NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JUNE/JULY EXAMINATION

COURSE CODE: MTH 411
COURSE TITLE: MEASURE AND INTEGRATION (3units)
TIME ALLOWED:3 HOURS
INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5)
QUESTIONS BEAR FULL MARKS

1(a) If $E_1, E_2, E_3 \ldots, E_n$ are pairwise disjoint measurable subsets of **R’,** show that $\mu^*(X \ \text{intersect} \left(\underset{j=1}{\overset{n}{¿}} E_j \right)) = \sum_{j=1}^{n} \mu^{¿}\left(X \cap E_j \right)$ for every subset X of **R’** 7marks

1(b) Show that the function $\mu^{*}¿¿$,is translation invariant i.e for every $\gamma \varepsilon$ **R’** and E $\subset$ **R’** we have $\mu^*\{ T_r(E)\} = \mu^*(E)$; where the translation function T_r is defined by $T_r(x) = x + r$ for every x ε **R’.** 7marks

2(a) Show that a subset E of **R’** is Lebesque measurable if and only if every subset X of **R’** we have $\mu^{¿}(X) = \mu^{¿}(X \cap E) + \mu^{¿}(X ¿)$ 9marks

2(b) Prove that if a subset E of **R’** is measurable, then so is its complement 5marks

3(a) Explain carefully what is meant by the LebesqueOuter Measure $\mu^*(E)$ of a subset E of the real line **R** 5marks

3(b) Prove that for any two subsets A and B of **R**, if A $\subset$ B, then $\mu^*(A) \leq \mu^*(B)$ 9marks

4(a) Prove that if two subsets A and B of the real line are measurable, then so is $A \ \text{intersect} B$
7mark

4(b) Prove that for every countable family $\{E_n\}_{n=1}^{\infty}$

of subsets of **R,** we have μ* $\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} \mu*\left(E_n\right)$ 7marks

5(a) Find the length of the set $\bigcup_{k=1}^{\infty}\left\{x : \frac{1}{k+1} \leq x \leq \frac{1}{k}\right\}$ 7marks

5(b) Prove that if E is any countable set of real numbers, the $\mu*(E)$ = 0 7marks

6(a) Define a set with measure Zero. 7marks

6(b) Suppose f = g almost everywhere. Show that $\int_{\chi} f d\mu = \int_{\chi} g d\mu$ 7marks

7(a) Let $E_1, E_2, E_3, \ldots, E_n$ be disjoint measurable subset of E with $\mu(E) < \infty$, then

Every linear combination S = $\sum_{k=1}^{m} C_k \chi_{EK}$ with real coefficient $a_1, a_2, a_3, \ldots a_m$ is

measurable simple function and $I_E(s) = \sum_{k=1}^{m} a_k \mu\left(E_k\right)$ .7marks

7(b) Prove that a monotonic increasing sequence of measurable sets

in **R**'satisfies the relation $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} \mu\left(A_n\right)$ 7marks