



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**  
**SCHOOL OF SCIENCE AND TECHNOLOGY**  
**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH382**

**COURSE TITLE: MATHEMATICAL METHOD IV (3 units)**

**TIME ALLOWED: 3 HOURS**

**INSTRUCTION: ANSWER ANY 4 QUESTIONS**

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

1.(a) solve the legendre equation

with singular point

$$1-x^2=0$$

$$x=\pm 1$$

o r

**-10 marks**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(x^2 + e^{2x})$$

(b) Solve the differntial equation

**-7 ½ marks**

2.(a) Solve the differential equation by variation of parameters the equation

$$y'' + y = \sec^3 x$$

**- 7 ½ marks**

$$(1+x^2)y'' - 4xy' + 6y = 0$$

(b) Solve the differential equation

**-10 marks**

3. (a) .Determine the regular singular point of the differential equation **- 10 marks**

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, n = 1, 2, 3, \dots$$

(b) Given that Rodrigues formula, show that

$$P_2(x) = \frac{1}{2} (3x^2 - 1)$$

**-7 ½ marks**

4.(a) Show that  $\Gamma(n+1) = n!$

**-7 ½ marks**

$$(b) \text{ Evaluate (i) } \frac{\Gamma(3)\Gamma\left(\frac{5}{2}\right)}{\Gamma(5.5)} \quad (ii) \quad \frac{\Gamma\left(\frac{-5}{2}\right)}{\Gamma(5)}$$

**-10 marks**

5.(a) Use the definition of Gamma function, evaluate  $\int_0^\infty x^6 e^{-2x} dx$

**-7 ½ marks**

$$(b) \text{ Prove that } \Gamma\left(\frac{-1}{2}\right) = \sqrt{\pi}$$

**-10 marks**

6.(a) Solve the Bessel equation  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \nu^2) y = 0$

**- 10marks**

$$(b) \text{ Evaluate } \int_0^1 x^4 (1-x)^3 dx$$

**-7 ½ marks**