**NATIONAL OPEN UNIVERSITY OF NIGERIA**
**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**
**SCHOOL OF SCIENCE AND TECHNOLOGY**
**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH301**
**COURSE TITLE: METRIC SPACES (3 units)**
**TIME ALLOWED:3 HOURS**
**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE(5)**
**QUESTIONS BEAR FULL MARKS**

1(a) What is a metric space? Give one example of a metric space. -4marks

1(b) What is a topological space? Give an example of a topological space.
-4marks

1(c) Define the length or norm of a vector x ε $R^3$-6marks

2(a) Let X ε $R^n$.Show that the set B(X, ε ) is open.-6marks

2(b) Let X be a complete metric space and {$O_n$} is countable collection of dense open subset of X. Show that ¿ $O_n$is not empty. -8marks

3 Let f and g be real-valued functions with Domain f = Range(g) = D ⊂ $R^n$.

Let $x_0$ be a point of accumulation on D. If the $\underset{n \to \chi}{Lim} f(x) = \ell$ and

$\underset{x \to x_0}{Lim} g(x)$ **=** n.

i) If for α, β ε **R**, show that $\underset{x \to x_0}{Lim}$ (αf + βg)(x) = $\alpha\ell + \beta m$
-4marks

ii) Show also that $\underset{x \to 0x}{Lim}(fg)(x) = \ell m$

-4marks

iii) If g(x) $\pm$ 0; for X ε D and $m \pm 0.$

Show that $\underset{x \to x_0}{Lim}\left(f/g\right)(x) = l/m = \dfrac{\lim_{x \to x_0} f(x)}{\lim_{x \to x_0} g(x)}$

-4marks

4 Let (X,d) and (Y,d) be metric spaces and f a mapping of X into Y. Let $\tau_1$ and $\tau_2$ be the topologies determined by d and d1 respectively. Then f(X, τ) $\to$ (y, τ) is continuous if and only if $X_n \to x \to f(Xn, \tau) \to f(x)$ ; that is if $x_1$, $x_2$, . . . , $x_n$, . . . , is a sequence of points in (X, d) converging to x, show that the sequence of points $f(x_1)$, $f(x_2)$, . . . , $f(x_n)$, . . . in (Y, d) converges to x. -14marks

5(a) Prove that for any y, z ε $\Re$ , max(y,z) = ½[y+z+ |y-z|], min(y,z) = ½[y+z- |y-z|].

-6marks

5(b) Let f,g: $\Re \to \Re$ be continuous at aε $\Re$ . Show that h,k: $\Re \to \Re$ defined through

h(x) = $\max_{x\varepsilon\Re}\left(f(x),g(x)\right)$ ,k(x) = $\min_{x\varepsilon\Re}\left(f(x),g(x)\right)$ are continuous at a.

-8marks

6 Let M = { A, d} be a metric space. Given any four points x, y, z, t ε A. Prove that
d(x, z) + d(y, t) ≥ |d(x, y)- d(z, t)| -14marks

7 Show that the mapping f **R** $\to$ $\mathbf{R}^+$defined by f(x) = $e^x$is a homeomorphism from **R**onto$R^+$(A homeomorphism from one

topological space to another is a bijective function)
-14marks