

$$L7Z1^* \quad \theta_1 \leq \theta \leq \theta_2$$

$$r = r(\theta)$$

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

$$\sigma'(\theta) = (\cos(\theta) r'(\theta) - r(\theta) \sin(\theta), \sin(\theta) r'(\theta) + r(\theta) \cos(\theta))$$

$$\int_{\sigma} f ds = \int_{\sigma} f(x, y) ds = \int_{\theta_1}^{\theta_2} f(\sigma(\theta)) \|\sigma'(\theta)\| d\theta =$$

$$= \int_{\theta_1}^{\theta_2} f(r(\theta) \cos \theta, r(\theta) \sin \theta) \|\sigma'(\theta)\| d\theta =$$

$$= \int_{\theta_1}^{\theta_2} f(r(\theta) \cos \theta, r(\theta) \sin \theta) \sqrt{r^2(\theta) + r'^2(\theta)} d\theta$$

$$\begin{aligned} * \quad \|\sigma'(\theta)\| &= \left(\cos^2(\theta) r'^2(\theta) - 2 \sin \theta \cos \theta r'(\theta) r(\theta) + r^2(\theta) \sin^2(\theta) + \sin^2(\theta) r'^2(\theta) + 2 \sin \theta \cos \theta r(\theta) r'(\theta) + r^2(\theta) \cos^2(\theta) \right)^{\frac{1}{2}} = \\ &= \left(r'^2(\theta) (\sin^2 \theta + \cos^2 \theta) + r^2(\theta) (\cos^2 \theta + \sin^2 \theta) \right)^{\frac{1}{2}} = \sqrt{r^2(\theta) + r'^2(\theta)} \end{aligned}$$

Przykład (b) dla $r = 1 + \cos \theta$ $0 \leq \theta \leq 2\pi$

$$r'(\theta) = -\sin \theta$$

$$\begin{aligned} L(\sigma) &= \int_{\sigma} ds = \int_{\theta_1}^{\theta_2} \|\sigma'(\theta)\| d\theta = \int_0^{2\pi} \sqrt{r^2(\theta) + r'^2(\theta)} d\theta = \int_0^{2\pi} \sqrt{1 + 2\cos \theta + \cos^2 \theta + \sin^2 \theta} d\theta = \\ &= \int_0^{2\pi} \sqrt{2(1 + \cos \theta)} d\theta = \sqrt{2} \int_0^{2\pi} \sqrt{1 + \cos \theta} d\theta = \end{aligned}$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{2 \cos^2 \left(\frac{\theta}{2}\right)} d\theta = 2 \int_0^{2\pi} |\cos \frac{\theta}{2}| d\theta$$

$$= 2 \int_0^{2\pi} |\cos \frac{\theta}{2}| d\theta = 8$$



2 WYKRESU