Alg Top: Informal Reading Group
Meeting 1.

Tom Sawada

Updated: October 6, 2022

Problem (3.C.4). ***H*-spaces.** Let (X, e, μ) be an H-space. We claim that the universal cover $p : \tilde{X} \to X$ has an H-space structure. Let $\tilde{e}$ be any lift of e. Since $\tilde{X} \times \tilde{X}$ is simply connected, we can lift the map

$$\mu \circ (p \times p) : \tilde{X} \times \tilde{X} \to \tilde{X} \times \tilde{X} \tag{1}$$

so that the diagram commutes:

$$\begin{tikzcd} & \tilde{X} \times \tilde{X} \arrow[d, "p"] \\ \tilde{X} \times \tilde{X} \arrow[ur, dashed, "\tilde{\mu}"] \arrow[r, "\mu \circ (p \times p)"'] & X \end{tikzcd}$$

To show that $\tilde{\mu}$ satisfies the axioms of an H-space, apply the homotopy lifting property to the map

$$F : \tilde{X} \to X \tag{2}$$

$$\tilde{x} \mapsto \tilde{\mu}(\tilde{e}, p(\tilde{x})) \tag{3}$$

$$\begin{tikzcd} & \tilde{X} \arrow[d, "p"] \\ \tilde{X} \arrow[ur, dashed, "\tilde{x} \mapsto \tilde{\mu}(\tilde{e}, \tilde{x})"] \arrow[r, "\tilde{x} \mapsto \mu(\tilde{e}, p(\tilde{x}))"'] & X \end{tikzcd}$$

Since the map $y \mapsto \mu(e, y)$ is homotopic to the identity, the map $\tilde{x} \mapsto \tilde{\mu}(\tilde{e}, \tilde{x})$ is also homotopic to the identity. $\square$