CODE: **MTH 305**   **TIME: 3 HOURS**
TITLE: **COMPLEX ANALYSIS II**   **TOTAL: 70 MARKS**
CREDIT UNIT   **3**
INSTRUCTION: **ANSWER ANY 5 QUESTIONS**

(1)   **Show that** $u(x,y)=x^3y-y^3x$ **is an harmonic function and find the**

function $v(x,y)$ **that ensures that** $f(z)=u(x,y)+jv(x,y)$ **is analytic**

(2)   **Evaluate the integral** $\int_c f(z)dz$ **where** $f(z)=(z-j)^2$ **and c is the**

straight line joining $A(z=0)$ **to** $B(z=1+j2)$ **.**

(3)   **Verify Cauchy's theorem by evaluating the integral** $\int_c (z^2+1)dz$

    (a) along the arc from A to B
    (b) along BO and OA

(4) (i)   **Consider the mapping of the circle** $|z|=1$ **under the transformation**

$w=z+\dfrac{4}{z}$ **onto the w-plane**

   (ii)   **Evaluate** $\int_{-\infty}^{\infty}\dfrac{\cos kx}{a^2+z^2}dx$ **where** $a>0$ **and** $k>0$ **.**

(5) **Determine the image in the w-plane of the circle** $|z|=2$ **in the z-plane**

under the transformation $w=\dfrac{z+j}{z-j}$ **and show the region in the w-plane**
onto which the region within the circle is mapped.

(6) Consider the integral $\oint_c f(z)dz$ **where** $f(z)=\dfrac{1}{z}$ **, evaluated round a**
closed contour in the z-plane

(7) Expand $f(z)=\dfrac{1}{z+1}$ **in a Taylor series about the point** $z=1$ **and find the values of z for which the expansion is valid.**