



NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MAY/JUNE 2012 EXAMINATION

MTH 421 ORDINARY DIFFERENTIAL EQUATION
TIME ALLOWED: 3 HRS

INSTRUCTION: ANSWER FOUR FROM SEVEN QUESTIONS. EQUATION ONE IS COMPOUSORY.

1. a. Solve the differential equation. $2xy \frac{dy}{dx} = y^2 - x^2.$

7 marks

- b. Solve the initial valued problem

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} + 9.04 y = 0, y(0) = 0, \frac{dy(0)}{dx} = 3$$

7 marks

- 2a. State and prove the foundermental theorem for the homogenous liner ordinary differential equation

7 marks

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 25 y = 0, y(0) = -3, \frac{dy(0)}{dx} = -1$$

- 2b. Solve the initial value problem

7 marks

3. Solve the initial value problem

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 5y = e^{5x} + 40 \cos 10x - 190 \sin 10x.$$

$$, y(0) = 0.16, \frac{dy(0)}{dx} = 40.08$$

14marks

$$\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

4a. Solve the equation

10marks

4b. Show that the solution to question [4a] are linearly independent .

4marks

5a. Find non trivial solution of the sturm-Liouville's problem

$$\frac{d^2 y}{dx^2} + \lambda y = 0, y(0) = 0, \frac{dy(\pi)}{dx} = 0$$

7marks

5b. Solve completely the differential equation

$$\frac{d^2 x}{dt^2} + x = 0 \dots \text{for} \dots x(0) = 0, \frac{dx(0)}{dt} = 1$$

7marks

6a. Reduce the equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$ to canonical form and find its general solution

7marks

6b. Prove that $u = F(xy) + xG\left(\frac{y}{x}\right)$ is the general solution of $x^2 u_{xx} - y^2 u_{yy} = 0$ 7marks

7a. Derive the solution to the Cauchy problem

$$u_{tt} = a^2 u_{xx} + \cos x, u(x, 0) = \sin x, u_t(x, 0) = 1 + x$$

7marks

7b. Solve this initial value problem

$$u_{tt} - c^2 u_{xx} = 0, u(x, 0) = x^3, u_t(x, 0) = \sin x$$

Let $g(x) = x^3, h(x) = \sin x$ By applying D'Alembert's formula.

7marks