L7 Z1* $\theta_1 \le \theta \le \theta_2$

$r = r(\theta)$

$x = r\cos\theta$
$y = r\sin\theta$

$\sigma'(\theta) = (\cos(\theta) r'(\theta) - r(\theta)\sin(\theta),\ \sin(\theta) r'(\theta) + r(\theta)\cos(\theta))$

$\|\sigma'(\theta)\| = ?$

$$\int_\sigma f\,ds = \int_\sigma f(x,y)\,ds = \int_{\theta_1}^{\theta_2} f(\sigma(\theta))\,\|\sigma'(\theta)\|\,d\theta =$$

$$= \int_{\theta_1}^{\theta_2} f(r(\theta)\cos\theta,\ r(\theta)\sin\theta)\cdot\|\sigma'(\theta)\|\,d\theta =$$

$$\overset{*}{=} \int_{\theta_1}^{\theta_2} f(r(\theta)\cos(\theta),\ r(\theta)\sin\theta)\sqrt{r^2(\theta) + r'^2(\theta)}\,d\theta$$

$$* \quad \|\sigma'(\theta)\| = \left(\cos^2(\theta) r'^2(\theta) - 2\sin\theta\cos\theta\, r'(\theta) r(\theta) + r^2(\theta)\sin^2(\theta) + \sin^2(\theta) r'^2(\theta) + 2\sin\theta\cos\theta\, r(\theta) r'(\theta) + r^2(\theta)\cos^2(\theta)\right)^{\frac{1}{2}} =$$

$$= \left(r^2(\theta)(\sin^2\theta + \cos^2\theta) + r'^2(\theta)(\cos^2\theta + \sin^2\theta)\right)^{\frac{1}{2}} = \sqrt{r^2(\theta) + r'^2(\theta)}$$

Policz $l(\sigma)$ dla $r = 1 + \cos\theta$ $0 \le \theta \le 2\pi$

$r'(\theta) = -\sin\theta$

$$l(\sigma) = \int_\sigma ds = \int_{\theta_1}^{\theta_2} \|\sigma'(\theta)\|\,d\theta = \int_0^{2\pi} \sqrt{r^2(\theta) + r'^2(\theta)}\,d\theta = \int_0^{2\pi} \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}\,d\theta =$$

$$= \int_0^{2\pi} \sqrt{2(1+\cos\theta)}\,d\theta = \sqrt{2}\int_0^{2\pi} \sqrt{1+\cos\theta}\,d\theta =$$

$$= \sqrt{2}\cdot\int_0^{2\pi} \sqrt{2\cos^2\left(\frac{\theta}{2}\right)}\,d\theta =$$

$$= 2\int_0^{2\pi} |\cos\tfrac{\theta}{2}|\,d\theta = 8$$

↑ z wykresu