zad 1

$$\begin{cases} x^2+y^2+z^2=1 \\ \sqrt{3}\,(x^2+y^2)^{1/2}=z \end{cases}$$

$R=1$

$\varphi(u,v) = (\sin u\cos v, \sin u\cos v, \cos v)$

$\rho(x,y,z) = |x|+y^2+z^4$

$\to$ ~~$x^2+y^2+z^2=1$~~

$3(1-z^2)=z^2$

$\Downarrow$

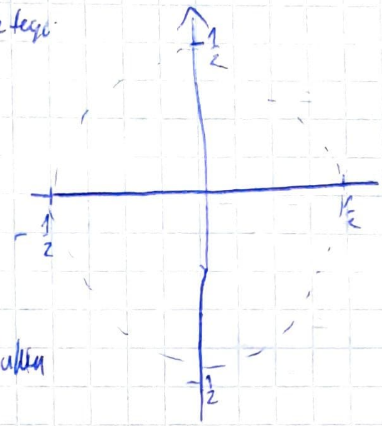
$3=4z^2$

$z=\frac{\sqrt{3}}{2}$

$\to y^2 = 1-z^2-x^2 = 1-\frac{3}{4}-x^2$

$y=\sqrt{\frac{1}{4}-x^2} \quad \vee \quad y=-\sqrt{\frac{1}{4}-x^2}$

z tego:




Wobec ~~[...]~~ powyższego całkujemy po okręgu o środku $O$ i $R=\frac{1}{2}$

~~[...]~~ przechodzimy na wsp. biegunowe dla ułatwienia rachunków.

$x = r\cos\theta$
$y = r\sin\theta \qquad r=\frac{1}{2}$

$\rho(x,y,z) = \rho\left(r\cos\theta, r\sin\theta, \frac{\sqrt{3}}{2}\right) = \rho\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{2}, \frac{\sqrt{3}}{2}\right)$

$$M = \int_0^{2\pi} \rho\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{2}, \frac{\sqrt{3}}{2}\right) d\theta = \int_0^{2\pi} \left|\frac{\cos\theta}{2}\right| + \left(\frac{\sin\theta}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^4 d\theta =$$

$$= \frac{1}{2}\int_0^{2\pi} |\cos\theta| + \frac{\sin^2\theta}{2} + \frac{9}{8}\, d\theta = \frac{1}{16}\int_0^{2\pi} 8|\cos\theta| + 4\sin^2\theta + 9\, d\theta =$$

$$= \frac{1}{16}\left(8\int_0^{2\pi} |\cos\theta|\, d\theta + 4\cdot\int_0^{2\pi} \sin^2\theta\, d\theta + 9\int_0^{2\pi} 1\, d\theta\right) =$$

$$= \frac{1}{16}\left(8\cdot 4 + 4\cdot\pi + 9\cdot 2\pi\right) = \frac{1}{16}(32+22\pi)$$

bo

$\bullet \int_0^{2\pi} |\cos\theta|\, d\theta \overset{\text{z wykresu}}{=\!=} 4\cdot\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta = 4\cdot \sin\theta\Big|_0^{\frac{\pi}{2}} = 4$

$\bullet \int_0^{2\pi} \sin^2\theta\, d\theta = \frac{1}{2}\int_0^{2\pi}(1-\cos(2\theta)) = \frac{1}{2}\int_0^{2\pi} 1 - \frac{1}{2}\int_0^{2\pi}\cos(2\theta)\, d\theta =$

$= \pi - \frac{1}{4}\int_0^{4\pi}\cos(t)\, dt = \pi - 0 = \pi$

Zad. 2

$$\begin{cases} x+y+z=1 \\ x,y,z \geq 0 \end{cases} \qquad \iint_S x^2+2xy \, dS \qquad \to x,y,z \in [0,1] \; *$$

$*$ z tego: $z = 1-x-y = f(x,y)$ dla $x,y,z \geq 0$ mamy

→ ze wzoru ze wstępu

$$\iint_S x^2+2xy \, dS = \iint x^2+2xy \cdot \sqrt{1+(1^2+1^2)} \, dxdy = \sqrt{3} \iint_D x^2+2xy \, dxdy =$$

→ 2 *

$$= \sqrt{3} \int_0^1 \int_0^{1-x} x^2+2xy \, dxdy = \sqrt{3} \int_0^1 yx^2+xy^2 \Big|_0^{1-x} dx =$$

$$= \sqrt{3} \int_0^1 (1-x)x^2 + x(1-x)^2 \, dx = \sqrt{3} \int_0^1 x^2-x^3+x-2x^2+x^3 \, dx = \sqrt{3}$$

$$= \sqrt{3} \int_0^1 x-x^2 \, dx = \sqrt{3} \; \frac{x^2}{2}-\frac{x^3}{3} \Big|_0^1 = \sqrt{3} \left(\frac{1}{2}-\frac{1}{3}\right) = \sqrt{3} \cdot \frac{1}{6}$$

Zad 3

$V \in \mathbb{R}^3 \qquad \sigma = x^2 + 4y^2 = 1$

$$\int_\sigma \cos(\angle(v,n))\, ds$$

mamy punkt wzór należe $\qquad a = \ldots$ gdy $y=0$ więc $a=1$

$\qquad b$ gdy $x=0$ więc $b = \frac{1}{2}$

$$\sigma(t) = (a\cos t,\ b\sin t) = (\cos t,\ \tfrac{1}{2}\sin t)$$

$$\sigma'(t) = (-\sin t,\ \tfrac{1}{2}\cos t)$$

$$n_\sigma(t) = (\tfrac{1}{2}\cos t,\ \sin t) \qquad V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\int_\sigma \cos(\angle(v,n))\,ds = \int_0^{2\pi} \frac{n_\sigma(t) \circ V}{\|V\| \cdot \|n_\sigma(t)\|}\,dt = \int_0^{2\pi} \frac{V_1 \cdot \frac{1}{2}\cos t + V_2 \sin t}{1 \cdot \sqrt{\frac{1}{4}\cos^2 t + \sin^2 t}}\,dt =$$

$$= \int_0^{2\pi} \frac{\frac{V_1}{2}\cos t + V_2 \sin t}{\sqrt{1 - \frac{3}{4}\cos^2 t}}\,dt = 0$$

bo

$$\int_0^{2\pi} \frac{\cos t}{\sqrt{4 - 3\cos^2 t}}\,dt = \int_0^{2\pi} \frac{\cos t}{\sqrt{3\sin^2 t + 1}} = \left| \begin{matrix} u = \sin t \\ \frac{du}{dx} = \cos t \end{matrix} \right| = \int_0^{0} \frac{1}{\sqrt{3u^2+1}}\,du = 0$$

z sin jedne