Zadanie 9

13.01.2021

Rozważamy problem z zadania **M12.7**:

$$y'(t) = \lambda y(t) \quad y(0) = 1 \tag{1}$$

Korzystamy ze wzoru:

$$y_{n+1} = y_n + \frac{h}{2}[f_n + f(t_{n+1}, y_n + hf_n)] \tag{2}$$

Z $y' = f(t, y)$ mamy $f_n = f(t_n, y_n) = \lambda y(t_n) = \lambda y_n$ mamy więc:

$$y_{n+1} = y_n + \frac{h}{2}[\lambda y_n + f(t_{n+1}, y_n + h\lambda y_n)] \tag{3}$$

Dalej $f(t_{n+1}, y_n + h\lambda y_n) \approx \lambda(y_n + h\lambda y_n)$ zatem:

$$y_{n+1} = y_n + \frac{h}{2}[\lambda y_n + f(t_{n+1}, y_n + h\lambda y_n)] \approx y_n + \frac{h}{2}[\lambda y_n + \lambda(y_n + h\lambda y_n)] \tag{4}$$

Uproszczając otrzymujemy ostatecznie:

$$y_{n+1} = y_n + h\lambda y_n + \frac{h^2\lambda^2}{2} y_n = y_n \left(1 + h\lambda + \frac{h^2\lambda^2}{2}\right) \tag{5}$$