NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

COURSE CODE: PHY 309 CREDIT UNIT: 3
COURSE TITLE: Quantum Mechanics I TIME: 3 Hours
INSTTRUCTION: Answer any five questions.

1 (a) (i) Show that the set $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\2\\1\end{pmatrix}\right\}$ is linearly independent

(b) Find the inner product of the following vectors:

(i) $\begin{pmatrix}i\\-2\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\3\end{pmatrix}$

(ii) A , $B\in M_{mn}$ if $A=\begin{bmatrix}2 & 1 & 1\\-1 & 3 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -1 & 2\\1 & 3 & 1\end{bmatrix}$.

(c) Find the norm of the following:

(i) $\begin{pmatrix}2i\\-1\\3\end{pmatrix}$ (ii) ix^2+2 , $0\le x\le 1$

(ii) Normalise the vector **a,** given that $a=\begin{pmatrix}1\\0\\1\end{pmatrix}$

2. (a) (i) Given that the quantum operators $\hat{A}=\hat{x}$ and $\hat{B}=\hat{p}_x=-i\frac{d}{dx}$ do not commute, evaluate $\hat{A}\hat{B}-\hat{B}\hat{A}$ and comment on your result.

(ii) If $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$, with n = 2, and $0 < x < L$, calculate the expectation value <x> of position x and <p> of momentum p.

(b) (i) Calculate the expectation value of a matrix operator, $\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{pmatrix}$ in state $\begin{pmatrix} 2i \\ 1 \\ -1 \end{pmatrix}$

(ii) Given the kinetic energy operator

$$\hat{T} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$$

and the linear momentum operator

$$\hat{p} = -i\hbar \frac{d}{dx}$$

evaluate $[\hat{T}, \hat{p}]$.

3(a) (i) Briefly discuss two experimental basis of the inadequacy of classical mechanics.

(ii) Find the change in wavelength if a photon is scattered at an angle of 23^0 after its collision with an electron initially at rest.

(b)(i) What is the wavelength of the wave associated with an electron moving at $10^6 m/s$?

(ii) What value does Rayleigh-Jeans formula predict for the radiation of frequency $6 \times 10^{13} Hz$ emitted by a blackbody per unit time, per unit area at 2500 ^{0}K. Compare this value with that predicted by Planck.

4(a)(i) Write down the one-dimensional time-independent Schroedinger equation.

(ii) Briefly discuss the interpretation of the Schroedinger equation and its solutions.

(b) (i) By solving the time-dependent Schroedinger equation for a free particle (*V* = 0), find the condition imposed on the angular frequency and the wave number.

(ii) What would the potential function be if $\psi(x) = \left(\frac{x}{x_0}\right)^n e^{-2x/x_0}$ is an eigenfunction of the Schroedinger equation? Assume that when $x \to \infty$, $V(x) \to 0$.

5. (a)(i) State the postulates of quantum mechanics

(ii) State the correspondence principle

(b). A particle of mass m is confined within a one-dimensional box of length $L/2$, subject to a potential:

$$V(x) = ¿\{0,\ 0 \le x \le L/2 ¿¿¿¿$$

If at t_0 , the wavefunction is $\psi(x) = Ax(L - x/2)$, i.e., $\psi(x,0) = Ax(L - x/2)$,

(i) normalise ψ , and hence, determine the value of *A*.

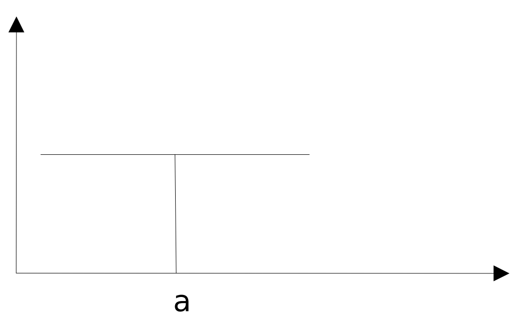
(ii) write $\psi(x,t)$ as a series, where $t > 0$.

6. (a)(i) State two conditions which apply to a system in bound state.

(ii) What are the allowable eigenfunctions and energy eigenvalues of the infinite potential well?

$$V(x) = ¿\{0,\ -L \le x \le L ¿¿¿¿$$

(b) A particle of mass *m* is incident from the left on the potential step shown in the figure.



Potential step

Find the probability that it will be scattered backward by the potential if

(i) $E > V_0$, and

(ii) $E < V_0$

7 (a) Given that the ladder operators $a=\sqrt{\frac{m\omega}{2\hbar}}\left(x+\frac{ip}{m\omega}\right)$, $a^{+}=\sqrt{\frac{m\omega}{2\hbar}}\left(x-\frac{ip}{m\omega}\right)$

and the position and momentum operators are given by $\hat{x}=\sqrt{\frac{m\omega}{\hbar}}\,x$ and $\hat{p}=\sqrt{\frac{1}{m\hbar\omega}}\,p$,

(i) What is the value of the commutator $[\hat{x},\hat{p}]$.

(ii) Show that $\hat{x}=\frac{1}{\sqrt{2}}\left(a+a^{+}\right)$, $\hat{p}=\frac{-i}{\sqrt{2}}\left(a-a^{+}\right)$.

(b) (i)For the ground state in question 7(a), find $¿\hat{x}^2>¿¿$ and $¿\hat{p}^2>¿¿$.

(ii) Given that the expectation of the position and the momentum operators under consideration are zero in the ground state of the oscillator, prove that the following expression holds: $¿x^2><p^2>¿¿$ = $\frac{1}{4}|<[x,p]>|^2$.

13021410 ??