NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MARCH/APRIL 2015 EXAMINATION

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: **MTH 341**
COURSE TITLE: **REAL ANALYSIS II** **TOTAL: 70**
TIME: **3 HOURS**

CREDIT UNIT: 3

INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS

1(a) Define the derivative or the differential coefficient of a function f at any point. 2 marks

(b) Let $h: R \to R$ be defined by

$$h(y) = ¿\, y ¿¿¿¿ \qquad \begin{matrix} 0 \le y \\ y > 0 \end{matrix}$$

find (i) $h'(0-)$,
(ii) $h'(0+)$

6 marks

(c) Prove that a function $g: R \to R$ defined by

$$g(x) = ¿ \left\{ x \sin \frac{1}{x} ¿¿¿¿ \right. \qquad \begin{matrix} x \ne 0 \\ elsewhere \end{matrix}$$

and $f(0) = 0$, is continuous but not derivable at the origin. 6 marks

2. (a) Define and explain a Monotonic Functions.
5marks

(b) Separate the intervals in which the function, f, defined on R by $f(x) = x^3 - 6x^2 + 9x + 4$, $x \in \Re$, is increasing or decreasing. 5marks

(c) Show that the function f, defined on R by $f(x) = x^3 - 6x^2 + 9x + 4$,

$\forall x \in \Re$ is increasing in every interval. 4marks

3(a) Find the point that satisfies the mean value theorem on the function $f(x) = \sin(x)$ and the interval $[0, \pi]$. 4 marks

(b) State and Prove Cauchy's Mean Value theorem. 4 marks

(c) Using the mean value theorem, prove that $e^x > x + 1$ for all x>0. (note: you can take $[0, x]$ as your interval of consideration). 6 marks

4(a) Discus with examples the relationship between differentiability and continuity of a function . 5marks

(b) Let f and g be two functions both defined on an interval *I*. If these are derivable at $c \in I$ then show that f/ g is also derivable at x = c. 5marks

(c) Find the derivative at a point y_0 of the domain of the inverse function of the function f, where f(x) = cos x, $x \varepsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. 4marks

5 (a) State the Rolle's theorem and give its geometrical interpretation. 6 marks

(b) Verify mean value theorem for the function $f(x) = (x-4)(x-6)(x-8)$ in $[4, 10]$ 8 marks

6. (a) Show that a necessary condition for f(c) to be an extreme value of a function f is that f'(c) = 0, in case it exists. 4marks

(b) Examine the function f given by $f(x) = (x-2)^4 (x+1)^5$; $\forall x \in \Re.$ for extreme values. 4marks

(c) Examine the polynomial function given by $f(x) = 10x^6 - 24x^5 + 15x^4 - 40x^3 + 108$ $\forall x \in \Re.$ for local maximum and minimum values. 6marks

7 (a) Using Maclaurin's theorem, prove that

$$Cos x \geq 1 - \frac{x^2}{2}, \qquad \forall \quad x \in \Re$$

4marks

(b) Find the Maclaurin Series expansion of

(i) e^x (ii) $Cos x$ (iii) $\log(1+x)$

10marks