Tim Savas
Homework 10

1 Reviewish: Audio Summer

Solution on Falstad: https://tinyurl.com/y3c4vkrn

Overview

From a high-level perspective, the hardest part of this problem is that we have to use a single supply. At an initial glance, I actually thought I would tackle this problem pretty easily. But it gets complicated having to deal with the one-arm-tied-around-back situation of the single supply. Note for instance, that you can't simply amplify a signal centered on zero with an op-amp whose negative supply is tied to ground. I walk through my explanation below, from left to right in my diagram.

I would also like to note at the top that solving this problem was a collective effort between a few classmates and I, in case my solution looks strikingly similar to either of theirs. We all contributed something to this problem! We also worked hard to understand each piece on our own, as I think you'll see below!

$R_{out-signalsource}$: unknown, for both sources

Naturally I'd like to add a follower to resolve this. Speaking to my above point though, the follower itself cannot be powered by a split supply. A split supply would be convenient here because as the question states, our signal is centered on ground. So before scrubbing the signal through this follower stage, we have to bias it up. How far up? We center it halfway between ground and our supply, so 7.5V. Therefore we need a biasing voltage divider of half. That explains the 1M || 1M on the signals' way into the followers. It also necessitates the 1μF C. That's a blocking cap, to keep the 15V we just added from shooting over to the AC source. It "blocks" it from that side of the circuit.

gain wanted: for one input, 5; for the other input, 10

The next stage solves this. It is a summer that both combines—"sums"—the signals and amplifies them according to the specs. The $R_{feedback}$ of 100K is the numerator to the respective 20K and 10K denominators, delivering gains of 5 and 10x.

Per the summing function of this stage, we sum currents, not voltages. So the 20K and 10K do that work as well, allowing us to work with the V leaving our followers as currents entering the summer.

Again though, we have to maintain a center on 7.5V. Notice we have powered the summer with just the +15V supply, going to ground. So instead of a "virtual ground," we create a "virtual 7.5V" by tying the non-inverting amp input of the summer to half of our supply. We get the half by way of another R || R voltage divider. Equal Rs in a voltage divider simply split a Vin in half.

try to make circuit stable despite assumed capacitive loading by the speaker
load: drive an 8-ohm speaker

At the final stage there is still some work to do. First of all, the gain has been achieved, but there is still a nagging offset we've maintained in order to do this work with one supply. On the output of the summer, the summed signal is at 6-9V centered on our 7.5V. A speaker won't like that. We run this through a push-pull change the offset and also to ensure proper impedance, Rout going into the 8 Ohm load of the speaker. The 470 R there just ensures reliable current going into the base of the transistor. See this discussion on Piazza for a good reference on this: https://piazza.com/class/kehgp441t735l?cid=72

The 10ohm and 100nF at the bottom is the snubber. It's for stability. Page 378 is good reference for this setup.

signal frequencies: 100Hz and above

We need a high-pass filter in here then, according to this spec. For high-pass, we can put $f_{\mathbf{3db}}$ at ½ our desired frequency of 100Hz, which ensure less than 10% attenuation at that desired frequency, and above of course. Using the R_{load} then, we can determine a capacitor value from our f_{3db} equation. It's a low R_{load}, and so it's a pretty big capacitor!

$$f_{3db} = 1/(2\pi RC)$$
$$C = 1/(2\pi f_{3db} R)$$
$$C = 400\mu F$$

R	f_{3db}	C
8	50	4.0E-04

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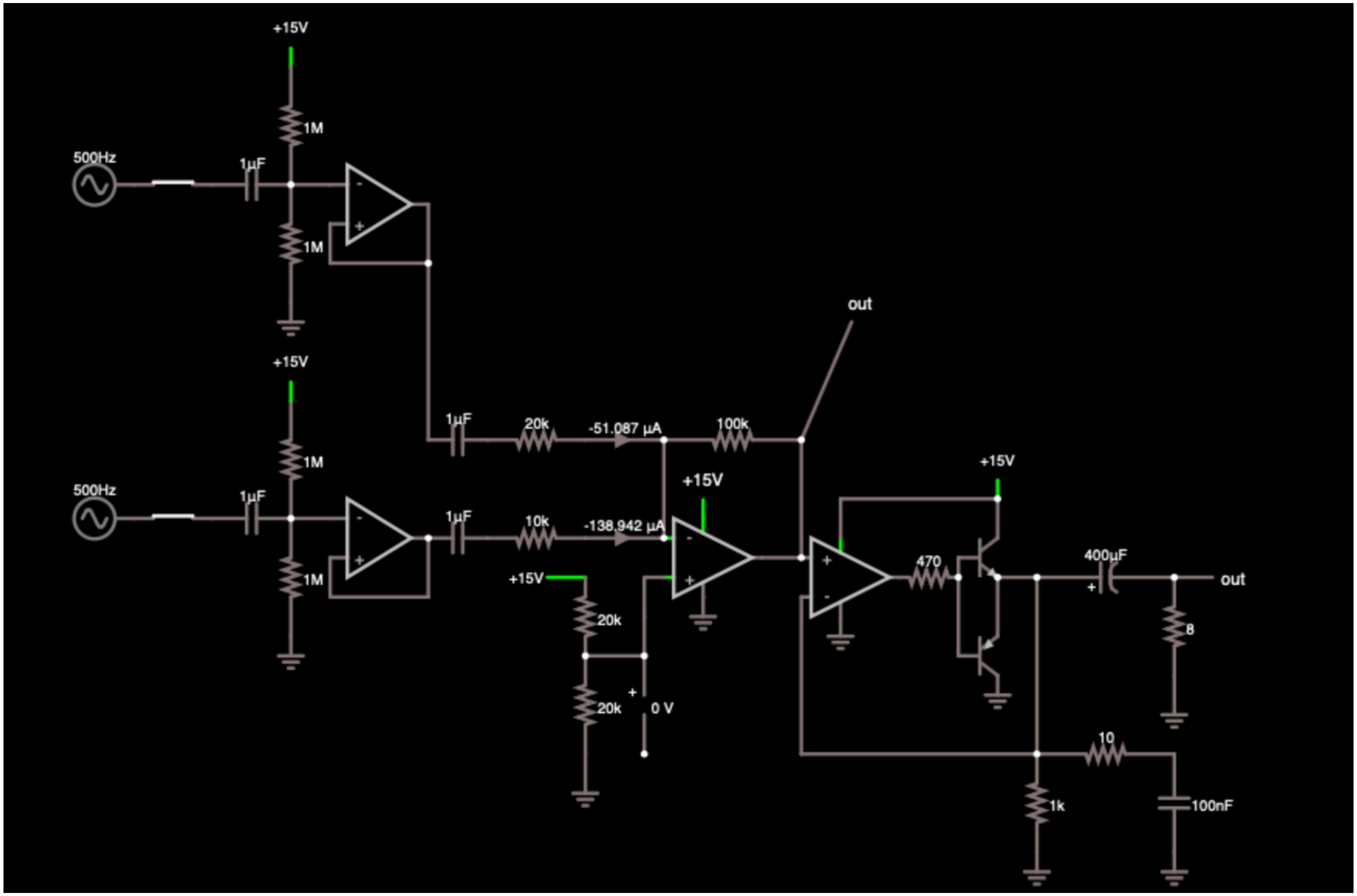


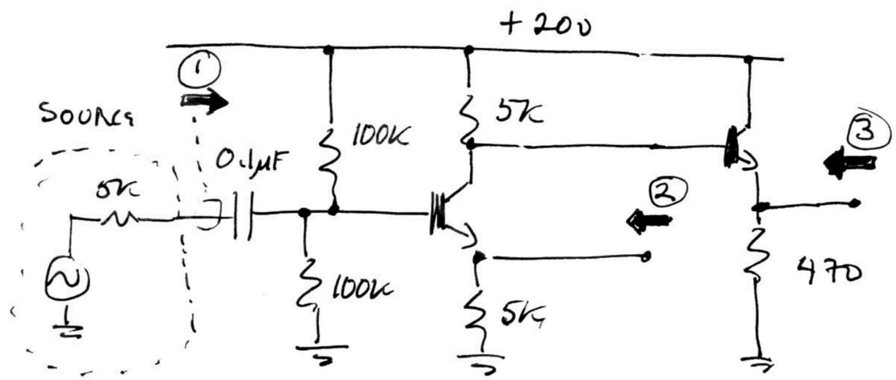
2 Impedances

I would like to just write out some more of this problem as practice for understanding V, I, and R in a phase splitter and transistors generally.

Feature	Value	Description
V_B	10	20V supply through a R || R divider splits it in half, delivering 10V into the base of the transistor
V_E	9.4	10V minus Vdrop gives 9.4V at the emitter
I_E	0.002	Voltage at the emitter going over that 5K R
I_C	0.002	Same as IE
r_e	13.298	25/IC (in mA)
R_C	5000	given
V_C	9.400	Current at the collector through the R_C
β	100	given
R_E	5000	given
R_{in_C}	45455	R_{Thev} of the divider || R_{in_base}
R_{in_base}	500000	beta x RE
R_1	100000	given
R_2	100000	given
$R_{Thev\_divider}$	50000	products over sums

Point 1: $R_{in\text{-}base}$

Value: 45K



This is R_{Thev} of the divider in parallel with R_{in} at the base. R_{Thev} of the divider is easy to figure out of course. An R || R setup gives an R_{Thev} of 1/2R, so it is 50K for that value. R_{in_base} is $\beta \times R_E$, which is 500K. At this point, we see R_{in} of 45K, which is just under 10x the 5K Rout of the previous stage. That works.

Note though that from a high level, R_{in_base} is also 10x bigger than R_{Thev} of the divider. Good design. Considering that, without getting into the above nitty gritty calculation, that 10x design allows us to not even have to look that far ahead at the R_{in_base}. We can actually just ignore it. And this checks out. The R_{Thev} of the divider is 50K, which is 10x the Rout of the source.

An Aside on the C

We're only dealing with signal frequencies so the Z_C doesn't matter. At signal frequencies the C has super low R. It's passing signal right through. Still I'd like to run an exercise on why that C value was chosen. I believe this is a high pass filter setup. It looks strikingly like the 4W.2 problem. We know C and R so we can figure out what signal frequency is for this circuit. First of all R would be the above, what I call R_{in_C}, 50K roughly. We can determine f_{3db} using these values. Multiply it by 2x to get a signal frequency, since it's high-pass.

$$f_{3db} = 1/(2\pi RC)$$

R	C	f_{3db}	fsignal
50000	0.0000001	31.8	63.7

Point 2: Rout-emitter, Transistor 1
Value: 63 Ohm

> Here, we do see the transistor's 'lens' effect, and to determine R_{out} at the emitter we 'look through' the transistor toward the signal source… R_{out} is RE in parallel with the other path, through the emitter. Let's write this out, first, with painful rigor. Then we'll throw out what doesn't matter much.

So the book describes what to toss out and what to keep. I'd like to explain a few of them in my own words:

Feature	**Value**	**Description**
RE	5000	By proper design, the 10x design specifically, can be ignored for this situation.
RThev_bias_divider	50000	Can also be ignored because defines Zin for the whole circuit and so it's large compared to Rout-signal.
re	12.5	25/IC (mA). This should be compared with the value of Rout-signal/Beta which is 50. Yes then we keep this. This is 9.4/5K, 2mA which gives 25/2, which gives 12.5 for re.
Rout-source	5000	Yes we must consider it.
Rout-emitter	62.5	Rout_emitter = re + Rsource/Beta. Nice! It's low like we want it.

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Point 3: $R_{out\text{-}emitter}$, Transistor 2
1 Ohm

Overall this whole question is probing us on when to consider re and RE for impedances at various points of a transistor. Page 184 offers an excellent review of this material. For the first transistor you can get a sense of whether re matters. If the source was really high, re wouldn't matter. But 5K/beta is only 50 Ohms. Therefore re is probably going to come into play here.

So what is little re? Well first we need to figure out what the voltage is at the base of the second transistor, which will take us to I_E and I_C. Here's a walkthrough on how I got here:

Feature	Value	Description
V_{B_01}	10	20V across a R || R voltage divider splits it in half.
V_E	9.4	VB minus a voltage drop.
R_E	5000	Given.
I_E	0.00188	VB across RE, so 9.4V across 5K.
I_C	0.00188	IE is the same as IC.
R_C	5000	Given.
V_{B_02}	9.4	IC across RC, so 2mA through 5K. Let's call it 10V.

Now we can start to figure out re of the second transistor, and ultimately what the Rout_emitter equation should consider it.

Feature	Value	Description
V_{B_02}	10	We just figured this out above.
V_E	9.4	VB minus a voltage drop.
R_E	470	Given.

I_E	0.02	VB across RE, so 9.4V across 470.
r_e	1.25	25/IC in mA.
R_{source}	5000	This is the 5K attached to the base of this second transistor. When you have a R at the collector, that becomes the output impedance at the collector.
$R_{out\_emitter}$	1	Rout_emitter = RE || (re + Rsource/Beta). We don't ignore RE this time, unlike the other transistor. Conversely, re is tiny so we can ignore it this round. This is a good value, it is low!

3 Stability

Figure 2: Transistor on right is used as photodiode

3.1 amp; I-to-V

Circuit A is less vulnerable to oscillation than circuit B here, for a few reasons. First of all, it has a big $R_{Feedback}$ in proportion to the R attached to its inverting input. This is an inverting amplifier by the way. The higher that $R_{Feedback}$ to Rin proportion is, the less vulnerable the circuit is to oscillations. If $R_{Feedback}$ was equal or close to equal to the Rin, then we would resolve it by adding a small R on the output of the op-amp.

The other reason is that we know the setup of Circuit B here is vulnerable due to lagging phase shifts, or low-pass effects that creep in. The book has a nice explanation of this on pg 361, which is actually not terribly intuitive. The large $R_{Feedback}$ of 4.7M drives stray capacitance at the site of virtual ground, creating a low-pass filter affect. If frequency increases, phase shift increases, which is how we would get parasitic oscillation. The remedy is tacking on a little capacitor across $R_{Feedback}$. We tune it such that above a certain frequency, the $R_{Feedback}$ gets bypassed, no low-pass filter affect occurs, therefore no phase shift and oscillation occur.

Remedy: https://tinyurl.com/y3j3vuzg

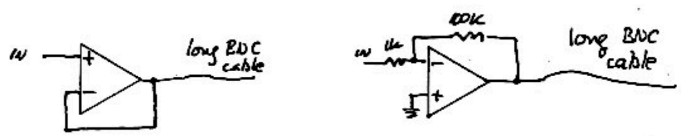

Figure 3: Long coax cable as load

3.2 Long coaxial cable as load

The inverting amp is feeding back a smaller fraction of the output, so it will do better here. Remember the principle I described in 3.1: the higher that $R_{feedback}$ to R_{in} proportion is, the less vulnerable the circuit is to oscillations. It's 100x here which is pretty high. Put another way, whatever noise there might be sneaking into the circuit through that super long cable, it's only getting amplified according to the feedback ratio, and so I believe the noise here is getting squished by 1/100th on its way back into the input. Oscillation should be minimal.

Meanwhile that follower is the opposite. There's no reduction of the noise that's making it into the cable. A follower by nature just feeds all of the output back into its input. Yikes! Okay so what's the remedy? See page 362-364 for a few options, and also Tom goes over this in the lecture, for my future reference.

1) One is to turn this follower into an amplifier, for reasons stated above.
2) Another would be to tack on a small R at the output of the amp. This relocates the phase shifting outside the feedback loop of the op-amp. A downside of this is that the Rout of the follower goes down—which is a bummer because this is a follower after all and we probably wanted high Rout.
3) The best solution is to add that R, along with a C and another R as I have done in the Falstad. This is split feedback. "Place the 'crossover' frequency—at which most of the signal passes through the capacitor, bypassing the phase-shifting network—safely below the frequency at which the circuit oscillates." This is one of those situations where we exploit capacitors and f3db to route the output signal either through the cap or the phase-shifting network depending on its frequency. Clever!

Remedies: https://tinyurl.com/y55vctnf

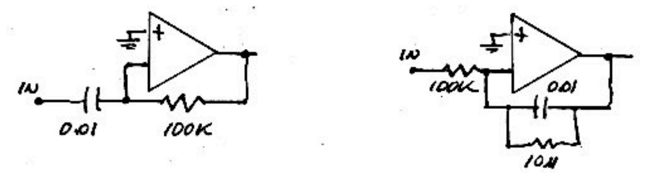

Figure 4: Two circuits that include caps

3.3 Two feedback circuits that include capacitors

First of all we know these two circuits. A is a differentiator and B is an integrator. See pages 307 and 303 respectively, for my future reference. For the differentiator, there's an internal phase shift. Then looking at the output back toward the input we see a high-pass filter, which then undoes that internal phase shift. So that thing should be super stable.

Circuit B, the integrator, should be super stable. Tom explains this according to a great question asked by Scott at the end of lecture. So we have some integration that's happening inside of the op-amp by necessity. We know that integration can be bad because it can cause negative phase shift. But then from the output looking back toward the input, the output sees a high-pass filter (the noted perspective is important, remember). High-pass filters cause a positive phase shift. So what we have here are two phase shifts that cancel each other out. That's handy! I've transcribed Tom's exact response for my own record keeping:

> Scott's question: the differentiator, to make a practical one with an opamp, you needed to make it an integrator. If I remember correctly, you didn't have to take those precautions with the integrator. And if internally in the opamp there's some integration going on, it would seem that this would be unstable, too.
>
> The problem is that: the instability with integration. There's already a built in integrator in the op-amp. So if we add an external integration, aren't we making things worse? The answer is, it sounds that way, but no. What is the op-amp doing? What function of the op-amp is being fed back? Look at the element in the feedback loop: it's a high-pass filter. It's an integrator and a low-pass from the input side, but in the feedback network we've added a high-pass. So why is that extremely safe? A low-pass filter has a lagging phase-shift. A high-pass is a jumping circuit, it's sensitive to changes. And its phase shift is positive. It's the opposite of a low-pass. So by adding this high-pass into the path, it actually undoes some of the internal phase-shift. So it's stabilizing. It's better than a piece of wire. It's rock solid.

So the differentiator is the one that may be vulnerable. The solution is to turn it into an integrator at high-frequencies by adding a C in parallel with the $R_{Feedback}$. Nice reference to this on page 307.

Remedy: https://tinyurl.com/y2urfl3m