NATIONAL OPEN UNIVERSITY OF NIGERIA
14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JUNE/JULY EXAMINATION

COURSE CODE: MTH312
COURSE TITLE: Groups and Rings
TIME ALLOWED:3 HOURS
INSTRUCTION: ANSWER ANY 4 QUESTIONS

1.(a) Prove that every subgroup of Z is normal in Z - **7 ½ marks**

(b) Let H be a subgroup of a group G. Show that following statement are equivalent

(i) H is normal in G

(ii) $g^{-1}Hg \subseteq H \forall g \in G$

(iii) $g^{-1}Hg = H \forall g \in G$ **10marks**

2.(a)(i) Show that $A_3 \Delta S_3$ **7½ marks**

(b) Write out the cayley tables for addition in Z6, the set of non-zero elements of Z6. **10 marks**

3. (a) Consider the set G $= \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} a, b \in R \right\}$ Ten g is a group with respect to matrix addition.Show that

$$f : G \to C : f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix} \right) = a + ib$$

is an isomorphism **10 marks**

(b).(i) Write out the cayley tables for multiplication in Z6, the set of non-zero elements of Z6. **7½ marks**

4.(a) Consider the set $M_2(R) = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} | a_{11}, a_{12}, a_{21}, a_{22} \right.$ are real numbers. Show that M2 (R) is a ring with respect to

Addition **10 marks**

(b) Show that $S=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a,b \in z\right\}$ is a subring of $R=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a,b \in R\right\}$ **7½ marks**

5. (a) Prove that $(S_n,\circ)$ is a non commutative group for $n\geq 3$ Hint use $A=\begin{pmatrix} 1\,2\,3 \\ 2\,3\,1 \end{pmatrix}, B=\begin{pmatrix} 1\,2\,3 \\ 3\,2\,1 \end{pmatrix}$

10mrks

(b) Find the principal ideals of Z10 generated by 3and 5

7 **½ marks**

6.(a) Let X be a non-empty set $\rho(x)$ be the collection of all subset of X and denote the symmetric difference operation.

Show that $(\rho(x),\Delta,\cap)$ is a ring. **10 marks**

(b) Consider the ring $\rho(x)$ and Let Y be a non-empty subset of x . f is defined $\rho(x)$ $\rho(y)$ by $f(A)=A\cap Y\,\forall\, A$

in $\rho(x)$ .Show that f is a hormorphism.**7 ½ marks**