



**NATIONAL OPEN UNIVERSITY OF NIGERIA  
JABI, ABUJA  
FACULTY OF SCEINCE**

**SEPTEMBER/OCTOBER 2016 EXAMINATION**

**COURSE CODE: PHY 313**  
**COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II**  
**TIME ALLOWED: (3 HRS)**  
**INSTRUCTION: Answer any 4 questions**

**QUESTION ONE**

1. (a) What are the necessary conditions for a function  $f(z)$  to be analytic at a point  $z_0$  in the region  $R$  of the  $z$  plane?

(b) Show that the function  $\bar{z}$  is **not analytic** at any point.

(c) Show that  $f(z): \rightarrow \mathbb{C}$  defined by  $f(z) = e^z$  is analytic in  $\mathbb{C}$  (ii)  $\frac{de^z}{dz} = e^z$ .

2.(a) Evaluate the integral  $\int_C f(z) = (z - i)^2$  and  $C$  is a straight line joining

(b) Show that  $\int_0^{\frac{\pi}{2}} e^{t+it} dt = \frac{1}{2} \left( e^{\frac{\pi}{2}} - 1 \right) + \frac{i}{2} \left( e^{\frac{\pi}{2}} + 1 \right)$

3.(a) Write  $f(z) = z^4$  in the form  $f(z) = u(x, y) + iv(x, y)$

(b) Express  $f(z) = 4x^2 + i4y^2$  by a formula involving the variables  $z$  and  $\bar{z}$

(c)  $f(z) = z^5 + 4z^2 - 6$  in polar form

4. (a) A circle in the  $z$ -plane has its centre at  $z = 3$  and a radius of 2 units. Determine its image in the  $w$ -plane.

(b) The ellipse centered at the origin with a horizontal major axis of 4 units and vertical minor axis of 2 units. Obtain the parametric equation that represents the ellipse.

(c) Show that the image of the right half plane  $Re(z) = x > 1$  under the linear transformation  $w = (-1 + i)z - 2 + 3i$  is the half plane  $u > 7$ .

5. (a) Using Cauchy's formula for derivatives, evaluate:

$$\int_C e^{5x} / (z + i)^4 dz \text{ where } C: |z| = 3$$

(b)  $\int_C [(z^4)/(z + 1)(z - i)^2] dz$ , where  $C$  is the ellipse  $9x^2 + 4y^2 = 36$

6. (a) Expand  $f(z) = \cos z$  in Taylor's series about  $z = (\pi/3)$

(b) Find the Laurent series for  $f(z) = [(z^2 - 1)/(z^2 + 5z + 6)]$  in the region  $2 < |z| < 3$

7. (a) Find the principal part and the residues at the pole of the function

$$f(z) = [(2z + 3)/(z + 2)^2]$$

(b) Find the principal point and the residues for the function

$$f(z) = [z/(z^4 + a^4)]$$