

L6 Z11*

L7 1-6
L6 10-14 L8 1-2

$\varphi(x)$ jest ~~ciągła~~ różniczkalna w przedziale otwartym na a, b i $\varphi'(x) \neq 0$

wtedy
 $\forall_{x \in (a,b)} \varphi'(x) < 0$ lub $\forall_{x \in (a,b)} \varphi'(x) > 0$

I) Dla $a < b$ $\varphi(a) > \varphi(b)$

$$\int_{[\varphi(a), \varphi(b)]} f(x) dx = \int_{\varphi(a)}^{\varphi(b)} f(x) dx = - \int_{\varphi(b)}^{\varphi(a)} f(x) dx = - \int_a^b f(\varphi(u)) \varphi'(u) du = - \int_a^b f(\varphi(u)) \cdot (-1 \varphi'(u)) du = \int_a^b f(\varphi(u)) |\varphi'(u)| du$$

II) Dla $a < b$ $\varphi(a) < \varphi(b)$

$$\int_{[\varphi(a), \varphi(b)]} f(x) dx = \int_{\varphi(a)}^{\varphi(b)} f(x) dx = \int_a^b f(\varphi(u)) |\varphi'(u)| du$$

Z II):

$$\int_{[\varphi(a), \varphi(b)]} f(x) dx = \int_a^b f(\varphi(u)) |\varphi'(u)| du$$