NATIONAL OPEN UNIVERSITY OF NIGERIA
14-16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS
SCHOOL OF SCIENCE AND TECHNOLOGY
JANUARY/FEBRUARY 2013 EXAMINATION

CODE: MTH 423 **TIME: 3 HOURS**
TITLE: INTEGRAL EQUATION **TOTAL: 70%**
CREDIT UNIT: 3

INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS

1(a) Find an integral formulation for the problem defined by

y"+ 4y = f(x), $0 \le x \le \frac{\pi}{4}$, y = 0 at x = 0 and y = 0 at x = $\frac{\pi}{4}$
-7marks

1(b) Transform the problem defined through y" + λy = 0 when y = 0 at x = 0 and
y' = 0 at x = 1 into integral equation form.
-7marks

2(a) Solve the integral equation

$$Q(x) = x^3 + \int_o^x e^{3(x-y)} Q(y)\, dy$$

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5marks

2(b) Solve the integral equation

$$\varphi(x) = \lambda \int_o^1 (1+xt)\, \varphi(t)\, dt \quad o \le x \le 1$$

-9marks

3 Find the eigenvalues and eigenfunction of the system defined by

$$\varphi(x) = \lambda \int_o^1 (1+xt)\, \varphi(t)\, dt + f(x)$$

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14marks

4(a) With proper integration and differentiation, convert the understated differential equation into integral equation.
$y''(x) + b_1(x) + b_2(x)y(x) = f(x)$ with the initial condition $y(0) = 0$; $y(0) = y_1$ -7marks

4(b) Using appropriate method, form the integral equation corresponding to
$Y'' + 2xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$.
-- 7marks

5 Solve the integral equation

$$Q(x) = x + 1 + \int_0^x (1+2(x-y))\, d(y)\, dy$$

-14marks

6(a) Solve the integral equation $\int_0^x Q(x-y)[Q(y)-2\sin ay]dy = x\cos ax$ - 7marks

6(b) Solve the integral equation $3\sin x + 2\cos x = \int_{-\infty}^{\infty} \sin(x+y)Q(y)dy$

- 7marks

7 Let (φ_n) be an orthogonal system, and let f be continuous. Set $\alpha_n = \int_I f(x)\,\varphi_n(x)\,dx$ .Show that, $\sum \alpha_n^2 \le \int_I f^2(x)\,dx$

and α_n^{1s} are known as the Fourier's coefficient. -14marks