# Assignment VII - DSAA(H)

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## # Question 7.1 (0.25 marks)

Illustrate the operation of CountingSort$(A, B, 3)$ on the array

| 0 | 2 | 0 | 1 | 3 | 1 | 1 |
|---|---|---|---|---|---|---|

with input elements from the set $\{0, 1, 2, 3\}$.

1. For each of the first three for loops, write down the contents of array $C$ after the loop has ended.
2. For the last for loop, write down the contents of the arrays $B$ and $C$ at the end of each iteration of the loop.

```
COUNTINGSORT(A, B, k)
 1: let C[0...k] be a new array
 2: for i = 0 to k do
 3:         C[i] = 0
 4: for j = 1 to A.length do
 5:         C[A[j]] = C[A[j]] + 1
 6: for i = 1 to k do
 7:         C[i] = C[i] + C[i − 1]
 8: for j = A.length downto 1 do
 9:         B[C[A[j]]] = A[j]
10:         C[A[j]] = C[A[j]] − 1
```

Sol:

1. Loop I: `0, 0, 0, 0`.
2. Loop II: `2, 3, 1, 1`.
3. Loop III: `2, 5, 6, 7`.
4. Loop IV:
    1. B: `_, _, _, _, 1, _, _`, C: `2, 4, 6, 7`.
    2. B: `_, _, _, 1, 1, _, _`, C: `2, 3, 6, 7`.
    3. B: `_, _, _, 1, 1, _, 3`, C: `2, 3, 6, 6`.
    4. B: `_, _, 1, 1, 1, _, 3`, C: `2, 2, 6, 6`.
    5. B: `_, 0, 1, 1, 1, _, 3`, C: `1, 2, 6, 6`.
    6. B: `_, 0, 1, 1, 1, 2, 3`, C: `1, 2, 5, 6`.
    7. B: `0, 0, 1, 1, 1, 2, 3`, C: `0, 2, 5, 6`.

# # Question 7.2 (0.25 marks)

**Question 7.2** (0.25 marks) Prove that after the **for** loop in lines 6-7 (in the pseudo-code provided at lecture) of CountingSort the array C contains in each position C[i] the number of elements less than or equal to $i$ (You can assume the previous two **for** loops are correct).

PF:

**Loop Invariant**: After j-th iteration of for loop in lines 6-7, the subarray $C[1, i]$ contains in each position $C[i], i \in [1, j]$ the number of elements less than or equal to i.

**Initialization**: Before entering the loop, $i = 0$, then the empty subarray is trivially satisfy the loop invariant.

**Maintenance**: Before the i-th loop, $C[i-1]$ contains the number of elements $\leq i-1$, and $C[i] \leftarrow C[i-1] + C[i]$, which adds the elements equal to $i$ with $\leq i-1$, therefore $C[i]$ represents the elements that $\leq i$.

**Termination**: After k-th iteration, the loop terminates. Then the whole array $C$ contains in each position $C[i]$ the number of elements less than or equal to i.

`Q.E.D.`.

# # Question 7.3 (0.25 marks)

**Question 7.3** (0.25 marks) Suppose that we were to rewrite the last **for** loop header of CountingSort as

"**for** $j = 1$ to $A$.length".

Then the algorithm:

1. Will not be stable and will not sort the numbers
2. Will be stable but will not sort the numbers
3. Will not be stable but will sort the numbers
4. Will be stable and will sort the numbers

Justify your answer.

Sol: The algorithm will not be stable but will sort the numbers.

The target range of elements equal to $i$ is still $B[C[i-1]+1, C[i]]$, i.e., all these equal elements will still be placed at the same range but different order, thus it's still **ordered**.

But for the stability, we are using $C[i] \leftarrow C[i]-1$ in the loop, thus the elements leftside will be placed to right for the iteration is from 1 to $A.length$, thus the equal elements are reversed, which is **unstable**.

# # Question 7.4 (marks:0.5)

Describe an algorithm CountingRange$(A, k, a, b)$ that given $n$ integers in the range 0 to $k$, preprocesses its input and and then answers any query about how many integers are present in the range $[a : b]$ in $O(1)$ time. The algorithm should use $O(n+k)$ preprocessing time.

Sol:

**Algorithm 1:** CountingRange-Init$(A, k)$

```
for i ← 0 to k do
    C[i] ← 0
end
for j ← 1 to A.length do
    C[A[j]] ← C[A[j]] + 1
end
S[0] ← C[0]
for i ← 1 to k do
    S[i] ← S[i − 1] + C[i]
end
return S
```

**Algorithm 2:** CountingRange-Query$(S, a, b)$

```
if a > b then
    return 0
end
if a = 0 then
    return S[b]
end
return S[b] − S[a − 1]
```

Explaination: We are using `CountingRange-Init` to preprocess the prefix summations of $C[i]$, obviously its runtime is $\Theta(n+k)$. For queries, use `CountingRange-Query`, which can return the answer by subtraction on prefix summations $S$ in $\Theta(1)$ runtime.

# # Question 7.5 (marks 0.25)

Illustrate the operation of RadixSort on the following list of English words:

COW, DOG, TUG, ROW, MOB, BOX, TAB, BAR, CAR, TAR, PIG, BIG, WOW

Sol:

1. StableSort by 3rd letter:

Bucket: $B < G < R < W < X$.

Result: MOB, TAB, DOG, TUG, PIG, BIG, BAR, CAR, TAR, COW, ROW, WOW, BOX.

2. StableSort by 2nd letter:

Bucket: $A < I < O < U$.

Result: TAB, BAR, CAR, TAR, PIG, BIG, MOB, DOG, COW, ROW, WOW, BOX, TUG.

3. StableSort by 1st letter:

Bucket: $B < C < D < M < P < R < T < W$.

Result: BAR, BIG, BOX, CAR, COW, DOG, MOB, PIG, ROW, TAB, TAR, TUG, WOW.

# # Question 7.6 (0.25 marks)

State which of the following algorithms are stable and which are not:

1. InsertionSort
2. MergeSort
3. HeapSort
4. QuickSort

For those that are stable argue why. For those that are not stable provide an example of an input that shows instability.

Sol:

1. **Stable**.

Take ascendent as example, the process of insertion is from right to left, and insert the element $A[i]$ when meeting the first $A[j] <= A[i]$, which make sure that the later elements will be placed righter, which holds stability.

2. **Stable**.

In MergeSort, we'll choose the elements in the left subarray first, which make sure the earlier elements will be placed lefter, which holds stability.

3. **Unstable**.

Example: $1, 2_1, 2_2$.

First we implement `Build-Max-Heap` which will implement `Max-Heapify(A, 1)`, then $2_2$ will be swaped to 1 as the largest. Eventually by `HeapSort` we'll get $2_2, 2_1, 1$ which is obviously unstable.

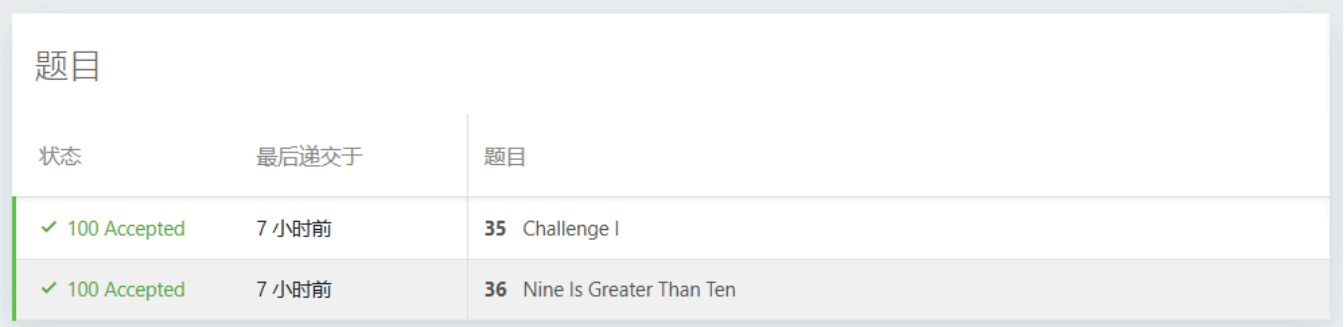
4. **Unstable**.

Example: $2_1, 2_2, 1$.

1 will be chosen as pivot, and $2_1$ will be swaped with 1, leading $1, 2_2, 2_1$, and when dealing with $2_2, 2_1$, the pivot $2_1$ will be swaped with itself, leading the final result: $1, 2_2, 2_1$, which is obviously unstable.

# # Question 7.7 (0.25 marks)

**Question 7.7** (0.25 marks) Implement the 'Challenge I' and 'Nine Is Greater than Ten' on the OJ system.



题目

| 状态 | 最后递交于 | 题目 |
| --- | --- | --- |
| ✓ 100 Accepted | 7 小时前 | 35 Challenge I |
| ✓ 100 Accepted | 7 小时前 | 36 Nine Is Greater Than Ten |

```cpp
using namespace std;
auto Sort = [](u32 *A, int N)->void{
    vector < basic_string < u32 > > buc(1 << 9);
    for(int base = 1; base <= 4; ++base){
        for(int i = 0; i < N; ++i)
            buc[(A[i] >> (8 * (base - 1))) & ((1u << 9) - 1)] += A[i];
        int cur(0);
        for(auto &b : buc){
            for(const auto &v : b)
                A[cur++] = v;
            b.clear();
        }
    }
};
```

```cpp
int main(){
    vector < string > A;
    int N = read();
    int mxlen(0);
    for(int i = 1; i <= N; ++i){
        string S; cin >> S; A.emplace_back(S);
        mxlen = max(mxlen, (int)S.length());
    }
    for(auto &s : A)s += string(mxlen - s.length(), '.');
    sort(A.begin(), A.end());
```

```
    for(const auto &s : A){
        for(auto c : s)if(c != '.')printf("%c", c);
        printf("\n");
    }

    // fprintf(stderr, "Time: %.6lf\n", (double)clock() /
CLOCKS_PER_SEC);
    return 0;
}
```