

# 1 Line Noise Filter

The noise is low frequency, and so we should design a high-pass filter.

The signal has a range. I think we should design a high-pass filter that preserves the lowest-possible frequency of the signal. The remaining signal frequencies will land in the bandpass.

## Determining frequencies from periods:

$$P_{\text{signal}} = .05\text{ms}, f_{\text{signal}} = 20\text{kHz}$$

$$P_{\text{noise}} = 16\text{ms}, f_{\text{noise}} = 62.5\text{Hz}$$

$$f = 1/t$$

Type	Period (ms)	Period (s)	Frequency (Hz)
Signal	0.05	0.00005	20000
Noise	16	0.016	62.5

## Determining R

Next I choose a resistor according to the load of 50k Ohms. There could be many different frequencies we have to deal with. Let's only focus on the worst case scenario.

When you put high frequencies in, the C behaves like a short (remember the equation  $Z_{\text{cap}} = 1/2\pi fc$ ). So the source would just see the resistor. This makes the impedance (resistance) of the R our worst case scenario. Considering that, using our 10x rule, we will make the R 1/10th the resistance of the load.

$$R_{\text{load}} = 50\text{k Ohm}$$

$$R_R = R_{\text{load}}/10$$

$$R_R = 5\text{k Ohm}$$

## Determining f3db

For a high-pass filter, the attenuation of the signal is about 10% at  $2f_{3\text{db}}$

We know we want to keep 20kHz and above (it's a high-pass filter!).

We will put  $f_{3\text{db}}$  at  $\frac{1}{2}$  of 20kHz, so  $f_{3\text{db}} = 10\text{KHz}$ .

## Determining the C

$$f_{3\text{db}} = 1/2\pi RC$$

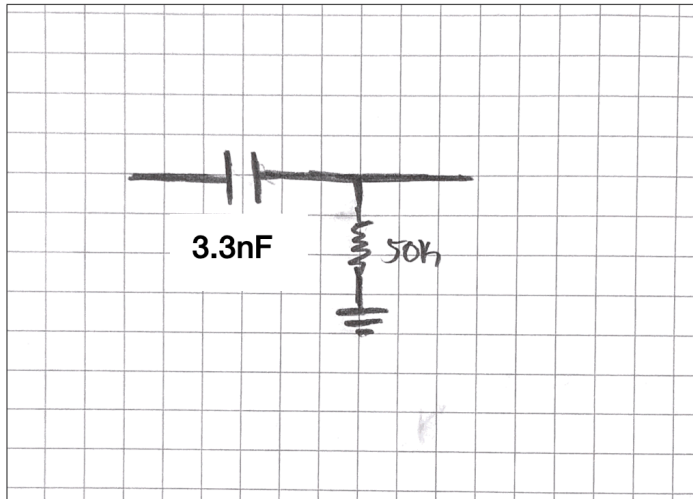
$$C = 1/2\pi Rf_{3\text{db}}$$

$$C = 1/(2*3.14*5000*10000)$$

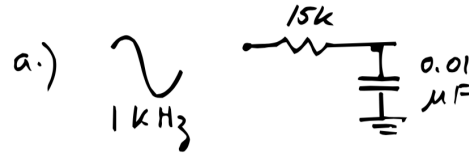
$$C = 3.18\text{nF}$$

This table says that there are 3.3nF capacitors available. I will use one of those for this application. If these were not available, I would find two that were and put them in parallel since  $C_{\text{total}} = C_1 \parallel C_2$

**Circuit Design:**

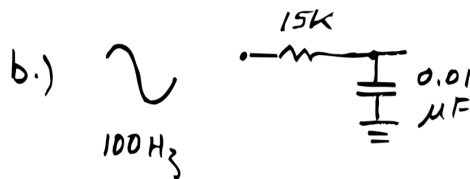


## Problem 2 Phase Shifts



a) See table for proof. ~1kHz frequency in is obviously quite comparable to the 1kHz  $f_{3db}$ . Phase shift for a low-pass filter is roughly  $-45^\circ$  at  $f_{3db}$ .

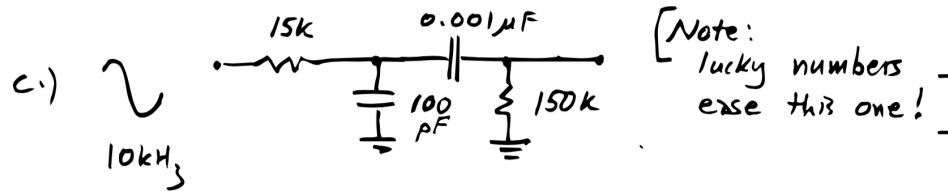
...more severe shifts occur over much of the possible input frequency range, shown in Fig. 2N.29. At many frequencies, phase shift is considerable:  $-45^\circ$  at  $f_{3db}$ , and more than that as frequency climbs, approaching  $-90^\circ$  [for a low-pass filter]. We don't worry because those shifts are applied to what we consider not signal but noise.



b) See table for proof. 100Hz is substantially lower than the filter's  $f_{3db}$  of ~1kHz. Actually it is about ten times less. For a high pass filter, that means this frequency will incur a negligible phase shift of about  $-6^\circ$ . More confirmation from the text:

If the amplitude out is close to amplitude in, you will see little or no phase shift. If the output is much attenuated, you will see considerable shift ( $90^\circ$  is maximum)...

Where the filter is passing a signal, the filter does not impose much phase shift... Both attenuation and phase shift will be less at lower frequencies.



c) This is a low-pass filter preceding a high pass filter. I have done out some calculations to see how  $f_{3db}$  compares to the  $f_{in}$  and thus deciding on the phase shift. I've put an equation in the table but it's all based on the equation which you can kind of do in your head for this problem:

$$f_{3db} = 1/2\pi RC$$

My reasoning for the  $-6^\circ$  phase shift of the low-pass filter in example c is the same as in example b. 10kHz is a much lower frequency than the 106kHz  $f_{3db}$ , so it won't experience much phase shift. It should be about  $-6^\circ$  since it's 1/10 of the  $f_{3db}$ .

Meanwhile the 10kHz signal is 10x higher than the filter's 1.06kHz  $f_{3db}$ , so there should be a similar and acute amount of phase shift in the other direction.

The phase shift's cancel out to zero for this problem.

problem	Filter Type	R (Ohm)	C (F)	$f_{in}$ (Hz)	$f_{3db}$ (Hz)	$f_{in}/f_{3db}$	Phase shift, deg
a	low-pass	15000	1E-08	1000	1061.6	94%	$-45^\circ$
b	low-pass	15000	1E-08	100	1061.6	9%	about $-6^\circ$
c	low-pass	15000	1E-10	10000	106157.1	9%	about $-6^\circ$
c	high-pass	150000	1E-09	10000	1061.6	942%	about $+6^\circ$

### 3 Little RC Problems

#### 3.1 You don't like the DC Level—output centered



#### 3.2 You don't like the DC Level—output offset

Ran out of time!

## **Problem 4 X100 Probe**

Ran out of time!

## Problem 5 Phase Shift: Troubleshoot

I've plotted out phase shift as it compares to frequency input and  $V_{out}/V_{in}$ . Note that this is for a high-pass filter. If it were a low-pass filter, these graphs would be flipped horizontally, see page 69.

If the filter were designed correctly, with a  $f_{3db}$  at 1kHz, then the phase shift of an input signal of 2kHz, 2 $f_{3db}$ , should certainly be less than  $45^\circ$ . Judging by the plot I would say the phase shift should be something like  $35^\circ$ .

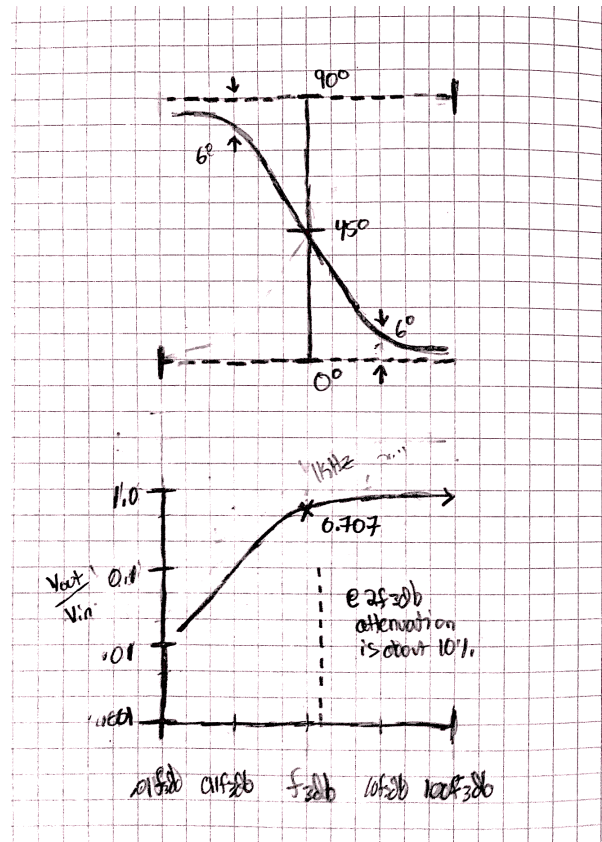
So what is happening here is that the filter is not filtering as we expect. There being phase shift from a signal means that signal is not making it into the passband of the filter.

So the problem could be coming from a poor RC design, one that does not in fact have the right  $f_{3db}$  that we're looking for. So one way to test this is to swap out some components of the RC filter, specifically the R or the C.

Better yet, probably just swap out your C, considering R needs to share that 10x relationship with the impedance of impedance in and out, respectively. No reason either of those should be changing so the R shouldn't need to be changed. So play with some different C values would be a good step.

Maybe you did the get R wrong though and you could swap that out, too.

Another thing you could do is input a sine wave into the filter using a function generator. Then you could compare  $V_{in}$  vs.  $V_{out}$  using two sets of scope probes. Turning the frequency knob on the function generator, watch for meaningful  $V_{out}/V_{in}$  moments. One good moment to look for would be when  $V_{out}$  is .70 of  $V_{in}$ , or when  $V_{out}$  is .9 of  $V_{in}$ . This would tell you what the actual  $f_{3db}$  (or 2 $f_{3db}$ , respectively) of the RC at hand is. From there you could determine a better capacitor or resistor for the filter, your R or your C. If you're working with a fixed load, then you'll just want to tweak the C.



## Problem 6 Filter design: Lab Exercise, recapitulated

Tips:

you can probably ignore the 50ohm resistor because it's just 5%

The first two are in series

Whatever load you hook up to will have  $Z_{in}$  10x the  $Z_{out}$  of the filter.

It's not in these specs.

We are given  $Z_{out}$  of the source.

As long as we have  $Z_{out}$  of the source or  $Z_{in}$  of the load, we can determine

If you're going forward, you need to go 10x up in R

If you're going down, you need to go 10x down in R

The transformer is putting an AC wave around some coils. There is a turns ratio in the transformer, it steps the voltage down by a factor of 2x. If you have half the number of turns, it steps voltage down by a factor of 2. If you have 1 turn on the secondary coil for every 10 on the primary coil, it will step the voltage down by a factor of 10. You can think of the function generator and transformer

The line noise is the 60Hz wave, and the function generator is the little blippers.

Signal: 60Hz

Noise is 2kHz to 20kHz.

set the function generator initially to 10kHz.

### Determining signal and noise frequencies:

If I understand the question correctly, below are the signal and noise frequencies. This scenario calls for a low-pass filter. Also the noise has a range, so I will calculate for the lowest possible noise frequency. Frequencies above that will be attenuated by the filter.

$$f_{\text{signal}} = 60\text{Hz}$$

$$f_{\text{noise}} = 2\text{kHz}$$

### Determining R

We do not know the impedance of the load, unlike other class problems. That's okay, we know impedance out of the source, or  $Z_{out}$  of the source. It's not frequency dependent so we can even call it  $R_{out}$  (pg. 83). This will help us choose a resistor for the filter, based on our "worst case" scenario and "10x" rules from chapters 2 and 1, respectively.

One catch is that 1K is in series with  $R_{out}$ .



$$R = 1k$$
$$R_{out} = 50$$

$$R_T = R + R_{out}$$
$$R_T = 1050 \text{ Ohms}$$

\*Note that the  $R_{out}$  is only 5% of the 1k R, so you can probably ignore it and just work with the 1k R.

So according to our 10x rule, R for the filter should be 10k.

$$R_{filter} = 10k$$

### Determining $f_{3db}$

For a low-pass filter, the attenuation of the signal is about 10% at  $\frac{1}{2}f_{3db}$

We know we want to keep 60Hz and below.

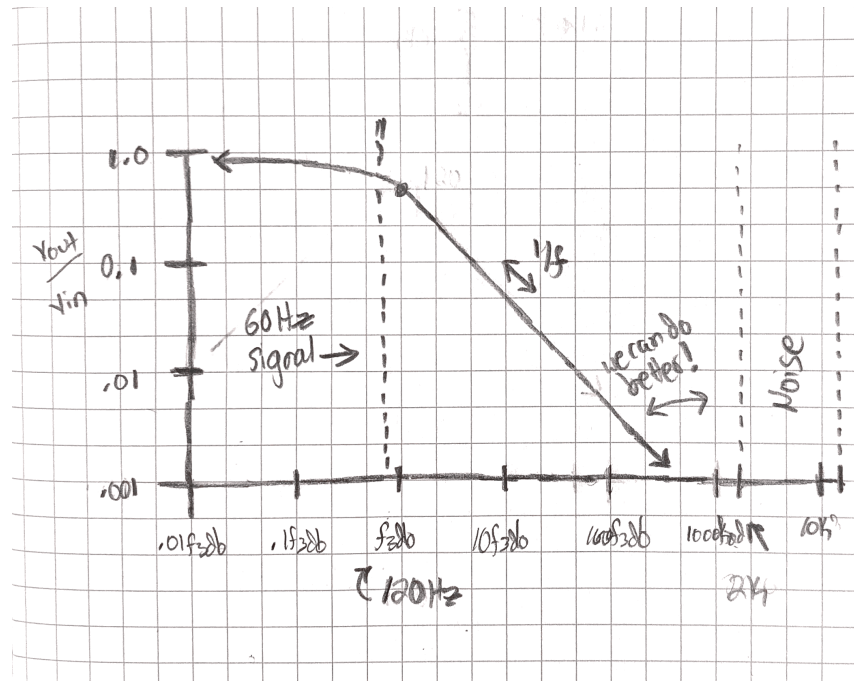
We will put  $f_{3db}$  at 2x of 60Hz, so  $f_{3db} = 120\text{Hz}$

Here's a table to speak to the question of gains and losses with different  $f_{3db}$  scenarios.

$f_{3db}$ Placement	Results
$2f_{signal}$ (120Hz)	11% attenuation of the 60Hz $f_{signal}$ . Less than $-45^\circ$ phase shift, which is reasonable. Frequencies higher than 120Hz attenuate at a healthily steep 1/f slope. This is okay.
$f_{signal}$ (60Hz)	30% (too much!) attenuation of the desired frequency. That's just not going to fly.
$f > 2f_{signal}$	Place $f_{3db}$ higher than $2f_{signal}$ and you risk having some high frequency noise come through the filter. If there's an upside, it's that the $f_{signal}$ gets less attenuation and less phase shift.

Taking another look at the last option though, it seems like you can actually push  $f_{3db}$  higher than  $2f_{signal}$  for this application. First off I actually think my log-log plot is close to being at scale between  $f_{3db}$  and higher. We know slope fall off is about 1/f after  $f_{3db}$  and so I've drawn it this way. You can see that 2kHz noise is well past the cut off for this filter setup.

20dB = amplitude ratio of 10. This comes up in the alternative description of an RC filter: a low-pass, for example, falls at “-20dB per decade,” meaning “cutting amplitude by a factor of ten for each 10-fold multiplication of frequency.”



So far I've approached this problem from the perspective of the passband, thinking I should "keep the signal" and chosen my  $f_{3db}$  according to the  $2f_{signal}$  rule. Noise is definitely out of the picture in this scenario, but there's still 11% attenuation of the signal.

So you could also approach the problem from the standpoint of the stopband, thinking "get rid of enough noise." There's another nice rule on this. For a low-pass, set the  $f_{3db}$  at 1/10th the high noise frequency and it will attenuate it by a factor of 10x compared to what it would have been if it were  $f_{3db}$ , so .07x attenuation ultimately.

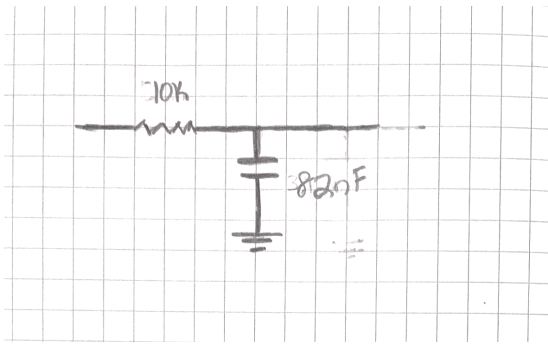
To further make the case for this, 2kHz is yet the lowest frequency of the net noise band. So it's not even all of the noise. Noise frequencies higher than 2kHz would be attenuated even more than .07 in this scenario. And to reiterate, there is even less attenuation of  $f_{signal}$  as compared to the 11% from the other  $f_{3db}$  option ( $2f_{signal}$ ).

So this is the right way to go.

$$f_{3db} = 200\text{Hz}$$

### Determining the C

$$\begin{aligned} f_{3db} &= 1/2\pi RC \\ C &= 1/2\pi R f_{3db} \\ C &= 1/(2 \cdot 3.14 \cdot 10000 \cdot 200) \\ C &= \sim 80\text{nF} \end{aligned}$$



For my initial choice of  $f_{3db}$  @  $2f_{signal}$  (120Hz)

signal attenuated by  $2/\sqrt{5}$

noise attenuated by something more than  $(1/10)/\sqrt{2}$  (ran out of time)

For my ultimate choice of  $f_{3db}$  @  $1/10 f_{noise}$  (200Hz):

signal attenuated by something less than  $2/\sqrt{5}$  (ran out of time)

noise (at least the 20kHz) attenuated by  $(1/10)/\sqrt{2}$