same time the upward one gets enough water but the downstream one is not watered sufficiently.

However, if they don't water their crops at the same time, both farms will suffer from a pest problem.

The game is about the timing of watering. Assume two possible times for watering, A and B.

Let x represent the utility loss for the farmer who gets reduced water and let y represent the loss in utility due to pests. Assume that when there is no crop loss due to lack of water or a pest problem, then the payoff is equal to 1.

Game Theory -2

1. Draw the payoff matrix using A and B as actions. Use x and y to define the payoffs for the upward and downward farms.

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Farm-down

Farm-up		A	B
	A	1, 1 – x	1 – y, 1 – y
	B	1 – y, 1 – y	1, 1 – x

Game Theory - 3

2. Under which conditions does a strongly dominant strategy equilibrium arise in this problem?

Farm-down

Farm-up	A	B
A	1, 1 – x	1 – y, 1 – y
B	1 – y, 1 – y	1, 1 – x

- There is no strongly dominant strategy equilibrium in this problem.
- Assume x,y are positive numbers. Then (B,A) cannot be a dominant strategy equilibrium (DSE) as row-agent could get more payoff by choosing action A.
- The same holds for (A,B). For (A,A) to be a DSE, A would have to be the dominant strategy for the up-farmer.
- This means that $1 > 1-y$ and $1-y > 1$ must be true at the same time.
- Thus (A,A) cannot be a DSE. The same reasoning holds for (B,B) meaning there is no DSE for any value of x or y.
- Actually none of the agents have a dominant strategy.

Logic - 1

Somebody stole the Christmas lights of Mr. Olsen in Trondheim. Inspector Harry Hole thinks one of the five famous burglars, Arne, Bernt, Cristoffer, Dina, Edna, in the town must have stolen the lights. He interviews each of them to find out the guilty one.

The following is the result of the interviews – two statements from each thief. It is well known that exactly one of the two statements of each thief is a lie:

Arne: *It was not Edna. It was Bernt.*

Bernt: *It was not Cristoffer. It was not Edna.*

Cristoffer: *It was Edna. It was not Arne.*

Derek: *It was Cristoffer. It was Bernt.*

Edna: *It was Dina. It was not Arne.*

Use the following propositional variables:

A=It was Arne. **B**=It was Bernt. **C**=It was Cristoffer. **D**=It was Dina. **E**=It was Edna

Logic - 2

1. Translate the evidence from each thief (i.e. from the statements of thieves taking also into account that exactly one of two statements of each statement of each thief is a lie) into propostional logic representation . i.e.,

   Arne: ……..

   Bernt:….

   ..

   ..

   Edna: …..

2. It is well known that one thief stole the lights alone. Represent this information as a set of implications.
3. Your data base consists of the statements you wrote in 1) and 2). Convert the statements in the KB to conjunctive normal form.
4. Using this KB, apply resolution refutation in order to infer "It was Cristoffer". If you cannot, then you must have made a mistake somewhere. Use the following structure to show your proof – add as many lines as you need:

   Resolve ………….. and …………….. to produce ……………..

   Resolve ………….. and …………….. to produce ……………..

   ………….

1) Translate the evidence (i.e. from the statements of thieves taking also into account that exactly one of the two statements of each thief is a lie.

Arne: $(E \land B) \lor (\neg E \land \neg B)$

Bernt: $(C \land \neg E) \lor (\neg C \land E)$

Cristoffer: $(\neg E \land \neg A) \lor (E \land A)$

Dina: $(\neg C \land B) \lor (C \land \neg B)$

Edna: $(\neg D \land \neg A) \lor (D \land A)$

Logic - 4

2) Representation of the information that one thief has stolen the lights alone:

$$(A \Rightarrow \neg B \land \neg C \land \neg D \land \neg E)$$
$$(B \Rightarrow \neg A \land \neg C \land \neg D \land \neg E)$$
$$(C \Rightarrow \neg A \land \neg B \land \neg D \land \neg E)$$
$$(D \Rightarrow \neg A \land \neg B \land \neg C \land \neg E)$$
$$(E \Rightarrow \neg A \land \neg B \land \neg C \land \neg D)$$

Logic - 5

3) The KB:

1. $(E \wedge B) \vee (\neg E \wedge \neg B)$
2. $(C \wedge \neg E) \vee (\neg C \wedge E)$
3. $(\neg E \wedge \neg A) \vee (E \wedge A)$
4. $(\neg C \wedge B) \vee (C \wedge \neg B)$
5. $(\neg D \wedge \neg A) \vee (D \wedge A)$
6. $(A \Rightarrow \neg B \wedge \neg C \wedge \neg D \wedge \neg E)$
7. $(B \Rightarrow \neg A \wedge \neg C \wedge \neg D \wedge \neg E)$
8. $(C \Rightarrow \neg A \wedge \neg B \wedge \neg D \wedge \neg E)$
9. $(D \Rightarrow \neg A \wedge \neg B \wedge \neg C \wedge \neg E)$
10. $(E \Rightarrow \neg A \wedge \neg B \wedge \neg C \wedge \neg D)$

Converted to CNF:

$(E \vee \neg B) (\neg E \vee B)$ (using distributivity of $\vee$ over ^ on 1.)
$(C \vee E) (\neg C \vee \neg E)$
$(\neg E \vee A) (E \vee \neg A)$
$(\neg C \vee \neg B) (C \vee B)$
$(\neg D \vee A) (D \vee \neg A)$
$(\neg A \vee \neg B) (\neg A \vee \neg C)$ (implic. elim. +distributivity for 6-10)
$(\neg A \vee \neg D) (\neg A \vee \neg E)$
$(\neg B \vee \neg C) (\neg B \vee \neg D)$
$(\neg B \vee \neg E) (\neg C \vee \neg D)$
$(\neg C \vee \neg E) (\neg D \vee \neg E)$

Logic - 6

4) Add ¬C to the KB and obtain { }

Resolve ¬B ∨ ¬E and C ∨ E to produce ¬B ∨ C
Resolve ¬B ∨ C and C ∨ B to produce C ∨ C = C
Resolve C and ¬C to produce { }

t is possible to do it in other ways as well. For example:

Resolve ¬ C and C ∨ E to give E
Resolve ¬ C and C ∨ B to give B
Resolve E and ¬ B ∨ ¬ E to give ¬ B
Resolve B and ¬ B to give {}

Converted to CNF:

(E ∨ ¬B) (¬E ∨ B)
(C ∨ E) (¬C ∨ ¬E)
(¬E ∨ A) (E ∨ ¬A)
(¬C ∨ ¬B) (C ∨ B)
(¬D ∨ A) (D ∨ ¬A)
(¬A ∨ ¬B) (¬A ∨ ¬C)
(¬A ∨ ¬D) (¬A ∨ ¬E)
(¬B ∨ ¬C) (¬B ∨ ¬D)
(¬B ∨ ¬E) (¬C ∨ ¬D)
(¬C ∨ ¬E) (¬D ∨ ¬E)