



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS**  
**SCHOOL OF SCIENCE AND TECHNOLOGY**  
**MAY/JUNE 2012 EXAMINATION**

MTH 305 COMPLEX ANALYSIS II  
 TIME ALLOWED: 3 HOURS

TOTAL: 70 MARKS  
 INSTRUCTION: ANSWER ANY 5 QUESTIONS

1. (a) Evaluate each of the following using theorems on limits (i)  $\lim_{z \rightarrow 1+i} (z^2 - 5z + 10)$   
 (ii)  $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{(z^2 - 2z + 4)}$

-8 marks

- (b) Prove that  $\lim_{z \rightarrow 0} \frac{z}{z}$

-6 marks

2. (a) Prove that the function  $U = 2x(1-y)$  is harmonic -6 marks  
 (b) Find a function  $V$  such that  $f(z) = u + iv$  and express  $f(z)$  in terms of  $z$  - 8 marks

3. (a) Prove that  $f(z) = z^2$  is uniformly continuous in the region  $|z| < 1$  -6 marks

- (b) Using the definition, find the derivative of  $w = f(z) = z^3 - 2z$  at the point where (a)  $z = z_0$  (b)  $z = -1$  -8 marks

4. (a) Expand  $f(z) = \frac{1}{z-3}$  is a Laurent series valid for (i)  $|z| < 3$  (ii)  $|z| > 3$

6 marks

- (b) Find the value of the integral  $I_1 = \int_c z^2 dz$ , where  $c_1$  is the line segment from  $z = 0$  to  $z = 2+i$  -8 marks

5. (a) Expand  $f(z) = \cos z$  in Taylor series about  $z = \frac{\pi}{4}$  and determine its region of convergence -6 marks

(b) Find the value of  $\oint_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$ , where C is a circle  $|z| = 1$  -8 marks

6. (a) Find the Laurent series for the function  $f(z) = (z-3) \sin \frac{1}{z+1}$  about  $z = -2$ . Also state that type of singularity and the region of convergence for the series. -6 marks

(b) Evaluate  $\int_C \frac{5z-2}{z(z-1)} dz$  -8 marks

7. (a) Verify that the real and imaginary parts of the function  $f(z) = z^2 + 5iz + 3 = i$  satisfy Cauchy-Riemann equation and deduce the analyticity of the function. -6 marks

(b) For each of the following functions, determine the poles and residues at the poles

(i)  $\frac{2z+1}{z^2-z-2}$  (ii)  $\left(\frac{z+1}{z-1}\right)^2$  -8 marks