NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JUNE/JULY EXAMINATION

COURSE CODE: STT311
COURSE TITLE: Probability Distribution II
TIME ALLOWED: 3 HOURS (3 units)
INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5)
QUESTIONS BEAR FULL MARKS

1(a) Define probability Measure indicating relevant properties.
8marks

1(b) Let $\left| A_n \right|$ be a sequence of independent measurable sets, show that

(i) If $\sum P(A_n) < \infty$ then $P(A_i) = 0$

(ii) If $\sum P(A_n) = \infty$ then $P(A) = 1$

where A_i = Lim sup A_n

12marks

2(a) Define a continuous random variable on a probability space
8marks

2(b) The length of life measure in hours of a certain rare type of insect is a random variable x with portability density function

$$f(x) = \begin{cases} \frac{3}{4}(2x - x^2) & 0 < x < 2 \\ 0 & elsewhere \end{cases}$$

If the amount of food measured in milligrams consumed in a life time by such an insect defined by the function $g(x) = x^2$, *where x is the length of life measured in hours,* find the expected amount of food that will be consumed by an insect of this type.
12marks

3(a) Find the constant C such that the function

$$f(x) = \begin{cases} Cx^2 & 0 < x < 3 \\ 0 & elsewhere \end{cases}$$

is a density function.
10marks

(b) Find the probability function corresponding to the random variable x (number of head that turns up) when a coin is tossed trice, assuming that the coin is fair.
10marks

4(a) Define Central limit theorem for independently and identically distributed (iid) random variable X and determine its moment generating function(Mgf). 5marks

(b) The joint density function of two continuous random variables x and y is

$$f(x, y) = \begin{cases} Cxy & 0<x<4, 1<y<5 \\ 0 & otherwise \end{cases}$$

(a). Find the value of the constant C
5marks

(b). Find $P(1 < x < 2, 2 < y < 3)$
5marks

(c). Find $P(x \geq 3, y \leq 3)$.
5marks

5 A pair fair dice is tossed. We obtain the finite equiprobable space consisting of the 36 ordered pairs of numbers between 1 and 6, given as
S = ((1,1), (1,2), . . . , (6,6)}. Let X assign to each point (a,b) in S, the maximum of its numbers i.e X(a,b) = max(a,b). Then

i) Show that X is a random variable with the image set X(S) = {1, 2, . . . , 6}
ii) Compute the distribution f(x)
iii) Compute also the expected value of X
iv) Compute the expected value of Y, if Y assigns to each point (a,b) in S, the sum of its numbers a+b.
v) Indicate the g(y) graphically.

4marks each

6 (a) What Is Expectation of Random Variables?
5marks

(b) Let X and Y be random variables on the same sample space S. Show that

E(X + Y) = E(X) + E(Y).

10marks

(c) Define rth moment of a random variable X about the mean μ.
5marks

7 (a) State and prove CHEBYSHEV'S INEQUALITY for a continuous random variable.

10marks

(b) State and prove DEMOVRE'S THEOREM

10marks