zad. 2

I) Stosujemy kwadratury Newtona – Cotesa:

$$I(f) = \int_a^b L_n(x)\,dx = \int_a^b \sum_{k=0}^{n} f(x_k) \cdot \prod_{\substack{j=0 \\ j\neq k}}^{n} \frac{x-x_j}{x_k-x_j}\,dx = \sum_{k=0}^{n} f(x_k) \int_a^b \prod_{\substack{j=0 \\ j\neq k}}^{n} \frac{x-x_j}{x_k-x_j}\,dx$$

współczynniki:

$$A_k^{(n)} = \int_a^b \prod_{j=0 \wedge j\neq k}^{n} \frac{x-x_j}{x_k-x_j}\,dx = \left| \begin{matrix} x = a+th \\ dx = h\cdot dt \end{matrix} \right| = \int_0^n \prod_{\substack{j=0 \\ j\neq k}}^{n} \frac{h(t-j)}{h(k-j)}\,h\,dt =$$

$$= h \prod_{\substack{j=0 \wedge \\ j\neq k}}^{n} \frac{1}{(k-j)} \int_0^n \prod_{\substack{j=0 \\ j\neq k}}^{n} (t-j)\,dt = \qquad \ulcorner \prod_{\substack{j=0 \wedge \\ j\neq k}}^{n} (k-j) = k!\,(n-k)!\,(-1)^{n-k}$$

$$\Downarrow$$

$$= h\,(-1)^{n-k} \frac{1}{k!\,(n-k)!} \int_0^n \prod_{\substack{j=0 \\ j\neq k}}^{n} (t-j)\,dt \quad \square \qquad \prod_{\substack{j=0 \wedge \\ j\neq k}}^{n} \frac{1}{(k-j)} = (-1)^{n-k} \frac{1}{k!\,(n-k)!} \lrcorner$$

II) a) $B_k^{(n)} = \dfrac{A_k^{(n)}}{b-a} = \dfrac{A_k^{(n)}}{nh}$ ← wymierne; ← całkowite } wymierne

← całka całki z wielomianu o współczynnikach całkowitych

b) $B_{n-k}^{(n)} = (-1)^k \dfrac{1}{n\cdot k!\,(n-k)!} \displaystyle\int_0^n \prod_{\substack{j=0 \\ j\neq n-k}}^{n} (t-j)\,dt$ ← przeciwny dowodzimy B_k

$$B_{n-k}^{(n)} = \left| \begin{matrix} t = n-s \\ dt = -ds \end{matrix} \right| = \frac{(-1)^k}{n\cdot k!\,(n-k)!} \int_0^n \prod_{\substack{j=0 \\ j\neq n-k}}^{n} (n-s-j)\,ds\,(-1) =$$

$$= \frac{(-1)^k(-1)^n}{n\cdot k!\,(n-k)!} \int_0^n \prod_{\substack{j=0 \\ j\neq n-k}}^{n} (s-(n-j))\,ds \cdot (-1) = \frac{(-1)^{n+k}}{n\cdot k!\,(n-k)!} \int_0^n \prod_{\substack{j=0 \\ j\neq n-k}}^{n} (s-(n-j))\,ds =$$

(·(−1): zmiana granic całk.)

(zmiana zmiennej na przeciwną)

$$= \frac{(-1)^{k+n}}{n\cdot k!\,(n-k)!} \int_0^n \prod_{\substack{w=0 \\ w\neq k}}^{n} (s-w)\,ds = \frac{(-1)^{n-k}}{n\,k!\,(n-k)!} \int_0^n \prod_{\substack{w=0 \\ w\neq k}}^{n} (s-w)\,ds = B_k^{(n)}$$

■