



NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

CODE: MTH 382

TIME: 3 HOURS

TITLE: MATHEMATICAL METHOD IV TOTAL: 70 MARKS

CREDIT UNIT: 3

INSTRUCTION: ANSWER ANY 5 QUESTIONS

1. (a) The gamma function of x is defined as $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$. Show that $\Gamma(x+1) = x\Gamma(x)$.
-4 marks

- (b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
-10 marks

2. (a) The beta function of x is defined as $B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$. Prove that $B(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$.
6 marks

- (b) Use the definition of beta function of x in 2(a), evaluate (i) $\int_0^1 x^5 (1-x)^6 dx$.
3 marks

- (ii) $\int_0^1 x^4 \sqrt{1-x^2} dx$.
-5 marks

3. (a) The Rodrigues formula is given as $p_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$ where $n=1, 2, 3, \dots$. Evaluate $p_4(x)$.
- 7 marks

(b) Solve the second order differential equation $\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = 2x^2 + 5x$ **-7 marks**

4. (a) Find a series solution of the differential equation $y'' + y = 0$, **-8 marks**

(b) Determine the singular points of the differential equation

$$2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$$

and classify them as regular or irregular **-6 marks**

5. (a) Given the Bessel equation as $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \nu) y = 0$, Solve the Bessel equation of order

zero **-10 marks**

(b) Find the solution of the initial value problem $y'' + y' - 6y = 0$, $y(0) = 2, y'(0) = 3$. **4 marks**

6. (a) Solve the hypergeometric equation of form $x(1-x)y'' + [t - (r+s+1)x]y' - rsy = 0$ **-10 markssss**

(b) Solve the differential equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 4x$ **-4 marks**

7.(a) Solve the differential equation by variation of parameters the equation

$$y'' + y = \sec^3 x$$

-6 marks

$$(1+x^2)y''-4xy'+6y=0$$

(b) Solve the differential equation

-8 marks

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