NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

CODE: **MTH 412** **TIME: 3 HOURS**

TITLE: **NORMED LINEAR SPACES** **TOTAL: 70%**

CREDIT UNIT: **3**

INSTRUCTION: **COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS**

1(a) Let X = $\mathbf{R}^2$. For each vector $\bar{x}$ = (x_1, x_2) ε X. Define $||.||_2 : X \to R$ by $||x||_2 = (\sum_{k=1}^{2} x_k^2)^{½}$. Show that ||.|| is a norm on X. -7marks

1(a) The surface of a unit sphere centered around the origin of a linear space with the ℓ^p-norm is the locus of points $\{(x_1, x_2, \cdots\}$. Show

$$\left(\sum_{k=1}^{\infty} |x_k|^p\right)^{\frac{1}{p}} = 1 .$$

that -7marks

2(a) What is a Convex set? If $x \varepsilon R^n$ and if r >0; show that the ball $B(s^*, r) = \{ t \varepsilon R^n : ||s - t^*|| < r\}$ centred at s* of radius r is a convex set -7marks

2(b) Let s, and t be vectors in $\mathbf{R}^n$, show that the line L through x in the direction of t given by $L = \{s+\alpha t : \alpha \varepsilon \mathbf{R} \}$ is a convex set. -7marks

3(a) Let (X, ρ) be a metric space. Define Cauchy sequence. -5marks

3(b) Let (X, ρ) be a complete metric space, and let E $\subset$ X. Show that (E, ρE) is complete if and only if it is closed. (Where ρE is the subspace metric induced byρ). -9marks

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4(a) Let S be a linear space over a scalar field T = (**R** or **C**)**.** When is a function ||.|| : S $\rightarrow$ R said to be a norm (in S)? -5marks

4(b) Show that the real line **R** becomes a normed linear space if you set ||S|| =|S| for every number Sε**R.** -9marks

5(a) Let S = c[0,1] = T, where c[0,1] is endowed with the supnorm.
Let D = {f εc'[0,1] : f'ε c[0,1]} where the prime denotes differentiation.
Let T : c[0,1] $\rightarrow$ c[0,1] be a map with domain D defined by Tf = f' (i.e. differentiation operator). Show that:

i) S is linear -2½marks

ii) S is closed. -2½marks

5(b) Show that an inner product space E becomes a normed linear space when equipped with the norm $\|t\| = \sqrt{\langle t,t \rangle}$ for all tε E. -9marks

6(a) Define the convergence of a sequence { s_n} of elements of S to a point sεS?

-5marks

6(b) Let P = [-4, 4] with $\|f\|_2 = \left(\int_{-4}^{4} \left| f(t) \right|^2\right)$ show that P is not complete.

-9marks

7(a) Define Linear Maps. -5marks

7(b) Let $X = l_2$. For each $\bar{x} = (x_1, x_2, x_3, \ldots x_k, \ldots)$ in l_2. Show that if $T\bar{x} = T(x_1, x_2, x_3, \ldots x_k, \ldots) = (0, x_1, x_2/2, x_3/3, \ldots x_k/k, \ldots)$, then T is a linear map on l_2.

9marks

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