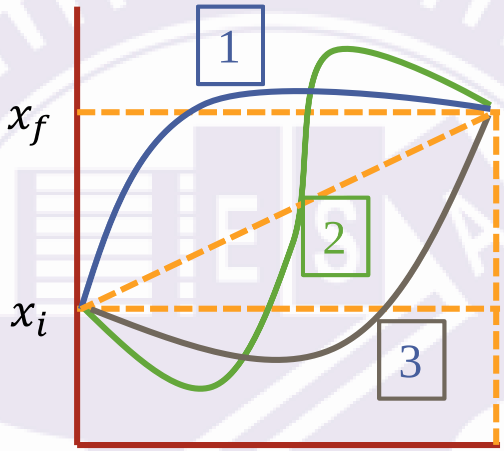
Chapter 5 – motion Planning

5.1 Introduction

- Main Consideration:
  - Easy to specify
  - Smoothness
  - Configuration
  - Singularity
  - Joint Motion & Cartesian Motion
- Path Description:
  - Initial, final and via way points.
  - Needs to specify both the position and the orientation for these points.

5.2 Polynomial Path Planning

- For a given duration an initial (x_i) and end (x_f) points.



- Cubic polynomial: can give a path connecting two points with first derivative continuity.

5.2 Polynomial Path Planning

- Assume it is specified that the initial and final velocity are zero.

$$\begin{cases} x(0) = x_i \\ x(t_f) = x_f \\ \dot{x}(0) = 0 \\ \dot{x}(t_f) = 0 \end{cases}$$

- Four constraints can be satisfied by polynomial of at least third degrees.

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{x}(t) = 2a_2 + 6a_3 t$$

5.2 Polynomial Path Planning

$$\begin{cases} x_i = a_0 \\ x_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ 0 = a_1 \\ 0 = a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{cases}$$

$$\begin{cases} a_0 = x_i \\ a_1 = 0 \\ a_2 = \dfrac{3}{t_f^2}(x_f - x_i) \\ a_3 = -\dfrac{2}{t_f^3}(x_f - x_i) \end{cases}$$

5.2 Polynomial Path Planning

- With via points: usually wish to pass through a via point without stopping.

- Then $\dot{x}(0) = \dot{x}_i, \;\; \dot{x}(t_f) = \dot{x}_f$

$$a_0 = x_i$$

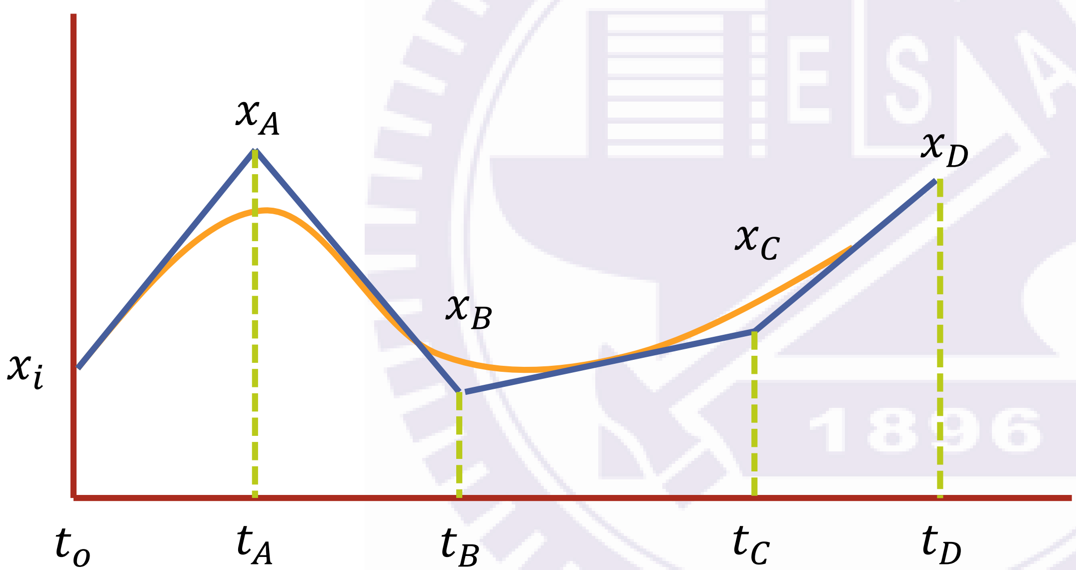
$$a_1 = \dot{x}_i$$

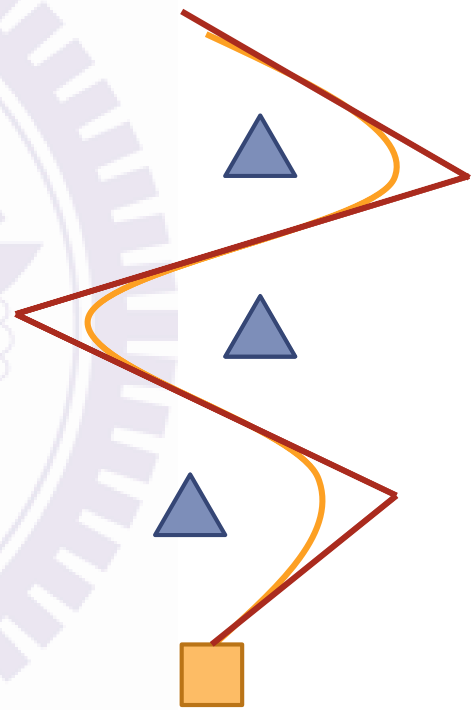
$$a_2 = \frac{3}{t_f^2}(x_f - x_i) - \frac{2}{t_f}\dot{x}_i - \frac{1}{t_f}\dot{x}_f$$

$$a_3 = -\frac{2}{t_f^3}(x_f - x_i) + \frac{1}{t_f^2}(\dot{x}_f + \dot{x}_i)$$

5.2 Polynomial Path Planning

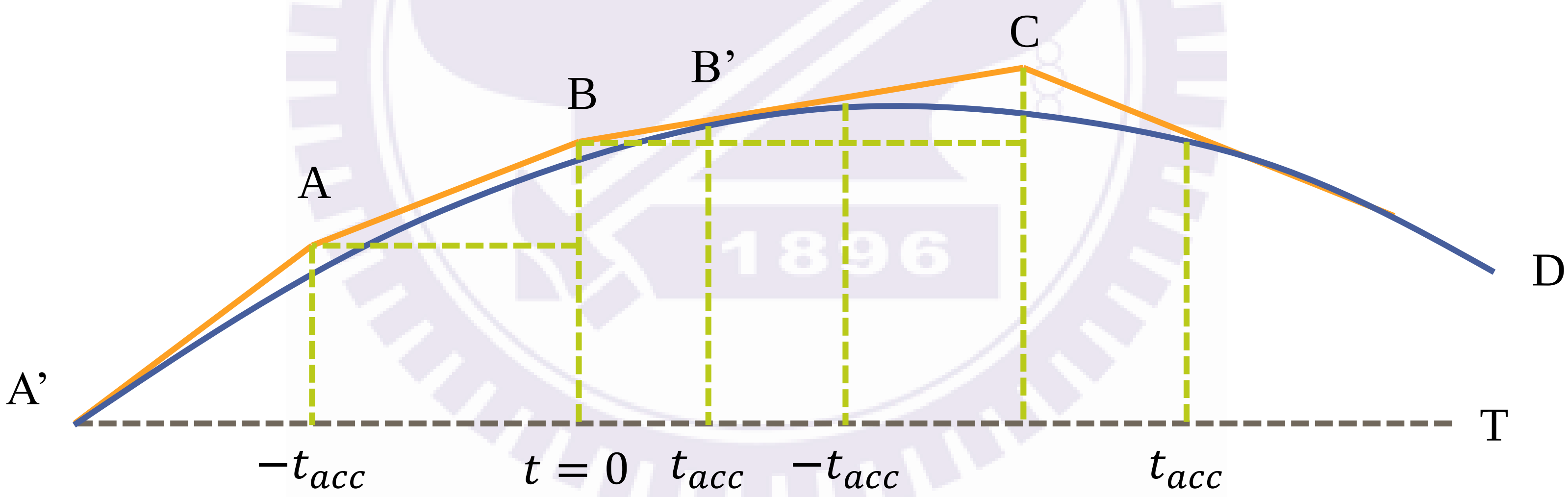
- Path planning: $\begin{cases} \mathit{shortest\ path} \\ \mathit{minimum} - \mathit{time\ trajectory} \end{cases}$





5.2 Polynomial Path Planning

- Need to render a path with acceleration into consideration.
- High order polynomial
  - To add in the constraint of acceleration, it seems like we need to raise the order of polynomial up to 5.
  - However, if the acceleration profile of the planned path is symmetrical, the path can be described by a 4th order polynomial.



5.2 Polynomial Path Planning

- Six boundary conditions from A to B'

$$q_A(t^+) = q_A(t^-) \qquad q_{B'}(t^+) = q_{B'}(t^-)$$

$$\dot{q}_A(t^+) = \dot{q}_A(t^-) \qquad \dot{q}_{B'}(t^+) = \dot{q}_{B'}(t^-)$$

$$0 = \ddot{q}_A(t^+) = \ddot{q}_A(t^-) \qquad \ddot{q}_{B'}(t^+) = \ddot{q}_{B'}(t^-) = 0$$

- Symmetry in acceleration can reduce one restriction to a 4th order which is enough

$$q(t) = a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$$\dot{q}(t) = 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1$$

$$\ddot{q}(t) = 12a_4 t^2 + 6a_3 t + 2a_2 \qquad -t_{acc} \le t \le t_{acc}$$

5.2 Polynomial Path Planning

Let $\begin{cases} \Delta C = C - B \\ \Delta B = A - B \end{cases}$

$$q(h) = [(\Delta C \frac{t_{acc}}{T} + \Delta B)(2-h)h^2 - 2\Delta B]h + B + \Delta B$$

$$\dot{q}(h) = [(\Delta C \frac{t_{acc}}{T} + \Delta B)(1.5-h)2h^2 - \Delta B]\frac{1}{t_{acc}}$$

$$\ddot{q}(h) = [(\Delta C \frac{t_{acc}}{T} + \Delta B)(1-h)]\frac{3h}{t_{acc}{}^2}$$

Where $h = \dfrac{t + t_{acc}}{2t_{acc}}$ for $-t_{acc} \le t \le t_{acc}$

5.2 Polynomial Path Planning

- For linear portion

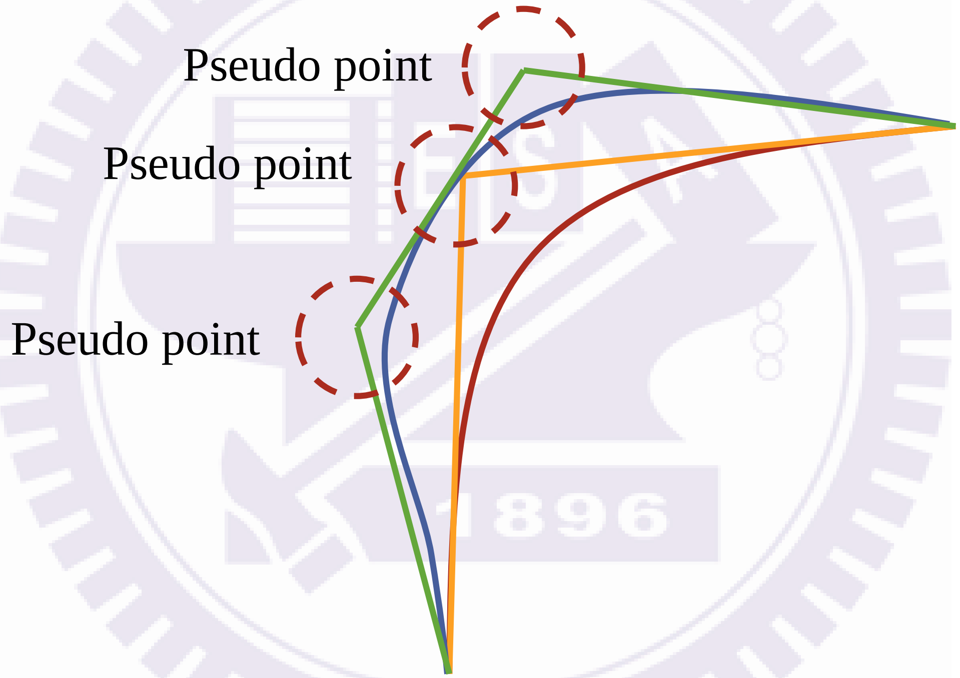
$$\begin{cases} q = \Delta C \cdot h + B \\ \dot{q} = \dfrac{\Delta C}{T} \\ \ddot{q} = 0 \end{cases} \qquad h = \frac{t}{T}, t_{acc} \le t \le T - t_{acc}$$

- Transition of different segments

$$T \leftarrow T_{new}$$
$$A, B, C \leftarrow B, C, D$$
$$\Delta B, \Delta C \leftarrow \Delta C, \Delta D$$

5.2 Polynomial Path Planning

- How to do it when the via points need to be passed?



5.3 Joint and Cartesian Motions

- Joint Motion:
  - Advantage: efficient in computation, no singularity problem, no configuration problem, minimum time planning.
  - Disadvantage: the corresponding Cartesian locations may be complicated.
- Note: the maximum velocity is limited by joint acceleration and velocity.
- Assume known:
  1. J: the current joint positions at the time T-tacc
  2. Jc: the joint positions of point C at t=T
- Planning:
  1. Evaluate joint positions at point D, J_D
  2. The time to move to joint D

$$t_i = \left| J_{D_i} - J_{C_i} \right| / V_i, i = 1, 2, \ldots, n$$

5.3 Joint and Cartesian Motions

- In order to have coordinate motion

$$T_{new} = \max\{t_i \mid i = 1, 2, \ldots, 6|, 2t_{acc}\}$$

- Cartesian Motion:
  - Advantage: motion between path segments and points is well defined. Different constraints, such as smoothness and shortest path, etc., can be imposed upon.
  - Disadvantage:
    1. Computational load is high
    2. The motion breaks down when singularity occurs i.e. $\dot{x} = J\dot{\theta}$, J is not invertible

5.3 Joint and Cartesian Motions

- Both position and orientation need to be planned.
- For positon planning, the similar approach as the joint motion can be used for both linear and transition segments. However, for orientation planning, no similar method is present.

- In Paul's book: two angle rotations:
  1. One rotation to align two a vectors
  2. One rotation to align two n, o vectors

5.3 Joint and Cartesian Motions

- Including translation, three transformations are planned
  - ① Find the corresponding T matrices for the initial (POS1) and final points (POS2)

$$T_6^1 = BASE^{-1} * POS1 * TOOL^{-1}$$
$$T_6^2 = BASE^{-1} * POS2 * TOOL^{-1}$$

  - ② Plan the motion from POS1 to POS2

$$T_6^1 = BASE^{-1} * POS1 * TOOL^{-1}$$
$$T_6^2 = BASE^{-1} * POS2 * TOOL^{-1}$$
$$T_6 = BASE^{-1} * POS1 * D(r) * TOOL^{-1}$$
$$0 \le r = \frac{t}{T} \le 1$$

$$r = 0, \quad D(0) = I$$
$$T_6 = BASE^{-1} * POS1 * TOOL^{-1} = T_6^1$$
$$r = 1, \quad POS2 = POS1 * D(1)$$
$$T_6 = BASE^{-1} * POS2 * TOOL^{-1} = T_6^2$$

5.3 Joint and Cartesian Motions

Let $POS\,1 = \begin{pmatrix} {}^1n & {}^1o & {}^1a & {}^1p \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$POS\,2 = \begin{pmatrix} {}^2n & {}^2o & {}^2a & {}^2p \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$D(1) = POS1^{-1} * POS2$$

$$= \begin{pmatrix} {}^1n \cdot {}^2n & {}^1n \cdot {}^2o & {}^1n \cdot {}^2a & {}^1n \cdot ({}^2p - {}^1p) \\ {}^1o \cdot {}^2n & {}^1o \cdot {}^2o & {}^1o \cdot {}^2a & {}^1o \cdot ({}^2p - {}^1p) \\ {}^1a \cdot {}^2n & {}^1a \cdot {}^2o & {}^1a \cdot {}^2a & {}^1a \cdot ({}^2p - {}^1p) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5.3 Joint and Cartesian Motions

③ Choose intermediate values of $D(r)$ to represent the translation and rotation, and let both translation and rotation be directly proportional to r, r varies linearly with respect to time with one translation and two rotations

$$D(r) = T_r(r) * Ra(r) * Ro(r)$$

$$T_r(r) = translation\ along\ the\ line\ from\ P1\ to\ P2$$

$$T_r(r) = \begin{pmatrix} 1 & 0 & 0 & r_x \\ 0 & 1 & 0 & r_y \\ 0 & 0 & 1 & r_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where x, y, z are the distances between POS1 and POS2

5.3 Joint and Cartesian Motions

- The first rotation Ra will align the approach vector a of POS1 and POS2. This rotation is about a vector k, obtained by rotating the y-axis of POS1, and angle ψ about the z-axis.

$$K=\begin{pmatrix} C\psi & -S\psi & 0 & 0 \\ S\psi & C\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}=\begin{pmatrix} -S\psi \\ C\psi \\ 0 \\ 1 \end{pmatrix}$$

- Then $Ra(r)$ represents a rotation of θ about K, which can be expressed as follows:

$$Ra(r)=\begin{pmatrix} S\psi^2V(r\theta)+C(r\theta) & -S\psi C\psi V(r\theta) & C\psi S(r\theta) & 0 \\ -S\psi C\psi V(r\theta) & C\psi^2V(r\theta)+C(r\theta) & S\psi S(r\theta) & 0 \\ -C\psi S(r\theta) & -S\psi S(r\theta) & C(r\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $V(r\theta)=Vers(r\theta)=1-\cos(r\theta)$

5.3 Joint and Cartesian Motions

- The second rotation $Ro(r)$ will align the orientation vector O of POS1 and POS 2. This rotation is simply a rotation about z-axis

$$Ro(r) = \begin{pmatrix} C(r\phi) & -S(r\phi) & 0 & 0 \\ S(r\phi) & C(r\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad D(r) = \begin{pmatrix} n_x & (...) & C\psi S(r\theta) & r_x \\ n_y & (...) & S\psi S(r\theta) & r_y \\ n_z & (...) & C(r\theta) & r_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Six variables: $x, y, z, \theta, \phi, \psi$

5.3 Joint and Cartesian Motions

- Solution of x, y, z

$$r = 1, \quad D(1) = T(1) * Ra(1) * Ro(1)$$

$$T(1) = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix} = D(1) * Ro(1)^{-1} * Ra(1)^{-1}$$

$$x = {}^{1}n \cdot ({}^{2}p - {}^{1}p)$$

$$y = {}^{1}o \cdot ({}^{2}p - {}^{1}p)$$

$$z = {}^{1}a \cdot ({}^{2}p - {}^{1}p)$$

5.3 Joint and Cartesian Motions

- Solution of ψ

$$Ra(1) = T(1)^{-1} * D(1) * Ro(1)^{-1}$$

Third column $C\psi S\theta = {}^1n \cdot {}^2a$

$$S\psi S\theta = {}^1o \cdot {}^2a$$

$$C\theta = {}^1a \cdot {}^2a$$

$$\therefore \psi = \tan^{-1}\left(\frac{{}^1o \cdot {}^2a}{{}^1n \cdot {}^2a}\right)$$

Where $sin\theta > 0$
i.e. $0° \le \theta \le 180°$

- Solution of θ

$$\tan\theta = \frac{[({}^1n \cdot {}^2a)^2 + ({}^1o \cdot {}^2a)^2]^{\frac{1}{2}}}{{}^1a \cdot {}^2a}, 0^o \le \theta \le 180^o$$

5.3 Joint and Cartesian Motions

- Solution of ϕ

$$\begin{pmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = Ro(1) = Ra(1)^{-1} * T(1)^{-1} * D(1)$$

$$= \begin{pmatrix} (S\psi)^2 V\theta + C\theta & -S\psi C\psi V\theta & -C\psi S\theta & 0 \\ -S\psi C\psi V\theta & (C\psi)^2 V\theta + C\theta & -S\psi S\theta & 0 \\ C\psi S\theta & S\psi S\theta & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} * T(1)^{-1} * D(1)$$

$$\therefore S\phi = -S\psi C\psi V\theta({}^1n \cdot {}^2n) + [(C\psi)^2 V\theta + C\theta]({}^1o \cdot {}^2n) - S\psi S\theta({}^1a \cdot {}^2n)$$

$$C\phi = -S\psi C\psi V\theta({}^1n \cdot {}^2o) + [(C\psi)^2 V\theta + C\theta]({}^1o \cdot {}^2o) - S\psi S\theta({}^1a \cdot {}^2o)$$

$$\therefore \tan\phi = \frac{S\phi}{C\phi}, -\pi \le \phi \le \pi$$

5.3 Joint and Cartesian Motions

④ Planning for linear portion, ψ is fixed

$$q(r) = x(r), y(r), z(r), \phi(r), \theta(r),$$

$$r = \frac{t}{T}$$ T is the time from POS1 to POS2

$$\begin{cases} q = \Delta C \cdot \dfrac{t}{T} \\ \dot{q} = \dfrac{\Delta C}{T} \\ \ddot{q} = 0 \end{cases}$$

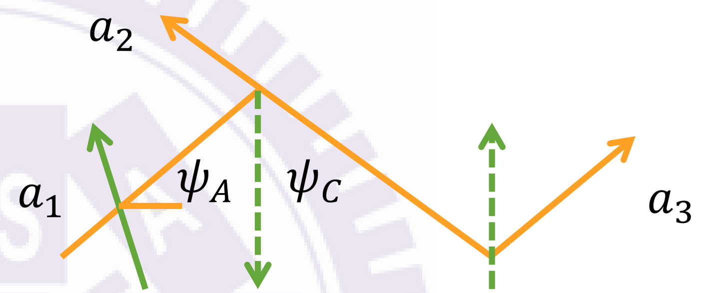
5.3 Joint and Cartesian Motions

⑤ Planning for transition portion

Similar to previous discussion, except ψ need to be planned as follows

$$\psi = (\psi_C - \psi_A)h + \psi_A$$

$$h = \frac{t + t_{acc}}{2t_{acc}}, -t_{acc} \le t \le t_{acc}$$

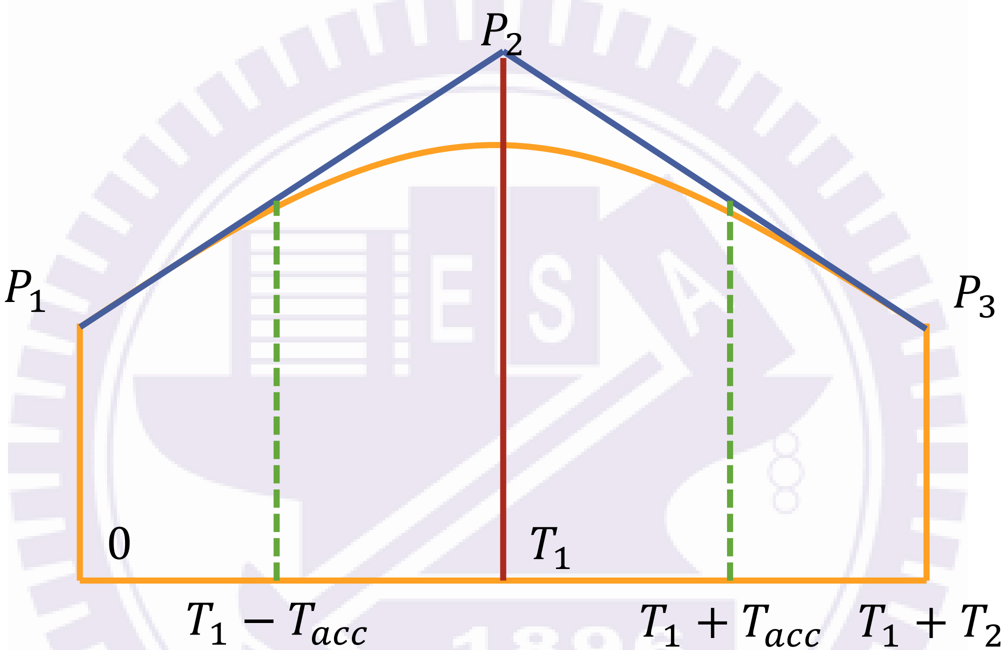


In order to minimize the effects of additional acceleration, caused by the rotation of ϕ, ψ, make sure that $|\psi_C - \psi_A| < 90°$

If $|\psi_C - \psi_A| > 90°$, then let $\begin{cases} \psi_A = \psi_A + 180^o \\ \theta_A = -\theta_A \end{cases}$

5.3 Joint and Cartesian Motions

- Example: Motion planning from P_1 *to* P_3 *via* P_2



$$P_1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_3 = \begin{pmatrix} -1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5.3 Joint and Cartesian Motions

- Planning $D(r)$ for P1 to P2
  - Find x, y, z ,θ, ϕ and ψ

$$x = {}^1n \cdot ({}^2p - {}^1p) = 0$$

$$y = {}^1o \cdot ({}^2p - {}^1p) = -10$$

$$z = {}^1a \cdot ({}^2p - {}^1p) = 0$$

$$\psi = \tan^{-1}\left(\frac{{}^1o \cdot {}^2a}{{}^1n \cdot {}^2a}\right) = \tan^{-1}\left(\frac{0}{-1}\right) = -180^o$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{({}^1n \cdot {}^2a)^2 + ({}^1o \cdot {}^2a)^2}}{{}^1a \cdot {}^2a}\right) = \tan^{-1}\left(\frac{1}{0}\right) = 90^o$$

$$\phi = \tan^{-1}\left(\frac{1}{0}\right) = 90^o$$

5.3 Joint and Cartesian Motions

- Let the transition starting from r=0.9 and robot position at r=0.9

$$A = P_1 * D(0.9) \qquad B = P_2 \qquad C = P_3$$

a) ΔB=A-B

$$X_A = 1, Y_A = 0,\ Z_A = 0 \qquad \psi_A = -90^\circ, \theta_A = 90^\circ,\ \phi_A = -90^\circ$$

b) ΔC=C-B

$$X_C = 0, Y_C = 0, Z_C = 10 \qquad \psi_C = 90^\circ,\ \ \theta_C = 90^\circ,\ \ \phi_C = 90^\circ$$

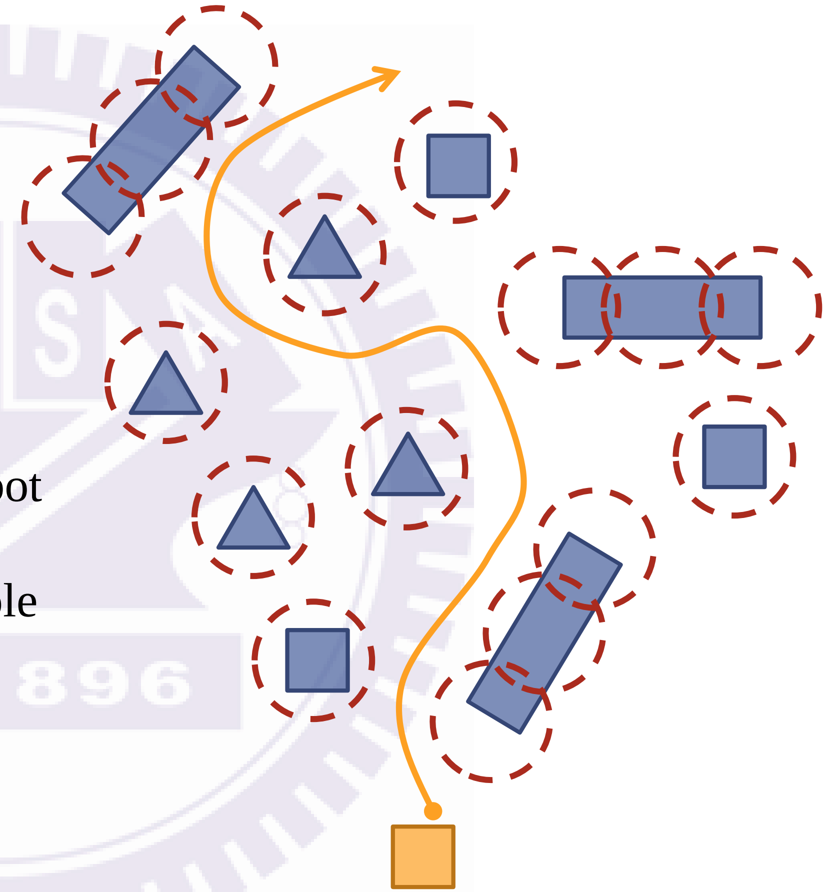
c) $|\psi_C - \psi_A| = 180° > 90°$

Let $\psi_A = \psi_A + 180^o = 90^\circ$

$$\theta_A = -\theta_A = -90^\circ,$$

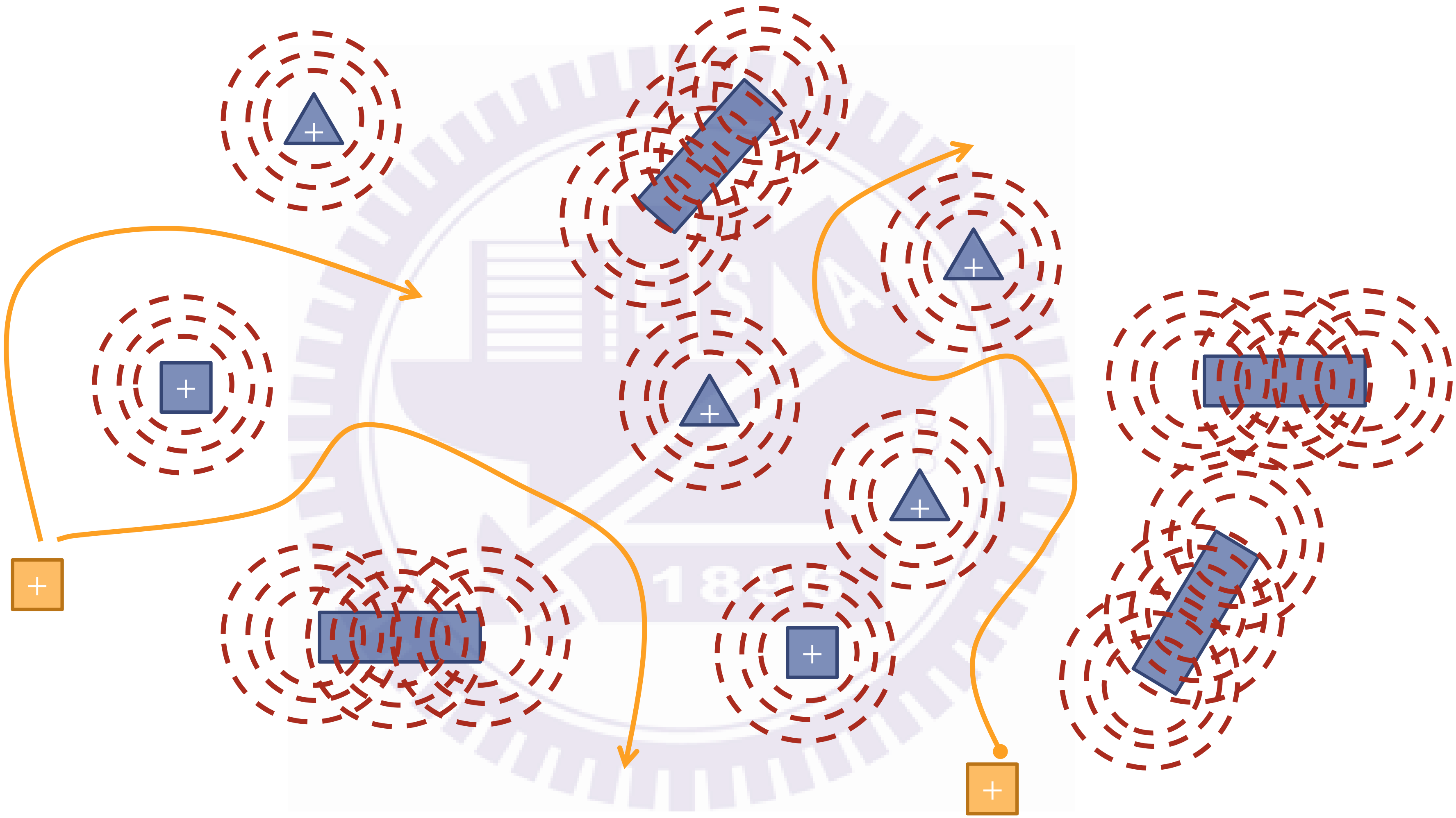
5.4 Configuration Space Approach

① Motion Planning
② Path Planning
③ Trajectory Planning



- Enlarge the obstacles and make the robot to be as small as a dot
- Large obstacles are divided into multiple circles

5.5 Potential Field Approach



5.6 Others

① Mobile robot

② Multi-DOF robot manipulator

- Configuration Space approach
- Potential–Field approach

- Map
- Intuition

- Static Obstacle
- Moving Obstacle

- Predictable Obstacle
- Unpredictable Obstacle

- Multiple Robot coordination
- Soft Robot
- Feasibility in task requirement, obstacle avoidance, kinematics and dynamics
- "On Human Performance in Telerobotics" V. Lumelsky, IEEE. Trans. Systems, Man and Cybernetics, Vol. 21(5), pp. 971-982, 1991.