Zad. 3

$p(x)$ st. n $\qquad q(x) = p[x, x_1, \dots, x_k]$

D-d ind:

1. Baza: $k=0$

$$q(x) = p[x] = p(x)$$

st $p(x) - 0 =$ st. $p(x)$, współczynniki te same

2. Krok ind.

Założenie indukcyjne: ~~$\forall_{k_0 \leq k-1}$ $p[x, x_1, x_2, \dots, x_{k_0}]$ st. $n-k_0$~~

Dla każdego $k_0 \leq k$ $\quad q(x) = p[x, x_1, \dots, x_{k_0}] = ax^{n-k} + \dots$

gdy: $p(x) = ax^n + \dots$

Dla k:

$$q(x) = p[x, x_1, \dots, x_k] \overset{\text{ze wzoru rekurencyjnego}}{=} \frac{p[x_1, x_2, \dots, x_k] - p[x, x_1, \dots, x_{k-1}]}{x_k - x} = \frac{W(x)}{x_k - x}$$

- $(x_k - x) \mid W(x)$ z tw. Bézouta bo $W(x_k) = p[x_1, \dots, x_k] - p[x_k, x_1, \dots, x_{k-1}] = p[x_1, \dots, x_k] - p[x_1, \dots, x_k] = 0.$

- Z zat. ind. $q_0(x) = p[x, x_1, \dots, x_{k-1}] = ax^{n-k+1} + \dots$ gdy $p(x) = ax^n + \dots$

$$W(x) = C - q_0(x)$$

$$\boxed{q(x) = \frac{W(x)}{x_k - x} = \frac{C - q_0(x)}{x_k - x} \begin{array}{l} \leftarrow \text{st. } n-k+1 \\ \leftarrow \text{st. } 1 \end{array} \quad \text{Zatem } q(x) \text{ st. } n-k}$$

~~Korzystając z dzielenia wielomianów W(x)~~ Zatem $q(x) \cdot (x_k - x) = W(x) = C - q_0(x)$

$$(a' x^{n-k-1} + \dots) \cdot (x_k - x) = -ax^{n-k-1} + \dots$$

gdy $p(x) = ax^n + \dots$

$$\Downarrow$$

$$a' = a$$