软件分析

南京大学 计算机科学与技术系 程序设计语言与 谭添

Static Program Analysis

Pointer Analysis Context Sensitivity (I)

Nanjing University

Tian Tan

2021

Problem of Context-Insensitive Pointer Analysis

```
void main() {
  Number n1, n2, x, y;
  n1 = new One(); // o_1
  n2 = new Two(); // o_2
  x = id(n1);
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  int i = x.get();
Number id(Number n) {
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interface Number {
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class One implements Number {
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int i = x.get(); Constant propagation: i = ?
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Problem of Context-Insensitive Pointer Analysis Context insensitivity

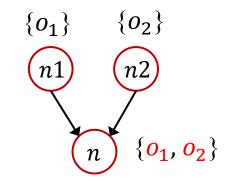
Context insensitivity, x.get():

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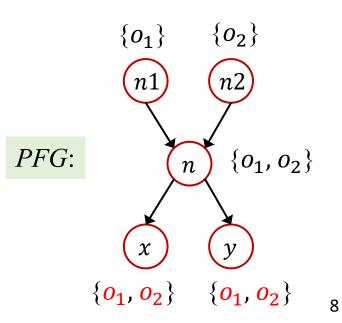


PFG:

Problem of Context-Insensitive Pointer Analysis

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Problem of Context-Insensitive Pointer Analysis

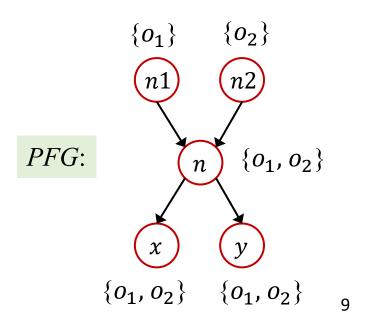
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Context insensitivity, x.get():

2 call targets

Constant propagation

i = NAC



Problem of Context-Insensitive Pointer Analysis

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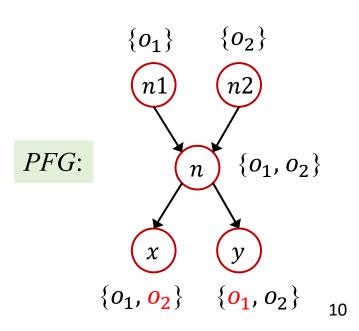
Context insensitivity, x.get():

- 2 call targets
- 1 false positive



Constant propagation

i = NAC



Via Context-Sensitive Pointer Analysis

Context sensitivity, x.get():

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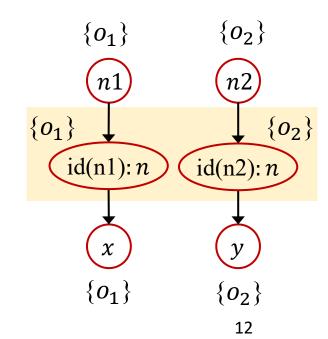
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void main() {

Context sensitivity, x.get():

PFG:

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Via Context-Sensitive Pointer Analysis

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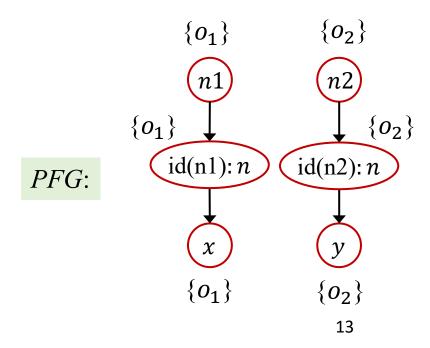
Context sensitivity, x.get():

- 1 call targets
- 0 false positive

Constant propagation

• i = 1





Contents

- 1. Introduction
- 2. Context Sensitive Pointer Analysis: Rules
- 3. Context Sensitive Pointer Analysis: Algorithms
- 4. Context Sensitivity Variants

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- 2. Context Sensitive Pointer Analysis: Rules
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Imprecision of Context Insensitivity (C.I.)

 In dynamic execution, a method may be called multiple times under different calling contexts

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x = id(n1);
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int i = x.get();

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Imprecision of Context Insensitivity (C.I.)

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 Under different calling contexts, the variables of the method may point to different objects

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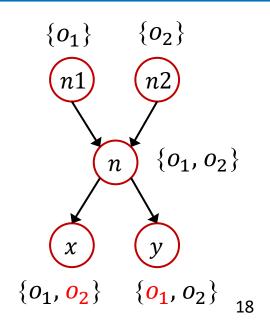
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Imprecision of Context Insensitivity (C.I.)

- In dynamic execution, a method may be called multiple times under different calling contexts
- Under different calling contexts, the variables of the method may point to different objects
- In C.I. pointer analysis, objects under different contexts are mixed and propagated to other parts of program (through return values or side-effects), causing spurious data flows

```
x = id(n1);
y = id(n2);
int i = x.get();

Number id(Number n) {
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 Context sensitivity models calling contexts by distinguishing different data flows of different contexts to improve precision

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- The oldest and best-known context sensitivity strategy is call-site sensitivity (call-string)
 - Which represents each context of a method as a chain of call sites, i.e., a call site of the method, a call site of the caller, a call site of caller of caller, etc.
 (abstract call stacks in dynamic execution)

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In call-site sensitivity, what are the contexts for method id(Number)

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In call-site sensitivity, method
id(Number) has two contexts:
[1] and [2]
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You will see other variants of context sensitivity in the next lecture

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The most straightforward approach to implement context sensitivity

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 The variables are also qualified by contexts (inherited from the method they are declared in)

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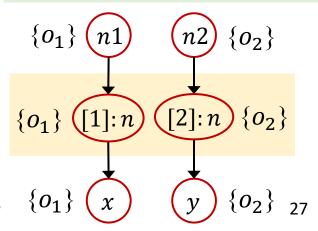
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Context-Sensitive Heap

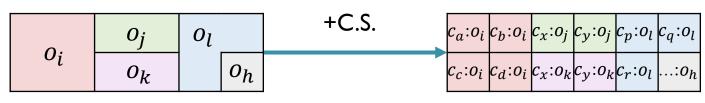
- OO programs (e.g., Java) are typically heap-intensive
- In practice, to improve precision, context sensitivity should also be applied to heap abstraction
 - The abstract objects are also qualified by contexts (called heap contexts)

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 - The abstract objects are also qualified by contexts (called heap contexts)
 The most common choice is to inherit contexts from the method where the object is allocated
 - Context-sensitive heap abstraction provides a finer-grained heap model over allocation-site abstraction



Allocation-site abstraction

Context-sensitive allocation-site abstraction

Why Context-Sensitive Heap Improve Precision?

 In dynamic execution, an allocation site can create multiple objects under different calling contexts

```
X newX(...) {
    X x = new X();
    ...
    return x;
}
```

Why Context-Sensitive Heap Improve Precision?

 In dynamic execution, an allocation site can create multiple objects under different calling contexts

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X newX(Y y) {
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 Different objects (allocated by the same site) may be manipulated with different data flows, e.g., stored different values to their fields

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- Different objects (allocated by the same site) may be manipulated with different data flows, e.g., stored different values to their fields
- ➤ In pointer analysis, analyzing such code without heap contexts may lose precision by merging the data flows of different contexts to one abstract object
- In contrast, distinguishing different objects from the same allocation site by heap contexts gains precision

Context-Sensitive Heap: Example

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1 \quad n1 = new One();
2 n2 = new Two();
3 \times 1 = \text{newX}(n1);
4 \quad x2 = newX(n2);
  n = x1.f;
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C.S., no C.S. heap

Variable	Object
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n2	02

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Variable	Object
n1	o_1
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3:x	08

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Variable	Object
n1	o_1
n2	02
3:p	o_1
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x1	08
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o ₈ .f	o_1

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Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	08
x1	08
4: p	02
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o_8 .f	o_1

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n1	o_1
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x1	08
4: p	02
4:x	08
x2	<i>0</i> ₈
Field	Object
o_8 .f	<i>0</i> ₁ , <u>0</u> ₂

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4:x	o_8
x2	08
n	2
Field	Object
o_8 .f	<i>o</i> ₁ , <i>o</i> ₂

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3:p	o_1
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4: p	o_2
4:x	o_8
x2	08
n	<i>o</i> ₁ , <i>o</i> ₂
Field	Object
o_8 .f	o_1, o_2

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3:x	08
x1	o_8
4: p	o_2
4:x	o_8
x2	08
n	<i>o</i> ₁ , <i>o</i> ₂
Field	Object
o_8 .f	<i>0</i> ₁ , <u>0</u> ₂

Spurious data flow / due to lack of C.S. heap

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Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4: p	o_2
4:x	o_8
x2	o_8
n	<i>o</i> ₁ , <i>o</i> ₂
Field	Object
o_8 .f	<i>o</i> ₁ , <u>o</u> ₂

C.S. + C.S heap

Variable	Object
n1	o_1
n2	02
3:p	o_1

Spurious data flow / due to lack of C.S. heap

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Variable	Object
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3 : p	o_1
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x1	08
4: p	02
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x2	08
n	<i>0</i> ₁ , <i>0</i> ₂
Field	Object
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4:x	08
x2	08
n	<i>0</i> ₁ , <i>0</i> ₂
Field	Object
o_8 .f	<i>0</i> ₁ , <i>0</i> ₂

C.S. + C.S heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	3: <i>o</i> ₈

Object with heap context

Spurious data flow / due to lack of C.S. heap

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Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	08
x1	08
4: p	02
4:x	08
x2	08
n	0 ₁ , 0 ₂
Field	Object
$o_8.f$	<i>o</i> ₁ , <i>o</i> ₂

Spurious data flow / due to lack of C.S. heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	3: <i>o</i> ₈



```
1 \quad n1 = new One();
2 n2 = new Two();
3 x1 = newX(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
  X newX(Number p) {
  X x = new X();
   x.f = p;
     return x;
10
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	08
x1	08
4: p	02
4:x	08
x2	08
n	<i>0</i> ₁ , <i>0</i> ₂
Field	Object
o_8 .f	<i>o</i> ₁ , <i>o</i> ₂

C.S. + C.S heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
Field	Object
3:0 ₈ .f	01

Spurious data flow / due to lack of C.S. heap

```
1 \quad n1 = new One();
2 n2 = new Two();
3 \times 1 = \text{newX}(n1);
4 \quad x2 = newX(n2);
   n = x1.f;
6
   X newX(Number p) {
     X x = new X();
  x.f = p;
10
     return x;
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	08
x1	08
4: p	02
4:x	08
x2	08
n	0 ₁ , 0 ₂
Field	Object
o_8 .f	<i>o</i> ₁ , <i>o</i> ₂

C.S. + C.S heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	02
4:x	4 : <i>o</i> ₈
Field	Object
3:0 ₈ .f	o_1

Spurious data flow / due to lack of C.S. heap

```
1 \quad n1 = new One();
2 n2 = new Two();
3 \times 1 = \text{newX}(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
   X newX(Number p) {
  X x = new X();
   x.f = p;
     return x;
10
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Object
o_1
o_2
o_1
08
08
o_2
<i>o</i> ₈
<i>o</i> ₈
<i>0</i> ₁ , <i>0</i> ₂
Object
<i>o</i> ₁ , <u>o</u> ₂

Spurious data flow / due to lack of C.S. heap

Variable	Object
n1	o_1
n2	o_2
3 : p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	o_2
4:x	4: <i>o</i> ₈
x2	4: <i>o</i> ₈
Field	Object
3:0 ₈ .f	o_1
4:0 ₈ .f	02

```
1 \quad n1 = new One();
2 n2 = new Two();
3 x1 = newX(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
  X newX(Number p) {
X \times = \text{new } X();
9 x.f = p;
10
     return x;
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Variable	Object
n1	01
n2	02
3:p	o_1
3:x	08
x1	08
4: p	o_2
4:x	08
x2	08
n	<i>0</i> ₁ , <i>0</i> ₂
Field	Object
$o_8.f$	<i>o</i> ₁ , <u>o</u> ₂

Spurious data flow / due to lack of C.S. heap

Variable	Object
n1	o_1
n2	o_2
3 : p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	o_2
4:x	4: <i>o</i> ₈
x2	4: <i>o</i> ₈
n	
Field	Object
3:0 ₈ .f	o_1
4:0 ₈ .f	02

```
1 \quad n1 = new One();
2 n2 = new Two();
3 x1 = newX(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
  X newX(Number p) {
8
 X x = new X();
9 x.f = p;
10
     return x;
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	02
3 : p	o_1
3:x	08
x1	08
4: p	02
4:x	08
x2	08
n	0 ₁ , 0 ₂
Field	Object
o_8 .f	<i>0</i> ₁ , <u>0</u> ₂

Spurious data flow / due to lack of C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	02
4:x	4: <i>o</i> ₈
x2	4: <i>o</i> ₈
n	o_1
Field	Object
3:0 ₈ .f	o_1
4:0 ₈ .f	o_2

```
1 \quad n1 = new One();
2 n2 = new Two();
3 \times 1 = \text{newX}(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
   X newX(Number p) {
X \times = \text{new } X();
9 x.f = p;
10
   return x;
11 }
12 class X {
13
      Number f;
14 }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	02
3:p	o_1
3:x	08
x1	08
4: p	02
4:x	08
x2	08
n	<i>0</i> ₁ , <i>0</i> ₂
Field	Object
o_8 .f	<i>o</i> ₁ , <i>o</i> ₂

Spurious data flow / due to lack of C.S. heap

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	02
4:x	4: <i>o</i> ₈
x2	4: <i>o</i> ₈
n	o_1
Field	Object
3:0 ₈ .f	o_1
4:0 ₈ .f	02

Context-sensitive heap improves precision

```
1 \quad n1 = new One();
2 n2 = new Two();
3 \times 1 = \text{newX}(n1);
4 \quad x2 = newX(n2);
5 n = x1.f;
6
 X newX(Number p) {
X x = \text{new } X();
9 x.f = p;
10 return x;
11 }
12 class X {
13 Number f;
14 }
```

C.I. + C.S. heap

Variable	Object
n1	o_1
n2	o_2
р	o_1, o_2
X	$3:o_8, 4:o_8$
x1	$3:o_8, 4:o_8$
x2	$3:o_8, 4:o_8$
n	0 ₁ , 0 ₂
Field	Object
3:0 ₈ .f	0 ₁ , 0 ₂
4:0 ₈ .f	0 ₁ , 0 ₂

Without C.S., C.S. heap cannot improve precision

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3: <i>o</i> ₈
x1	3: <i>o</i> ₈
4: p	02
4:x	4: <i>o</i> ₈
x2	4: <i>o</i> ₈
n	o_1
Field	Object
3:0 ₈ .f	o_1
4:0 ₈ .f	02

Context-sensitive heap improves precision



- 1. Introduction
- 2. Context Sensitive Pointer Analysis: Rules
- 3. Context Sensitive Pointer Analysis: Algorithms
- 4. Context Sensitivity Variants

Domain and Notations

In context-sensitive analysis, program elements are qualified by contexts

Context: $c, c', c'' \in C$

Context-sensitive methods: $c: m \in C \times M$

Context-sensitive variables: $c: x, c': y \in C \times V$

Context-sensitive objects: $c: o_i, c': o_j \in \mathbb{C} \times \mathcal{O}$

Fields: $f,g \in F$

Instance fields: $c: o_i.f, c': o_j.g \in \mathbb{C} \times \mathcal{O} \times \mathcal{F}$

Context-sensitive pointers: $CSPointer = (C \times V) \cup (C \times O \times F)$

Points-to relations: $pt: CSPointer \rightarrow \mathcal{P}(\mathbb{C} \times O)$

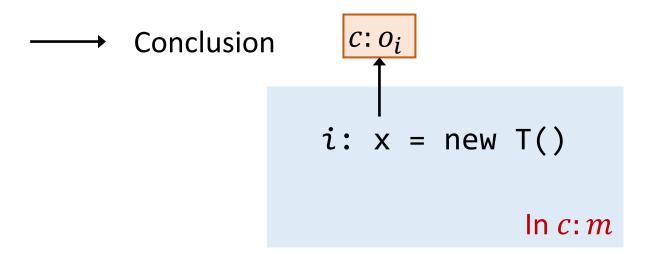
Kind	Statement	Rule (under context c)
New	i: x = new T()	$\overline{c:o_i\in pt(c:x)}$
Assign	x = y	$\frac{c': o_i \in pt(c:y)}{c': o_i \in pt(c:x)}$
Store	x.f = y	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c:y)}{c'': o_j \in pt(c':o_i.f)}$
Load	y = x.f	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c':o_i.f)}{c'': o_j \in pt(c:y)}$

Kind	Statement	Rule
New	i: x = new T()	$o_i \in pt(x)$
Assign	x = y	$\frac{o_i \in pt(y)}{o_i \in pt(x)}$
Store	x.f = y	$o_i \in pt(x), o_j \in pt(y)$ $o_j \in pt(o_i, f)$
Load	y = x.f	$o_i \in pt(x), o_j \in pt(0, f)$ $o_j \in pt(y)$

Kind	Statement	Rule (under context c)
New	i: x = new T()	$\overline{c:o_i\in pt(c:x)}$
Assign	x = y	$\frac{c': o_i \in pt(c:y)}{c': o_i \in pt(c:x)}$
Store	x.f = y	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c:y)}{c'': o_j \in pt(c':o_i.f)}$
Load	y = x.f	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c':o_i.f)}{c'': o_j \in pt(c:y)}$

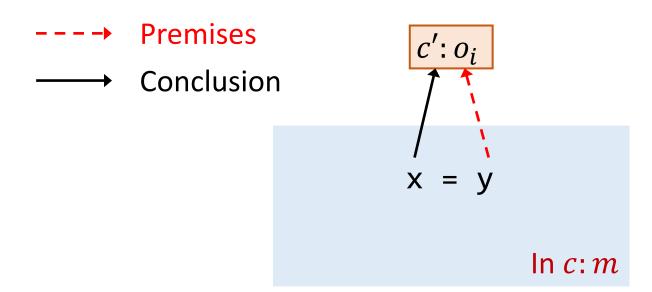
Rule: New

$$\overline{c : o_i \in pt(c : x)}$$



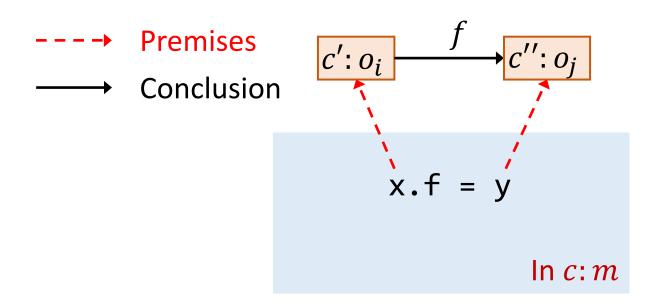
Rule: Assign

$$\frac{c':o_i \in pt(c:y)}{c':o_i \in pt(c:x)}$$



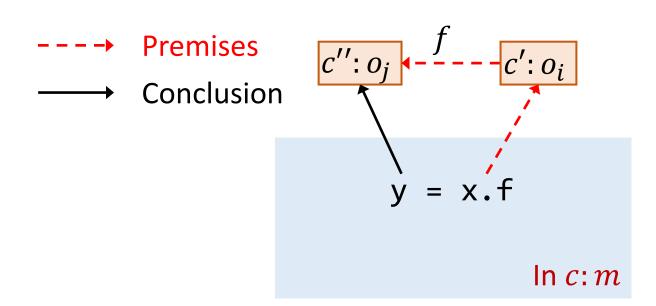
Rule: Store

$$\frac{c':o_i \in pt(c:x), c'':o_j \in pt(c:y)}{c'':o_j \in pt(c':o_i.f)}$$



Rule: Load

$$\frac{c':o_i \in pt(c:x), c'':o_j \in pt(c':o_i.f)}{c'':o_j \in pt(c:y)}$$



---→ Premises

→ Conclusion

Kind	Kind Rule Illustra	
New	$\overline{c:o_i\in pt(c:x)}$	$c: o_i$ $i: x = new T()$
Assign	$\frac{c':o_i \in pt(c:y)}{c':o_i \in pt(c:x)}$	$c': o_i$ $x = y$
Store	$\frac{c':o_i \in pt(c:x), c'':o_j \in pt(c:y)}{c'':o_j \in pt(c':o_i.f)}$	$c': o_i \xrightarrow{f} c'': o_j$ $x.f = y$
Load	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c':o_i.f)}{c'': o_j \in pt(c:y)}$	$c'': o_j c': o_i$ $y = x.f$

Caller context: c

Rule: Call

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_i \in pt(c:x),$ $m = \text{Dispatch}(o_i, \mathbf{k}), c^t = \text{Select}(c, l, c': o_i)$ $c'': o_u \in pt(c: aj), 1 \leq j \leq n$ $c''': o_v \in pt(c^t: m_{ret})$ $c': o_i \in pt(c^t: m_{this})$ $c'': o_u \in pt(c^t: m_{pj}), 1 \leq j \leq n$ $c''': o_v \in pt(c:r)$

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c:aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c:r)$

ightharpoonup Dispatch(o_i , k): resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)

Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_i \in pt(c:x),$ $m = \text{Dispatch}(o_i, k), c^t = \text{Select}(c, l, c': o_i)$ $c'': o_u \in pt(c: aj), 1 \leq j \leq n$ $c''': o_v \in pt(c^t: m_{ret})$ $c': o_i \in pt(c^t: m_{this})$ $c'': o_u \in pt(c^t: m_{pj}), 1 \leq j \leq n$ $c''': o_v \in pt(c: r)$

- Dispatch (o_i, k) : resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- ightharpoonup Select($c, l, c': o_i$): selects context for target method m, based on the information available at call site l

Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c: aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c: r)$

Callee context: 2

```
Caller context: c

1 ...
2 x = id(n1);
3 y = id(n2);
4 ...

Select(...) = 3

Number id(Number n) {
    return n;
}

Callee context: 3

Number id(Number n) {
    return n;
}
```

Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c: aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c: r)$

- Dispatch (o_i, k) : resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- Select $(c, l, c'; o_i)$: selects context for target method m, based on the information available at call site l
- $\succ c^t$: m_{this} : this variable of c^t : m

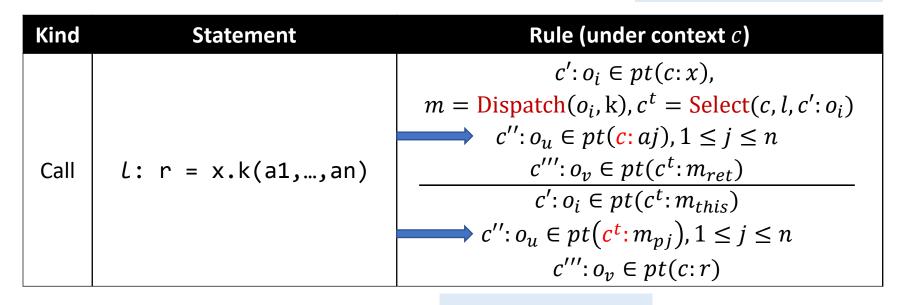
Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c: aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c: r)$

- Dispatch (o_i, k) : resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- Select $(c, l, c'; o_i)$: selects context for target method m, based on the information available at call site l
- c^t : m_{this} : this variable of c^t : m
- $ightharpoonup c^t$: the *j*-th parameter of c^t : m

Rule: Call

Caller context: cCallee context: c^t



Callee context: 2

Caller context: c

1 ...
2 x = id(n1);
3 y = id(n2);
4 ...

Select(...) = 3

Number id(Number n) {
 return n;
 }

Callee context: 3

Number id(Number n) {
 return n;
 }

Number id(Number n) {
 return n;
 }

Rule: Call

Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c:aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c:r)$

- Dispatch (o_i, k) : resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- Select $(c, l, c': o_i)$: selects context for target method m, based on the information available at call site l
- c^t : m_{this} : this variable of c^t : m
- c^t : m_{pj} : the *j*-th parameter of c^t : m
- $ightharpoonup c^t$: m_{ret} : the variable that holds the return value of c^t : m

Rule: Call

Caller context: cCallee context: c^t

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c: aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c: r)$

Callee context: 2

```
Caller context: c

1 ...
2 x = id(n1);
3 y = id(n2);
4 ...

Select(...) = 3

Number id(Number n) {
    return n
}

Callee context: 3

Number id(Number n) {
    return n
}
```

The X You Need To Understand in This Lecture

- Concept of context sensitivity (C.S.)
- Concept of context-sensitive heap (C.S. heap)
- Why C.S. and C.S. heap improve precision
- Context-sensitive pointer analysis rules

注意注意! 划重点了!



Static Program Analysis

Pointer Analysis Context Sensitivity (II)

Nanjing University

Tian Tan

2021

Review: C.S. Pointer Analysis Rules

Kind	Statement	Rule (under context c)
New	i: x = new T()	$\overline{c:o_i\in pt(c:x)}$
Assign	x = y	$\frac{c': o_i \in pt(c:y)}{c': o_i \in pt(c:x)}$
Store	x.f = y	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c:y)}{c'': o_j \in pt(c':o_i.f)}$
Load	y = x.f	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(c':o_i.f)}{c'': o_j \in pt(c:y)}$
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(c: aj), 1 \leq j \leq n$ $c''': o_{v} \in pt(c^{t}: m_{ret})$ $c': o_{i} \in pt(c^{t}: m_{this})$ $c'': o_{u} \in pt(c^{t}: m_{pj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(c: r)$



- 1. Introduction
- 2. Context Sensitive Pointer Analysis: Rules
- 3. Context Sensitive Pointer Analysis: Algorithms
- 4. Context Sensitivity Variants

How to Implement Context-Sensitive Pointer Analysis

Recall context-insensitive pointer analysis

1. Build pointer flow graph (PFG)

Mutually dependent

2. Propagate points-to information on PFG

How to Implement Context-Sensitive Pointer Analysis

Recall context-insensitive pointer analysis

1. Build pointer flow graph (PFG)
Mutually dependent
2. Propagate points-to information on PFG

Context-sensitive pointer analysis

Build pointer flow graph (PFG with C.S.)
 Mutually dependent
 Propagate points-to information on PFG with C.S.

Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

• Nodes: CSPointer = $(C \times V) \cup (C \times O \times F)$

A node *n* represents a <u>context-sensitive</u> variable or a <u>field of</u> a <u>context-sensitive</u> abstract object

With C.S., the nodes (pointers) are qualified by contexts

Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

• Nodes: CSPointer = $(C \times V) \cup (C \times O \times F)$

A node *n* represents a <u>context-sensitive</u> variable or a <u>field of</u> a <u>context-sensitive</u> abstract object

With C.S., the nodes (pointers) are qualified by contexts

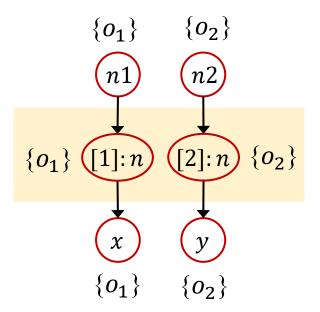
• Edges: CSPointer × CSPointer

An edge $x \rightarrow y$ means that the objects pointed by pointer x may flow to (and also be pointed to by) pointer y

e.g., edge $c: a \to c': b$ means that objects in pt(c: a) may flow to pt(c': b)

An Example

```
1 x = id(n1);
2 y = id(n2);
3 int i = x.get();
4
5 Number id(Number n) {
6  return n;
7 }
```



The PFG contains two nodes for variable n in method id(), one node per context

Pointer Flow Graph: Edges

 PFG edges are added according to the statements of the program and the corresponding rules

Kind	Statement	Rule (under context c)	PFG Edge
New	i: x = new T()	$\overline{c:o_i\in pt(c:x)}$	N/A
Assign	x = y	$\frac{c': o_i \in pt(\boldsymbol{c}: \boldsymbol{y})}{c': o_i \in pt(\boldsymbol{c}: \boldsymbol{x})}$	$c: x \leftarrow c: y$
Store	x.f = y	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(\boldsymbol{c}:\boldsymbol{y})}{c'': o_j \in pt(\boldsymbol{c}': \boldsymbol{o_i}.\boldsymbol{f})}$	$c': o_i.f \leftarrow c: y$
Load	y = x.f	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(\mathbf{c}': \mathbf{o_i}. \mathbf{f})}{c'': o_j \in pt(\mathbf{c}: \mathbf{y})}$	$c: y \leftarrow c': o_i. f$

Pointer Flow Graph: Call

Kind	Statement	Rule (under context c)
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, \mathbf{k}), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(\mathbf{c}: \mathbf{aj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(\mathbf{c}^{t}: \mathbf{m_{ret}})$ $c': o_{i} \in pt(c^{t}: \mathbf{m_{this}})$ $c'': o_{u} \in pt(\mathbf{c}^{t}: \mathbf{m_{pj}}), 1 \leq j \leq n$ $c''': o_{v} \in pt(\mathbf{c}: \mathbf{r})$

Caller context: c

Callee context: c^t

```
C x = new T();
...
r = x.foo(a1, a2);

class T ... { ...
B foo(A p1, A p2) {
    this...
    return ret;
}}
```

PFG Edge

 $c: a1 \rightarrow c^t: m^{p1}$... $c: an \rightarrow c^t: m_{pn}$ $c: r \leftarrow c^t: m_{ret}$

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           for each x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.S. reachable methods CG C.S. call graph edges
```

```
AddReachable(c:m)
   if c: m \notin RM then
      add c: m to RM
     S \cup = S_m
      foreach i: x = \text{new } T() \in S_m \text{ do}
         add \langle c: x, \{c: o_i\} \rangle to WL
      foreach x = y \in S_m do
        AddEdge(c: y, c: x)
ProcessCall(c: x, c': o_i)
   foreach l: r = x.k(a1,...,an) \in S do
     m = Dispatch(o_i, k)
     c^t = \text{Select}(c, l, c': o_i)
      add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
     if c: l \rightarrow c^t: m \notin CG then
         add c: l \to c^t: m to CG
        AddReachable(c^t: m)
         foreach parameter p_i of m do
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
```

```
Solve(m^{entry})
  WL=[], PFG=\{\}, S=\{\}, RM=\{\}, CG=\{\}\}
  AddReachable m^{entry}
  while WL is not empty do
     remove \langle n, pts \rangle from WL
    \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable
                               x then
       foreach o_i \in \Delta do
          foreach x.f = y \in S do
            AddEdge(y, o_i. f)
          foreach y = x.f \in S do
            AddEdge(o_i. f, y)
          ProcessCall(x, o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.I. reachable methods CG C.I. call graph edges
```

```
AddReachable( m)
  if m \notin RM then
     add m to RM
     S \cup = S_m
     foreach i: x = \text{new } T() \in S_m \text{ do}
       add \langle x, \{o_i\} \rangle to WL
     foreach x = y \in S_m do
       AddEdge(y, x)
ProcessCall(x, o_i)
  foreach l: r = x.k(a1,...,an) \in S do
     m = Dispatch(o_i, k)
     add \langle m_{this}, \{ o_i \} \rangle to WL
     if l \rightarrow m \notin CG then
       add l \rightarrow m to CG
       AddReachable(:m)
       foreach parameter p_i of m do
          AddEdge(a_i, p_i)
       AddEdge(m_{ret}, r)
```

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.S. reachable methods CG C.S. call graph edges
```

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.S. reachable methods CG C.S. call graph edges
```

Callee context: c^t

```
class T ... { ...
  B foo(A p1, A p2) {
    this...
    return ret;
  }}
```

 c^t : T. foo(A, A) $\in RM$

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.S. reachable methods CG C.S. call graph edges
```

```
c^t: T. foo(A, A) \in RM

c: 2 \rightarrow c^t: T. foo(A, A) \in CG
```

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
AddReachable(c:m)

if c:m \notin RM then

add c:m to RM

S \cup = S_m

foreach i: \times = \text{new } T() \in S_m do

add \langle c: x, \{c: o_i\} \rangle to WL

foreach \times = y \in S_m do

AddEdge(c: y, c: x)
```

Kind	Statement	Rule (under context c)	PFG Edge
New	i: x = new T()	$\overline{c:o_i\in pt(c:x)}$	N/A
Assign	x = y	$\frac{c': o_i \in pt(\boldsymbol{c}: \boldsymbol{y})}{c': o_i \in pt(\boldsymbol{c}: \boldsymbol{x})}$	$c: x \leftarrow c: y$

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
AddEdge(s, t)
   if s \rightarrow t \notin PFG then
      add s \rightarrow t to PFG
      if pt(s) is not empty then
         add \langle t, pt(s) \rangle to WL
Propagate(n, pts)
   if pts is not empty then
      pt(n) \bigcup = pts
      foreach n \rightarrow s \in PFG do
         add \langle s, pts \rangle to WL
```

S Set of reachable statements S_m Set of statements in method m RM Set of **C.S.** reachable methods CG **C.S.** call graph edges

Exactly same as in C.I. analysis

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
AddEdge(s, t)

if s \to t \notin PFG then

add s \to t to PFG

if pt(s) is not empty then

add \langle t, pt(s) \rangle to WL

Propagate(n, pts)

if pts is not empty then

pt(n) \cup = pts

foreach n \to s \in PFG do

add \langle s, pts \rangle to WL
```

Kind	Statement	Rule (under context c)	PFG Edge
Store	x.f = y	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(\boldsymbol{c}:\boldsymbol{y})}{c'': o_j \in pt(\boldsymbol{c}': \boldsymbol{o_i}.\boldsymbol{f})}$	$c': o_i.f \leftarrow c: y$
Load	y = x.f	$\frac{c': o_i \in pt(c:x), c'': o_j \in pt(\mathbf{c}': \mathbf{o_i}.\mathbf{f})}{c'': o_j \in pt(\mathbf{c}:\mathbf{y})}$	$c: y \leftarrow c': o_i. f$

```
Solve(m^{entry})
  WL=[], PFG=\{\}, S=\{\}, RM=\{\}, CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
ProcessCall(c: x, c': o_i)

foreach L: r = x. k(a1,...,an) \in S do

m = \text{Dispatch}(o_i, k)

c^t = \text{Select}(c, l, c' : o_i)

\text{add } \langle c^t : m_{this}, \{c' : o_i\} \rangle \text{ to } WL

if c: l \to c^t : m \notin CG then

\text{add } c: l \to c^t : m \text{ to } CG

AddReachable(c^t : m)

foreach parameter p_i of m do

AddEdge(c: a_i, c^t : p_i)

AddEdge(c^t : m_{ret}, c: r)
```

Kind	Statement	Rule (under context c)	PFG Edge
Call	l: r = x.k(a1,,an)	$c': o_{i} \in pt(c:x),$ $m = \text{Dispatch}(o_{i}, k), c^{t} = \text{Select}(c, l, c': o_{i})$ $c'': o_{u} \in pt(\boldsymbol{c}: \boldsymbol{aj}), 1 \leq j \leq n$ $c''': o_{v} \in pt(\boldsymbol{c}^{t}: \boldsymbol{m_{ret}})$ $c': o_{i} \in pt(c^{t}: \boldsymbol{m_{this}})$ $c'': o_{u} \in pt(\boldsymbol{c^{t}}: \boldsymbol{m_{pj}}), 1 \leq j \leq n$ $c''': o_{v} \in pt(\boldsymbol{c}: \boldsymbol{r})$	$c: a1 \rightarrow c^{t}: m^{p1}$ $c: an \rightarrow c^{t}: m_{pn}$ $c: r \leftarrow c^{t}: m_{ret}$

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           for each x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
S Set of reachable statements S_m Set of statements in method m RM Set of C.S. reachable methods CG C.S. call graph edges
```

```
AddReachable(c:m)
   if c: m \notin RM then
      add c: m to RM
      S \cup = S_m
      foreach i: x = \text{new } T() \in S_m \text{ do}
        add \langle c: x, \{c: o_i\} \rangle to WL
      foreach x = y \in S_m do
        AddEdge(c: y, c: x)
ProcessCall(c: x, c': o_i)
   foreach l: r = x.k(a1,...,an) \in S do
     m = Dispatch(o_i, k)
     c^t = \text{Select}(c, l, c': o_i)
      add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
     if c: l \to c^t: m \notin CG then
         add c: l \to c^t: m to CG
        AddReachable(c^t: m)
        foreach parameter p_i of m do
           AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t; m_{ret}, c; r)
```



- 1. Introduction
- 2. Context Sensitive Pointer Analysis: Rules
- 3. Context Sensitive Pointer Analysis: Algorithms
- 4. Context Sensitivity Variants

Context Sensitivity Variants

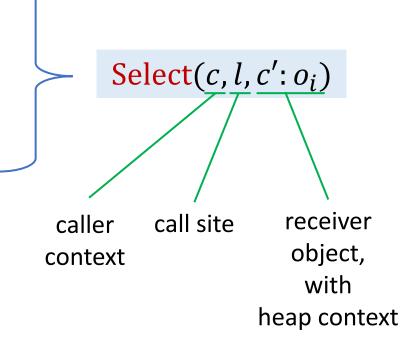
- Call-site sensitivity
- Object sensitivity
- Type sensitivity

•

Context Sensitivity Variants

- Call-site sensitivity
- Object sensitivity
- Type sensitivity

•



Context Sensitivity Variants

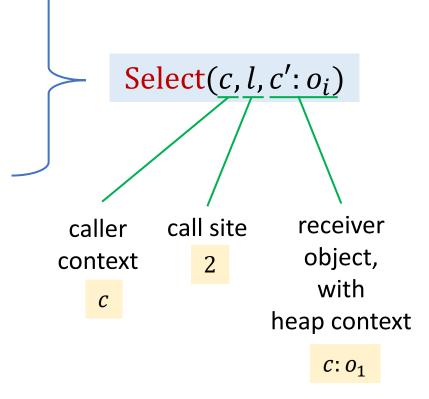
- Call-site sensitivity
- Object sensitivity
- Type sensitivity
- •

```
Caller context: c
```

```
1 C x = new T();
2 r = x.foo(a1, a2);
```

Callee context: c^t

```
class T ... {
    B foo(A p1, A p2) {
     this...
    return ret;
    }}
```



Context Insensitivity

 Can be seen as a special case of context sensitivity in C.S. analysis framework

Call-Site Sensitivity*

- Each context consists of a list of call sites (call chain)
 - At a method call, append the call site to the caller context as callee context
 - Essentially the abstraction of call stacks

```
\begin{aligned} \text{Select}(\boldsymbol{c}, \boldsymbol{l}, \underline{\ }) &= [\boldsymbol{l'}, \dots, \boldsymbol{l''}, \boldsymbol{l}] \\ \text{where } c &= [\boldsymbol{l'}, \dots, \boldsymbol{l''}] \end{aligned}
```

Also called call-string sensitivity, or *k*-CFA

* Olin Shivers, 1991. "Control-Flow Analysis of Higher-Order Languages". Ph.D. Dissertation. Carnegie Mellon University.

```
void main() {
2
     a.foo();
4
     •••
5
6
   void foo() {
8
     b.bar();
9
10
11 }
12
13 void bar() {
14
15
16
17 }
```

```
void main() {    Context:[]
2
     a.foo();
4
     •••
5
6
   void foo() {
8
     b.bar();
9
10
11 }
12
13 void bar() {
14
15
16
17 }
```

```
void main() {
                    Context: []
2
     a.foo();
4
5
6
                    Context: [3]
   void foo() {
8
     b.bar();
9
10
11 }
12
13 void bar() {
14
15
16
17 }
```

```
void main() {
                    Context: []
2
     a.foo();
4
5
6
                     Context: [3]
   void foo() {
8
     b.bar();
9
10
11 }
12
                     Context: [3,9]
13 void bar() {
14
15
16
17 }
```

```
void main() {
                    Context: []
     a.foo();
4
5
6
                    Context: [3]
   void foo() {
8
     b.bar();
9
10
11 }
12
                    Context: [3,9]
13 void bar() {
14
     bar(); ?
15
16
17 }
```

Call-Site Sensitivity: Example

```
void main() {
                     Context: []
2
     a.foo();
4
5
6
   void foo() {
                      Context: [3]
8
      b.bar();
9
10
11 }
12
                      Context: [3,9]
13 void bar()
14
                               [3,9,15]
     bar();
15
                               [3,9,15,15]
16
                               [3,9,15,15,15]
17 }
                               [3,9,15,15,15, ...]
```

k-Limiting Context Abstraction

- Motivation
 - Ensure termination of pointer analysis
 - Avoid too many contexts (long call chains) in real-world programs blow up the pointer analysis

k-Limiting Context Abstraction

- Motivation
 - Ensure termination of pointer analysis
 - Avoid too many contexts (long call chains) in real-world programs blow up the pointer analysis
- Approach: set an upper bound for length of contexts, denoted by k
 - For call-site sensitivity, each context consists of the last k
 call sites of the call chains
 - In practice, k is a small number (usually ≤3)
 - Method contexts and heap contexts may use different k
 - e.g., k=2 for method contexts, k=1 for heap contexts

• 1-call-site/1-CFA

$$Select(_, l, _) = [l]$$

```
• 1-call-site/1-CFA
                                           void main() {
                                                             Context: []
                                             a.foo();
          Select(\_, l, \_) = [l]
                                                             Context: [3]
                                           void foo() {
                                             b.bar();
                                        10
                                        11
                                        12
                                                             Context: ???
                                        13 void bar()
                                        14
                                             bar();
                                        15
                                        16
```

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```
• 1-call-site/1-CFA
                                            void main() {
                                                             Context: []
          Select(\_, l, \_) = [l]
                                              a.foo();
                                                             Context: [3]
                                            void foo() {
                                              b.bar();
                                        10
                                        11
                                        12
                                                             Context: [9]
                                        13 void bar()
                                        14
                                                                      [15]
                                              bar();
                                        15
                                        16
```

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• 1-call-site/1-CFA

$$Select(_, l, _) = [l]$$

• 2-call-site/2-CFA

```
Select(c, l, _) = [l'', l]
where c = [l', l'']
```

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```
Solve(m^{entry})
   WL=[],PFG={},S={},RM={},CG={}
   AddReachable([]: m^{entry})
   while WL is not empty do
      remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
      Propagate(n, \Delta)
      if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
            AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
            AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
1 class C {
                                 void m() {
                            10
    static void main() {
                           11
                                    Number n1, n2, x, y;
      C c = new C();
                       13 n2 = new Two();
   c.m();
                            14 x = this.id(n1);
                                y = this.id(n2);
                            15
    Number id(Number n) { 16
                                x.get();
      return n;
                            17
                            18 }
```

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
interface Number {
  int get(); }
class One implements Number {
  public int get() { return 1; }}
class Two implements Number {
  public int get() { return 2; }}
```

```
void m() {
1 class C {
                        10
                       11
   static void main() {
                               Number n1, n2, x,
                   12 	 n1 = new One();
     C c = new C();
                        13 n2 = new Two();
   c.m();
                        14 x = this.id(n1);
                        15
                           y = this.id(n2);
   Number id(Number n) { 16
                           x.get();
     return n;
                        17
                        18 }
```

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
interface Number {
  int get(); }
class One implements Number {
  public int get() { return 1; }}
class Two implements Number {
  public int get() { return 2; }}
```

```
void m() {
1 class C {
                      10
   static void main() { 11
                            Number n1, n2, x,
                  C c = new C();
                      13 n2 = new Two();
   c.m();
                      14 x = this.id(n1);
                         y = this.id(n2);
                      15
   Number id(Number n) { 16
                            x.get(); >
     return n;
                      17
                      18 }
```

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
     Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
          ProcessCall(c: x, c': o_i)
```

```
interface Number {
  int get(); }
class One implements Number {
  public int get() { return 1; }}
class Two implements Number {
  public int get() { return 2; }}
```

```
1 class C {
                              void m() {
                         10
   static void main() {
                         11
                                Number n1, n2, x, y
                    12
     C c = new C();
                                n1 = new One();
   c.m();
                         13 n2 = new Two();
                                x = this.id(n1);
                         14
                                y = this.id(n2);
                         15
   Number id(Number n) { 16
                                x.get(); >
     return n;
                         17
                         18 }
```

For simplicity, we do not apply C.S. heap and omit this variable of C.id(Number)

```
1-Call-Site: Example
```

WL: []

```
Solve(m^{entry})
  >WL=[],PFG={},S={},RM={},CG={}
                                                 Processing:
   AddReachable([]: m^{entry})
   while WL is not empty do
      remove \langle n, pts \rangle from WL
      \Delta = pts - pt(n)
                                             PFG:
      Propagate(n, \Delta)
      if n represents a variable c: x then
        foreach c': o_i \in \Delta do
          foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
          foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
                                                    CG: {}
          ProcessCall(c: x, c': o_i)
1 class C {
                                   void m() {
                             10
    static void main() {
                             11
                                     Number n1, n2, x, y;
                        12
      C c = new C();
                                     n1 = new One();
                             13 n2 = new Two();
    c.m();
                                     x = this.id(n1);
                                                         RM: \{\}
                             14
                                 y = this.id(n2);
                             15
    Number id(Number n) { 16
                                 x.get();
      return n;
                             17
                             18 }
```

```
1-Call-Site: Example
```

return n;

WL: []

```
Solve(m^{entry})
    WL=[],PFG={},S={},RM={},CG={}
                                                 Processing:
  \rightarrowAddReachable([]: m^{entry})
   AddReachable(c:m)
     if c: m \notin RM then
                                             PFG:
        add c: m to RM
       S \cup = S_m
       foreach i: x = \text{new } T() \in S_m \text{ do}
          add \langle c: x, \{c: o_i\} \rangle to WL
       foreach x = y \in S_m do
          AddEdge(c: y, c: x)
                                                    CG: {}
1 class C {
                                   void m() {
                             10
    static void main() {
                             11
                                      Number n1, n2, x, y;
      C c = new C();
                             n1 = new One();
                             13 n2 = new Two();
      c.m();
                             14 x = this.id(n1);
                                                          RM: \{\}
                             15
                                 y = this.id(n2);
    Number id(Number n) {
                                  x.get();
                             16
```

17

18 }

WL: []

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
                                                        Processing:
  AddReachable([]: m^{entry})
 AddReachable(c:m)
    if c: m \notin RM then
                                                   PFG:
   \Longrightarrow add c: m to RM
       S \cup = S_m
       foreach i: x = \text{new } T() \in S_m \text{ do}
          add \langle c: x, \{c: o_i\} \rangle to WL
       foreach x = y \in S_m do
          AddEdge(c: y, c: x)
                                                           CG: {}
```

```
x,y;
```

```
1 class C {
                             void m() {
                         10
   static void main() {
                         11
                                Number n1, n2, x, y;
     C c = new C();
                         n1 = new One();
                         13 n2 = new Two();
     c.m();
                         14 x = this.id(n1);
                            y = this.id(n2);
                         15
   Number id(Number n) {
                            x.get();
                         16
     return n;
                         17
                         18 }
```

```
WL: [\langle []:c, \{o_3\}\rangle]
```

```
Solve(m^{entry})
  WL=[],PFG={},S={},RM={},CG={}
  AddReachable([]: m^{entry})
  AddReachable(c:m)
    if c: m \notin RM then
       add c: m to RM
       S \cup = S_m
       foreach i: x = \text{new } T() \in S_m \text{ do}
     \longrightarrow add \langle c: x, \{c: o_i\} \rangle to WL
       foreach x = y \in S_m do
          AddEdge(c: y, c: x)
```

Processing:

PFG:

CG: {}

```
1 class C {
                               void m() {
                          10
   static void main() {
                          11
                                 Number n1, n2, x, y;
   \Longrightarrow C c = new C();
                          n1 = new One();
                          13 n2 = new Two();
     c.m();
                          14 x = this.id(n1);
                          15
                              y = this.id(n2);
   Number id(Number n) {
                              x.get();
                          16
      return n;
                          17
                          18 }
```

```
WL:
```

```
Solve(m^{entry})
    WL=[],PFG={},S={},RM={},CG={}
                                                   Processing: \langle [ ]:c, \{o_3\} \rangle
   AddReachable([]: m^{entry})
    while WL is not empty do
    \Rightarrow remove \langle n, pts \rangle from WL
      \Delta = pts - pt(n)
                                               PFG:
      Propagate(n, \Delta)
      if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
                                                      CG: {}
           ProcessCall(c: x, c': o_i)
1 class C {
                                    void m() {
                               10
    static void main() {
                              11
                                       Number n1, n2, x, y;
      C c = new C();
                              12
                                       n1 = new One();
                               13 n2 = new Two();
    c.m();
                                                            RM: { []:C.main() }
                                       x = this.id(n1);
                               14
                                   y = this.id(n2);
                               15
    Number id(Number n) {
                                   x.get();
                              16
       return n;
                               17
                              18 }
```

```
WL: []
```

```
Solve(m^{entry})
   WL=[],PFG={},S={},RM={},CG={}
                                                           Processing: \langle [ ]:c, \{o_3\} \rangle
  AddReachable([]: m^{entry})
  while WL is not empty do
     remove \langle n, pts \rangle from WL
                                                                  {0<sub>3</sub>}
     \Delta = pts - pt(n)
                                                     PFG:
    \triangleright Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           foreach x.f = y \in S do
              AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
              AddEdge(c': o_i. f, c: y)
                                                               CG: {}
           ProcessCall(c: x, c': o_i)
```

```
1 class C {
                              void m() {
                         10
   static void main() {
                         11
                                Number n1, n2, x, y;
     C c = new C();
                         12
                               n1 = new One();
                         13 n2 = new Two();
   c.m();
                               x = this.id(n1);
                         14
                            y = this.id(n2);
                         15
   Number id(Number n) {
                            x.get();
                         16
     return n;
                         17
                         18 }
```

```
WL: []
```

```
ProcessCall(c: x, c': o_i)
   foreach l: r = x.k(a1,...,an) \in S do
     m = Dispatch(o_i, k)
     c^t = \text{Select}(c, l, c': o_i)
     add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
     if c: l \rightarrow c^t: m \notin CG then
         add c: l \to c^t: m to CG
        AddReachable(c^t: m)
         foreach parameter p_i of m do
            AddEdge(c: a_i, c^t: p_i)
        AddEdge(c^t: m_{ret}, c: r)
    \longrightarrow ProcessCall(c: x, c': o_i)
```

```
Processing: \langle [ ] : c, \{o_3\} \rangle
\{o_3\}
PFG: [ ] : c
```

```
ProcessCall(c: x, c': o_i)
```

```
CG: {}
```

```
1 class C {
                               void m() {
                          10
   static void main() {
                          11
                                 Number n1, n2, x, y;
   C c = new C();
                     12 	 n1 = new One();
                          13 n2 = new Two();
4 \longrightarrow c.m();
                          14 x = this.id(n1);
                          15
                             y = this.id(n2);
   Number id(Number n) {
                             x.get();
                          16
     return n;
                          17
                          18 }
```

```
WL:
```

```
ProcessCall(c: x, c': o_i)
  foreach l: r = x.k(a1,...,an) \in S do
     m = \text{Dispatch}(o_i, k) m = \text{C.m}()

ightharpoonup c^t = Select(c, l, c': o_i) c^t = ?
     add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                    PFG:
     if c: l \to c^t: m \notin CG then
        add c: l \to c^t: m to CG
        AddReachable(c^t: m)
        foreach parameter p_i of m do
           AddEdge(c: a_i, c^t: p_i)
        AddEdge(c^t: m_{ret}, c: r)
           ProcessCall(c: x, c': o_i)
```

```
Processing: \langle [ ]:c, \{o_3\} \rangle
         \{o_3\}
```

CG: {}

```
1 class C {
                              void m() {
                         10
   static void main() {
                         11
                                Number n1, n2, x, y;
  C c = new C();
                    12 	 n1 = new One();
                         13 n2 = new Two();
4 \longrightarrow c.m();
                         14 x = this.id(n1);
                            y = this.id(n2);
                         15
   Number id(Number n) { 16
                             x.get();
     return n;
                         17
                         18 }
```

```
RM: { []:C.main() }
```

```
WL: []
```

```
ProcessCall(c: x, c': o_i)
  foreach l: r = x.k(a1,...,an) \in S do
     m = \text{Dispatch}(o_i, k) m = \text{C.m}()
 \Rightarrow c^t = \text{Select}(c, l, c': o_i) c^t = [4]
     add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                     PFG:
     if c: l \to c^t: m \notin CG then
        add c: l \to c^t: m to CG
        AddReachable(c^t: m)
        foreach parameter p_i of m do
           AddEdge(c: a_i, c^t: p_i)
        AddEdge(c^t: m_{ret}, c: r)
           ProcessCall(c: x, c': o_i)
```

```
Processing: \langle [ ] : c, \{o_3\} \rangle
\{o_3\}
G: [ ] : c
```

CG: {}

```
1 class C {
                              void m() {
                         10
   static void main() {
                         11
                                Number n1, n2, x, y;
  C c = new C();
                     12 	 n1 = new One();
                         13 n2 = new Two();
4 \longrightarrow c.m();
                         14 x = this.id(n1);
                         15
                            y = this.id(n2);
   Number id(Number n) { 16
                             x.get();
     return n;
                         17
                         18 }
```

ProcessCall($c: x, c': o_i$)

```
WL: [\langle [4]:C.m_{this}, \{o_3\}\rangle]
                                                                 Processing: \langle [ ]:c, \{o_3\} \rangle
foreach l: r = x.k(a1,...,an) \in S do
   m = \text{Dispatch}(o_i, k) m = \text{C.m}()
   c^t = \text{Select}(c, l, c': o_i) c^t = [4]
                                                                         \{o_3\}
\Rightarrow add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                           PFG:
```

AddReachable(c^t : m) **foreach** parameter p_i of m do AddEdge($c: a_i, c^t: p_i$) AddEdge(c^t : m_{ret} , c: r)

if $c: l \rightarrow c^t: m \notin CG$ then

add $c: l \to c^t$: m to CG

ProcessCall($c: x, c': o_i$)

```
CG: {}
```

```
1 class C {
                              void m() {
                          10
   static void main() {
                         11
                                 Number n1, n2, x, y;
  C c = new C();
                     12 	 n1 = new One();
                          13 n2 = new Two();
4 \longrightarrow c.m();
                          14 x = this.id(n1);
                             y = this.id(n2);
                          15
6
   Number id(Number n) { 16
                             x.get();
     return n;
                          17
                          18 }
```

```
ProcessCall(c: x, c': o_i)
foreach l: r = x.k(a1,...,an) \in S do

m = Dispatch(o_i, k)
m = C.m()
c^t = Select(c, l, c': o_i)
c^t = [4]
add c^t: m_{this}, \{c': o_i\} \} to WL

if c: l \rightarrow c^t: m \notin CG then

add c: l \rightarrow c^t: m to CG

AddReachable(c^t: m)
foreach parameter p_i of m do

AddEdge(c: a_i, c^t: p_i)
```

ProcessCall($c: x, c': o_i$)

AddEdge(c^t : m_{ret} , c: r)

```
CG: \{ [ ]:4 \rightarrow [4]:C.m() \}
```

 $[\langle [4]:C.m_{this}, \{o_3\}\rangle]$

WL:

```
1 class C {
                            void m() {
                        10
   static void main() {
                        11
                              Number n1, n2, x, y;
  C c = new C();
                   12 	 n1 = new One();
                        13 n2 = new Two();
4 c.m();
                        14 x = this.id(n1);
                           y = this.id(n2);
                        15
6
   Number id(Number n) { 16
                           x.get();
     return n;
                        17
                        18 }
```

```
ProcessCall(c: x, c': o_i)
   foreach l: r = x.k(a1,...,an) \in S do
     m = \text{Dispatch}(o_i, k) m = \text{C.m}()
     c^t = \text{Select}(c, l, c': o_i) c^t = [4]
      add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
     if c: l \rightarrow c^t: m \notin CG then
         add c: l \to c^t: m to CG
  \longrightarrow AddReachable(c^t: m)
        foreach parameter p_i of m do
           AddEdge(c: a_i, c^t: p_i)
        AddEdge(c^t: m_{ret}, c: r)
           ProcessCall(c: x, c': o_i)
```

```
Processing: \langle [ ]:c, \{o_3\} \rangle
           AddReachable(c: m)
PFG:
              if c: m \notin RM then
              \Rightarrow add c: m to RM
                 S \cup = S_m
                 foreach i: x = \text{new } T() \in S_m \text{ do}
            \longrightarrow add \langle c: x, \{c: o_i\} \rangle to WL
                 foreach x = y \in S_m do
                    AddEdge(c: y, c: x)
```

 $[\langle [4]:C.m_{this}, \{o_3\}\rangle,$

 $\langle [4]:n1, \{o_{12}\} \rangle, \langle [4]:n2, \{o_{13}\} \rangle]$

WL:

```
void m() {
1 class C {
                             10
    static void main() {
                             11
                                     Number n1, n2, x, y;
      C c = new C();
                             12 \longrightarrow n1 = new One();
                                  \rightarrow n2 = new Two();
4 c.m();
                             13
                                     x = this.id(n1);
                             14
                             15
                                     y = this.id(n2);
    Number id(Number n) {
                                    x.get();
                             16
                             17
      return n;
                             18 }
```

RM: { []:C.main(), [4]:C.m() }

AddReachable([]: m^{entry})

while WL is not empty do

 $\Delta = pts - pt(n)$

 \triangleright Propagate (n, Δ)

remove $\langle n, pts \rangle$ from WL

foreach $c': o_i \in \Delta$ do

Solve (m^{entry})

```
WL:
                                                                     [\langle [4]:n1, \{o_{12}\}\rangle, \langle [4]:n2, \{o_{13}\}\rangle]
WL=[],PFG={},S={},RM={},CG={}
                                                           Processing: \langle [4]: C.m_{this}, \{o_3\} \rangle
                                                                  \{o_3\}
                                                     PFG:
  if n represents a variable c: x then
                                                                  {0<sub>3</sub>}
         foreach x.f = y \in S do
                                                         [4]:C.m_{this}
           AddEdge(c: y, c': o_i. f)
        foreach y = x.f \in S do
           AddEdge(c': o_i. f, c: y)
```

 $CG: \{ []:4 \rightarrow [4]:C.m() \}$

```
ProcessCall(c: x, c': o_i)
1 class C {
                                 void m() {
                            10
    static void main() {
                            11
                                    Number n1, n2, x, y;
      C c = new C();
                            12
                                   n1 = new One();
                                   n2 = new Two();
    c.m();
                            13
                                   x = this.id(n1);
                            14
                                   y = this.id(n2);
6
                            15
    Number id(Number n) {
                                   x.get();
                            16
      return n;
                            17
                            18 }
```

RM: { []:C.main(), [4]:C.m() }

```
ProcessCall(c: x, c': o_i)
                                                               Processing: \langle [4]: C.m_{this}, \{o_3\} \rangle
   foreach l: r = x.k(a1,...,an) \in S do
      m = Dispatch(o_i, k)
      c^t = \text{Select}(c, l, c'; o_i)
                                                                       \{o_3\}
      add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                         PFG:
      if c: l \rightarrow c^t: m \notin CG then
         add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                                      {o_3}
         foreach parameter p_i of m do
                                                             [4]:C.m_{this}
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                                   CG: \{ []:4 \rightarrow [4]:C.m() \}
    \longrightarrow ProcessCall(c: x, c': o_i)
```

WL:

```
1 class C {
                               void m() {
                          10
    static void main() {
                          11
                                  Number n1, n2, x, y;
      C c = new C();
                          n1 = new One();
                             n2 = new Two();
   c.m();
                          13
                          14 \longrightarrow x = this.id(n1);
                          15
                              y = this.id(n2);
6
   Number id(Number n) {
                          16
                              x.get();
      return n;
                          17
                          18 }
```

RM: { []:C.main(), [4]:C.m() }

6

```
WL:
                                                                  [\langle [4]:n1, \{o_{12}\} \rangle, \langle [4]:n2, \{o_{13}\} \rangle]
 ProcessCall(c: x, c': o_i)
                                                         Processing: \langle [4]:C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
   \Rightarrow c^t = \text{Select}(c, l, c'; o_i) c^t = [14]
                                                               \{o_3\}
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                    PFG:
       if c: l \to c^t: m \notin CG then
          add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                               {o_3}
          foreach parameter p_i of m do
                                                       [4]:C.m_{this}
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                             CG: \{ []:4 \rightarrow [4]:C.m() \}
            ProcessCall(c: x, c': o_i)
1 class C {
                                        void m() {
                                  10
     static void main() {
                                  11
                                           Number n1, n2, x, y;
       C c = new C();
                            12 n1 = new One();
                                  13 n2 = new Two();
    c.m();
                                                                   RM: { []:C.main(), [4]:C.m() }
                                  14 \longrightarrow x = this.id(n1);
                                      y = this.id(n2);
                                  15
     Number id(Number n) { 16
                                      x.get();
       return n;
                                  17
                                                                                                134
                                  18 }
```

```
ProcessCall(c: x, c': o_i)
                                                         Processing: \langle [4]:C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
       c^t = \text{Select}(c, l, c': o_i) c^t = [14]
                                                                \{o_3\}
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                    PFG:
       if c: l \to c^t: m \notin CG then
   \longrightarrow add c: l \rightarrow c^t: m to CG
   \longrightarrow AddReachable(c^t: m)
                                                               {o_3}
         foreach parameter p_i of m do
                                                       [4]:C.m_{this}
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                             CG: \{ []:4 \rightarrow [4]:C.m(), \}
            ProcessCall(c: x, c': o_i)
                                                                     [4]:14 \rightarrow [14]:C.id(Number)
1 class C {
                                        void m() {
                                  10
                                           Number n1, n2, x, y;
     static void main() {
                                  11
       C c = new C();
                                  12 n1 = new One();
                                      n2 = new Two();
    c.m();
                                  13
                                                                    RM: { []:C.main(), [4]:C.m(),
                                  14 \longrightarrow x = this.id(n1);
                                                                             [14]:C.id(Number) }
                                  15
                                      y = this.id(n2);
6
     Number id(Number n) { 16
                                      x.get();
       return n;
                                  17
                                                                                                135
                                  18 }
```

WL:

```
ProcessCall(c: x, c': o_i)
                                                        Processing: \langle [4]:C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
       c^t = \text{Select}(c, l, c': o_i) c^t = [14]
                                                               \{o_3\}
                                                                          {}
                                                                                      { }
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                   PFG:
       if c: l \rightarrow c^t: m \notin CG then
                                                                        [4]:n1
         add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                               {o_3}
         foreach parameter p_i of m do
                                                       [4]:C.m_{this}
      \longrightarrow AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                            CG: \{ []:4 \rightarrow [4]:C.m(), \}
            ProcessCall(c: x, c': o_i)
                                                                    [4]:14 \rightarrow [14]:C.id(Number)
                                        void m() {
1 class C {
                                  10
                                           Number n1, n2, x, y;
     static void main() {
                                  11
       C c = new C();
                                 n1 = new One();
                                  13
                                     n2 = new Two();
    c.m();
                                                                   RM: { []:C.main(), [4]:C.m(),
                                  14 \longrightarrow x = this.id(n1);
                                      y = this.id(n2);
                                                                            [14]:C.id(Number) }
                                  15
6
    Number id(Number n) {
                                 16
                                      x.get();
       return n;
                                  17
                                                                                               136
                                  18 }
```

WL:

```
ProcessCall(c: x, c': o_i)
                                                        Processing: \langle [4]:C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
       c^t = \text{Select}(c, l, c': o_i) c^t = [14]
                                                               \{o_3\}
                                                                          {}
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                   PFG:
       if c: l \rightarrow c^t: m \notin CG then
                                                                        [4]:n1
                                                                                      [14]:n
         add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                               {o_3}
         foreach parameter p_i of m do
                                                       [4]:C.m_{this}
            AddEdge(c: a_i, c^t: p_i)

ightharpoonup AddEdge(c^t: m_{ret}, c: r)
                                                            CG: \{ []:4 \rightarrow [4]:C.m(), \}
            ProcessCall(c: x, c': o_i)
                                                                     [4]:14 \rightarrow [14]:C.id(Number)
                                        void m() {
1 class C {
                                  10
                                           Number n1, n2, x, y;
     static void main() {
                                  11
       C c = new C();
                                  n1 = new One();
                                      n2 = new Two();
    c.m();
                                  13
                                                                   RM: { []:C.main(), [4]:C.m(),
                                  14 \longrightarrow x = this.id(n1);
                                       y = this.id(n2);
                                                                            [14]:C.id(Number) }
                                  15
6
     Number id(Number n) {
                                  16
                                       x.get();
       return n;
                                  17
                                                                                               137
                                  18 }
```

WL:

```
ProcessCall(c: x, c': o_i)
                                                        Processing: \langle [4]: C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
       c^t = \text{Select}(c, l, c': o_i) c^t = [15]
                                                               \{o_3\}
                                                                          {}
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                    PFG:
                                                                        [4]:n1
       if c: l \rightarrow c^t: m \notin CG then
                                                                                      [14]:n
         add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                               {o_3}
         foreach parameter p_i of m do
                                                       [4]:C.m_{this}
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                            CG: \{ []:4 \rightarrow [4]:C.m(), \}
            ProcessCall(c: x, c': o_i)
                                                                     [4]:14 \rightarrow [14]:C.id(Number),
1 class C {
                                        void m() {
                                  10
                                                                     [4]:15 \rightarrow [15]:C.id(Number)
                                           Number n1, n2, x, y;
     static void main() {
                                  11
       C c = new C();
                                  12
                                           n1 = new One();
                                  13
                                      n2 = new Two();
    c.m();
                                                                   RM: { []:C.main(), [4]:C.m(),
                                           x = this.id(n1);
                                  14
                                  15 \longrightarrow y = this.id(n2);
                                                                             [14]:C.id(Number),
6
     Number id(Number n) {
                                           x.get();
                                  16
                                                                             [15]:C.id(Number) }
       return n;
                                  17
                                                                                               138
                                  18 }
```

WL:

```
ProcessCall(c: x, c': o_i)
                                                         Processing: \langle [4]: C.m_{this}, \{o_3\} \rangle
    foreach l: r = x.k(a1,...,an) \in S do
       m = \text{Dispatch}(o_i, k) m = \text{C.id}(\text{Number})
       c^t = \text{Select}(c, l, c': o_i) c^t = [15]
                                                                \{o_3\}
                                                                           {}
                                                                                        { }
       add \langle c^t : m_{this}, \{c' : o_i\} \rangle to WL
                                                    PFG:
       if c: l \rightarrow c^t: m \notin CG then
                                                                         [4]:n1
                                                                                       14:n
          add c: l \to c^t: m to CG
         AddReachable(c^t: m)
                                                                {o_3}
          foreach parameter p_i of m do
                                                       [4]:C.m_{this}
                                                                         [4]:n2
            AddEdge(c: a_i, c^t: p_i)
         AddEdge(c^t: m_{ret}, c: r)
                                                             CG: \{ []:4 \rightarrow [4]:C.m(), \}
            ProcessCall(c: x, c': o_i)
                                                                      [4]:14 \rightarrow [14]:C.id(Number),
1 class C {
                                         void m() {
                                  10
                                                                      [4]:15 \rightarrow [15]:C.id(Number)
     static void main() {
                                  11
                                            Number n1, n2, x, y;
       C c = new C();
                                  12
                                            n1 = new One();
                                           n2 = new Two();
     c.m();
                                  13
                                                                            { []:C.main(), [4]:C.m(),
                                                                    RM:
                                            x = this.id(n1);
                                  14
                                  15 \longrightarrow y = this.id(n2);
                                                                             [14]:C.id(Number),
6
     Number id(Number n) {
                                            x.get();
                                  16
                                                                              [15]:C.id(Number) }
        return n;
                                  17
                                                                                                139
                                  18 }
```

WL:

```
Solve(m^{entry})
   WL=[],PFG=\{\},S=\{\},RM=\{\},CG=\{\}\}
  AddReachable([]: m<sup>entry</sup>)
  while WL is not empty do
     remove \langle n, pts \rangle from WL
     \Delta = pts - pt(n)
    Propagate(n, \Delta)
     if n represents a variable c: x then
        foreach c': o_i \in \Delta do
           foreach x.f = y \in S do
             AddEdge(c: y, c': o_i. f)
           foreach y = x.f \in S do
             AddEdge(c': o_i. f, c: y)
           ProcessCall(c: x, c': o_i)
```

```
WL:
                         [\langle [4]:n1, \{o_{12}\}\rangle, \langle [4]:n2, \{o_{13}\}\rangle]
        Processing: ...
                                   {o<sub>12</sub>}
                   {o<sub>3</sub>}
                                                        {0<sub>12</sub>}
                                                                            {o<sub>12</sub>}
PFG:
                                   [4]:n1
                                    {o<sub>13</sub>}
                   {o_3}
                                                        {o<sub>13</sub>}
                                                                            {o<sub>13</sub>}
     [4]:C.m_{this}
                                                        [15]:n
                                  [4]:n2
```

```
void m() {
1 class C {
                            10
    static void main() {
                            11
                                   Number n1, n2, x, y;
      C c = new C();
                            12
                                   n1 = new One();
                                   n2 = new Two();
      c.m();
                            13
                                   x = this.id(n1);
                            14
                                   y = this.id(n2);
                            15
    Number id(Number n) {
                                   x.get();
                            16
      return n;
                            17
                            18 }
```

CG: { []:4 \rightarrow [4]:C.m(), [4]:14 \rightarrow [14]:C.id(Number), [4]:15 \rightarrow [15]:C.id(Number) }

> > 140

```
WL: []
```

```
Solve(m^{entry})
    WL=[],PFG={},S={},RM={},CG={}
                                                          Processing: \langle [4]: x, \{o_{12}\} \rangle
    AddReachable([]: m^{entry})
    while WL is not empty do
       remove \langle n, pts \rangle from WL
                                                                 {o_3}
                                                                          \{o_{12}\}
                                                                                       \{o_{12}\}
                                                                                                    \{o_{12}\}
       \Delta = pts - pt(n)
                                                     PFG:
                                                                          [4]:n1
                                                                                        [14]:n
       Propagate(n, \Delta)
       if n represents a variable c: x then
          foreach c': o_i \in \Delta do
                                                                                        \{o_{13}\}
                                                                 \{o_3\}
                                                                           \{o_{13}\}
                                                                                                    \{o_{13}\}
            foreach x.f = y \in S do
                                                        [4]:C.m_{this}
                                                                          [4]:n2
                                                                                       [15]:n
               AddEdge(c: y, c': o_i. f)
            foreach y = x.f \in S do
               AddEdge(c': o_i. f, c: y)
                                                              CG: \{ []:4 \rightarrow [4]:C.m(), \}
       \longrightarrow ProcessCall(c: x, c': o<sub>i</sub>)
                                                                       [4]:14 \rightarrow [14]:C.id(Number),
1 class C {
                                         void m() {
                                   10
                                                                       [4]:15 \rightarrow [15]:C.id(Number),
     static void main() {
                                   11
                                            Number n1, n2, x, y;
                                                                       [4]:16 \to [16]:One.get() 
       C c = new C();
                                   12
                                            n1 = new One();
                                            n2 = new Two();
       c.m();
                                   13
                                                                     RM:
                                                                             { []:C.main(), [4]:C.m(),
                                            x = this.id(n1);
                                   14
                                                                              [14]:C.id(Number),
                                   15
                                            y = this.id(n2);
6
     Number id(Number n) { 16=
                                         ⇒x.get();
                                                                              [15]:C.id(Number),
        return n;
                                   17
                                                                               [16]:One.get() } <sub>141</sub>
                                   18 }
```

```
WL:
```

```
Solve(m^{entry})
    WL=[],PFG={},S={},RM={},CG={}
    AddReachable([]: m<sup>entry</sup>)
                                                 Final results
    while WL is not empty do
      remove \langle n, pts \rangle from WL
                                                             \{o_3\}
      \Delta = pts - pt(n)
                                                  PFG:
      Propagate(n, \Delta)
      if n represents a variable c: x then
         foreach c': o_i \in \Delta do
                                                             \{o_3\}
            foreach x.f = y \in S do
                                                     [4]:C.m_{this}
              AddEdge(c: y, c': o_i. f)
            foreach y = x.f \in S do
              AddEdge(c': o_i. f, c: y)
            ProcessCall(c: x, c': o_i)
                                       void m() {
1 class C {
                                 10
     static void main() {
                                 11
                                          Number n1, n2, x, y;
       C c = new C();
                                 12
                                          n1 = new One();
                                          n2 = new Two();
       c.m();
                                 13
```

14

15

16

17

18 }

Number id(Number n) {

return n;

6

Algorithm finishes

x = this.id(n1);

y = this.id(n2);

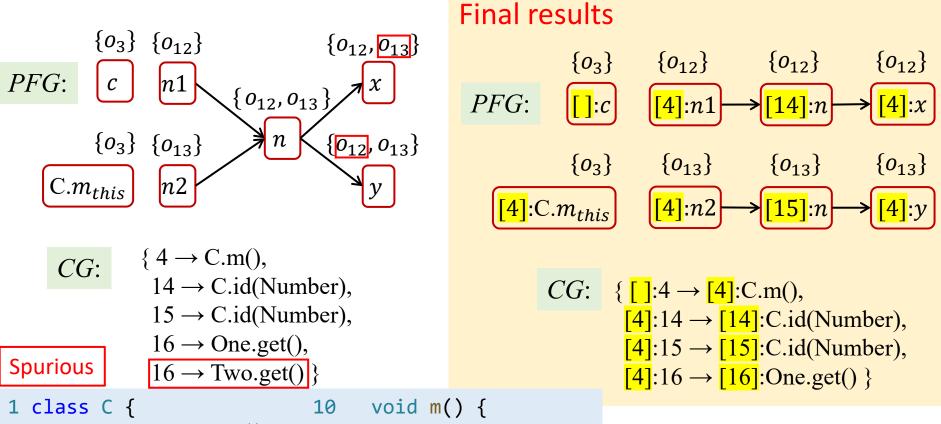
x.get();

```
\{o_{12}\}
                 \{o_{12}\}
                                  \{o_{12}\}
[4]:n1
                  14]:n
 \{o_{13}\}
                  \{o_{13}\}
                                  \{o_{13}\}
[4]:n2
```

```
CG: \{ []:4 \rightarrow [4]:C.m(), \}
         [4]:14 \rightarrow [14]:C.id(Number),
         [4]:15 \rightarrow [15]:C.id(Number),
         [4]:16 \to [16]:One.get()
```

```
{ []:C.main(), [4]:C.m(),
RM:
         [14]:C.id(Number),
         [15]:C.id(Number),
         [16]:One.get() } <sub>142</sub>
```

C.I. vs. C.S. (1-Call-Site)



```
static void main() {
                            11
                                    Number n1, n2, x, y;
      C c = new C();
                            12
                                    n1 = new One();
                                    n2 = new Two();
      c.m();
                            13
                                    x = this.id(n1);
                            14
                                   y = this.id(n2);
6
                            15
    Number id(Number n) {
                            16
                                    x.get();
                            17
      return n;
                            18 }
```

Object Sensitivity*

- Each context consists of a list of abstract objects (represented by their allocation sites)
 - At a method call, use the receiver object with its heap context as callee context
 - Distinguish the operations of data flow on different objects

Select(
$$_$$
, $_$, c' : o_i) = [o_j , ..., o_k , o_i]
where c' = [o_j , ..., o_k]

Essentially "allocation-site sensitivity"

* Ana Milanova, Atanas Rountev, and Barbara G. Ryder. "Parameterized Object Sensitivity for Points-to and Side-Effect Analyses for Java". ISSTA 2002.

```
a1 = new A();
  a2 = new A();
3 b1 = new B();
4 b2 = new B();
5 a1.set(b1);
6 a2.set(b2);
  x = a1.get();
8
  class A {
   B f;
10
   void set(B b) {
12
       this.doSet(b);
13
  void doSet(B p) {
14
15
       this.f = p;
16
17
    B get() {
       return this.f;
18
19
20 }
```

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
5 a1.set(b1);
6 a2.set(b2);
  x = a1.get();
8
  class A {
10 B f;
void set(B b) {
12
       this.doSet(b);
13 }
14 void doSet(B p) {
15
       this.f = p;
16
17 B get() {
   return this.f;
18
19
20 }
```

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
 a1.set(b1); \
6 a2.set(b2);
   x = a1.get();
8
  class A {
10
   B f;
   void set(B b) {
        this.doSet(b);
12
13 }
14 void doSet(B p) {
        this.f = p;
15
16
     B get() {
17
        return this.f;
18
19
20 }
```

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
 a1.set(b1); \
6 a2.set(b2);
   x = a1.get();
8
  class A {
10
   B f;
   void set(B b) {
        this.doSet(b);
12
13 }
14 void doSet(B p) {
        this.f = p;
15
16
     B get() {
17
        return this.f;
18
19
20 }
```

Variable	Object
$[o_1]$: set_{this}	01
$[o_1]$: b	03

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
 a1.set(b1); \
6 a2.set(b2);
  x = a1.get();
8
  class A {
   B f;
10
   void set(B b) {
       this.doSet(b); c^t = ?
12
13 }
   void doSet(B p) {
14
       this.f = p;
15
16
     B get() {
17
18
       return this.f;
19
20 }
```

Variable	Object
$[o_1]$: set _{this}	o_1
$[o_1]$: b	<i>o</i> ₃

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
 a1.set(b1); \
6 a2.set(b2);
  x = a1.get();
8
   class A {
   B f;
10
   void set(B b) {
       this.doSet(b); c^t = [o_1]
12
13 }
   void doSet(B p) {
14
        this.f = p;
15
16
     B get() {
17
18
       return this.f;
19
20 }
```

Variable	Object
<mark>[o₁]</mark> : set _{this}	o_1
[<mark>0₁]</mark> : b	03
$[o_1]$: $doSet_{this}$	<i>o</i> ₁
$[o_1]$: p	03
Field	Object
o_1 .f	03

```
a1 = new A(); a1 \rightarrow o_1
2 a2 = new A(); a2 \rightarrow o_2
3 b1 = new B(); b1 \rightarrow o_3
4 b2 = new B(); b2 \rightarrow o_4
  a1.set(b1); \
6 ra2.set(b2);
 x = a1.get();
  class A {
11 ▶ void set(B b) {
    this.doSet(b); c^t = [o_1]
    void doSet(B p)
        this.f = p;
16 }
17 B get() {
18
       return this.f;
19
20 }
```

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set_{this}	02
$[o_2]$: b	04
$[o_2]$: $doSet_{this}$	02
$[o_2]$: p	04
Field	Object
$o_1.f$	03
o_2 .f	04

```
a1 = new A(); a1 \rightarrow o_1
       2 a2 = new A(); a2 \rightarrow o_2
       3 b1 = new B(); b1 \rightarrow o_3
       4 b2 = new B(); b2 \rightarrow o_4
         a1.set(b1);
       6 ra2.set(b2);
      7 x = a1.get();
c^t = [o_2] \setminus \text{class A } \{
       11 ▶ void set(B b) {
       this.doSet(b); c^t = [o_1]
            void doSet(B p) {
               this.f = p;
       16 }
       17 B get() {
       18
               return this.f;
       19
       20 }
```

Variable	Object
$[o_1]$: set _{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set _{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
$[o_2]$: p	04
x	03
Field	Object
$o_1.f$	03
<i>o</i> ₂ .f	o_4



1-call-site

1	a1 = new A(); $a1 \rightarrow o_1$
2	a2 = new A(); $a2 \rightarrow o_2$
3	b1 = new B(); $b1 \rightarrow o_3$
4	b2 = new B(); $b2 \rightarrow o_4$
5	a1.set(b1);
6	a2.set(b2);
7	x = a1.get();
8	
9	<pre>class A {</pre>
10	B f ;
11	<pre>void set(B b) {</pre>
12	<pre>this.doSet(b);</pre>
1 3	}
14	<pre>void doSet(B p) {</pre>
1 5	<pre>this.f = p;</pre>
16	}
17	B get() {
18	return this.f;
19	}
20	}

Variable	Object
$[o_1]$: set _{this}	o_1
$[o_1]$: b	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set _{this}	o_2
$[o_2]$: b	04
$[o_2]$: $doSet_{this}$	02
$[o_2]$: p	04
х	03
Field	Object
$o_1.f$	03
$o_2.f$	04

1-call-site

1	a1 = new A(); $a1 \to o_1$
2	a2 = new A(); $a2 \rightarrow o_2$
3	b1 = new B(); $b1 \to o_3$
4	b2 = new B(); $b2 \rightarrow o_4$
5	a1.set(b1); \
6	a2.set(b2); `\
7	x = a1.get(); \ ,
8	$ c^t = [5]$
9	class A {
10	B f;
11	<pre>void set(B b) {</pre>
12	<pre>this.doSet(b);</pre>
13	}
14	<pre>void doSet(B p) {</pre>
15	this.f = p;
16	}
17	<pre>B get() {</pre>
18	return this.f;
19	}
20	}

Variable	Object
$[5]$: set_{this}	o_1
[5]: b	<i>o</i> ₃

Variable	Object
$[o_1]$: set _{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[o ₂]: p	04
x	03
Field	Object
$o_1.f$	03
<i>o</i> ₂ .f	04

1-call-site

1 a1 = new A(); $a1 \rightarrow o_1$ 2 a2 = new A(); $a2 \rightarrow o_2$ 3 **b1** = new B(); $b1 \rightarrow o_3$ 4 b2 = new B(); $b2 \rightarrow o_4$ 5 a1.set(b1); \ 6 a2.set(b2); x = a1.get();8 class A { 10 B **f**; void set(B b) { this.doSet(b); $c^t = [12]$ 12 **1**3 void doSet(B p) { 14 15 this.f = p; 16 17 B get() { return this.f; 18 19 20 }

Variable	Object
[5]: set _{this}	o_1
[5]: b	03
$[12]$: $doSet_{this}$	<i>o</i> ₁
[12]: <i>p</i>	<i>o</i> ₃
Field	Object
o_1 .f	<i>o</i> ₃

Variable	Object
$[o_1]$: set_{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
[<i>o</i> ₁]: <i>p</i>	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[o ₂]: p	04
x	03
Field	Object
o_1 .f	03
<i>o</i> ₂ .f	04

1-call-site

1 a1 = new A(); $a1 \rightarrow o_1$ 2 a2 = new A(); $a2 \rightarrow o_2$ 3 b1 = new B(); $b1 \rightarrow o_3$ 4 b2 = new B(); $b2 \rightarrow o_4$ a1.set(b1); \ 6 ra2.set(b2); x = a1.get(); $c^t = 6 \setminus \text{class A}$ { 10\ B f; 11 ≯void set(B b) { this.doSet(b); $c^t = [$ 12 **13** } void doSet(B p) 14 this.f = p; 15 **16** } 17 B get() { 18 return this.f; 19 20 }

Variable	Object
[5]: set _{this}	o_1
[5]: b	03
[6]: set _{this}	02
[6]: b	04
$[12]$: $doSet_{this}$	o_1
[12]: p	03
[2] Field	Object
o_1 .f	o_3

Variable	Object
$[o_1]$: set _{this}	o_1
$[o_1]$: b	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set_{this}	02
$[o_2]$: b	04
$[o_2]$: $doSet_{this}$	02
$[o_2]$: p	o_4
x	03
Field	Object
<i>o</i> ₁ .f	03
o ₂ .f	04

1-call-site

1	a1 = new A(); $a1 \rightarrow o_1$
2	a2 = new A(); $a2 \rightarrow o_2$
3	b1 = new B(); $b1 \rightarrow o_3$
4	b2 = new B(); $b2 \to o_4$
5	a1.set(b1); <
6	ra2.set(b2); ``.
7	x = a1.get(); \
8	$c^t = [5]$
$c^t = [6]$	class A {
	B f;
11	<pre> yound set(B b) { t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t</pre>
12	this.doSet(b); $c^t = [1]$
$c^t = 713$	()
14	<pre>▶ void doSet(B p) {</pre>
15	this.f = p;
16	}
17	<pre>B get() {</pre>
18	return this.f;
19	}
20	}

Variable	Object
$[5]$: set_{this}	o_1
[5]: b	03
[6]: set _{this}	02
[6]: b	o_4
[12]: $doSet_{this}$	o_1
[12]: <i>p</i>	03
[2] Field	Object
o_1 .f	03

Variable	Object
$[o_1]$: set_{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
[<i>o</i> ₁]: <i>p</i>	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[o ₂]: p	04
x	03
Field	Object
o_1 .f	03
<i>o</i> ₂ .f	04

1-call-site

	1	a1 =	= new	A();	$a1 \rightarrow$	o_1
	2	a2 =	= new	A();	$a2 \rightarrow$	02
	3	b1 =	= new	B();	$b1 \rightarrow$	03
	4	b2 =	= new	B();	$b2 \rightarrow$	o_4
	5	a1.	set(b	1);、		
	6	a2.	set(b	2); `		
	7	x =	a1.g	et();	\\	
	8				C^{ι}	=[5]
$c^t = [6]$	<mark>)</mark>	clas	ss A	{	الممر	
	10	В	f;	A		
	11	> V(oid s	et(B	b) {	
	12		this	.doSe	t(b);	
	13	}		1	c^t	= <mark>[12]</mark>
	14	V	oid d	oSet(Bp)	{
	15		this	.f =	p;	
	16	}				
	17	В	get() {		
	18		retu	rn th	is.f;	
	19	}				
	20	}				

Variable	Object
[5]: set _{this}	o_1
[5]: b	03
<mark>[6]</mark> : set _{this}	02
[6]: b	o_4
$[12]$: $doSet_{this}$	<i>o</i> ₁ ,
[12]: <i>p</i>	0 ₃ , <mark>0₄</mark>
Field	Object
$o_1.f$	0 ₃ , 0 ₄
o_2 .f	03,04

Variable	Object
[o ₁]: set _{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[<i>o</i> ₂]: <i>p</i>	04
x	03
Field	Object
<i>o</i> ₁ .f	03
<i>o</i> ₂ .f	04

1-call-site

	1	a1 =	new	A();	a1 -	$\rightarrow o_1$
	2	a2 =	new	A();	a2 -	$\rightarrow o_2$
	3	b1 =	new	B();	<i>b</i> 1 -	$\rightarrow o_3$
	4	b2 =	new	B();	b2 -	$\rightarrow o_4$
	5	a1.se	et(bi	L); 🔪		
	6	a2.se	et(b2	2); ``		
	7	X = 3	a1.ge	et();		+
	8				C	t = [5]
$c^t = [e^t]$	<mark>5]</mark>	class	s A	[الممر	
	10	В	f;	4		
	11	> ∨0:	id se	et(B	b) {	
	12	1	this	doSe	t(b)	;
	13	}		1	c^t	= [12]
	14	VO	id do	Set(B p)	{
	15	+	this	.f =	p;	
	16	}				
	17	В	get()) {		
	18	1	retur	n th	is.f	;
	19	}				
	20	}				

Variable	Object
[5]: set _{this}	o_1
[5]: b	03
<mark>[6]</mark> : set _{this}	02
[6]: b	o_4
$[12]$: $doSet_{this}$	o_1, o_2
[12]: <i>p</i>	o_3, o_4
x	o_3, o_4
Field	Object
$o_1.f$	o_3 , o_4
o_2 .f	o_3 , o_4

Variable	Object
[o ₁]: set _{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[<i>o</i> ₂]: <i>p</i>	04
x	03
Field	Object
<i>o</i> ₁ .f	03
<i>o</i> ₂ .f	04

1-call-site

	1	a1 =	= nev	v A()	; a	1 →	o_1
	2	a2 =	= nev	v A()	; a	$2 \rightarrow$	o_2
	3	b1 =	= nev	v B()	; b	1 →	03
	4	b2 =	= nev	v B()	; b	2 →	o_4
	5	a1.9	set(l	1);、	`		
	6	a2.	set(l	02);			
	7	x =	a1.8	get()	; ``\	\	E = 3
	8					C^{ι}	= [5]
$c^t = [0]$	<mark>6]</mark> \	clas	ss A	{	مم	,	
	10	В	f;	_			
	11	> V(oid s	set(B	b)	{	
	12		this	. doS	Set(b);	
	13	}		<i>'</i>		c^t =	= <mark>[12]</mark>
	14	V	oid (doSet	(B	p) ·	{
	15		this	.f =	p;		
	16	}					
	17	В	get	() {			
	18		reti	ırn t	his	.f;	
	19	}					
	20	}					

Object
o_1
03
02
o_4
o_1, o_2
o_3, o_4
o_3, o_4
Object
o_3 , o_4
0_{3} , 0_{4}

Spurious



Variable	Object
$[o_1]$: set_{this}	o_1
[<i>o</i> ₁]: <i>b</i>	03
$[o_1]$: $doSet_{this}$	o_1
[<i>o</i> ₁]: <i>p</i>	03
$[o_2]$: set_{this}	02
[<i>o</i> ₂]: <i>b</i>	04
$[o_2]$: $doSet_{this}$	02
[<i>o</i> ₂]: <i>p</i>	04
x	03
Field	Object
o_1 .f	03
<i>o</i> ₂ .f	o_4

```
1-call-site
                                                                1-object
  a1 = new A();
  a2 = new A();
                                  []:main()
                                                               []:main()
  b1 = new B();
                            a1.set(b1) | a2.set(b2)
                                                         a1.set(b1)
                                                                     a2.set(b2)
  b2 = new B();
  a1.set(b1);
  a2.set(b2);
   x = a1.get();
                           [5]:A.set(B)
                                                        [o_1]:A.set(B)
8
                                                       this.doSet(b)
   class A {
                          this.doSet(b)
10
     B f;
11
     void set(B b) {
                                                                   [o_2]:A.set(B)
                                       [6]:A.set(B)
       this.doSet(b);
12
                                                                   this.doSet(b)
                                      this.doSet(b)
13
     void doSet(B p) {
14
15
       this.f = p;
16
                             [12]:A.doSet(B)
                                                      [o_1]:A.doSet(B)
17
     B get() {
       return this.f;
18
                                                                  [o_2]:A.doSet(B)
19
20 }
```

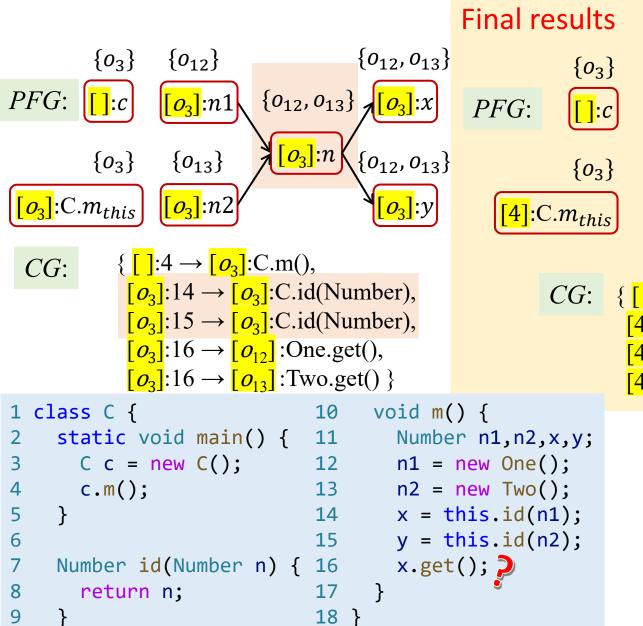
C.S. (1-Object)

vs. C.S. (1-Call-Site)

```
Final results
              \{o_3\}
                       {o<sub>12</sub>}
                                         \{o_{12}\}
                                                        \{o_{12}\}
                                                        [4]:x
                         [4]:n1
 PFG:
                                       \rightarrow [14]:n
              [ ]:C
              {o_3}
                           \{o_{13}\}
                                         {o_{13}}
                                                        \{o_{13}\}
    [4]:C.m_{this}
                          [4]:n2
                                         [15]:n
           CG: \{ []:4 \rightarrow [4]:C.m(), 
                      [4]:14 \rightarrow [14]:C.id(Number),
                      [4]:15 \rightarrow [15]:C.id(Number),
                      [4]:16 \rightarrow [16]:One.get() }
```

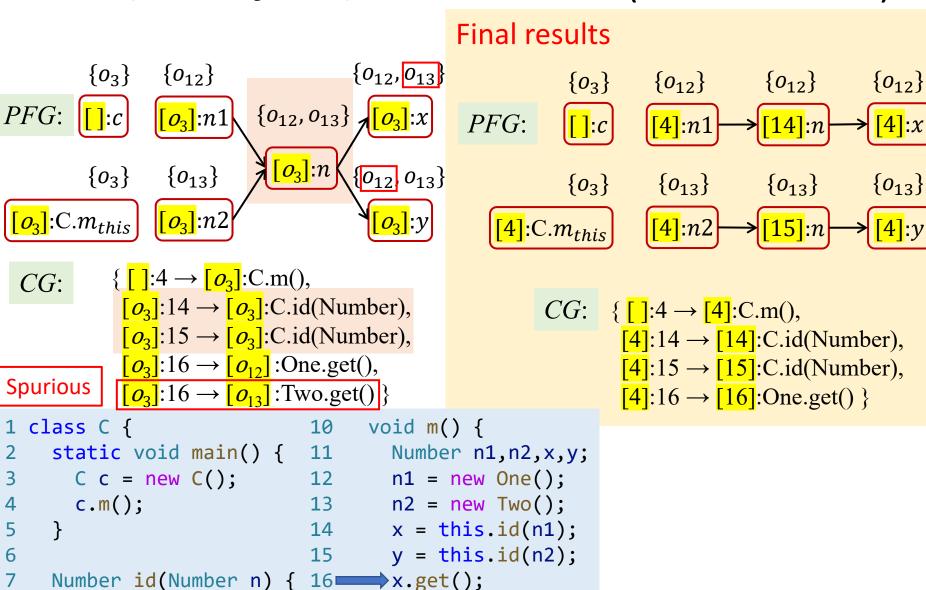
```
class C {
                                 void m() {
                            10
    static void main() {
                            11
                                   Number n1, n2, x, y;
      C c = new C();
                            12
                                   n1 = new One();
                                   n2 = new Two();
      c.m();
                            13
                                   x = this.id(n1);
                            14
                                   y = this.id(n2);
6
                            15
                                   x.get(); >
    Number id(Number n) {
                            16
      return n;
                            17
                            18 }
```

C.S. (1-Object) vs. C.S. (1-Call-Site)



```
\{o_{12}\}
                              \{o_{12}\}
                                             \{o_{12}\}
              [4]:n1
               {o_{13}}
                              \{o_{13}\}
                                             \{o_{13}\}
              [4]:n2
CG: \{ []:4 \rightarrow [4]:C.m(), 
          [4]:14 \rightarrow [14]:C.id(Number),
          [4]:15 \rightarrow [15]:C.id(Number),
          [4]:16 \to [16]:One.get()
```

C.S. (1-Object) vs. C.S. (1-Call-Site)



return n;

17

18 }

• In theory, their precision is incomparable

• In theory, their precision is incomparable

• In practice, object sensitivity generally outperforms call-site sensitivity for OO languages (like Java)

	Time	(s)	#may-fai	l-cast	#call-gra	ph-edge
	2-call	2-obj	2-call	2-obj	2-call	2-obj
batik	6,886	3,300	2,452	1,606	94,211	76,807
checkstyle	2,277	2,003	863	581	54,171	48,809
sunflow	5,570	1,208	2,504	1,837	100,701	89,866
findbugs	3,812	2,661	2,056	1,409	72,118	65,836
јрс	3,343	559	1,855	1,392	89,677	81,030
eclipse	1,896	146	886	546	42,872	38,151
chart	2,705	282	1,481	883	59,691	52,374
fop	5,503	1,200	1,975	1,446	79,524	71,408
xalan	1,927	1,093	919	533	48,763	44,871
bloat	5,712	3,525	1,699	1,193	58,696	53,143

For all numbers, lower is better (in terms of efficiency or precision)

	Time ((s)	#may-fai	l-cast	#call-graph-edge		
	2-call	2-obj	2-call	2-obj	2-call	2-obj	
batik	6,886	3,300	2,452	1,606	94,211	76,807	
checkstyle	2,277	2,003	863	581	54,171	48,809	
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chart	2,705	282	1,481	883	59,691	52,374	
fop	5,503	1,200	1,975	1,446	79,524	71,408	
xalan	1,927	1,093	919	533	48,763	44,871	
bloat	5,712	3,525	1,699	1,193	58,696	53,143	

$$A a = (A) o;$$

For all numbers, lower is better (in terms of efficiency or precision)

	Time	(s)	#may-fai	l-cast	#call-gra	ph-edge
	2-call	2-obj	2-call	2-obj	2-call	2-obj
batik	6,886	3,300	2,452	1,606	94,211	76,807
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bloat	5,712	3,525	1,699	1,193	58,696	53,143

For all numbers, lower is better (in terms of eff

In general

- Precision: object > call-site
- Efficiency: object > call-site

Type Sensitivity*

- Each context consists of a list of types
 - At a method call, use the type containing the allocation site
 of the receiver object with its heap context as callee context
 - A coarser abstraction over object sensitivity

Select(
$$_$$
, $_$, c' : o_i) = [t' , ..., t'' , InType(o_i)] where c' = [t' , ..., t'']

^{*} Yannis Smaragdakis, Martin Bravenboer, and Ondrej Lhoták. "Pick Your Contexts Well: Understanding Object-Sensitivity". POPL 2011.

Type Sensitivity*

- Each context consists of a list of types
 - At a method call, use the type containing the allocation site
 of the receiver object with its heap context as callee context
 - A coarser abstraction over object sensitivity

```
\frac{\mathsf{Select}(\_,\_,\boldsymbol{c}';\boldsymbol{o_i}) = [t',...,t'',\mathsf{InType}(o_i)]}{\mathsf{where}\ c' = [t',...,t'']}
```

* Yannis Smaragdakis, Martin Bravenboer, and Ondrej Lhoták. "Pick Your Contexts Well: Understanding Object-Sensitivity". POPL 2011.

Under the same *k*-limiting, the precision of type sensitivity is **no better than** object sensitivity

Type sensitivity is a coarser abstraction over object sensitivity

```
class X {
    void main() {
3
      Y y1 = new Y();
    y1.foo();
4
5
      Y y2 = new Y();
6
  y2.foo();
      Y y3 = new Y();
8
      y3.foo();
9
10 }
                     Contexts of foo()
11 class Y {
                     Object-sensitivity: ?
    void foo() {}
13 }
```

Under the same *k*-limiting, the precision of type sensitivity is **no better than** object sensitivity

Type sensitivity is a coarser abstraction over object sensitivity

```
class X {
     void main() {
       Y y1 = new Y();
3
4
     y1.foo();
5
       Y y2 = new Y();
6
  y2.foo();
       Y y3 = new Y();
8
       y3.foo();
9
10 }
                        Contexts of foo()
11 class Y {
                        \triangleright Object-sensitivity: [o_3], [o_5], [o_7]
     void foo() {}
13 }
                        Type-sensitivity:
```

Under the same *k*-limiting, the precision of type sensitivity is **no better than** object sensitivity

Type sensitivity is a coarser abstraction over object sensitivity

```
class X {
     void main() {
3
       Y y1 = new Y();
     y1.foo();
4
5
       Y y2 = new Y();
6
  y2.foo();
     Y y3 = new Y();
8
       y3.foo();
9
10 }
                       Contexts of foo()
11 class Y {
                        \triangleright Object-sensitivity: [o_3], [o_5], [o_7]
     void foo() {}
13 }
                        Type-sensitivity: [X]
```

Under the same *k*-limiting, the precision of type sensitivity is **no better than** object sensitivity

Type sensitivity is a coarser abstraction over object sensitivity

```
class X {
     void main() {
3
       Y y1 = new Y();
       y1.foo();
5
       Y y2 = new Y();
6
      y2.foo();
      Y y3 = new Y();
8
       y3.foo();
9
10 }
11 class Y {
     void foo() {}
13 }
```

Compared to object sensitivity, type sensitivity trades precision for better efficiency, by merging the allocation sites in the same type in contexts

 In practice, type sensitivity is less precise but more efficient than object sensitivity

```
Contexts of foo()
```

- \triangleright Object-sensitivity: $[o_3]$, $[o_5]$, $[o_7]$
- Type-sensitivity: [X]

Call-Site vs. Object vs. Type Sensitivity

	Time (s)			Time (s) #may-fail-cast			st	#call-graph-edge		
	2-call	2-obj	2-type	2-call	2-obj	2-type	2-call	2-obj	2-type	
batik	6,886	3,300	378	2,452	1,606	1,938	94,211	76,807	77,337	
checkstyle	2,277	2,003	125	863	581	695	54,171	48,809	49,274	
sunflow	5,570	1,208	197	2,504	1,837	2,247	100,701	89,866	90,967	
findbugs	3,812	2,661	265	2,056	1,409	1,683	72,118	65,836	66,443	
јрс	3,343	559	128	1,855	1,392	1,599	89,677	81,030	81,527	
eclipse	1,896	146	57	886	546	665	42,872	38,151	38,337	
chart	2,705	282	84	1,481	883	1,155	59,691	52,374	52,965	
fop	5,503	1,200	251	1,975	1,446	1,753	79,524	71,408	71,847	
xalan	1,927	1,093	99	919	533	729	48,763	44,871	45,444	
bloat	5,712	3,525	74	1,699	1,193	1,486	58,696	53 , 143	54,279	

For all numbers, lower is better (in terms of efficiency or precision)

Call-Site vs. Object vs. Type Sensitivity

	Time (s)			Time (s) #may-fail-cast			st	#call-graph-edge		
	2-call	2-obj	2-type	2-call	2-obj	2-type	2-call	2-obj	2-type	
batik	6,886	3,300	378	2,452	1,606	1,938	94,211	76,807	77,337	
checkstyle	2,277	2,003	125	863	581	695	54,171	48,809	49,274	
sunflow	5,570	1,208	197	2,504	1,837	2,247	100,701	89,866	90,967	
findbugs	3,812	2,661	265	2,056	1,409	1,683	72,118	65,836	66,443	
јрс	3,343	559	128	1,855	1,392	1,599	89,677	81,030	81,527	
eclipse	1,896	146	57	886	546	665	42,872	38,151	38,337	
chart	2,705	282	84	1,481	883	1,155	59,691	52,374	52,965	
fop	5,503	1,200	251	1,975	1,446	1,753	79,524	71,408	71,847	
xalan	1,927	1,093	99	919	533	729	48,763	44,871	45,444	
bloat	5,712	3,525	74	1,699	1,193	1,486	58,696	53 , 143	54,279	

For all numbers, lower is better (in terms

In general

- Precision: object > type > call-site
- Efficiency: type > object > call-site

The X You Need To Understand in This Lecture

- Algorithm for context-sensitive pointer analysis
- Common context sensitivity variants
- Differences and relationship among common context sensitivity variants

注意注意! 划重点了!

