

Zad. 1

$\text{egds}_1 := \text{Kranche } G_1$

$e_{\text{ges}2} = \text{Krawinkel } G_2$

```
for  $e_1$  in edges:
    flag = False
    for  $e_2$  in edges:
        if  $e_1 == e_2$ :
            del  $e_2$ 
    flag = True
    break
```

ist! Flag
retem Fabe

refem time

Zad. 2

a)



$$\sum_{i=0}^n \deg(v_i) = 2m \quad \text{mit } m \leq 10 \quad \text{z. Lemma 1:}$$


~~11~~ Ist keine m.e.D. i. $Z_m = 1+2+3+3 = 11$


Wher we istige tali opat mosh

b) Nie istnieje: V_i dla którego $\deg(V_i) = 4$ musi być potęgą 2, ponieważ 3 warunków st. 1 już nie mogą być potęgą 2 innymi. Zostaje jeden warunek st. 3 który ma st. 1 i nie możemy dodać żadnego kraw.

c) 3 pyadi

- \emptyset ~~Orze~~ Wszystkie wielkości st. 2 a nie 1

-  Nie mamy tak wężej hordy. ~~5/2~~ ~~10/2~~ ~~15/2~~ ~~20/2~~ ~~25/2~~ ~~30/2~~ ~~35/2~~ ~~40/2~~ ~~45/2~~ ~~50/2~~ ~~55/2~~ ~~60/2~~ ~~65/2~~ ~~70/2~~ ~~75/2~~ ~~80/2~~ ~~85/2~~ ~~90/2~~ ~~95/2~~ ~~100/2~~ ~~105/2~~ ~~110/2~~ ~~115/2~~ ~~120/2~~ ~~125/2~~ ~~130/2~~ ~~135/2~~ ~~140/2~~ ~~145/2~~ ~~150/2~~ ~~155/2~~ ~~160/2~~ ~~165/2~~ ~~170/2~~ ~~175/2~~ ~~180/2~~ ~~185/2~~ ~~190/2~~ ~~195/2~~ ~~200/2~~ ~~205/2~~ ~~210/2~~ ~~215/2~~ ~~220/2~~ ~~225/2~~ ~~230/2~~ ~~235/2~~ ~~240/2~~ ~~245/2~~ ~~250/2~~ ~~255/2~~ ~~260/2~~ ~~265/2~~ ~~270/2~~ ~~275/2~~ ~~280/2~~ ~~285/2~~ ~~290/2~~ ~~295/2~~ ~~300/2~~ ~~305/2~~ ~~310/2~~ ~~315/2~~ ~~320/2~~ ~~325/2~~ ~~330/2~~ ~~335/2~~ ~~340/2~~ ~~345/2~~ ~~350/2~~ ~~355/2~~ ~~360/2~~ ~~365/2~~ ~~370/2~~ ~~375/2~~ ~~380/2~~ ~~385/2~~ ~~390/2~~ ~~395/2~~ ~~400/2~~ ~~405/2~~ ~~410/2~~ ~~415/2~~ ~~420/2~~ ~~425/2~~ ~~430/2~~ ~~435/2~~ ~~440/2~~ ~~445/2~~ ~~450/2~~ ~~455/2~~ ~~460/2~~ ~~465/2~~ ~~470/2~~ ~~475/2~~ ~~480/2~~ ~~485/2~~ ~~490/2~~ ~~495/2~~ ~~500/2~~ ~~505/2~~ ~~510/2~~ ~~515/2~~ ~~520/2~~ ~~525/2~~ ~~530/2~~ ~~535/2~~ ~~540/2~~ ~~545/2~~ ~~550/2~~ ~~555/2~~ ~~560/2~~ ~~565/2~~ ~~570/2~~ ~~575/2~~ ~~580/2~~ ~~585/2~~ ~~590/2~~ ~~595/2~~ ~~600/2~~ ~~605/2~~ ~~610/2~~ ~~615/2~~ ~~620/2~~ ~~625/2~~ ~~630/2~~ ~~635/2~~ ~~640/2~~ ~~645/2~~ ~~650/2~~ ~~655/2~~ ~~660/2~~ ~~665/2~~ ~~670/2~~ ~~675/2~~ ~~680/2~~ ~~685/2~~ ~~690/2~~ ~~695/2~~ ~~700/2~~ ~~705/2~~ ~~710/2~~ ~~715/2~~ ~~720/2~~ ~~725/2~~ ~~730/2~~ ~~735/2~~ ~~740/2~~ ~~745/2~~ ~~750/2~~ ~~755/2~~ ~~760/2~~ ~~765/2~~ ~~770/2~~ ~~775/2~~ ~~780/2~~ ~~785/2~~ ~~790/2~~ ~~795/2~~ ~~800/2~~ ~~805/2~~ ~~810/2~~ ~~815/2~~ ~~820/2~~ ~~825/2~~ ~~830/2~~ ~~835/2~~ ~~840/2~~ ~~845/2~~ ~~850/2~~ ~~855/2~~ ~~860/2~~ ~~865/2~~ ~~870/2~~ ~~875/2~~ ~~880/2~~ ~~885/2~~ ~~890/2~~ ~~895/2~~ ~~900/2~~ ~~905/2~~ ~~910/2~~ ~~915/2~~ ~~920/2~~ ~~925/2~~ ~~930/2~~ ~~935/2~~ ~~940/2~~ ~~945/2~~ ~~950/2~~ ~~955/2~~ ~~960/2~~ ~~965/2~~ ~~970/2~~ ~~975/2~~ ~~980/2~~ ~~985/2~~ ~~990/2~~ ~~995/2~~ ~~1000/2~~ ~~1005/2~~ ~~1010/2~~ ~~1015/2~~ ~~1020/2~~ ~~1025/2~~ ~~1030/2~~ ~~1035/2~~ ~~1040/2~~ ~~1045/2~~ ~~1050/2~~ ~~1055/2~~ ~~1060/2~~ ~~1065/2~~ ~~1070/2~~ ~~1075/2~~ ~~1080/2~~ ~~1085/2~~ ~~1090/2~~ ~~1095/2~~ ~~1100/2~~ ~~1105/2~~ ~~1110/2~~ ~~1115/2~~ ~~1120/2~~ ~~1125/2~~ ~~1130/2~~ ~~1135/2~~ ~~1140/2~~ ~~1145/2~~ ~~1150/2~~ ~~1155/2~~ ~~1160/2~~ ~~1165/2~~ ~~1170/2~~ ~~1175/2~~ ~~1180/2~~ ~~1185/2~~ ~~1190/2~~ ~~1195/2~~ ~~1200/2~~ ~~1205/2~~ ~~1210/2~~ ~~1215/2~~ ~~1220/2~~ ~~1225/2~~ ~~1230/2~~ ~~1235/2~~ ~~1240/2~~ ~~1245/2~~ ~~1250/2~~ ~~1255/2~~ ~~1260/2~~ ~~1265/2~~ ~~1270/2~~ ~~1275/2~~ ~~1280/2~~ ~~1285/2~~ ~~1290/2~~ ~~1295/2~~ ~~1300/2~~ ~~1305/2~~ ~~1310/2~~ ~~1315/2~~ ~~1320/2~~ ~~1325/2~~ ~~1330/2~~ ~~1335/2~~ ~~1340/2~~ ~~1345/2~~ ~~1350/2~~ ~~1355/2~~ ~~1360/2~~ ~~1365/2~~ ~~1370/2~~ ~~1375/2~~ ~~1380/2~~ ~~1385/2~~ ~~1390/2~~ ~~1395/2~~ ~~1400/2~~ ~~1405/2~~ ~~1410/2~~ ~~1415/2~~ ~~1420/2~~ ~~1425/2~~ ~~1430/2~~ ~~1435/2~~ ~~1440/2~~ ~~1445/2~~ ~~1450/2~~ ~~1455/2~~ ~~1460/2~~ ~~1465/2~~ ~~1470/2~~ ~~1475/2~~ ~~1480/2~~ ~~1485/2~~ ~~1490/2~~ ~~1495/2~~ ~~1500/2~~ ~~1505/2~~ ~~1510/2~~ ~~1515/2~~ ~~1520/2~~ ~~1525/2~~ ~~1530/2~~ ~~1535/2~~ ~~1540/2~~ ~~1545/2~~ ~~1550/2~~ ~~1555/2~~ ~~1560/2~~ ~~1565/2~~ ~~1570/2~~ ~~1575/2~~ ~~1580/2~~ ~~1585/2~~ ~~1590/2~~ ~~1595/2~~ ~~1600/2~~ ~~1605/2~~ ~~1610/2~~ ~~1615/2~~ ~~1620/2~~ ~~1625/2~~ ~~1630/2~~ ~~1635/2~~ ~~1640/2~~ ~~1645/2~~ ~~1650/2~~ ~~1655/2~~ ~~1660/2~~ ~~1665/2~~ ~~1670/2~~ ~~1675/2~~ ~~1680/2~~ ~~1685/2~~ ~~1690/2~~ ~~1695/2~~ ~~1700/2~~ ~~1705/2~~ ~~1710/2~~ ~~1715/2~~ ~~1720/2~~ ~~1725/2~~ ~~1730/2~~ ~~1735/2~~ ~~1740/2~~ ~~1745/2~~ ~~1750/2~~ ~~1755/2~~ ~~1760/2~~ ~~1765/2~~ ~~1770/2~~ ~~1775/2~~

- 
 Tak samo jak wyci
 Wartość dodatnia

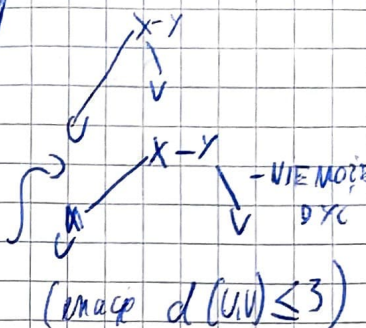
Zad. 3

• Zał. $k = d(G) > 3$ • $d(u,v) \neq k$, \bar{d} - odległość w \bar{G}

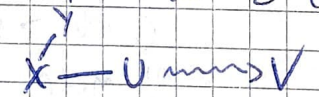
Wczy dowód $x, y \in V(G)$, ~~$x \neq y$ nie mogą być bliźniaczymi w G~~

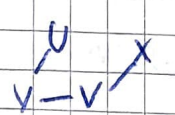
~~$d(u,v) = 3$~~

$1^0 \quad d(x,y) \neq 1 \Rightarrow \exists \{x,y\} \in E(\bar{G}) \Leftrightarrow \bar{d}(x,y) = 1$

$2^0 \quad d(x,y) = 1$
 x nie łączy z u lub y
 y nie łączy z u lub y albo

 (mając $d(u,v) \leq 3$)

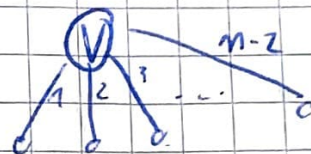
I) $\forall x$ nie jest łączy z u i v , wtedy w \bar{G} są łączy: ~~$\bar{d}(x,y) = 2$~~

II) ~~x nie łączy z~~ Jest jedna krawędź z $\{x,y\} \times \{u,v\}$
 BSO x łączy z u : 
~~Wtedy nie mogą być bliźniaczymi w G~~
 ~~x łączy z v i u~~

Wtedy w \bar{G}  $d(x,y) = 2$

Zad. 4 $d(G)=2$ $\max\{\deg(v) : v \in V(G)\} = n-2$

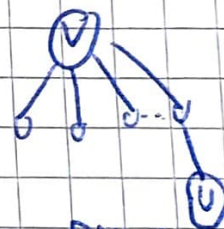
Niech v t.e. $\deg(v) = n-2$



Rys. 1

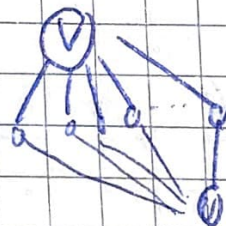
- Aby ~~zrealizować~~ v musi być miareczką pomiędzy z dektate. jeden miareczkiem, onay go v .

Wtedy v musi być połączony z jednym z sąsiadów v (G nie n)



Rys. 2

- W oparciu z Rys. 2 możemy zobaczyć z u do sąsiadów v d.t. 3 a $d(G)=2$ więc musimy połączyć u z resztą sąsiadów v .



Rys. 3

Mał więc odcinek: $(n-2) + (n-2)$ krawędzi.

$$2n-4 \leq n$$



Zad 8

Dla ~~określ~~ i, j wyznaczyć $B^T B$ o tych współrzędnych B :

$$(B^T B)_{ij} = \begin{cases} 1 - e_i \cdot e_j - \text{wszystkie} & i \neq j \\ 2 - 1 = 1 & i = j \\ 0 - \text{wszystkie} & i, j > n \end{cases} \cdot \begin{bmatrix} 0 & \dots & 1 & 1 \\ \vdots & & & \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 1 \\ \vdots \end{bmatrix}_m = 1 \Leftrightarrow \text{KA są takie same} \quad \text{jedna}$$

$$(B^T B - 2I)_{ij} = \begin{cases} 1 - e_i \cdot e_j - \text{wszystkie} & i \neq j \\ 0 - \text{wszystkie} & i = j \end{cases}$$

$$B^T B - 2I = C \quad \square$$

Zad 11

• Liczba drzew Cayleya liczba drzew indeksu $\{1 \dots n\}$ k^{n-2} .

• ~~Liczba drzew z 1 wierzchołkiem~~ Wszystkie drzewa $\{2 \dots n\}$ mogą być wierzchołkami (każdy wierzchołek może być z 1).

Wzrost

$$(n-1) \cdot (n-1)^{n-3}$$

Wszystkich drzew: n^{n-2}

$$\text{Ostatecznie: } p_n = \frac{n(n-1)^{n-2}}{n^{n-2}}$$

Liczba granice

$$\lim_{n \rightarrow \infty} p(n) = \lim_{n \rightarrow \infty} \left[\frac{n(n-1)^{n-2}}{n^{n-2}} = \left(1 - \frac{1}{n}\right)^{n-2} \right] = e^{-1}$$

Zad 5

a) • $r(G) \leq d(G)$ typowe $\min SS \leq \max SS$

• $d(G) \leq 2 \cdot r(G)$

niech $d(u,v) = d(G)$

niech $c \in W$ centrum

~~$d(G)$~~

$d(G) = d(u,v) \leq d(u,c) + d(c,v) \leq 2 \cdot r(G)$

b)

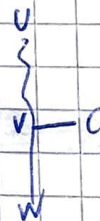
$\left. \begin{array}{l} \{ \\ \} \end{array} \right\} d(G)$

lemat: w. centrum, c jest na najdłuższej ścieżce
dł. $d(G)$

pod: Jeżeli c jest

poza $u \rightarrow w$ to

$r(v) < r(c)$



U.c) z Lematu wiemy, że centrum jest na najdłuższej ścieżce. Jeżeli zbiór

wszystkich centrów zawiera nie 2 to dla pewnych i, j $r(c_i) < r(c_j)$, $r(c_j) < r(c_i)$

Algem: Wypiszmy indeksy najdłuższej ścieżki w grafie.