NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
MAY/JUNE 2012 EXAMINATION

MTH 312 GROUPS AND RINGS
TIME: 3 HOURS

TITLE: TOTAL: 70 MARKS

INSTRUCTION: ANSWER ANY 5 QUESTIONS

1.(a) Prove that every subgroup of Z is normal in Z -4 marks

(b) Let H be a subgroup of a group G. Show that following statement are equivalent

(i) H is normal in G

(ii) $g^{-1}Hg \subseteq H \, \forall \, g \in G$

(iii) $g^{-1}Hg = H \, \forall \, g \in G$ - 10marks

2.(a)(i) Show that $A_3 \Delta S_3$ -4 marks

(b) Write out the cayley tables for addition in Z6, the set of non-zero elements of Z6. -8 marks

3. (a) Consider the set G $= \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} a, b \in R \right\}$ Ten g is a group with respect to matrix addition.Show that

$$f : G \rightarrow C : f\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right) = a + ib$$

is an isomorphism -7 marks

(b).(i) Write out the cayley tables for multiplication in Z6, the set of non-zero elements of Z6. -8 marks

4.(a) Consider the set $M_2(R)=\left\{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} | a_{11}, a_{12}, a_{21}, a_{22}\right.$ are real numbers. Show that M2 (R) is a ring with respect to

Addition -10 marks

(b) Show that $S=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} | a, b \in z\right\}$ is a subring of $R=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} | a, b \in R\right\}$ - 4 marks

5. (a) Prove that $(S_n, \circ)$ is a non commutative group for $n \geq 3$ Hint use

$A=\begin{pmatrix} 1\,2\,3 \\ 2\,3\,1 \end{pmatrix}, B=\begin{pmatrix} 1\,2\,3 \\ 3\,2\,1 \end{pmatrix}$

-7 marks

(b) Find the principal ideals of Z10 generated by 3and 5 -6 marks

6.(a) Let X be a non-empty set $\rho(x)$ be the collection of all subset of X and denote the symmetric difference operation.

Show that $(\rho(x), \Delta, \cap)$ is a ring.-8 marks

(b) Consider the ring $\rho(x)$ and Let Y be a non-empty subset of x . *f* is defined $\rho(x)$ $\rho(y)$ by $f(A)=A \cap Y \forall A$

in $\rho(x)$ .Show that *f* is a hormorphism. -6 marks

7.(a) Let A and B be two rings. Show that the projection map $P: A \times B \rightarrow A : P(x, y)=X$ is homorphism.what are ker P

And Im P ?-8 marks

(b) Show that map $\varphi : C[0,1] R \times R : \varphi$ if *f(1)=(f(0),f(1))* is a homomorphism -6 marks