Alg Top: Informal Reading Group
Meeting 1.

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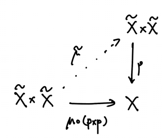
Problem (3.C.4)**.** *H***-spaces.** Let (X, e, μ) be an H-space. We claim that the universal cover $p : \tilde{X} \to X$ has an H-space structure. Let $\tilde{e}$ be any lift of e. Since $\tilde{X} \times \tilde{X}$ is simply connected, we can lift the map

$$\mu \circ (p \times p) : \tilde{X} \times \tilde{X} \to \tilde{X} \times \tilde{X} \tag{1}$$

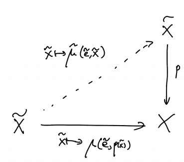
so that the diagram commutes:



To show that $\tilde{\mu}$ satisfies the axioms of an H-space, apply the homotopy lifting property to the map

$$F : \tilde{X} \to X \tag{2}$$

$$\tilde{x} \mapsto \tilde{\mu}(\tilde{e}, p(\tilde{x})) \tag{3}$$



Since the map $y \mapsto \mu(e, y)$ is homotopic to the identity, the map $\tilde{x} \mapsto \tilde{\mu}(\tilde{e}, \tilde{x})$ is also homotopic to the identity. $\square$

Problem (3.C.5)**.** Let (X, e, μ) be a Hopf space. Define the map

$$F : [0, 1]^2 \to X \tag{4}$$

$$(s,t) \mapsto \mu(f(s), g(t)) \tag{5}$$

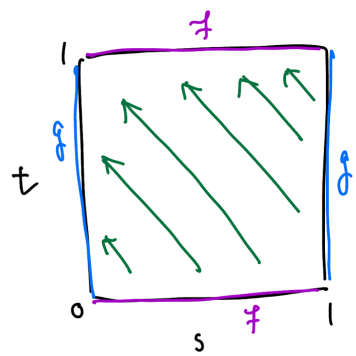
where f, g are loops based at e. Observe that

$$F(s,0) = F(s,1) = \mu(f(s), e) \simeq f(s) \tag{6}$$

$$F(0,t) = F(1,t) = \mu(e, g(t)) \simeq g(t) \tag{7}$$

Diagrammatically, we have:

Figure 1: Domain of the function F. F is homotopic to f on the top and bottom edges, and it is homotopic to g on the left and right edges. To obtain a homotopy between $f \cdot g$ and $g \cdot f$, just consider the straightline homotopy along the diagonal.



Then compose F with the straightline homotopy parallel to the line $y = -x$. This gives the desired homotopy. $\square$