Photometric Units of Aurora

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1 basic concept of aurora

Light is a narrow range of electromagnetic waves, which is made of photons in wavelengths between 350 to 700 nm. Aurora is mostly consist of visible light. It has a spectral range of 310 to 670nm where 558.7nm is the most crucial spectral line, which is known as atomic oxygen green line.

The energy of light can be measured either in energy units(J,eV) or in quantum units(quanta,einsteins). Note that quanta can also be shown as photons. The conversion between these units is based on the following equation:

$$E = \frac{h \cdot c}{\lambda}$$

where

$$h = 6.6 \times 10^{(-34)} J \cdot s \quad c = 3 \times 10^8 m/s$$

2 energy measurements

The radiant flux(ϕ_e) is a unit to express the radiant quantity. It is the flow of radient energy(Q_e) past a given point in a unit period, and is defined as follows:

$$\phi_e = \frac{dQ_e}{dt}\ (watts(J/s))$$

Define the radient intensityI_e:

$$I_e = \frac{d\phi_e}{d\Omega}\ (watts/steradians)$$

note that steraidan is the unit of solid angle $\frac{A}{r^2}$

Radience(L_e) is the radient intensity emitted in a certain direction from a radiant source, divided by unit area of an orthogrphically projected surface.

$$L_e = \frac{dI_e}{ds \cdot cos\theta}\ (watts/steradian/m^2)$$

3 luminous

The only difference between luminous flux and radiant flux is that luminous is realted to human's eyes.

$$\phi = km \int \phi_e(\lambda) v(\lambda) d\lambda\ (lumen)$$

where:

$\phi_e(\lambda)$:Spectral radiant density of a radient flux, or spectral radiant flux

km:Maximum sensitivity of the human eye(638lumens/watt)

$v(\lambda)$:Typical sensitivity of the human eye

Illuminance is defined as luminous flux incident per unit area of surface:

$$E = \frac{d\phi}{ds}(lumen/m^2 orlux)$$

Luminous intensity correspond to radient intensity and has the international unit of candelas(cd): $I = \frac{d\phi}{d\Omega}$And Luminance corresponds to radiance:$L = \frac{dI}{ds \cdot cos\theta}(candelas/m^2)$

4 quantum measurements

The aeronomists desired to have a radianve unit that would be a measure of rate at which photons coming down from a patch of the sky would strike each square centimeter of normal area. The reason for using photons per second instead of watts is that in gaseous photophysics the photon is conveniently treated statistically along with the concentrations of the other particles of the medium. The number of photons per second per square centimeter per steradina from the quiet night airglow is of the order of a million per angstorm.Thus the rayleigh has been defined in terms of megaphotons

$$1Rayleigh = \frac{10^6}{4\pi}(\frac{photons}{sec \cdot cm^2 \cdot steradian})$$

5 the relation between the energy measurement and quantumn measurement

5.1 solid angle factor

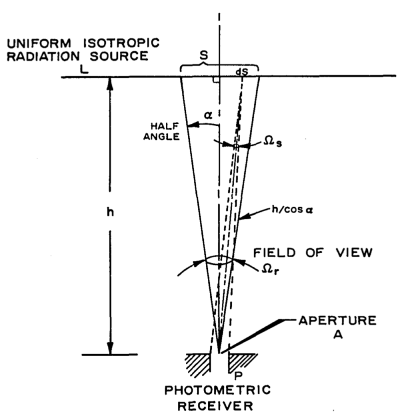

Figure1.Geometry for uniform extended radiation source viewed by photometric receiver with an ideal field of view.

Equivalance theorem: the power received by the photometric instrument is the triple product of the radiance seen $L_e = (\frac{W}{cm^2 steradian})$,the aperture area of the instrument A(cm^2)and the field of view$\Omega_r$which can be represented by:

$$P = L_e A \Omega_r$$

5.2 volume emission rates

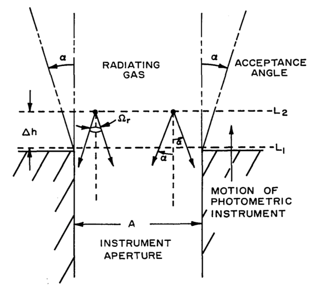

Fig. 2. Close-up geometry of an instrument acceptance aperture moving through a radiating gas.

Notice that a steradian stands for an area of a sphere equal to (r^2) , the acceptance solid angle of the instrument is$\frac{4\pi(sr)}{\Omega_r}$ And the total power per volume is:

$$u = \frac{\Delta P}{\Delta V} = \frac{4\pi}{\Omega_r} \cdot \frac{L_{e2} A \Omega_r - L_{e1} A \Omega_r}{A \Delta h} = -4\pi \frac{\Delta L_e}{\Delta h}(W/cm^3)$$

If the radiance function L is experimentally observed, the volume emission function is:

$$u(h) = -4\pi \frac{dL_e(h)}{dh}(W/cm^3)$$

If using Rayleigh(photons), the volume emission function is:

$$u(h) = -\frac{dR(h)}{dh} \times 10^6 (\frac{photons}{sec \cdot cm^3})$$

5.3 conversion formulas

Define the rayleigh spectral radiance $R_\lambda(rayleighs/\mu m)$and the power spectral radiance $L_{e\lambda}(\frac{watts}{cm^2 sr \mu m})$

The relation for the energy of a photon of wavelength λ inμm is

$$E_\lambda = \frac{hc}{\lambda \times 10^{-4}}$$

So we can get the relation by using the formula:

$$L_{e\lambda} = \frac{hc}{\lambda \times 10^{-4}} \frac{R_\lambda}{4\pi \times 10^{-6}}$$

than we can obtain

$$R_\lambda = 2\pi \lambda L_{e\lambda} \times 10^{13} (R/\mu m)$$

the inverse formula is:

$$L_{e\lambda} = \frac{R_\lambda}{2\pi\lambda} \times 10^{-13}$$

So for the monochromatic radiation, the conversion formula to rayleighs is

$$R = 2\pi\lambda L_e \times 10^{13}(R)$$

For the nonmonochromatic light(such as aurora), integration over the wavelengthλ is needed.

6 character and calculation of CCD

In a charged coupled device (CCD),the optical signal can be directly converted into analog current signal, and the current signal can be amplified and converted into analog digital signal to achieve image acquisition, storage, transmission, processing and reproduction. THe photon number recieved by CCD is

$$N = \frac{\eta \cdot \tau \times 10^4}{4\pi} \int_0^R V(h)dh(\frac{photons}{sec \cdot cm^2 sr})$$

where η is quantum efficiency of detector and τ is the transmittance of optical system.$V(h)$ is the volume emission rate Recall the volume emission rate in section5.3

$$dR(h) = -\frac{1}{4\pi}u(h)dh \times 10^6(\frac{photons}{sec \cdot cm^2 \cdot sr})$$

and the relation of R and L:

$$L_e = \frac{R}{2\pi\lambda} \times 10^{-13}$$

obtain the power radiance:

$$L_e = \int_0^R \frac{\eta \cdot \tau \times 10^{-9}}{8\pi^2 h} V(h)dh$$