Problem 4 [1.5p]

1. $\frac{\partial}{\partial \bar{\Phi}} \log \mathcal{L}(\bar{\Phi}) = \frac{\partial}{\partial \bar{\Phi}} \left[\log \binom{n}{n\bar{\varphi}} + n\bar{\varphi} \log \bar{\Phi} + (n - n\bar{\varphi}) \log(1-\bar{\Phi}) \right] =$

$= \frac{n\bar{\varphi}}{\bar{\Phi}} - \frac{n - n\bar{\varphi}}{1-\bar{\Phi}} = 0 \Rightarrow \bar{\varphi} = \bar{\Phi}$ From Lecture

2. $\frac{\partial}{\partial \bar{\Phi}} \log \left(\prod_{i=1}^{n} p(x_i \mid \bar{\Phi}) \right) = \frac{\partial}{\partial \bar{\Phi}} \sum \log(p(x_i \mid \bar{\Phi})) =$

$= \frac{\partial}{\partial \bar{\Phi}} \sum_{i=1}^{n} \log \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x_i-\mu)^2}{2\sigma^2}} \right) = \frac{\partial}{\partial \bar{\Phi}} \sum_{i=1}^{n} -\frac{1}{2}\log(2\pi\sigma^2) - \frac{(x_i-\mu)^2}{2\sigma^2} =$

$= \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu) = 0 \Rightarrow \mu = \frac{1}{n} \sum x_i$

3. $L(\lambda; x_1 \ldots x_n) = \prod_{j=1}^{n} \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}$

$\log(L) = -n\lambda - \sum_{j=1}^{n} \log(x_j!) + \log(\lambda) \sum_{j=1}^{n} x_j$

$\frac{dl}{d\lambda} = -n + \frac{1}{\lambda} \sum_{j=1}^{n} x_j = 0 \Rightarrow \lambda = \frac{1}{n} \sum_{j=1}^{n} x_j$

Problem 5.

$$\cancel{M} \; L(x_1 \dots x_n \mid \mu, \lambda) = \prod_{i=1}^{n} \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}$$

$$\log(L) = \sum_{i=1}^{n} \log\left(\frac{1}{2b} \cdot e^{-\frac{|x-\mu|}{b}}\right) = \sum_{i=1}^{n} \left(\log\left(\frac{1}{2b}\right) - \frac{|x_i-\mu|}{b}\right)$$

$$\frac{d}{d\mu}\left(\sum_{i=1}^{n} \log\left(\frac{b}{2}\right) - \frac{|x_i-\mu|}{b}\right) = \frac{d}{d\mu}\left(-\frac{1}{b}\sum_{i=1}^{n} |x_i-\mu|\right)$$

$$\underset{\mu}{\operatorname{argmin}} = \underset{\mu}{\operatorname{argmin}} \sum_{i=1}^{n} \operatorname{sign}(x_i - \mu) = 0$$

Med

wtw liczba dodatnie = liczba ujemnych.

eng. Average absolute deviation - MAD

pol: suma odchyleń bezwzględnych - średnia

Problem 6.

1. Box 1: B x2 R x8 — Box 2: B x5 R x5

$$P(B_1 \mid Red) = \frac{P(Red \mid B_1)\,P(B_1)}{P(Red)} = \frac{\frac{8}{10}\cdot\frac{1}{2}}{\frac{13}{20}} = \frac{8}{13}$$

2. N/P negative / positive

TN/P - tested -||-

$$P(TP \mid N) = \frac{1}{100}$$

$$P(TN \mid P) = \frac{1}{100}$$

$$P(P) = \frac{1}{10^6}$$

$$P(N) = 1 - P(P) = \frac{10^{6}-1}{10^6}$$

A) ~~$P(TP \mid P) = \frac{P(P \mid TP)\,P(TP)}{}$~~

$$P(P \mid TP) = \frac{P(TP \mid P)\,P(P)}{P(TP)} = \frac{P(TP \mid P)\,P(P)}{P(TP \mid P)\,P(P) + P(TP \mid N)\,P(N)} = \frac{\frac{99}{100}\cdot\frac{1}{10^6}}{\frac{99}{100}\cdot\frac{1}{10^6} + \frac{1}{100}\cdot\frac{10^6-1}{10^6}} = 0.0001 = 0.01\%$$

B) $P(P|TP) = \dfrac{P(TP|P)\,P(P)}{P(TP|P)\,P(P) + P(TP|N)\cdot P(N)} = \dfrac{\frac{99}{100}\cdot\frac{1}{1000}}{\frac{99}{100}\cdot\frac{4}{100}+\frac{1}{100}\cdot\frac{999}{1000}} = 0.0902 = 9.02\%$

c) $P(TP) = P(TP|P)\,P(P) + P(TP|N)\,P(N) = \frac{80}{100}$

$$\left(\frac{1}{x} + \frac{999999}{(1-x)}\right) = \frac{80}{100}\cdot 10^6$$

$$999999 - 999998x = 8\cdot 10^5$$

$$\frac{999999 - 8\cdot 10^5}{999998} = x \approx 0.1999994 = 19.99994\%$$