NATIONAL OPEN UNIVERSITY OF NIGERIAN
SCHOOL OF SCIENCE AND TECHNOLOGY
END OF SEMESTER EXAMINATION MAY, 2012

CODE: MTH 402 TIME: 3 HOURS
TITLE: GENERAL TOPOLOGY II TOTAL: 70%

CREDIT UNIT: 3

INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS

1(a) Prove that the intersection $\tau = \underset{\alpha}{intersect\,\tau\alpha}$ of topologies { τ_α} $\alpha\varepsilon\Delta$ on X its itself a topology in X (where Δ is some indexing set) -7marks

1(b) Let X be a set, and let B be a basis for a topology on τ on X. Show τ equals the collection of all unions of elements of B. -7marks

2(a) Let B and B' be bases for the topology τ and τ' respectively on X. Show that the following are equivalent.

i) τ' is finer than τ
ii) For each x ε X and each basis element в ε B such that x ε B, we know that в ε τ by definition and that τ ⊂ τ', by condition (i)

therefore в ε τ' -7marks

2(b) Let d be a metric on the set X. show that the collection of all ε – balls $B_d(x, \varepsilon)$ for x ε X and ε > 0 is a basis for a topology on X, called the metric topology induced by d. -7marks

3(a) State the properties under which d is a metric on X, given a function d: $X \times X \to R$, for all x, y, z ε X.
-7marks

3(b) Let X and Y be two topological spaces. Let B be the collection of all sets of the form U x V, where U is an open subset of X and V is an open subset of Y i.e. B = { U x V: U is open in X and V is open in Y}. Show that B is the basis for a for a topology on X x Y. -14marks

4(a) Let Y be a subspace of X. If U is open in Y and Y is open in X.

Show that U is open in X. -7marks

4(b) Show that the mapping f: $R \to R^+$ defined by f(x) = e^x is a homeomorphism from R to R^+. (Recall that a homeomorpism from one topological space to another is a bijective function f such that f and f^{-1} are both continuous) -7marks

5(a) Give an example to show that every discrete space is Hausdorff.
-2½marks

5(b) What does it mean to say that a topological space is Hausdorff?
-2½marks

5(c) Let X be the Hausdorff space, then for all x ε X, show that the singleton

set {x} is closed. -9marks

6(a) Let X and Y be topological spaces. When is a function f: X → Y said to be

continuous? -
7marks

6(b) Let X be the subspace of R given by X = [0,1] U [2,4], Define f : X → R by

f(x) = Prove that f is continuous.

-7marks

7(a) Let R be endowed with standard topology. Show that for all x ε R, w = { (x – ε, x + ε) ε > 0} is a neighbourhood basis of x.
-7marks

7(b) Let X be a topological space and let x ε X. Suppose X is first countable, show that there exist a countable basis of x, say W = { W_n n ≥ 1 } such that $W_{n+1} \subset W_n$ -7marks