NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MARCH/APRIL 2014 EXAMINATION

COURSE CODE: PHY 313
COURSE TITLE: MATHEMATICAL METHODS IN PHYSICS II EXAMINATION
TIME ALLOWED: 3HOURS
INSTRUCTION:ANSWER QUESTION ANY FIVE QUESTIONS

QUESTION ONE

a. When is f (z) said to be continuous at z = a

b. Simplify :

   i. (5-j9) – (2-j6) +(3-j4)

   ii. (6-j3)(2+j5)(6-j2)

   iii. $(4-j3)^2$

   iv. (5-j4)(5+j4)

c. Show that

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

QUESTION TWO

a. What are the necessary and sufficient conditions for the derivative of the function f (z)=u+iv, where u and v are real-valued functions of x and y, to exist for all values of z in domain D?

b. Find the Laurent series of:

$$f(z) = \frac{1}{z(z-3)^3}$$

about the singularities $z = 0$ and $z = 2$ (separately). Hence verify that $z = 0$ is a pole of order 1 and $z = 2$ is a pole of order 3, and find the residue of $f(z)$ at each pole.

QUESTION THREE

a. State the Cauchy and Riemann Equations

b. If w = log z, find $\frac{dw}{dz}$ and determine the value of z at which function ceases to be analytic.

c. Evaluate :

$$I = \int_0^\infty \frac{dx}{\left(x^2+a^2\right)^4}; \text{ wherea is real.}$$

QUESTION FOUR

a. State the residue theorem

b. Express f(z) = $f(z)=z^5+4z^4-6$ in polar form

c. Show that for the analytic function f (z) = u+iv, the two families of curves u(x,y) =c1 and v(x,y) = c2 are orthogonal.

N:B (Two curves are said to be orthogonal if they intersect at right angle at each point of intersection. Mathematically,if the curves have slopes m_1 and m_2, then the curves are orthogonal if $m_1 m_2 = -1$.)

QUESTION FIVE

a.

i. Is 4π real or imaginary ?

ii. Find the modulus and the argument of the complex number z = 2− 3i

iii. Find the complex conjugate of z = a + 2i + 3ib.

b. A function f (z) is defined as follows:

$$f(z)=f(x)=\begin{cases} \frac{x^3-y^3}{x^2+y^2}+i\frac{x^3+y^3}{x^2+y^2}, \wedge z\neq 0 \\ 0, \wedge x=0 \end{cases}$$

Show that $f(z)$ is continuous and that Cauchy-Riemann equations are satisfied at the origin. Also show that f ′(0) does not exist.

QUESTION SIX

a. Prove that the function f (z) = $|z|^2$ is continuous everywhere.

b. Show that the function $f(z)=e^{-z^{-4}}(z\neq 0)$ and $f(0) = 0$ is not analytic at $z=0$

c. Given that (a+b) + j(a-b) = $(1+j)^2$ + j(2+j), obtain the values of a and b.

QUESTION SEVEN

a. What is mapping ?

b. If $\frac{R_1 + j\omega L}{R_3} = \frac{R_2}{R_4 - j\frac{1}{j\omega C}}$, where R_1, R_2, R_3, R_4, ω, L and C are real ,

show that

$$L = \frac{C R_2 R_3}{\omega^2 C^2 R_4{}^2 + 1}$$