Week 2

Working with boolean logic and arithmetic operations

Sarthak Tickoo, Feb 2023

What is boolean logic?

Worked on by George Bool. Hence the name.

- Branch of Algebra
- Works with truth values of a variable
- Typically used notations for values are “true” and “false”.
  - Shorthand - T/ F
  - Algebraically - 1/0
  - 1 = T, 0 = F

What can we do with it?

Many things…

- Uses the mathematical logic.
- Operations include
  - Conjunction - and. Symbol - $\wedge$
  - Disjunction - or. Symbol - $\vee$
  - Negation - not. Symbol - $\neg$
  - Exclusive OR - xor. Symbol $\oplus$
- Just like +, - , *, / help with numeric expressions, these operators help us work with logical expressions.

Boolean Algebra - AND

Where everything HAS to be true…

Boolean Algebra - OR

Where something HAS to be true…

Boolean Algebra - XOR

Likes when things are exclusive…

Boolean Algebra - NOT

Flipping things

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Truth Table - AND

X	Y	X && Y
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table - OR

X	Y	X || Y
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table - XOR

X	Y	X ^ Y
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table - NOT

X	!X
0	1
1	0

Some examples - AND

Some examples - OR

Some examples - XOR

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Some examples - NOT

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Extension of the logic to more than 2 variables

There is not much difference…

E.g. Truth Table with 3 variables - AND

X	Y	Z	X && Y && Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

E.g. Truth Table with 3 variables - OR

X	Y	Z	X || Y || Z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

And so on….

Composition of Logical Operators

Where things start getting interesting

- Rules of boolean logic we learnt earlier still apply.
- Remember PEDMAS/ BODMAS.
- AND, OR, NOT have precedence like DMAS.
    - First Negation
    - Second Conjunction
    - Third Disjunction
- Brackets have precedence.
- E.g. (A or B) and C is not the same as A or B and C
    - LHS and’s the result of A or B with C
    - RHS evaluates B and C and or’s that with A

Truth table for the previous expressions

(A or B) and C vs. A or B and C

A	B	C	(A or B) and C	A or B and C
FALSE	FALSE	FALSE	FALSE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE
FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE
TRUE	TRUE	TRUE	TRUE	TRUE

Some examples with multiple-variables

Order matters

Laws of Boolean Algebra

- Associativity of and, or
  - A or (B or C) = (A or B) or C
  - A and (B and C) = (A and B) and C
- Commutativity of and, or
  - A or B = B or A
  - A and B = B and A
- Distributivity of and over or
  - A and (B or C) = (A and B) or (A and C)
- Distributivity of or over and
  - A or (B and C) = (A or B) and (A or C)

Laws of Boolean Algebra

- A and 0 = 0, A or 0 = A
- A and 1 = A, A or 1 = 1
- Idempotence of and
  - A and A = A
- Idempotence of or
  - A or A = A
- A and not(A) = 0
- A or not(A) = 1
- not(not(A)) = A

DeMorgan’s Laws

- A and B = not(not(A) or not(B))
- A or B = not(not(A) and not(B))