



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS**  
**SCHOOL OF SCIENCE AND TECHNOLOGY**  
**MAY/JUNE 2012 EXAMINATION**

STT 311 STATISTICS  
 TIME ALLOWED: 3 HOURS

TOTAL: 100%

**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS**

1(a) Define probability Measure indicating relevant properties. 8marks

1(b) Let  $A_n$  be a sequence of independent measurable sets, show that

(i) If  $\sum P(A_n) < \infty$  then  $P(\bigcap_{n=1}^{\infty} A_n) = 0$

(ii) If  $\sum_{n=1}^{\infty} P(A_n) = \infty$  then  $P(\bigcup_{n=1}^{\infty} A_n) = 1$

where  $A_1 = \limsup A_n$

12marks

2(a) Define a continuous random variable on a probability space  
 8marks

2(b) The length of life measure in hours of a certain rare type of insect is a random, reliable  $x$  with portability density function

$$f(x) = \begin{cases} \frac{3(2x-x^2)}{4} & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

If the amount of food measured in milligrams consumed in a life time by such an insect defined by the function  $g(x) = \int_0^x f(t) dt$ , where  $x$  is the length of life measured in hours, find the expected amount of food that will be consumed by an insect of this type.-12marks

3(a) Find the constant  $C$  such that the function

$$f(x) = \begin{cases} Cx^2 & 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Elsewhere

Is a density function.

3(b) Find the probability density function  $f_x(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$  -10marks

4(a) Define Central limit theorem for independently and identically distributed (iid) random variable X and determine its moment generating function(Mgf). 10marks

4(b) A random variable x has the density function

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$$(1) \quad f(x) = \frac{C}{x^2 + 1} \quad \text{where } x \geq 0$$

a. Find the value of the constant C

b. Find the probability that  $X^2$  lies between  $1/3$  and  $1$  -10marks

5 A pair fair dice is tossed. We obtain the finite equiprobable space consisting of the 36 ordered pairs of numbers between 1 and 6, given as  $S = \{(1,1), (1,2), \dots, (6,6)\}$ . Let X assign to each point (a,b) in S, the maximum of its numbers i.e  $X(a,b) = \max(a,b)$ . Then

- Show that X is a random variable with the image set  $X(S) = \{1, 2, \dots, 6\}$
- Compute the distribution  $f(x)$
- Compute also the expected value of X
- Compute the expected value of Y, if Y assigns to each point (a,b) in S, the sum of its numbers  $a+b$ .
- Indicate the  $g(y)$  graphically. -4marks

6 The joint probability function of two discrete random variable X and Y - 1 is given by  $f(x, y) = C(2x + y)$ , where x and y can assume all integers such that  $0 \leq x \leq 2, 0 \leq y \leq 3$ , and  $f(x, y) = 0$  otherwise

a. find the value of the constant C

b. Find  $p(x = 2, y = 1)$ .

(c) find  $p(x > 1, y < 2)$

7 (a) What Is Expectation of Random Variables?

(b) Let X and Y be random variables on the same sample space S. Show that

$$E(X + Y) = E(X) + E(Y).$$

(c) Define rth moment of a random variable X about the mean  $\mu$ .