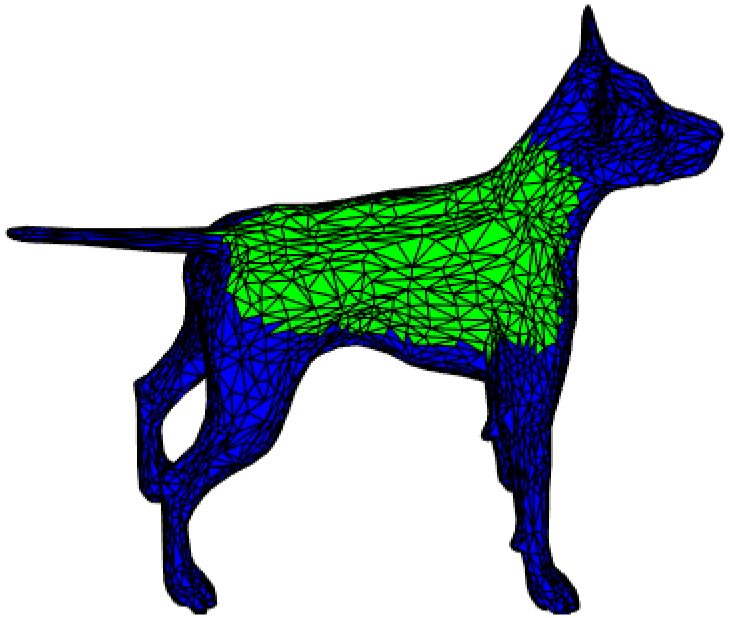
Implementation for

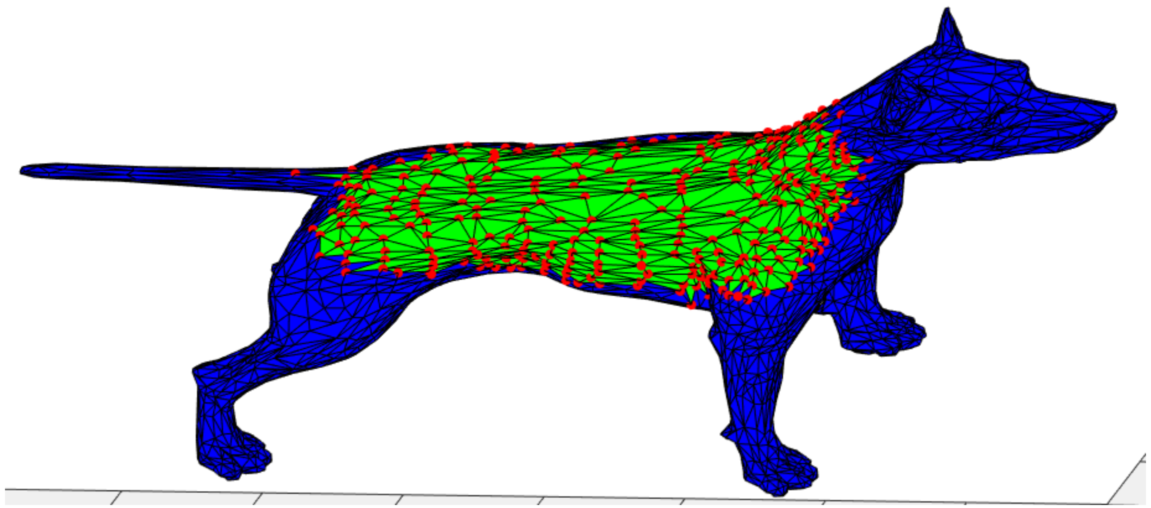
“Correspondence-Free Region Localization for Partial Shape Similarity via Hamiltonian Spectrum Alignment”



By Tsachi Blau

Goal

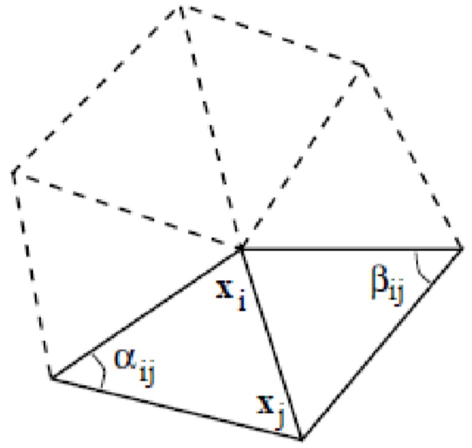
- We are given a full shape and a partial shape
- Find the corresponding vertices of the partial shape on the full shape



Background-Laplace Beltrami

- Encode the geometry of the domain
- Define as
  - $W_{ij} = \begin{cases} -\frac{1}{2}\left(\cot \alpha_{ij} + \cot \beta_{ij}\right) & i \neq j, j \in N_i \\ \sum_{k \in N_i} \frac{1}{2}(\cot \alpha_{ik} + \cot \beta_{ik}) & i = j \\ 0 & otherwise \end{cases}$

  - $A_{ij} = \begin{cases} A_{ij} & i \neq j, j \in N_i \\ 0 & otherwise \end{cases}$

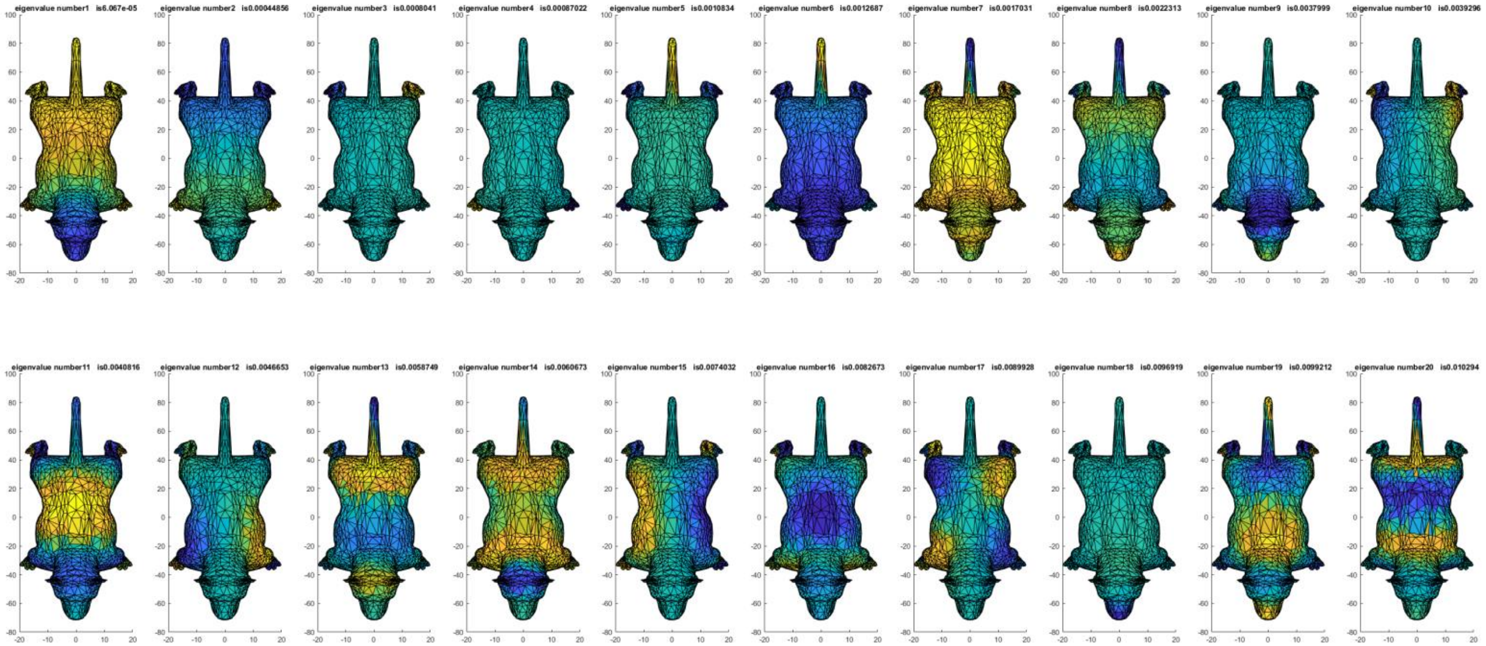


Background-Laplace Beltrami properties

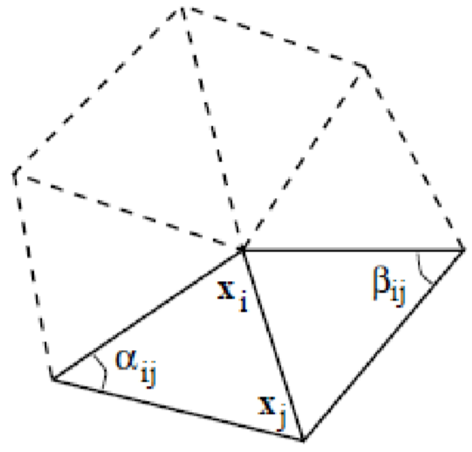
- $\Delta_x \phi_i = \lambda_i \phi_i$
- Eigenvalues of the Laplacian corresponding to the frequencies
- The eigenfunctions are orthogonal to each other
- The Laplacian Beltrami is isometry invariant

Background-Laplacian Beltrami

- Every eigenfunctions corresponding to some frequency
- Sorted in ascending order



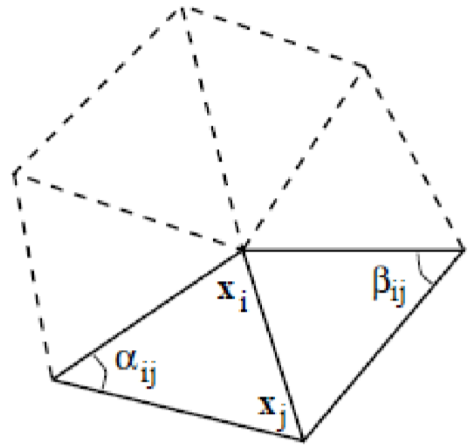
Background-Laplace Beltrami-with Dirichlet boundary condition



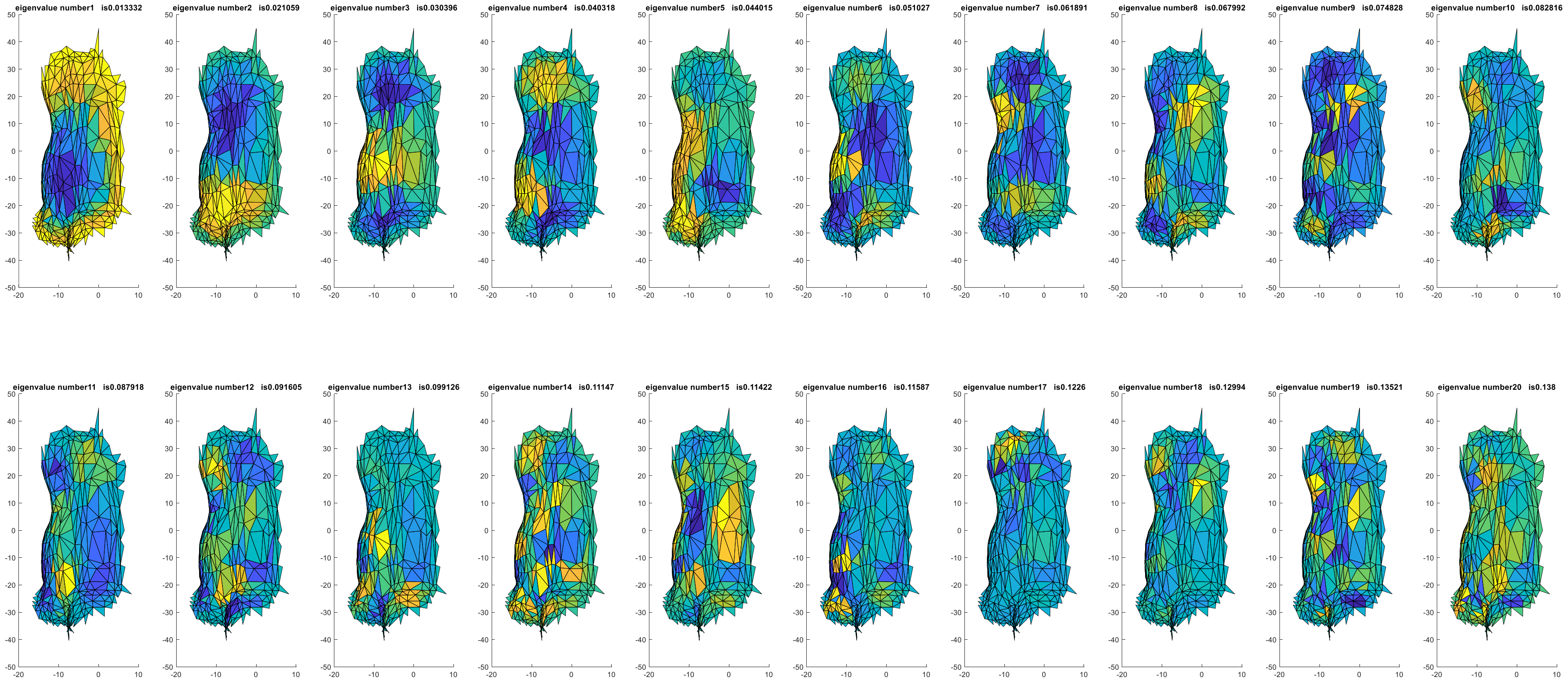
- We will use the Dirichlet boundary condition
- It will keep correct eigenfunctions and eigenvalues

Background-Laplacian Beltrami-with Dirichlet boundary condition

- Define as
  - $W_{ij} = \begin{cases} -\frac{1}{2}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right) & e_{ij} \in E_{int} \\ 0 & i \neq j \; and \; (i \in V_{bdr} \; o \\ \sum_{k\in N_i}\frac{1}{2}(\cot\alpha_{ik} + \cot\beta_{ik}) & i = j \; and \; v_i \in V_{int} \\ 1 & i = j \; and \; v_i \in V_{brd} \end{cases}$
  - $A_{ij} = \begin{cases} A_{ij} & v_i \in V_{int} \\ 0 & v_i \in V_{bdr} \end{cases}$



Background-Laplacian Beltrami-with Dirichlet boundary condition



Background-Hemiltonian

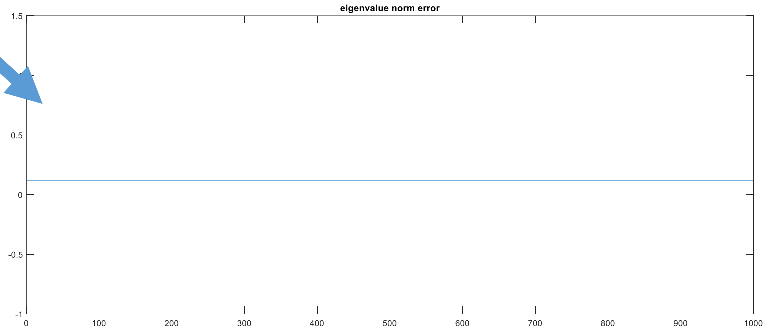
- Potential function
- $v_\tau = \begin{cases} 0 & x \in R \\ \tau & else \end{cases}$
- $H_x = \Delta_x + v_\tau$

Background-Hemiltonian

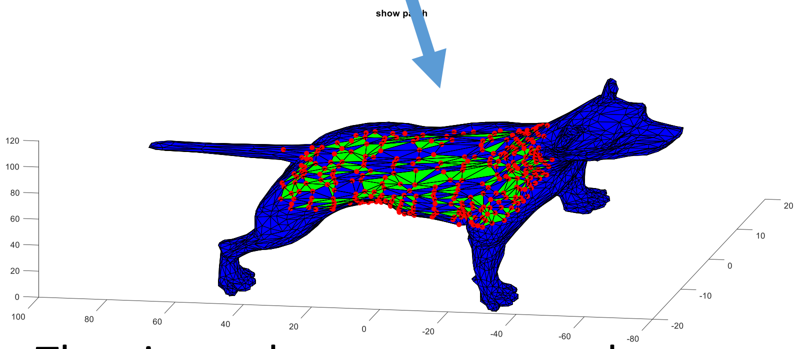
- λ_i is the eigenvalue of the Hemiltonian
- μ_i is the eigenvalue of the partial
- ψ_i is the eigenvalue of the Hemiltonian
- ϕ_i is the eigenvalue of the partial
- $\lambda_i \psi_i = (\Delta_x + v_\tau)\psi_i = \Delta_R \psi_i = \mu_i \phi_i$
    - Same eigenvalues !

Experiment – Sanity check

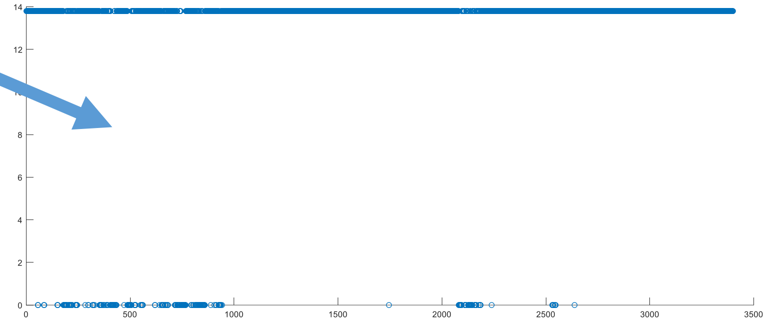
The error is not zero.. As expected.



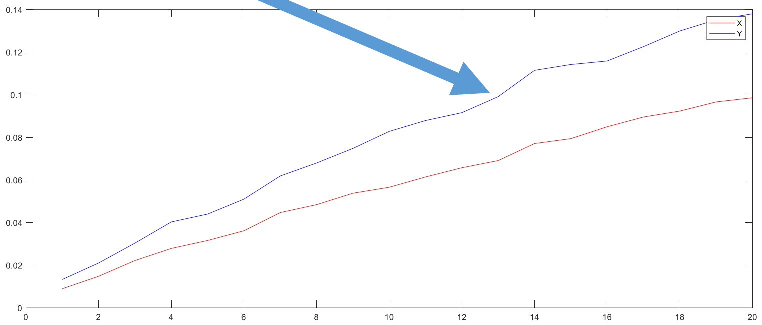
The point we choose remains the same



We don't change any point in v



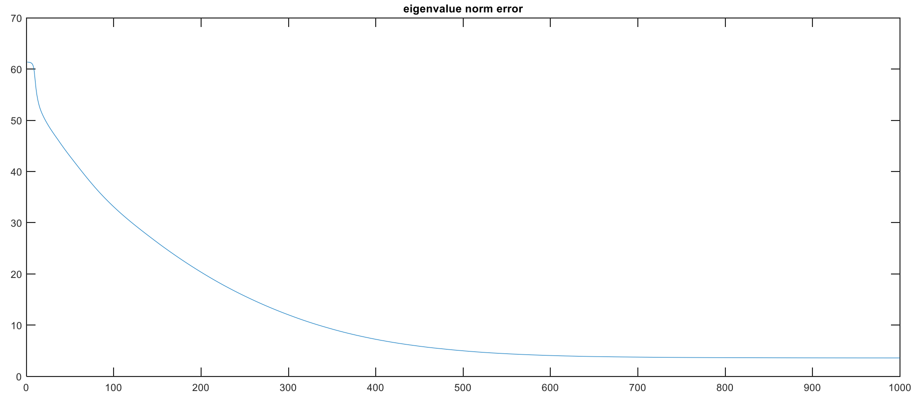
The eigenvalues are not equal

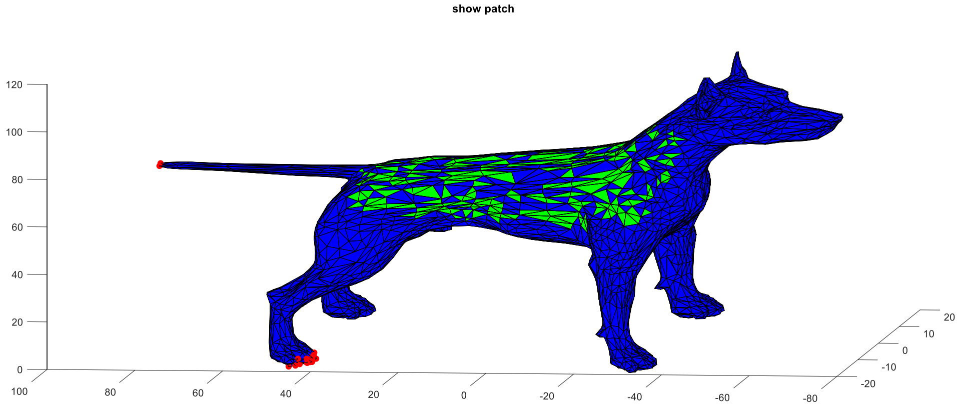


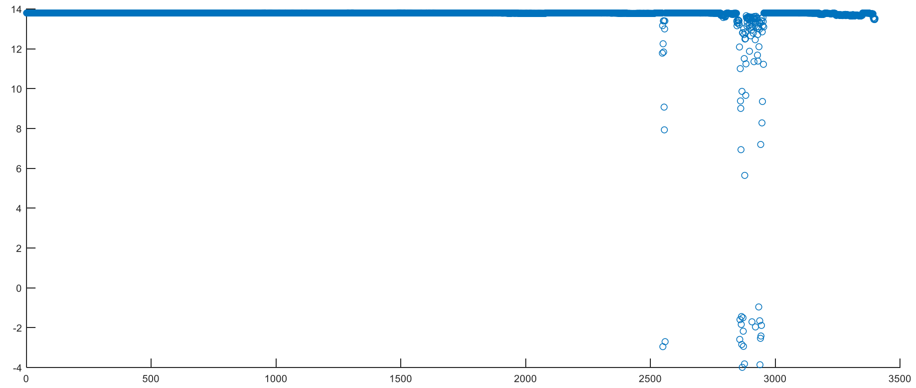
Probably cause we don't use Dirichlet boundary condition with consideration to the potential v

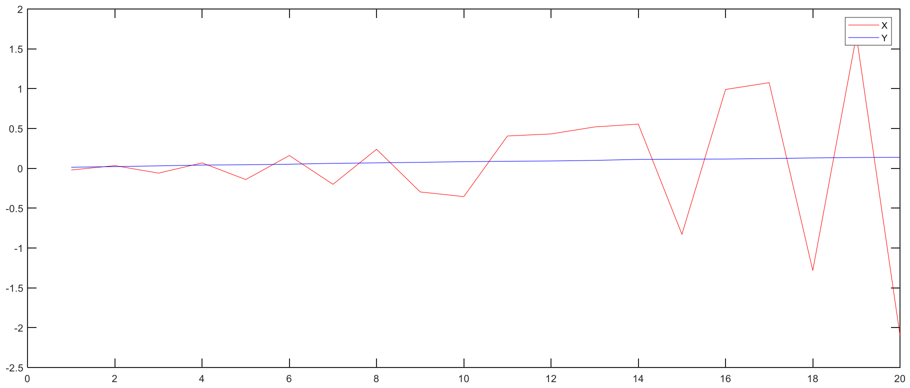
Experiment

- Start with constant V









Doesn’t works

- The article suggests to initialize v randomly and pick the best results it might improve the results

Future work

- Add another constraint on V
  - $\min_{v \geq 0} \left(\left\| \lambda\left(\Delta_x + diag(v)\right) - \mu \right\|_w^2 + \ \|v\|_2^2 \right)$

- Add scale invariant Laplacian

Questions