

# Probabilistic Techniques and Randomized Algorithms

Homework 2023-2024

## Problem 1 (1.0)

Find the sample points of the space  $G_{3,2/3}$  and the probability of each point.

## Problem 2 (1.0)

In  $G_{4,p}$  random graphs find a) the distribution and the average degree of any vertex b) the distribution and the average number of edges.

## Problem 3 (1.0)

Prove that there exists a two-coloring of the edges of the complete bipartite graph  $K_{m,n}$  with at most  $\binom{m}{a}\binom{n}{b}2^{1-ab} + \binom{n}{a}\binom{m}{b}2^{1-ab}$  monochromatic  $K_{a,b}$ .

## Problem 4 (1.0)

Let  $W(r, k)$  be the least  $N$  so that every coloring with  $r$  colors of the numbers  $1, 2, \dots, N$  contains a monochromatic arithmetic progression of length  $k$ . Show that :

$$W(r, k) > \sqrt{2} r^{\frac{k-1}{2}}$$

## Problem 5 (1.5)

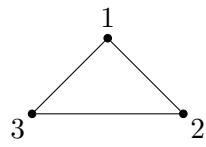
Find the threshold probability for the existence, with high probability, of a triangle in  $G_{n,p}$ .

## Problem 6 (1.5)

Prove that any  $k$ -SAT formula in which no variable appears in more than  $\frac{2^{k-2}}{k}$  clauses is satisfiable.

## Problem 7 (1.5)

A particle moves randomly on the vertices of the following triangle, starting from vertex 1, and at each discrete time step moves to the next (clockwise) vertex with a probability of  $1/3$  or to the previous (anticlockwise) vertex with a probability of  $2/3$ . a) Find the one-step transition matrix b) Find the stationary distribution c) On average, how many steps will it take for the particle to move from vertex 1 to vertex 3?



### Problem 8 (1.5)

Find the expected number of steps for a symmetric random walk starting from vertex  $u$  to reach vertex  $v$  for the first time in the following graph.

