NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MARCH/APRIL 2014 EXAMINATION

COURSE CODE: MTH 422
COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION
TIME ALLOWED: 2Hrs. 30mins
INSTRUCTION: INSTRUCTION: ANSWER ANY FOURQUESTIONS.

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1. Solve the vibration of an elastic string governed by the one dimensional wave equation.

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$$

subject to the boundary condition

$$u(0,t) = 0, \quad u(L,t) = 0. \forall t$$

14marks

2. Given $xp + yq = pq$

Find a. The initial element if $x = x_o,\ y = o$ and $z = {}^{x_o}/_{2}$ $z(x,o) = {}^{x}/_{2}$

5marks

b. The characteristics stripe containing the initial elements

5marks

c.The integral surface which contain the initial element. 4marks

State and Prove CAUCHY **KOVALEWASKI** Theorem.

3.

14marks

4a. Find the general solution of

$\left(Zx_i \;\; Zy_i - 1 \right)$ (A, B, C)

,

By method of Lagrange multiplier
7marks

4b.. Derive the solution to the Cauchy problem

$u_{tt} = a^2 u_{xx} + \cos x, u(x,0) = \sin x, u_t(x,0) = 1 + x$ 7marks

5. Prove that $u = F(xy) + xG\left(\frac{y}{x}\right)$ is the general solution of $x^2 u_{xx} - y^2 u_{yy} = 0$ 14marks

6a) Determine the characteristic equation, the characteristic curve and the canonical form of

$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xy u_x + y^2 u_y = 0$ 7marks

6 b) Prove that the equation in 6a above can be solved

7marks

7. By inspection, classify the following partial differential equations into the foolowing: non-linear,quasi-linearand linear. If linear, determine whether each is homogeneous or not

$u_{xx} + u_{yy} - 2u = x^2$

$u_x^2 + \log u = ¿ 2xy ¿$

¿

$$2u_{xx}-4u_{xy}+2u_{yy}+3u=0$$

3.5marks each= 14marks

The total obtainable marks is 70 marks. Good Luck.