

# Problem 4 [1.5p]

$$1. \frac{\partial}{\partial \bar{\phi}} \log \mathcal{L}(\bar{\phi}) = \frac{\partial}{\partial \bar{\phi}} \left[ \log \binom{n}{n\bar{\phi}} + n\bar{\phi} \log \bar{\phi} + (n - n\bar{\phi}) \log(1 - \bar{\phi}) \right] =$$

$$= \frac{n\bar{\phi}}{\bar{\phi}} - \frac{n - n\bar{\phi}}{1 - \bar{\phi}} = 0 \Rightarrow \bar{\phi} = \bar{\phi} \quad \text{From Lecture}$$

$$2. \mathcal{L}(\bar{\mu}) = \prod_{i=1}^n p(x_i | \bar{\mu})$$

$$\frac{\partial}{\partial \bar{\mu}} \log \left( \prod_{i=1}^n p(x_i | \bar{\mu}) \right) = \frac{\partial}{\partial \bar{\mu}} \sum \log(p(x_i | \bar{\mu})) =$$

$$= \frac{\partial}{\partial \bar{\mu}} \sum_{i=1}^n \log \left( \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} \right) = \frac{\partial}{\partial \bar{\mu}} \sum_{i=1}^n \left[ -\frac{1}{2} \log(2\pi\sigma^2) - \frac{(x_i - \mu)^2}{2\sigma^2} \right] =$$

$$= -\frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu) = 0 \Rightarrow \mu = \frac{1}{n} \sum x_i$$

$$B. \mathcal{L}(\lambda; x_1, \dots, x_n) = \prod_{j=1}^n \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}$$

$$\log(\mathcal{L}) = -n\lambda - \sum_{j=1}^n \log(x_j!) + \log(\lambda) \sum_{j=1}^n x_j$$

$$\frac{d\mathcal{L}}{d\lambda} = -n + \frac{1}{\lambda} \sum_{j=1}^n x_j = 0 \Rightarrow \lambda = \frac{1}{n} \sum_{j=1}^n x_j$$

# Problem 5.

$$L(x_1 \dots x_n | \mu, \sigma) = \prod_{i=1}^n \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{|x_i - \mu|^2}{2\sigma^2}}$$

$$\log(L) = \sum_{i=1}^n \log\left(\frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{|x_i - \mu|^2}{2\sigma^2}}\right) = \sum_{i=1}^n \left(\log\left(\frac{1}{\sigma \sqrt{2\pi}}\right) - \frac{|x_i - \mu|^2}{2\sigma^2}\right)$$

$$\frac{d}{d\mu} \left( \sum_{i=1}^n \log\left(\frac{1}{\sigma \sqrt{2\pi}}\right) - \frac{|x_i - \mu|^2}{2\sigma^2} \right) = \frac{d}{d\mu} \left( -\frac{1}{\sigma^2} \sum_{i=1}^n |x_i - \mu| \right)$$

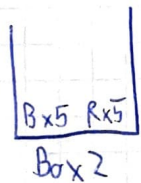
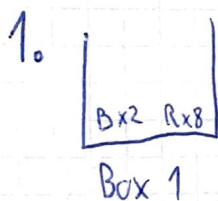
$$\arg\min_{\mu} = \arg\min_{\mu} \sum_{i=1}^n |x_i - \mu| = 0$$

wtw liana dodatni = liana ujemni.

emg. Average absolute deviation  $\sim$  MAD

rel: SUMA odg. liana bezg. liana - srednja

## Problem 6.



$$P(B_1 | Red) = \frac{P(Red | B_1) P(B_1)}{P(Red)} = \frac{\frac{8}{10} \cdot \frac{1}{2}}{\frac{13}{20}} = \frac{8}{13}$$

2. NP negative / positif  
TN/P - tested - 1 -

$$P(TP | N) = \frac{1}{100}$$

$$P(TN | P) = \frac{1}{100}$$

$$P(P) = \frac{1}{10^6}$$

$$P(N) = 1 - P(P) = \frac{10^6 - 1}{10^6}$$

A)  $P(TP | P) = \frac{P(P | TP) P(TP)}{P(TP)}$

$$P(P | TP) = \frac{P(TP | P) P(P)}{P(TP)} = \frac{P(TP | P) P(P)}{P(TP | P) P(P) + P(TN | P) P(N)} = \frac{\frac{99}{100} \cdot \frac{1}{10^6}}{\frac{99}{100} \cdot \frac{1}{10^6} + \frac{1}{100} \cdot \frac{10^6 - 1}{10^6}}$$

$$= \frac{\frac{99}{100} \cdot \frac{1}{10^6}}{\frac{99}{100} \cdot \frac{1}{10^6} + \frac{1}{100} \cdot \frac{10^6 - 1}{10^6}} = 0.0001 = 0.01\%$$



~~Part 1~~

$$b) P(P|TP) = \frac{P(TP|P) P(P)}{P(TP|P) P(P) + P(TP|N) P(N)} = \frac{\frac{99}{100} \cdot \frac{1}{1000}}{\frac{99}{100} \cdot \frac{4}{100} + \frac{1}{100} \cdot \frac{999}{1000}} = 0.0902 = 9.02\%$$

$$c) P(TP) = P(TP|P) P(P) + P(TP|N) P(N) = 80\% \cdot \frac{80}{100}$$
$$x(1 + \frac{999999}{1-x}) = \frac{80}{100} \cdot 10^6$$
$$x(1000000 - 999999x) = 80 \cdot 10^5$$
$$\frac{999999 - 8 \cdot 10^5}{999999} = x \approx 0.1999996 = 19.99996\%$$