



NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MAY/JUNE 2012 EXAMINATION

MTH 315 ANALYTICAL DYNAMICS
TIME ALLOWED: 3 HOURS

TOTAL: 70 MARKS
INSTRUCTION: ANSWER ANY 5 QUESTIONS

- 1.(a) (i) Define degrees of freedom and give two examples -4 marks
(ii) Find the number of degrees of freedom in each for a rigid body which can move freely in three dimensional -4marks
(b) A system of particles consists of a 3 gram mass located at (1,0,-1), a 5 gram mass at (-2,1,3) and 2 gram mass at (3,-1,1).
Find the coordinates of the center of mass -6 marks
- 2.(a) (i) Differentiate between holonomic and non-holonomic constraints -4 marks
(ii) In each of the following cases state whether the constraint is holonomic or non-holonomic :
a bead moving on a circular wire, a particle sliding down an inclined plane under the influence of gravity
and a particle sliding down a sphere from a point near the top under the influence of gravity - 3marks
(b) Prove that the total momentum of a system of particles can be found by multiplying the total mass M of the system by the velocity of the center of mass -7 marks
- 3.(a) Two particles having masses m_1 and m_2 are located on a frictionless double inclined curves and connected by an inextensible massless string passing over a smooth pulley. Use the principle of virtual work to show that for
$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{m_2}{m_1}$$
equilibrium we must have ,where α_1 and α_2 are the angles of the incline. -6 marks
(b) Use D'Alembert's principle to describe the motion of the masses in (a) -8 marks
4. (a) State the laws of motion given by Sir Isaac Newton which are considered the axioms of mechanics -6 marks
(b) Due to a force field, a particle of mass 5 units moves along a space curve whose position vector is given as a function

of time t by $r = (2t^3 + t)i + (3t^4 - t^2 + 8)j - 12t^2k$. Find (a) the velocity, (b) the momentum (c) the acceleration and (d) the force field at any time t . -8 marks

- 5.(a) Explain the following terms (i) Work (ii) Power (iii) Energy -6 marks
 (b) (i) A particle of constant mass m moves in space under the influence of a force field F .

Assuming that at times t_1 and

t_2 the velocity is v_1 and v_2 respectively, prove that the work done is the change in kinetic energy, ie

$$\int_{t_1}^{t_2} F \cdot dr = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- 4 marks

- (ii) Show that the force field F defined by

$F = (y^2z^3 - 6xz^2)i + 2xyz^3j + (3xy^2z^2 - 6x^2z)k$ is a conservative force field. -4 marks

- 6.(a) Explain the following terms (i) uniform force field (ii) uniformly accelerated motion (iii) freely falling bodies -6 marks

(b) A particle of mass m moves along a straight line under the influence of a constant force of magnitude F . If its initial

speed is v_0 find (i) the speed, (ii) the velocity and (iii) the distance travelled after time t . -8 marks

7. (a) Explain the following terms (i) moment of a force (ii) Moment of a Couple (iii) Equilibrium of a Particle -6 marks

(b) (i) A uniform beam is 24m long and has a mass 100kg and masses of 60kg and 80 kg are suspended from its ends; at

what point must the beam be supported so that it may rest horizontally? -6 marks

$$V = \frac{1}{2}kx^2, k > 0$$

(ii) particle moves along the x axis in a force field having potential
 Determine the point of equilibrium. -2 marks