NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JUNE/JULY EXAMINATION

COURSE CODE: MTH402
COURSE TITLE: GENERAL TOPOLOGY II (3 units)
TIME ALLOWED: 3 HOURS
INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS

1(a) Prove that the intersection $\tau = \underset{\alpha}{\mathrm{intersect}\ \tau\alpha}$ of topologies { τ_α} $\alpha\varepsilon\Delta$ on X its itself a topology in X (where Δ is some indexing set)
-7marks

1(b) Let X be a set, and let B be a basis for a topology on τ on X. Show τ equals the collection of all unions of elements of B.
-7marks

2(a) Let B and B’ be bases for the topology τ and τ’ respectively on X. Show that the following are equivalent.

i) τ’ is finer than τ
ii) For each x ε X and each basis element в ε B such that x ε B, we know that в ε τ by definition and that τ ⊂ τ’, by condition (i)

therefore в ε τ’ -7marks

2(b) Let d be a metric on the set X. show that the collection of all ε – balls $B_d(x, \varepsilon)$ for x ε X and ε > 0 is a basis for a topology on X, called the metric topology induced by d. -7marks

3(a) State the properties under which d is a metric on X, given a function d: X x X → R, for all x, y, z ε X.
-6marks

3(b) Let X and Y be two topological spaces. Let B be the collection of all sets of the form U x V, where U is an open subset of X and V is an open subset of Y i.e. B = { U x V: U is open in X and V is open in Y}. Show that B is the basis for a for a topology on X x Y. -8marks

4(a) Let Y be a subspace of X. If U is open in Y and Y is open in X.

Show that U is open in X. -7marks

4(b) Show that the mapping $f: R \to R^+$ defined by $f(x) = e^x$ is a homeomorphism from R to R^+. (Recall that a homeomorpism from one topological space to another is a bijective function f such that f and f^{-1} are both continuous) -7marks

5(a) Give an example to show that every discrete space is Hausdorff. -2½marks

5(b) What does it mean to say that a topological space is Hausdorff? -2½marks

5(c) Let X be the Hausdorff space, then for all $x \in X$, show that the singleton set $\{x\}$ is closed. -9marks

6(a) Let X and Y be topological spaces. When is a function $f: X \to Y$ said to be continuous? -7marks

6(b) Let X be the subspace of R given by $X = [0,1] \cup [2,4]$, Define $f : X \to R$ by

$f(x) =$. Prove that f is continuous. -7marks

7(a) Let R be endowed with standard topology. Show that for all $x \in R$, $w = \{ (x - \varepsilon, x + \varepsilon) \; \varepsilon > 0\}$ is a neighbourhood basis of x. -7marks

7(b) Let X be a topological space and let $x \in X$. Suppose X is first countable, show that there exist a countable basis of x, say $W = \{ W_n \; n \geq 1 \}$ such that $W_{n+1} \subset W_n$ -7marks