NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

CODE:MTH 301 **TIME: 3 HOURS**
TITLE: METRIC SPACES **TOTAL: 70%**
CREDIT UNIT: 3

INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE(5) QUESTIONS BEAR FULL MARKS

1(a) If X ε R^n. Prove that the set B(ε -x, x + ε) is open.
6marks

1(b) Let X be a complete metric space and {A_n} is countable collection of dense open subset of X. Show that ¿ $A_n$is not empty. -8marks

2(a) Define a metric space? Give one example of a metric space.
-4marks

2(b) What is meant by a topological space? Give an example of a topological space.
4marks

2(c) Define the length or norm of a vector x ε R^3-6marks

3 Let (S, d) and (T, d) be metric spaces and f a mapping of S into T. Let τ_1 and τ_2 be the topologies determined by d and d1 respectively. Then f(S, τ) → (T, τ) is continuous if and only if $S_n \rightarrow S \rightarrow f(S_n, \tau) \rightarrow f(s)$; that is if $s_1, s_2, \ldots, s_n, \ldots$, is a sequence of points in (S, d) converging to x, show that the sequence of points $f(s_1), f(s_2), \ldots, f(s_n), \ldots$ in (T, d) converges to x. -14marks

4 Let M = { A, d} be a metric space. Given any four points x, y, z, t ε A. Prove that
$d(x, z) + d(y, t) \geq |d(x, y) - d(z, t)|$ -14marks

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5 Let f and g be real-valued functions with Domain f = Range(g) = $D \subset R^n$.

Let x_0 be a point of accumulation on D. If the $\lim_{n \to \chi} f(x) = \ell$ and $\lim_{x \to x_0} g(x) = n$.

i) If for α, βε**R**, show that $\lim_{x \to x_0} (\beta g + \alpha f)(x) = \alpha\ell + \beta m$ -6marks

ii) If g(x) ± 0; for X ε D and $m \pm 0$.

Show that $\lim_{x \to x_0} \left(\frac{f}{g}\right)(x) = \frac{l}{m} = \frac{\lim_{x \to x_0} f(x)}{\lim_{x \to x_0} g(x)}$

-8marks

6 Show that the mapping f **R** → **R**$^+$ defined by f(x) = e^x is a homeomorphism from **R** onto R^+.
-14marks.

7(a) Prove that for any s, tε $\Re$, max(s,v) = ½[t+ v + |t-v|], min(t,v) = ½[t+v- |t-v|].
-6marks

7(b) Let $f,g: \Re \rightarrow \Re$ be continuous at $a \in \Re$. Show that $h,k: \Re \rightarrow \Re$ defined through

$h(x) = \max_{x \in \Re} (f(x), g(x))$, $k(x) = \min_{x \in \Re} (f(x), g(x))$ are continuous at a. -8marks

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