NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MARCH/APRIL 2015 EXAMINATION

COURSE CODE: **MTH422**
COURSE TITLE: **PARTIAL DIFFERENTIAL EQUATION**
TOTAL: 70%
TIME: **3 HOURS**

INSTRUCTION: INSTRUCTION: ANSWER ANY FIVE QUESTIONS

1a. Find the general solution of

$$\left(Zx_i \;\; Zy_i \;-1 \right) \qquad \left(A, B, C \right)$$

,

By method of Lagrange multiplier

7marks

.1b.. Derive the solution to the Cauchy problem

$u_{tt} = a^2 u_{xx} + \cos x, u(x,0) = \sin x, u_t(x,0) = 1 + x$

7marks

2. Solve the vibration of an elastic string governed by the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Where u(x, y) is the deflection of the string. Since the string is fixed at the ends x = 0 and $x = l$, we have the two **boundary conditions** thus

$u(0,t) = 0, \quad u(l,t) = 0$ for all t

The form of the motion of the string will depend on the initial deflection (deflection at t = 0) and on the initial velocity (velocity at t = 0). Denoting the initial deflection by f(x) and the initial velocity by g(x), the two **initial conditions** are

$u(x,0) = f(x) \qquad \frac{\partial u}{\partial t}\Big|_{t=0} = g(x)$

14marks

3 Given $xp + yq = pq$ Find

a. The initial element if $x = x_o,\ y = o$ and $z = {x_o}/{2}$ $z(x,o) = {x}/{2}$

5marks

b. The characteristics stripe containing the initial elements

5marks

c. The integral surface which contain the initial element. 4marks

4a. Reduce the equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$ to canonical form and find its general solution

7marks

4b. Prove that $u = F(xy) + xG\left(\frac{y}{x}\right)$ is the general solution of $x^2 u_{xx} - y^2 u_{yy} = 0$ 7marks

5a. Form the PDEs whose general solutions are as follow:

(i) $z = Ae^{-p^2 t}\cos px$ 4marks

5b. Separate $u_x + 2u_{tx} - 10u_{tt} = 0$ and the boundary conditions $u(0,t) = 0, u_x(L,t) = 0$

For $0 < x < L$ and $\forall t$ hence, solve completely. Hint: Let $u(x,y) = X(x)T(t)$ 8marks

6. By inspection, classify the following partial differential equations into the following: non-linear, quasi-linear and linear. If linear, determine whether each is homogeneous or not

$u_{xx} + u_{yy} - 2u = x^2$

$u_x^2 + \log u = ¿ 2xy ¿$

¿

$2u_{xx} - 4u_{xy} + 2u_{yy} + 3u = 0$ 3.5marks each= 14marks

7a. State and Prove Cauchy Kovalewaski theorem.

7b. Given that $y'' + 4y = 0$ and $y(0) = 2,$ $y(\pi) = 3$. Show that the following boundary value problem has no solutions.