NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JUNE/JULY EXAMINATION

COURSE CODE: PHY309
COURSE TITLE: QUANTUM MECHANICS
TIME ALLOWED:
INSTRUCTION: ANSWER QUESTION ANY FIVE QUESTIONS

QUESTION ONE

A particle trapped in the well

$$V=\begin{cases} 0, 0<x<a \\ \infty, elsewhere \end{cases}$$

is found to have a wave function

$$\frac{i}{2}\sqrt{\frac{2}{a}}\sin\left\{\frac{\pi x}{a}\right\}+\sqrt{\frac{2}{3a}}\sin\left\{\frac{3\pi x}{a}\right\}-\sqrt{\frac{2}{16a}}\sin\left\{\frac{3\pi x}{a}\right\}$$

i. If the energy is measured, what are the results and what is the probability of obtaining each result?
ii. What is the most probable energy for this particle?
iii. What is the average energy of the particle?

QUESTION TWO

A particle is confined within a one-dimensional region $0 \quad x \quad L$. At time $t = 0$, its wave function is given as $A\left[1-\cos\frac{\pi x}{L}\right]\sin\frac{\pi x}{L}$

i. Normalise the wave function.
ii. Find the average energy of the system at time $t = 0$ and at an arbitrary time t_0.
iii. Find the average energy of the particle.
iv. Write the expression for the probability that the particle is found within $0 \quad x \quad L/2$?

QUESTION THREE

a. What are the allowable eigenfunctions and energy eigenvalues of the infinite potential well?

$$V=\begin{cases} 0, -L\le x\le L \\ \infty, elsewhere \end{cases}$$

b. Show that the following row vectors are linearly dependent:

i. (1, 1, 0), (1, 0, 1), (3, 2, 1).

ii. Show the opposite for (1, 1, 0), (1, 0, 1), (0, 1, 1).

QUESTION FOUR

a. Compton scattering can be described in terms of a collision between a photon and a electron, as shown in Fig.1. The energy E and momentum P of a relativistic electron and the energy E and momentum p of a photon are related by:

$$E^2 - P^2c^2 \quad = \quad m^2_e c^4 \quad \text{and} \quad \mathcal{E} = pc.$$

Let E_i, P_i and E_f , P_f denote the initial and final energies and momenta of the electron, and let $\mathcal{E}_i$, p_i and $\mathcal{E}_f$, p_f denote the initial and final energies and momenta of the photon.

Assume that the electron is initially at rest, so that $E_i = m_ec^2$ and $P_i = 0$, and assume that the photon is scattered through an angle θ.

By considering the conservation of momentum shows that:

$$\mathcal{E}_i^2 - 2\,\mathcal{E}_i\,\mathcal{E}_f \cos\theta + \mathcal{E}_f^2 = E^2_f - m^2_e c^4$$

Write down the equation describing the conservation of relativistic energy in the collision and show that it may be rearranged to give

$$\mathcal{E}_i^2 - 2\,\mathcal{E}_i\,\mathcal{E}_f + \mathcal{E}_f^2 \quad = \quad E^2_f - 2\,E_f\, m^2_e c^4 + m^2_e c^4$$

Show that this equation implies that the increase in wavelength of the scattered photon is given by:

$$\lambda - \lambda' = \frac{h}{m_e c}(1 - cos\theta)$$

b. Find the change in wavelength if a photon is scattered at an angle of 46.6˚ after its collision with an electron initially at rest.

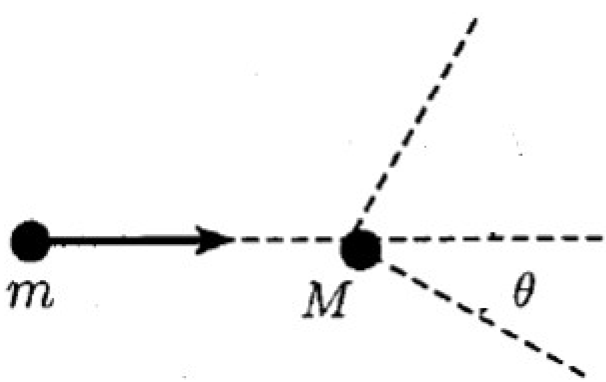

Fig 1

QUESTION FIVE

a. Use only the uncertainty principle to estimate the binding energy E_B of Hydrogen in terms of m_e ,e, ħ ,c (Evaluate the answer in terms of Electron Volts to at least 1 digit accuracy , using $m_e c^2$ = 5x105 eV and the known value of the fine structure constant, $\alpha = e^2/\hbar c$.

b. In a far off galaxy, mystery matter changes the Coulomb potential to

$$V(r) = e^2/r\ (d/r)^{\epsilon}$$

Where d is a new length scale and $\epsilon \ll 1$. Assuming that m_e does not change, show using uncertainty principle that first order in ϵ, the Bohr radius, r_B, changes to f x r_B where $f \approx 1- \epsilon\ \{1+\log(d/r_B)\}$.

Hint: $1/(1+ \epsilon) \approx 1- \epsilon$ and $x^{\epsilon} \approx 1+ \epsilon \log x$

QUESTION SIX

a. You are given the set

$$S\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\-1\end{pmatrix}\right\}$$

i. Are the linearly independent?
ii. Are they orthogonal?
iii. Are they normalized? If not, normalize them

b. Find the eigenvalues and the corresponding eigenfunctions of the matrix.

$$A=\begin{bmatrix}0 & 0 & 1\\0 & 0 & 0\\1 & 0 & 0\end{bmatrix}$$

c. If this matrix represents a physically observable attribute of a particle, what is the expectation value of the attribute in each of the possible states. Comment on your results.

QUESTION SEVEN

a. A quantum-mechanical oscillator of mass m moves in one dimension such that its energy eigenstate $\psi(x) = (y^2/\pi)^{1/4}\ \exp(- y^2x^2/2)$ with energy $E = \hbar^2 y^2/2m$

i. Find the mean position of the particle.
ii. Find the mean momentum of the particle

β. Normalise the eigenfunctions ĩx) = Aexp $\left[\frac{-m\omega}{2\hbar}x^2\right]$. Hence, find the probability that the particle subjected to harmonic oscillation lies in the range $0 \le x \le \frac{1}{2}$.