NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MARCH/APRIL 2015 EXAMINATION

SCHOOL OF SCIENCE AND TECHNOLOGY

Course Code:	**MTH 312**	
Course Title:	**Groups and Rings**	**TOTAL: 70**
TIME:	**3 HOURS**	
CREDIT UNIT:	**3**	

INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS

1. a) Let $G=\langle a\rangle$ be a finite cyclic group of order n, with generator a. Prove that a^k generates G if and only if $\gcd(k,n)=1$ (4 marks)

b) Suppose that $n=p^k$, where p is a prime and k is a positive integer. Find the number of generators of G (4 marks)

c) Let $f:G\rightarrow G^1$ be a group homomorphism. Prove that *ker f* is a normal subgroup of G. This is denoted by $f\nabla G$ (6 marks)

2. a) Define Group homomorphism (2 marks)

b) Show that every permutation in S_n ($n \geq 2$) can be written as a product of transpositions. (6 marks)

c) Show that if H and K be normal subgroups of a group G such that K Í H. then $(G/K)/(H/K)\simeq G/H$. (6 marks)

3. a) Let X be a non-empty set, $\rho(X)$ be the collection of all subsets of X and Δ denote the

symmetric difference operation. Show that $(\rho(X),\Delta,\cap)$ is a ring. (4 marks)

b) Show that the set $S=\{x+y\sqrt[3]{3}+z\sqrt[3]{9}: x,y,z\in Q\}$ is a ring with respect to addition and
multiplication on R. (5 marks)

c) Prove that the set $M=\{(a,b,c,d): a,b,c,d\in Q\}$ with addition and Multiplication
define by

$$(a,b,c,d)+(e,f,g,h)=(a+e,b+f,c+g,d+h)$$
$$(a,b,c,d)(e,f,g,h)=(ae+bg,af+bh,ce+dg,cf+dh)$$

for all $(a,b,c,d),(e,f,g,h)\in M.$ is a ring. (5 marks)

4. a) The direct product of two groups G and H is the group GxH with group operation $(g,h)(g^1,h^1)=(gg^1=hh^1)$ for $g,g^1\in G$ and $h,h^1\in H$. Suppose $G=\langle a\rangle$ and $H=\langle h\rangle$ are cyclic of orders m and n, respectively, and that $\gcd(m,n)=1.$
Prove that GxH is cyclic. (4 marks)

b) Let $s\in N$, show that the map $f:Z\to Z_s$ given by $f(m)=m$ for all $m\in Z$ is a
homomorphism. Obtain Kernel of f and Image of f . (4 marks)

c) Suppose G has two subgroups H, K with $K\nabla G$. Let $HK=\{hk: h\in H, k\in K\}$.
Prove that HK is a subgroup of G. (6 marks)

5. a) Define an Ideal in a ring ? (2 marks)

b) Show that the set of matrices $\left\{\begin{bmatrix} x & x \\ x & x\end{bmatrix} x\in\Re\right\}$ is a commutative ring with unity (6 marks)

c) Let X be a ring. Show that the identity map 1_X is a ring homomorphism. What are Ker 1_X and Im 1_X?

d) (6 marks)

6. a) Define Epimorphism or an onto homomorphism in a ring. (2 marks)

b) Let $r \in N$, show that the map $t: Z \rightarrow Z_r$ given by $t(m)=m$ t(m) = m for all $m \in Z$ is a homomorphism. Obtain Ker t and Im t. (6 marks)

c) Let A and B be two rings. Show that the projection map $P: AxB \rightarrow A: p(x,y)=x$ is a homomorphism. (6 marks)

7. a) Let G be a group with normal subgroups $H \nabla G$ and $K \nabla G$. Assume that $H \cap G=\{e\}$ and $HK=G$. Prove that $G \simeq HxK$. (6 marks)

b) Let G be a nonabelian group of order 2n, where $n \geq 3$. Suppose there exist elements $a,b \in G$ such that a has order 2, and $bab^{-1}=a^{-1}$. Prove that $G \simeq D_n$. (4 marks)

c) Let $f: R \rightarrow S$ be an onto ring homomorphism. Show that if J is an ideal of S, then $f(f^{-1}(J))=J.$ (4 marks)