

NATIONAL OPEN UNIVERSITY OF NIGERIA 14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS SCHOOL OF SCIENCE AND SCIENCES MAY/JUNE 2012 EXAMINATION

MTH 382 MATHEMATICAL METHOD IV

TIME: 3 HOURS

INSTRUCTION: ANSWER ANY 5 QUESTIONS

$$(1-x^2)y''-2xy'+n(n+1)y=0$$

1.(a) solve the legendre equation

with singular

$$1-x^2=0$$
 $x=\pm 1$ point or -8 marks

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4 y = 8(x^2 + e^{2x})$$

(b) Solve the differntial equation

-6marks

2.(a) Solve the differential equation by variation of parameters the equation

$$y$$
"+ $y = \sec^3 x$
-6 marks

$$(1+x^2)y''-4xy'+6y=0$$

(b) Solve the differential equation

-8 marks

- 3. (a) .Determine the regular singular point of the differential equation $\stackrel{\mp}{}$
 - 10 marks

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx} (x^2 - 1)^n, n = 1,2,3,...$$

(b) Given that Rodrigues formular

,show

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

that

-4 marks

4.(a) Show that $\Gamma(n+1)=n!$

(b)Evaluate (i)
$$\frac{\Gamma(3)\Gamma\left(\frac{5}{2}\right)}{\Gamma(5.5)}$$
 (ii)
$$\frac{\Gamma\left(\frac{-5}{2}\right)}{\Gamma(5)}$$

-8 marks

- 5.(a) Use the definition of Gamma function, evaluate $\int_0^\infty x^6 e^{-2x} dx$
 - (b) Prove that $\Gamma\left(\frac{-1}{2}\right) = \sqrt{\pi}$

-8 marks

 $x^{2} \frac{d^{2} y}{dx} + x \frac{dy}{dx} + (x^{2} - v^{2}) y = 0$ 6.(a) Solve the Bessel equation

- 10marks

-6 marks

- (b) Evaluate $\int_{0}^{1} x^{4} (1-x)^{3} dx$ -4 marks
- 7.(a) Solve the hypergeometric equation of form

$$x(1-x)y"+[t-(r+s+1)x]y'-rsy=0$$
 -10 marks

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 4x$$
-4 marks

(b) Solve the differential equation