Zad.1 MAURYCY BORKOWSKI ZAD: 1, 3, 4, 5, 6, 8, 12, 14

$\lfloor an \rfloor + \lfloor (1-a)n \rfloor = \lfloor an \rfloor + \lfloor n - an \rfloor = \lfloor an \rfloor + \lfloor n \rfloor + \lfloor -an \rfloor \overset{*}{=} n-1$

$* \quad \lfloor x \rfloor + \lfloor -x \rfloor = \begin{cases} 0, & x \in \mathbb{Z} \\ -1, & x \notin \mathbb{Z} \end{cases}$

$\lceil an \rceil + \lceil n \rceil + \lceil -an \rceil = n+1$

D-d dla $\lfloor \, \rfloor$:

$1^\circ \; x \in \mathbb{Z}$

$\lfloor x \rfloor = -\lfloor -x \rfloor$

$2^\circ \; x \notin \mathbb{Z}$

$x = a + k \quad a = \{x\} \quad a > 0$

$\lfloor a + k \rfloor = \lfloor k \rfloor$

$\lfloor -x \rfloor = \lfloor -k \rfloor + \lfloor -a \rfloor = -\lfloor k \rfloor - 1$

Zad.3

OKREŚLIĆ a_0, a_1

a) $a_n = n a_{n-2}$

b) $a_n = a_{n-1} + a_{n-3}$ OKREŚLIĆ a_0, a_1, a_2

c) $a_n = 2a_{\lfloor n/2 \rfloor} + n$

$a_0 = 2a_0 + 0 \iff a_0 = 0$

$a_1 = 2a_{\underset{0}{\lfloor \frac{1}{2} \rfloor}} + 1 = 1$

$a_2 = 2a_{\underset{1}{\lfloor \frac{2}{2} \rfloor}} + 2 = 4$

$\vdots$

możemy wyznaczyć wszystkie wartości

Zad.4

a) $f_n = f_{n-1} + 3^n \quad n > 1, \; f_1 = 3$ $\qquad f_n = \sum_{i=1}^{n} 3^i$

b) $h_n = h_{n-1} + (-1)^{n+1} n$ $\qquad h_n = \sum_{i=1}^{n} (-1)^{i+1} \cdot i$

c) $l_n = l_{n-1} l_{n-2}$ $\qquad l_n = 1 \quad 2^{F_n}$

Zad 5

a) $a_0 = 1$, $a_n = \frac{2}{a_{n-1}}$ $\quad a_n = \begin{cases} 1, & a \equiv_2 0 \\ 2, & \text{wpp} \end{cases}$

b) $b_0 = 0$, $b_n = \frac{1}{1+b_{n-1}}$ $\quad b_n = \frac{F_{n-1}}{F_n}$

c) $c_0 = 1$, $c_n = \sum_{i=0}^{n-1} c_i$ $\quad c_n = 2^{n-1}$ $\quad n > 0$

d) $d_0 = 1$, $d_1 = 2$

$d_n = \frac{d_{n-1}^2}{d_{n-2}}$ $\quad d_n = 2^n$

Zad. 6

a) $y_0 = y_1 = 1$ $\quad y_n = \frac{y_{n-1}^2 + y_{n-2}}{y_{n-1} + y_{n-2}}$ $\quad y_n = 1$

b) $z_0 = 1$, $z_1 = 2$ $\quad z_n = \frac{z_{n-1}^2 - 1}{z_{n-2}}$ $\quad z_n = n+1$

c) $t_0 = 0$, $t_1 = 1$ $\quad t_n = \frac{(t_{n-1} - t_{n-2} + 3)^2}{4}$ $\quad t_n = n^2$

Zad. 8

$a_n = \frac{1 + a_{n-1}}{a_{n-2}}$ $\quad a_0 = \alpha \quad a_1 = \beta$

$a_2 = \frac{1+\beta}{\alpha} = \frac{1}{\alpha} + \frac{\beta}{\alpha}$

$a_3 = \frac{1 + \frac{1+\beta}{\alpha}}{\beta} = \frac{1}{\beta} + \frac{1+\beta}{\alpha\beta} = \frac{1}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta} = \frac{\alpha+\beta+1}{\alpha\beta}$

$a_4 = \frac{1 + \frac{1}{\alpha} + \frac{\beta}{\alpha}}{\frac{1}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}} = \frac{\alpha+1}{\beta}$

$a_5 = \frac{1 + \frac{\alpha+1}{\beta}}{\frac{1+\alpha+\beta}{\alpha\beta}} = \alpha$

$a_6 = \frac{1+\alpha}{\frac{1+\alpha}{\beta}} = 1+\alpha = \frac{\beta}{1+\alpha} = \beta$

CYKL

$\begin{cases} \alpha \neq 0 \\ \alpha \neq -1 & a_0 \\ \beta \neq 0 \\ \beta \neq -1 & a_2 \\ \alpha+\beta \neq -1 & a_3 \end{cases}$

Zad. 12 Wzór: $\frac{n^3+5n+6}{6} = 1 + n + \binom{n}{2} + \binom{n}{3}$

D-d:

*N prostych może podzielić płaszczyznę na maksymalnie $\frac{n^2+n+2}{2}$ obszarów. $\frac{n^3+5n+6}{6}$

1. Podstawa indukcji:

- dla 1: $\frac{1+5+6}{6} = 2$ ✓

2: $\frac{8+10+6}{6} = 4$ ✓

2. Załóżmy: przestrzeń możemy podzielić n cięciami na części

n+1 płaszczyzn możemy podzielić:

$\frac{n^3+5n+6}{6} + \frac{n^2+n+2}{2} = \frac{(n+1)^3+5(n+1)+6}{6}$ ∎

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Zad. 14

a)

$F_0 + F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$

b) $F_1 + F_3 + \ldots + F_{2n-1} =$

Podstawa $F_0 = 0 = F_2 - 1$

Krok: F_{n+2}

$F_0 + \ldots F_{n-1} + F_n = (F_{n+1} - 1) + F_n = F_{n+2} - 1$ ∎

b) $F_1 + F_3 + \ldots + F_{2n-1} = F_{2n}$

Podstawa $F_1 = 1 = F_2$ ✓

Krok

$F_1 + \ldots + F_{2n-3} + F_{2n-1} = F_{2n-2} + F_{2n-1} = F_{2n}$ ∎

c) $F_0^2 + \ldots + F_n^2 = F_n F_{n+1}$

Podstawa $F_0^2 = 0^2 = 0 \cdot 1 = F_0 \cdot F_1$

Krok $F_0^2 + \ldots + F_{n-1}^2 + F_n^2 = F_{n-1} F_n + F_n^2 = F_n \cdot (F_{n-1} + F_n) = F_n \cdot F_{n+1}$ ∎

d) $F_n F_{n+2} = F_{n+1}^2 + (-1)^{n+1}$

Podstawa: $F_0 F_2 = 0 \cdot 1 = 1^2 - 1 = F_1^2 - (1)^1 = 0$

Krok: $F_n F_{n+2} = F_n(F_n + F_{n+1}) = F_n^2 + F_n F_{n+1} \overset{\text{zał.}}{=} F_{n-1} F_{n+1} - (-1)^n + F_n F_{n+1} = F_{n+1}(F_{n-1} + F_n) - (-1$

$= F_{n+1}^2 + (-1)^{n+1}$