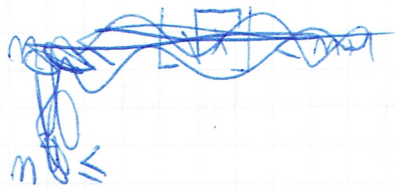
Zad. 14



$n \leq$

z def: $\quad * \lfloor \sqrt{\lfloor x \rfloor} \rfloor = n$

$\Leftrightarrow$ (1)

$n \leq \lfloor \sqrt{\lfloor x \rfloor} \rfloor < n+1$

Dalej z własności $\lfloor \rfloor$:

$n \leq \lfloor \sqrt{\lfloor x \rfloor} \rfloor \leq \sqrt{\lfloor x \rfloor} < n+1$

Podnosimy do kwadratu

$n^2 \leq \lfloor x \rfloor < (n+1)^2$

z wł. $\lfloor \rfloor$

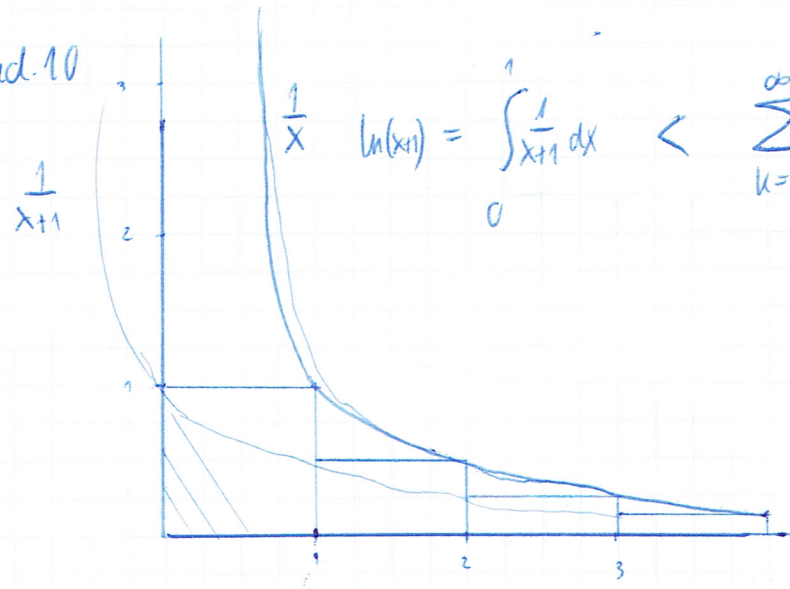
$n^2 \leq \lfloor x \rfloor \leq x < (n+1)^2$

$\sqrt{\ }$ obustronnie

$n \leq \sqrt{x} < (n+1)^2$

z def $\lfloor \rfloor$: $\lfloor \sqrt{x} \rfloor = n \overset{*}{=} \lfloor \sqrt{\lfloor x \rfloor} \rfloor$

Zad. 10



$\frac{1}{x+1}$

$\frac{1}{x}$

$$\ln(x+1) = \int_0^1 \frac{1}{x+1}\,dx < \sum_{k=1}^{\infty} \frac{1}{k} < \int_{0}^{\infty} \frac{1}{x}\,dx + 1 = \ln(x)+1$$

nadmiarowa w 0

Zad. 15

a) $x = k + \frac{1}{2} \quad k \in \mathbb{Z}$

$\lfloor x + \frac{1}{2} \rfloor = \lfloor k+1 \rfloor = k+1$

$\lfloor n \rfloor$

$\lceil x - \frac{1}{2} \rceil = \lceil k \rceil = k$