Project 1

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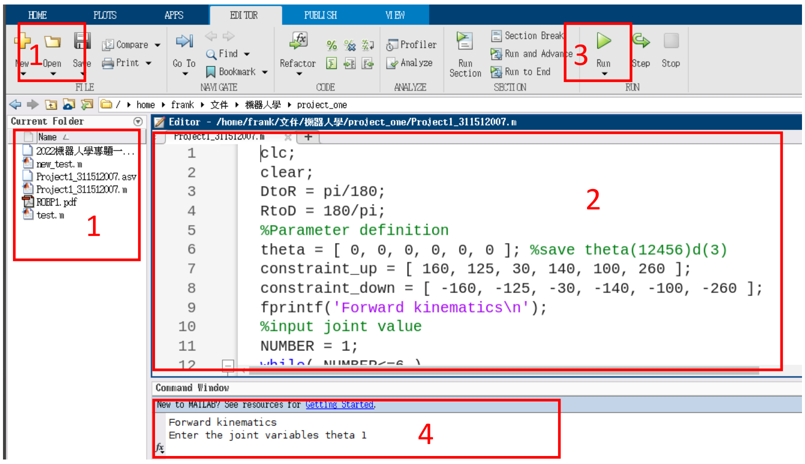
一. 介面說明

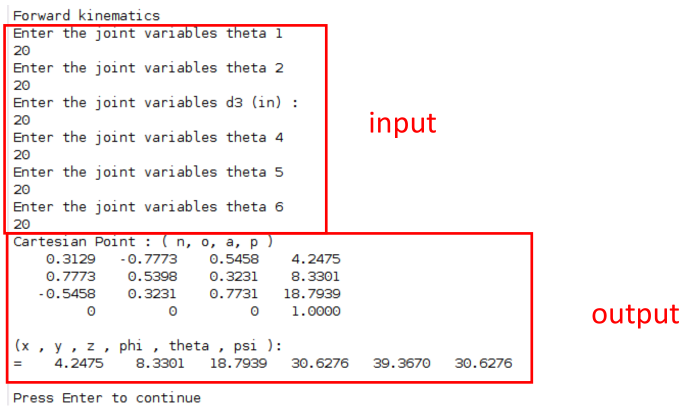
a. Matlab



1. 將程式碼檔案(.m)打開
2. 程式碼的編輯視窗
3. 執行程式碼
4. Matlab 終端機可以進行操作

b. 如何操作我的程式

1. 當出現 Enter the joint variables theta / d3 在操作介面鍵入其值，重複操作 6 次後會得到 n, o, a, p 還有(x, y, z, phi, theta, psi)，按下 enter 即可以繼續操作



2. 當出現 Please input T 請在操作介面鍵入 T（4x4 矩陣）

```
Inverse kinematic:
Please input T :
[0.3129    -0.7773    0.5458     4.2475
    0.7773     0.5398     0.3231     8.3301
   -0.5458     0.3231     0.7731    18.7939
         0          0          0     1.0000]
```

input

按下 enter 後會得到 8 組解

```
-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
   20.0000    20.0000    20.0000    19.9999    20.0000    20.0001

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
theta4 out of range!
   20.0000    20.0000    20.0000 -160.0001   -20.0000 -159.9999

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
theta2 out of range!
theta4 out of range!
theta5 out of range!
   20.0000 -160.0000   -20.0000   160.0001   160.0000 -159.9999

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
theta2 out of range!
theta5 out of range!
   20.0000 -160.0000   -20.0000   -19.9999 -160.0000    20.0001

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
  -74.0339   -20.0000    20.0000    79.5158    38.6125    62.6478

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
  -74.0339   -20.0000    20.0000 -100.4842   -38.6125 -117.3522

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
theta2 out of range!
theta5 out of range!
  -74.0339   160.0000   -20.0000   100.4842   141.3875 -117.3522

-----------------------------------------------
Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 )
theta2 out of range!
theta5 out of range!
  -74.0339   160.0000   -20.0000   -79.5158 -141.3875    62.6478
```

output

二. 程式架構說明

a. 順向運動學

```
DtoR = pi/180;
RtoD = 180/pi;
%Parameter definition
theta = [ 0, 0, 0, 0, 0, 0 ]; %save theta(12456)d(3)
constraint_up = [ 160, 125, 30, 140, 100, 260 ];
constraint_down = [ -160, -125, -30, -140, -100, -260 ];
```

在一開始我做了一些初始參數設定，DtoR & RtoD 向 deg 與 rad 單位可以互相轉換，初始化要輸入的 theta，並且用 array 存下每一個每一軸之最大值與最小值。

```
%input joint value
NUMBER = 1;
while( NUMBER<=6 )
    if NUMBER ~= 3
        fprintf('Enter the joint variables theta %d \n',NUMBER);
        joint=input('');
        if( constraint_down(NUMBER)<=joint&&joint<=constraint_up(NUMBER) )
            theta(NUMBER) = joint;
            NUMBER = NUMBER+1;
        else
            fprintf('theta%d is out of range\nPlease input again \n',NUMBER);
        end
    else
        fprintf('Enter the joint variables d%d (in) : \n',NUMBER);
        len_d=input('');
        if( constraint_down(NUMBER)<=joint&&joint<=constraint_up(NUMBER) )
            theta(NUMBER) = len_d;
            NUMBER = NUMBER+1;
        else
            fprintf('d%d is out of range\nPlease input again: \n',NUMBER);
        end
    end
end
```

我用 number 來代表每次輸入，因為第 3 次是要輸入 d3 之長度而不是角度，我用 else 把那次輸入獨立出來，用 input 這個 matlab 函式來讀我們鍵入的資料，而且如果數值並不在上下範圍以內，則會跳處 please input again 的提示，從新輸入直到符合範圍為止，最後所有資料會存在 theta 的 array 裡

```
%Forward kinematics
A1 = trnsformation(0, 0, -90*DtoR, theta(1)*DtoR);
A2 = trnsformation(6.375, 0, 90*DtoR, theta(2)*DtoR)
A3 = trnsformation(theta(3), 0, 0, 0);
A4 = trnsformation(0, 0, -90*DtoR, theta(4)*DtoR);
A5 = trnsformation(0, 0, 90*DtoR, theta(5)*DtoR);
A6 = trnsformation(0, 0, 0, theta(6)*DtoR);

T6=A1*A2*A3*A4*A5*A6;
nx = T6(1,1); ny = T6(2,1); nz = T6(3,1);
ox = T6(1,2); oy = T6(2,2); oz = T6(3,2);
ax = T6(1,3); ay = T6(2,3); az = T6(3,3);
px = T6(1,4); py = T6(2,4); pz = T6(3,4);
x = px; y = py; z = pz;
phi = atan2(ay, ax) * RtoD;
theta = atan2(sqrt(ax^2 + ay^2), az) * RtoD;
psi = atan2(oz, -nz) * RtoD;
p = [ x, y, z, phi,theta, psi];
```

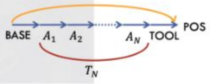
上面這邊的重點是我寫的 function transformation 來做出 6 個轉移矩陣，最後將其相乘後得到 T，及可以得到 x, y, z, phi, theta, psi

```
function A = trnsformation(d, a, alpha, theta)
    A = [ cos(theta)    -sin(theta)*cos(alpha)     sin(theta)*sin(alpha)     a*cos(theta)
    sin(theta)     cos(theta)*cos(alpha)      -cos(theta)*sin(alpha)     a*sin(theta)
    0          sin(alpha)           cos(alpha)                 d
    0          0            0                  1 ];
end
```

根據 DH model 之參數的公式

$$A_n = Rot(z,\theta_n) * Trans(0,0,d_n) * Trans(a_n,0,0) * Rot(x,\alpha_n)$$

$$= \begin{pmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



寫出以上 function，輸入 (d,a,alpha,theata)輸出 A

b. 逆向運動學

```
T = input('Please input T : \n');
nx = T(1,1);  ny = T(2,1);  nz = T(3,1);
ox = T(1,2);  oy = T(2,2); oz = T(3,2);
ax = T(1,3);  ay = T(2,3);  az = T(3,3);
px = T(1,4);  py = T(2,4); pz = T(3,4);
```

輸入 T 並將存到 n,o,a,p 之變數裡

```
%theta1 (2 solution)
P = sqrt(px*px+py*py);
fi = atan2(py, px);
lo1 = -6.375/P;
lo2 = (1-(6.375/P)^2)^0.5;
theta1 = [fi+atan2(lo1, lo2), fi+atan2(lo1, -lo2)]* RtoD;
%theta2 (4 solution)
%depend theta1 (2 solution) and atan2 (2 solution)
temp1 = (cos(theta1(1)*DtoR)*px+sin(theta1(1)*DtoR)*py);
temp2 = (cos(theta1(2)*DtoR)*px+sin(theta1(2)*DtoR)*py);
theta2 = [atan2(temp1,pz), atan2(-temp1,-pz), atan2(temp2,pz), atan2(-temp2,-pz)]* RtoD;
%d 3
%depend theta2 (4 solution)
temp1 = px.*cos(theta1(1)*DtoR)+py.*sin(theta1(1)*DtoR);
temp2 = px.*cos(theta1(2)*DtoR)+py.*sin(theta1(2)*DtoR);
d3 = [temp1/sin(theta2(1)*DtoR), temp1/sin(theta2(2)*DtoR), temp2/sin(theta2(3)*DtoR), temp2/sin(theta2(4)*DtoR)];
```

先將 theta1,theta2,d3 之值解出來

```
%%unify 4 theta solution of joint(123)
list1_theta = [theta1(1), theta2(1), d3(1)];
list2_theta = list1_theta;
list3_theta = [theta1(1), theta2(2), d3(2)];
list4_theta = list3_theta;
list5_theta = [theta1(2), theta2(3), d3(3)];
list6_theta = list5_theta;
list7_theta = [theta1(2), theta2(4), d3(4)];
list8_theta = list7_theta;
```

雖然解除前 3 軸只能得到 4 組解，但我們先用 8 個 array 存下來，之後再將後三軸之解補進去。

```
for i = 1:4
    if(i==1)
        T3 = A123(list1_theta, DtoR);
        T36 = inv(T3)*T6;
        theta4 = atan2(T36(2,3), T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list1_theta = [list1_theta, theta4, theta5, theta6];
        theta4 = atan2(-T36(2,3), -T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list2_theta = [list2_theta, theta4, theta5, theta6];
    elseif(i==2)
        T3 = A123(list3_theta, DtoR);
        T36 = inv(T3)*T6;
        theta4 = atan2(T36(2,3), T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list3_theta = [list3_theta, theta4, theta5, theta6];
        theta4 = atan2(-T36(2,3), -T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list4_theta = [list4_theta, theta4, theta5, theta6];
    elseif(i==3)
        T3 = A123(list5_theta, DtoR);
        T36 = inv(T3)*T6;
        theta4 = atan2(T36(2,3), T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list5_theta = [list5_theta, theta4, theta5, theta6];
        theta4 = atan2(-T36(2,3), -T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list6_theta = [list6_theta, theta4, theta5, theta6];
    elseif(i==4)
        T3 = A123(list7_theta, DtoR);
        T36 = inv(T3)*T6;
        theta4 = atan2(T36(2,3), T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list7_theta = [list7_theta, theta4, theta5, theta6];
        theta4 = atan2(-T36(2,3), -T36(1,3))*RtoD;
        [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD);
        list8_theta = [list8_theta, theta4, theta5, theta6];
    end
end
```

用 4 次迴圈將後三軸之所有解解出來並且填到之前解的變數裡，其中用到幾個我自己做的 fuction A123, implment456

```
function T3 = A123(list_theta,DtoR)
    A1 = trnsformation(0, 0, -90*DtoR, list_theta(1)*DtoR);
    A2 = trnsformation(6.375, 0, 90*DtoR, list_theta(2)*DtoR);
    A3 = trnsformation(list_theta(3), 0, 0, 0);
    T3 = A1*A2*A3;
end
```

帶入一組解，會得到前三軸矩陣相乘的 T3

```
function [theta5, theta6] = implment456(theta4,T36,DtoR,RtoD)
    A4 = trnsformation(0, 0, -90*DtoR, theta4*DtoR);
    T46 = inv(A4)*T36;
    theta5 = atan2(T46(1,3), -T46(2,3))*RtoD;
    A5 = trnsformation(0, 0, 90*DtoR, theta5*DtoR);
    T56 = inv(A5)*T46;
    A6 = T56;
    theta6 = atan2(-T56(1,2), T56(2,2))*RtoD;
end
```

帶入第四軸的解，可以得到第五與第六軸的解

```
function print_output(list_theta, constraint_up, constraint_down)
    fprintf('Corresponding variables ( theta1, theta2, theta3, theta4, theta5, theta6 ) \n');
    for i = 1:6
        if(list_theta(i) < constraint_down(i) || list_theta(i) > constraint_up(i) )
            fprintf('theta%d out of range!\n',i);
        end
    end
    disp(list_theta);
    fprintf('-------------------------------------------------\n');
end
```

最後輸出用此式子進行輸出，放入一組解與上下邊界範圍，輸出可以檢視哪一軸超出範圍

三. 數學運算說明

Joint	d(*in*)	a(*in*)	α	θ
1	0	0	-90°	0°
2	6.375	0	90°	0°
3	d_3	0	0°	0°
4	0	0	-90°	0°
5	0	0	90°	0°
6	0	0	0°	0°

$$\begin{pmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} n_x & n_y & n_z & -p \bullet n \\ o_x & o_y & o_z & -p \bullet o \\ a_x & a_y & a_z & -p \bullet a \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ s\theta_2 & 0 & -c\theta_2 & 0 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 A_6 = \begin{bmatrix} c_5 c_6 & -c_5 s_6 & s_5 & 0 \\ s_5 c_6 & -s_5 s_6 & -c_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 A_3 = \begin{bmatrix} c_2 & 0 & s_2 & s_2 d_3 \\ s_2 & 0 & -c_2 & -c_2 d_3 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 A_5 A_6 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & 0 \\ s_4 c_5 c_6 - c_4 s_6 & -s_4 s_5 s_6 + c_4 c_6 & s_4 s_5 & 0 \\ -s_5 c_6 & s_5 s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & c_1 s_2 d_3 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 s_2 d_3 \\ -s_2 & 0 & c_2 & c_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} & & & s_2 d_3 \\ & \cdots & & -c_2 d_3 \\ & & & 6.375 \\ & & & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} & P_x \\ f_{21} & f_{22} & f_{23} & P_y \\ f_{31} & f_{32} & f_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cdots & s_2 d_3 \\ \vdots & -c_2 d_3 \\ & 6.375 \\ & 1 \end{bmatrix}$$

$$-P_x s_1 + P_y c_1 = 6.375 \qquad (P_x = P\cos\phi,\ P_y = P\sin\phi)$$

$$\phi = \operatorname{atan2}\left(\frac{P_y}{P_x}\right) \qquad P = \sqrt{P_x^2 + P_y^2}$$

$$-P s_1 c_\phi + P c_1 s_\phi = 6.375$$

$$\begin{cases} \sin(\theta_1 - \phi) = \dfrac{6.375}{-P} = l_1 \\ \cos(\theta_1 - \phi) = \sqrt{1 - \left(\dfrac{6.375}{P}\right)^2} = l_2 \end{cases}$$

$$\theta_1 = \phi + \operatorname{atan}\left(\frac{6.375}{-P},\ \sqrt{1 - \left(\frac{6.375}{P}\right)^2}\right) \quad \#$$

$$c_1 P_x + s_1 P_y = s_2 d_3$$

$$+P_z = +c_2 d_3$$

$$\tan\theta_2 = \frac{c_1 P_x + s_1 P_y}{P_z}$$

$$\theta_2 = \operatorname{atan2}(c_1 P_x + s_1 P_y , P_z) \quad \#$$

$$d_3 = \frac{c_1 P_x + s_1 P_y}{s_2} \quad \#$$

$${}^3T_6 = A_4 A_5 A_6 = T_3^{-1} \cdot T_6 = \begin{bmatrix} \cdots & \cdots & {}^3T_6(1,3) & 0 \\ \vdots & & {}^3T_6(2,3) & 0 \\ \vdots & & & 0 \\ & & & 1 \end{bmatrix}$$

$$A_4 A_5 A_6 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & 0 \\ s_4 c_5 c_6 - c_4 s_6 & -s_4 s_5 s_6 + c_4 c_6 & s_4 s_5 & 0 \\ -s_5 c_6 & s_5 s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{s_4 s_5}{c_4 s_5} = \tan\theta_4 \quad \theta_4 = \operatorname{atan2}\left(\frac{{}^3T_6(2,3)}{{}^3T_6(1,3)}\right) \#$$

$$ {}^{4}T_6 = A_5 A_6 = A_4^{-1} \cdot {}^{3}T_6 $$

$$ A_5 A_6 = \begin{bmatrix} c_5 c_6 & -c_5 s_6 & s_5 & 0 \\ s_5 c_6 & -s_5 s_6 & -c_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cdots & & {}^{4}T_6(1,3) & 0 \\ \vdots & & {}^{4}T_6(2,3) & 0 \\ & & \vdots & 0 \\ & & & 1 \end{bmatrix} $$

$$ \theta_5 = \operatorname{atan2}\left(\frac{{}^{4}T_6(1,3)}{-{}^{4}T_6(2,3)}\right) \# $$

$$ A_6 = A_5^{-1} \cdot {}^{4}T_6 $$

$$ A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \vdots & A_6(1,2) & \cdots & \\ \vdots & A_6(2,2) & \vdots & \\ \vdots & & \vdots & \\ 0 & \vdots & \vdots & \end{bmatrix} $$

$$ \theta_6 = \operatorname{atan2}\left(\frac{-A_6(1,2)}{A_6(2,2)}\right) \# $$

四. 加分題

討論兩種逆向運動學(代數法，幾何法)的優缺點，解析解又可分為代數解和幾何解。商用的機械臂一般都會採用解析解，因為求解速度快且準確，

而不會採用本質迭代的數值解法。在這次的實驗中也發現，一組卡式座標可以算得 8 個逆運動學解，但並不是每個解都可以使用的，所以必須考慮的關節轉動角度的限制和物理限制。