NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

CODE: **MTH 402**

TITLE: **GENERAL TOPOLOGY II**

CREDIT UNIT: **3**

INSTRUCTION: **COMPLETE ANSWERS TO** **TIME: 3 HOURS**

ANY FIVE (5) QUESTIONS BEAR FULL MARKS
TOTAL: 70%

1(a) Let Y be a subspace of X. If U is open in Y and Y is open in X.

Show that U is open in X. -7marks

1(b) Show that the mapping $f:S \rightarrow S^+$ defined by $f(x) = e^x$ is a homeomorphism

from R to S^+ -7marks

2(a) Let R be endowed with standard topology. Show that for all x ε R,
w = { (x – ε, x + ε) ε> 0} is a neighbourhood basis of x. -7marks

2(b) Let Y be a topological space and let yεY. Suppose Y is first countable, show that there exist a countable basis of y, say V = { V_n n ≥ 1 } such that $V_{n+1} \subset V_n$-7marks

3(a) What does it mean to say that a topological space is Hausdorff? -2½marks

3(b) Give an example to show that every discrete space is Hausdorff. -2½marks

3(c) Let P be theHausdorff space, then for all pεP, show that the singleton

set {x} is closed. -9marks

4(a) State the properties under which d is a metric onS, given a function $d: S \times S \rightarrow R$, for all $x, y, z \in S$.
-6marks

4(b) Let S and T be two topological spaces. Let B be the collection of all sets of the form VxW, where V is an open subset of S andW is an open subset of Y i.e. B = { V x W: V is open in S and W is open in T}. Show that B is the basis for a for a topology on S x T. -8marks

5(a) Prove that the intersection $\tau = \underset{\alpha}{\text{intersect}\,\tau\alpha}$ of topologies $\{ \tau_\alpha \}$ $\alpha \varepsilon \Delta$ on X its itself a topology in X (where Δ is some indexing set)
-7marks

5(b) Let X be a set, and let B be a basis for a topology on τ on X. Show τ equals the collection of all unions of elements of B.
-7marks

6(a) Let X and Y be topological spaces. When is a function $f: X \rightarrow Y$ said to be continuous? -7marks

6(b) Let S be the subspace of R given by X = [0,1] U [2,4], Define $f : S \rightarrow R$ by
f(s) = . Prove that f is continuous. -7marks

7(a) Let d be a metric on the set X. show that the collection of all ε– balls $B_d(x, \varepsilon)$ for $x \varepsilon X$ and $\varepsilon > 0$ is a basis for a topology on X, called the metric topology induced by d. -7marks

7(b) Let B and B' be bases for the topology τ and τ' respectively on X. Show that the following are equivalent.

i) τ' is finer than τ
ii) For each $x \varepsilon X$ and each basis element $B \varepsilon B$ such that $x \varepsilon B$, we know that $B \varepsilon \tau$ by definition and that $\tau \subset \tau'$, by condition (i) therefore $B \varepsilon \tau'$
-7marks