13 $\quad \omega \in \Omega^k(V) \qquad \eta \in \Omega^l(V)$

$$f^*(\omega \wedge \eta) = f^*\left(\frac{(k+l)!}{k!\,l!}\,\mathrm{Alt}(\omega \otimes \eta)\right) = \frac{1}{k!\,l!} \cdot \sum_{\sigma \in S_{k+l}} \mathrm{sgn}(\sigma)\, f^* T(w \ldots v_k, \eta_{k+1} \ldots \eta_{k+l}) =$$

$$= \frac{1}{k!\,l!} \sum_{\sigma \in S_{k+l}} \mathrm{sgn}(\sigma)\, T(f(w_1) \ldots f(w_k), f(\eta_{k+1}) \ldots f(\eta_{k+l})) = \cancel{\frac{1}{k!\,l!} \sum_{\sigma \in S_{k+l}} \mathrm{sgn}}\ \frac{(k+l)!}{k!\,l!}\,\mathrm{Alt}(f^*(\omega) \otimes f^*(\eta)) =$$

$$= f^*(\omega) \wedge f^*(\eta)$$

∎

Zad. 14* $\omega \in \Omega^k(V)$ $\eta \in \Omega^j(V)$

$$\omega \wedge \eta = \frac{(k+j)!}{k!\,j!} Alt(\omega \otimes \eta) = \frac{(k+j)!}{k!\,j!} \cdot \frac{1}{(k+j)!} \cdot \sum_{\sigma \in S_{k+j}} sgn(\sigma) \cdot (\omega \otimes \eta)(v_{\sigma(1)} \dots v_{\sigma(k+j)})$$

permutacja przesuwa **

$$= \frac{1}{k!\,j!} \cdot \sum_{\sigma \in S_{k+j}} sgn(\sigma \circ \tau) (\overset{\eta}{\omega} \otimes \overset{w}{\eta})(v_{\sigma(1)} \dots v_{\sigma(k+j)}) = \frac{1}{k!\,j!} \sum_{\sigma \in S_{k+j}} sgn(\sigma) \overset{*}{sgn(\tau)} (\eta \otimes \omega)(v \dots)$$

$$\overset{*}{=} \frac{1}{k!\,j!} \sum_{\sigma \in S_{k+j}} (-1)^{k+j} sgn(\sigma)(\eta \otimes \omega)(v_{\sigma(1)} \dots v_{\sigma(k+j)}) = \frac{1}{k!\,j!} \cdot (k+j)!\, Alt(\eta \otimes \omega) \cdot (-1)^{kj} = (-1)^{kj} \eta \wedge \omega \blacksquare$$

* zmiana ciągu $1 \dots n$ w ciąg $n \dots 1$ wymaga $\underline{kn}$ zmian (transpozycji)

więc aby τ: $\underbrace{v_{\sigma(1)} \dots}_{\omega} \underbrace{\dots v_{\sigma(k+j)}}_{\eta}$ było $\underbrace{v_{\sigma(1)} \dots}_{\eta} \underbrace{\dots v_{\sigma(k+j)}}_{\omega}$ musisz wykonać kj inwersji
**