



**NATIONAL OPEN UNIVERSITY OF NIGERIA
SCHOOL OF SCIENCE AND TECHNOLOGY
END OF SEMESTER EXAMINATION 2012**

CODE:MTH 304

TITLE: COMPLEX ANALYSIS I

CREDIT UNIT: 3

INSTRUCTION: ANSWER ANY 5 QUESTIONS

TIME: 3 HOURS

TOTAL: 100 MARKS

1.(a) Find the following complex number in the form of $x + iy$

(i) $(4 - 7i)(-2 + 3i)$

$\frac{(5 + 2i)}{(1 + i)}$

(ii) $\frac{(5 + 2i)}{(1 + i)}$

(5 each = 10 marks)

(b) Evaluate each of the following using theorems on limits

$\lim_{z \rightarrow -1+i} (z^2 - 5z + 10)$

(i) $\lim_{z \rightarrow -1+i} (z^2 - 5z + 10)$

$\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{(z^2-2z+4)}$

(ii) $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{(z^2-2z+4)}$

(5 each = 10 marks)

2. (a) Find a polar form of $(1+i)(1+i\sqrt{3})$

6marks

(b) Find a function y that describes that part of the curve $y = 4x^3 + 1$ between $x=0$ and $x=-10$

7marks

(c) Find a function V such that $f(z) = u + iv$ and express $f(z)$ in terms of z

7marks

3. (a) Prove that $f(z) = z^2$ is uniformly continuous in the region $|z| < 1$

10marks

(b) Using the definition, find the derivative of $w = f(z) = z^3 - 2z$ at the point where

(i) $z = z_0$

(ii) $z = -1$

10marks

4. (a) Expand $f(z) = \frac{1}{z-3}$ in Laurent series about (i) $|z| < 3$ (ii) $|z| > 3$

10marks

(b) Find the value of the integral $I_1 = \int_c z^2 dz$, where c_1 is the line segment from $z = 0$ to $z = 2+i$ 10 marks

5. (a) Expand $f(z) = \cos z$ in Taylor series about $z = \frac{\pi}{4}$ and determine its region of convergence. 10 marks

(b) Let C be the ellipse $9x^2 + 4y^2 = 36$ traversed once in the counterclockwise direction. determine the function $g(z) = \int_C \frac{s^2 + s + 1}{s - z} ds$ by find $g(4i)$ 10marks

6. Suppose f is analytic inside and on the simple closed curve C . Show that

$$\int_C \frac{f'(z)}{z - w} dz = \int_C \frac{f(z)}{(z - w)^2} dz$$

for every $w \in C$. 10 marks

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$$\int_C \frac{5z - 2}{z(z - 1)} dz$$

(b) Evaluate 10

marks

7. (a) Verify that the real and imaginary parts of the function $f(z) = z^2 + 5iz + 3 = i$ satisfy Cauchy-Riemann equation 8 marks

(b) Find the value of $\oint_C \frac{\sin^6}{\left(z - \frac{\pi}{6}\right)^3} dz$, where C is a circle $|z| = 1$ 12

marks

Good Luck