TUBA ABBASI
STOCHASTIC COMPUTING

HW 1

Question 1

Suppose we have function f: N → N where N is the set of natural numbers. Prove that f (n) = n2 is injective where n ∈ N

Suppose f(a) = f(b) for f: N —> N
Therefore a^2 = b^2
Therefore √a^2 = √b^2
Therefore |a| = |b|
Therefore a = b
Q.E.D

Question 2

What is a function and how does it differ from a relation?

In both functions and relations, you have a set of inputs that are related to a set of outputs.

The difference however lies in the definition of a function. Each input in a function can have only one output, whereas in a relation each input can have multiple outputs.

In a sense, a function is a subset of relations.

Question 3

Prove that the cardinality of the set of naturals ***N*** *is the same as the cardinality of the set of rational numbers* ***Q***

In order to prove that two sets have the same cardinality we must show that there exists a one-to-one correspondence between the two sets (per George Cantor)

Therefore for all n ∈ N there is some q ∈ Q, such that f(q) = n

N	1	2	3	4	5	6	7	8	…
Q	0	1/1	-1/1	2/1	1/2	-1/2	-2/1	3/1	…

We would exclude values like 2/2, 3/3, 2/4, (and so forth) from this table because they are not unique Rationals. These are considered duplicates.

The table above shows a one-to-one correspondence between Naturals **N** and Rationals **Q**.
Q.E.D

Question 4

Which of these relations are equivalence relations?

An equivalence relation is “a relation between elements of a set that is reflexive, symmetric, and transitive.”

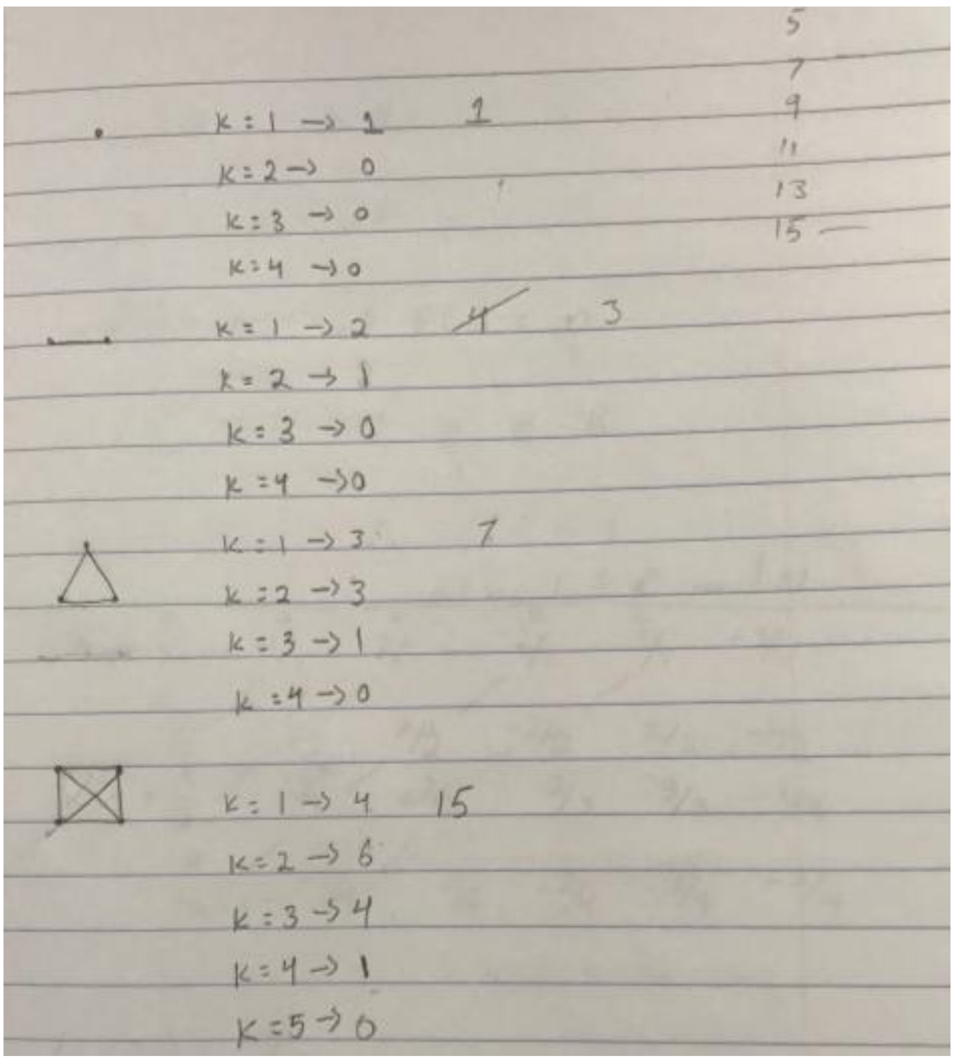
Only B has an equivalence relation.
It is reflexive due to the following values: (1,1), (2,2), (3,3), (4,4) — if a
It is symmetric due to the following values: (1,2), (2,1) — if (a,b) then (b,a)
It is transitive due to the following values: (1,2),(2,1),(1,1) — if (a,b) and (b,a), then (b,c)

(a) {(2, 2),(2, 3),(2, 4),(3, 2),(3, 3),(3, 4)} —> only reflexive

(b) {(1, 1),(1, 2),(2, 1),(2, 2),(3, 3),(4, 4)} —> reflexive, symmetric, transitive

(c) {(2, 4),(4, 2)} —> only symmetric

Question 5
Given the possible graphs you can construct from a set of vertices V, with cardinality n = | V |, how many k-cliques can be drawn where k=1...n



2, 4, 8… = 2^n ——> this is derived from the patterns drawn above.

Starting at k=1 we see that the number of k-cliques increases by 2^n where n=k.

Therefore the number of k cliques is the sum of