NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MAY/JUNE 2012 EXAMINATION

PHY 307 SOLID STATE PHYSICS I
TIME ALLOWED: 3 Hours

INSTTRUCTION: Answer any five questions.

PHYSICAL CONSTANTS:
Speed of light $c=2.9979 ms^{-1}$; mass of electro $m_e=0.9110\times10^{-31} kg$;
Electronic charge $e=1.6022\times10^{-19} C$; Avogadro's number $N_A=6.0221\times10^{26} kmol^{-1}$;
Boltzmann constant $k=1.3806\times10^{-23} J K^{-1}$; Plank's constant $h=6.6257\times10^{-34} Js$

1. (a) Define the following terms
(i) Unit cell 1 mark
(ii) Basis 1 mark
(ii) Bravias Lattice 2 marks

(ii) Show that the perpendicular distance between two adjacent planes of a set (hkl) in a cubic lattice of lattice constant *a* is

$$d_{hkl}=\frac{a}{\left(h^2+k^2+l^2\right)^{1/2}}$$

4 marks

(iii) If x, y and z axes intercept 3, 4, and 2 of a crystal plane, calculate the Miller indices for the plane.

2 marks

(b) (i) An X-ray Diffractometer recorder chart for an element, which has a cubic crystal structure, shows diffraction peaks at the following 2θ: 40, 58, 73, 86.8, 100.4 and 114.7 degrees. The wavelength of the incoming X-rays used was $1.540 Å$.
Determine the type of the cubic structure and the lattice constant of the element. 5 marks

(ii)An X-ray beam of energy 0.01 MeV is reflected at the (100) plane of sylvine crystal $\left(d_{100}=0.314\, nm\right)$. Calculate the glancing angle θ at which the first order Bragg's spectrum will be observed.
5 marks

2. (a) Explain the terms:
(i) reciprocal space lattice
1 mark
(ii) first Brillouin zone
2 marks

(iii) Wigner-Seitz primitive cell 2 marks

(b) The primitive translational vectors of the hexagonal space lattice may taken as

$$A=\left(3^{\left(\frac{1}{2}\right)}\frac{a}{2}\right)i+\left(\frac{a}{2}\right)j \;; \quad B=-\left(3^{\left(\frac{1}{2}\right)}\frac{a}{2}\right)i+\left(\frac{a}{2}\right)j \;; C=ck$$

(i) Show that the volume of the primitive cell is $\left(3^{\left(\frac{1}{2}\right)}\frac{a}{2}\right)a^2$ c 5 marks

(ii) Show that the primitive translations of the reciprocal lattice are

$$A=\left(\frac{2\pi}{3^{1/2}}\right)i+\left(\frac{2\pi}{a}\right)j;\; B=-\left(\frac{2\pi}{3^{1/2}}\right)i+\left(\frac{2\pi}{a}\right)j \;;\; C=\left(\frac{2\pi}{c}\right)k$$

So that the lattice is its own reciprocal, but with a rotation axes 6 marks

(iii) Describe and sketch the first Brillouin zone of the hexagonal space lattice. 4marks

3. (a) Briefly explain the following terms and give an example of each:

(i) ionic bond 3 marks

(ii) Metallic bond 3 marks

(iii) Van der Waals bond 3 marks

(b) (i) Sketch the potential energy curve representing the interaction between two atoms and use it to explain the variation of the potential energy U with interatomic distance r of the atoms. 6 marks

(ii) Demonstrate that the cohesive energy per ion of two ions at equilibrium separation in an ionic crystal is

$$U_0=\frac{-\alpha e^2}{4\pi\varepsilon_o r_0}\left(1-\frac{1}{n}\right),$$

where the symbols have their usual meaning. 5 marks

a) Define the following terms:

4. (

(i) Lattice vibration 2 marks

(ii) Phonons 2 marks

(b)(i) Briefly explain the assumptions made in the harmonic approximation and deduce the dispersion relation for a monatomic linear lattice. 8 marks

(ii) Sketch the dispersion curve within the first Brillouin zone of a one dimensional monatomic linear lattice. 4 marks

(iii) Discuss the long wavelength approximation of the dispersion equation for a one dimensional monatomic linear lattice. 4 marks

5. (a) (i) What do you understand by term “Lattice heat capacity” of a crystal? 2 marks

(ii) State the main assumptions of the Debye model of heat capacity of a crystalline solid

3 marks
(b) (i) Use Debye model to obtain an expression for the total phonon energy, hence, obtain an expression for the heat capacity at constant volume at very low temperatures of a crystalline solid.
8 marks
(ii) Write down the expression for Einstein's approximation of the thermal energy and use it to obtain the heat capacity at constant volume of a crystalline solid.
4 marks
(iii) Briefly, highlight the disagreement between Einstein's and Debye's models of lattice heat capacity at very low temperatures.
3 marks

6. (a) (i) State the basic assumptions of the free electron model of metals. 3 marks
(ii) Define the term Fermi energy and write down an expression for the Fermi energy of a one-dimensional system of N free electrons each of mass m confined to a length L by finite potential barriers.
3 marks
(iii) Write down the expression for Fermi distribution function and sketch the function at $T \approx 0$ K.
4 marks
(b) (i) Write down the Schrödinger's equation and its solution in three dimensions for free electrons confined to a cube of edge L.
4 marks
(ii) Estimate the Fermi energy and velocity for sodium (Na) and comment on the answer you obtain for Fermi velocity.
Hint: Sodium has BCC structure with lattice parameter $a=4.2\,Å$, and one valence electron per atom.
6 marks
7 (a) At room temperature, $k_B T/e=26\,mV$. A sample of cadmium sulfide displays a mobile carrier density of 10^{16} cm^{-3} and a mobility coefficient $\mu=10^2 cm^2/volt\ sec$
(i) Calculate the electrical conductivity of this sample 5 marks
(ii) If the charge carriers have an effective mass equal to 0.1 times the mass of a free electron, what is the average time between successive scatterings? 5 marks
(b) (i) Briefly discuss the term "superconductivity" and illustrate with a sketch of resistance versus temperature curve.
4 marks
(ii) Mention six irregularities in the appearance of superconductivity based on empirical data

6 marks