Assignment III - DSAA(H)

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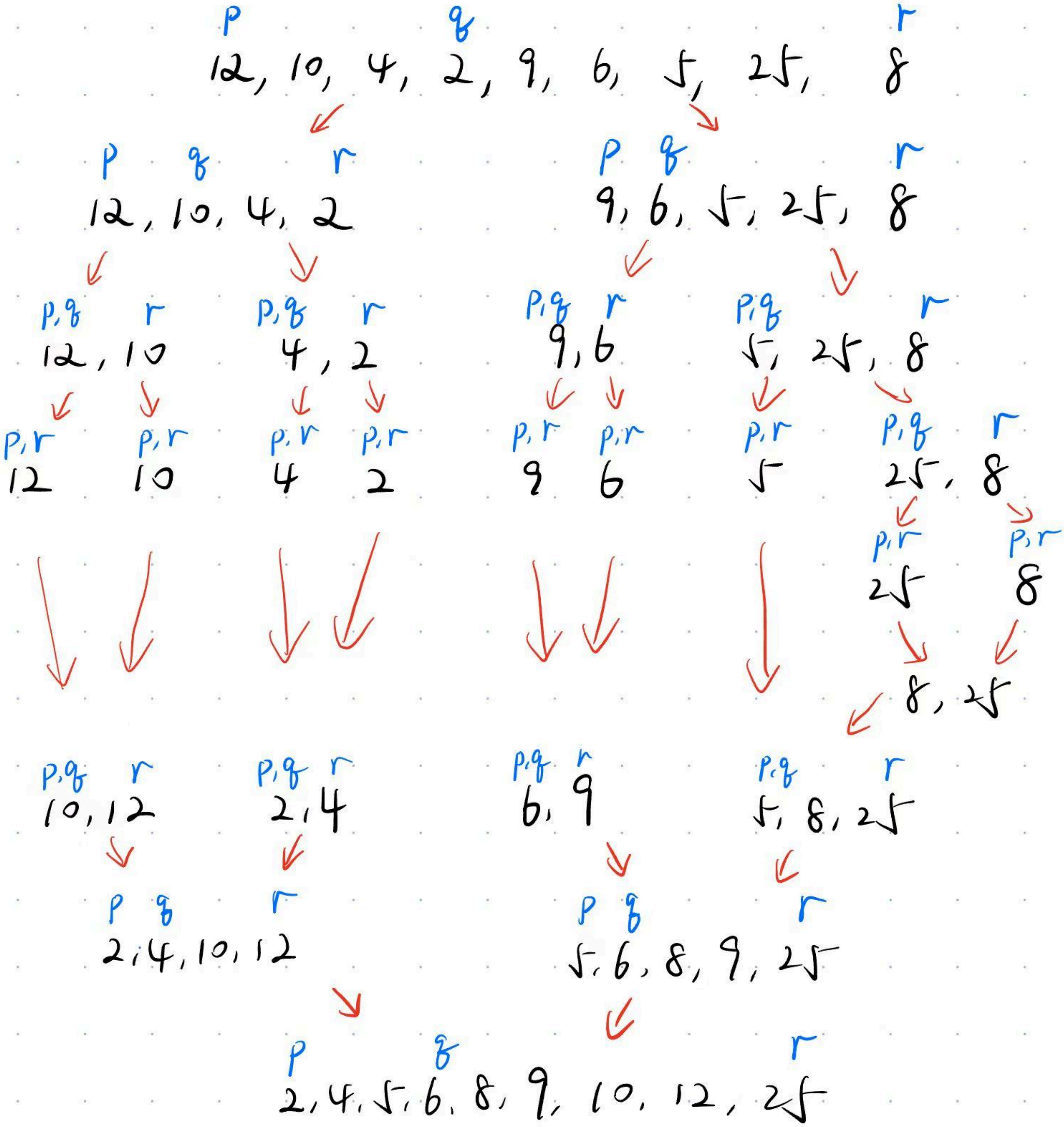
Question 3.1 (0.1 marks)

Consider the following input for MergeSort:

12	10	4	2	9	6	5	25	8

Illustrate the operation of the algorithm (follow how it was done in the lecture notes).

Sol: Firstly, divide: `[12, 10, 4, 2 | 9, 6, 5, 25, 8]`, then for the left, keep dividing: `[12, 10 | 4, 2]`, for the right: `[9, 6 | 5, 25, 8]`. And keep dividing: `[12, 10] -> [12 | 10]`, `[4, 2] -> [4 | 2]`, `[9, 6] -> [9 | 6]`, `[5, 25, 8] -> [5 | 25, 8] -> [5], [25 | 8]`. After dividing, implement conquest and combination step by step: `[25], [8] -> [8, 25]`, then, `[12], [10] -> [10, 12]`, `[4], [2] -> [2, 4]`, `[9], [6] -> [6, 9]`, `[5], [8, 25] -> [5, 8, 25]`, then, `[10, 12], [2, 4] -> [2, 4, 10, 12]`, `[6, 9], [5, 8, 25] -> [5, 6, 8, 9, 25]`, eventually, `[2, 4, 10, 12], [5, 6, 8, 9, 25] -> [2, 4, 5, 6, 8, 9, 10, 12, 25]` , at which we finished the MergeSort.



p q r
12, 10, 4, 2, 9, 6, 5, 25, 8
p q r
12, 10, 4, 2
p q r
9, 6, 5, 25, 8
p,q r
12, 10
p,q r
4, 2
p,q r
9, 6
p,q r
5, 25, 8
p,r
12
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10
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4
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2
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9
p,r
6
p,r
5
p,q r
25, 8
p,r
25
p,r
8
8, 25
p,q r
10, 12
p,q r
2, 4
p,q r
6, 9
p,q r
5, 8, 25
p q r
2, 4, 10, 12
p q r
5, 6, 8, 9, 25
p q r
2, 4, 5, 6, 8, 9, 10, 12, 25

Question 3.2 (0.45 marks)

Question 3.2 (0.45 marks) Prove using the substitution method the runtime of the MERGE-SORT Algorithm on an input of length n, as follows. Let n be an exact power of 2, $n = 2^k$ to avoid using floors and ceilings. Use mathematical induction over k to show that the solution of the recurrence involving positive constants $c, d > 0$

$$T(n) = \begin{cases} d & \text{if } n = 2^0 = 1 \\ 2T(n/2) + cn & \text{if } n = 2^k \text{ and } k \geq 1 \end{cases}$$

is $T(n) = dn + cn \log n$ (we always use log to denote the logarithm of base 2, so $\log = \log_2$).

Hint: you may want to rewrite the above by replacing n with 2^k. Then the task is to prove that $T(2^k) = d2^k + c2^k \cdot k$ using the recurrence

$$T(2^k) = \begin{cases} d & \text{if } k = 0 \\ 2T(2^{k-1}) + c2^k & \text{if } k \geq 1 \end{cases}$$

PF: Base: When $n = 2^0 = 1$, we have $T(n) = d = d \cdot 1 + c \cdot 1 \cdot \log 1$.

For any k s.t. $k \geq 1$, assuming we have $T(2^{k-1}) = d2^{k-1} + c2^{k-1} \cdot (k-1)$, i.e., the equivalence holds when $n = 2^{k-1}$.

Then, we have:

$$\begin{aligned} T(2^k) &= 2T(2^{k-1}) + c2^k \\ &= 2(d2^{k-1} + c2^{k-1} \cdot (k-1)) + c2^k \\ &= d2^k + 2^k \cdot (c(k-1) + c) \\ &= d2^k + c2^k \cdot k \end{aligned}$$

As we obtain $k = \log n$, thus $T(n) = dn + cn \log n$, i.e., the equivalence holds when $n = 2^k$.

With mathematical induction, we have proved that it holds for all $k \geq 0$.

`Q.E.D.`.

Question 3.3 (0.4 marks)

Question 3.3 (0.4 marks) Use the Master Theorem to give asymptotic tight bounds for the following recurrences. Justify your answers.

1. $T(n) = 2T(n/4) + 1$
2. $T(n) = 2T(n/4) + \sqrt{n}$
3. $T(n) = 2T(n/4) + \sqrt{n}\log^2 n$
4. $T(n) = 2T(n/4) + n$

Sol:

1. We have $a = 2, b = 4, f(n) = 1$, the watershed: $W(n) = n^{\log_b^a} = n^{\frac{1}{2}}$. Then, let $\epsilon = \frac{1}{2}$, $f(n) = 1 = O(n^{\log_b(\frac{1}{2}-\epsilon)})$, satisfy case 1, thus $T(n) = \Theta(n^{\frac{1}{2}})$.
2. We have $a = 2, b = 4, f(n) = n^{\frac{1}{2}}$, the watershed: $W(n) = n^{\log_b^a} = n^{\frac{1}{2}}$. Then, let $k = 0$, $f(n) = n^{\frac{1}{2}} = \Theta(n^{\frac{1}{2}} \cdot \lg^0 n)$, satisfy case 2, thus $T(n) = \Theta(n^{\frac{1}{2}} \log n)$.
3. We have $a = 2, b = 4, f(n) = n^{\frac{1}{2}} \log^2 n$, the watershed: $W(n) = n^{\log_b^a} = n^{\frac{1}{2}}$. Then, let $k = 2$, $f(n) = n^{\frac{1}{2}} \log^2 n = \Theta(n^{\frac{1}{2}} \cdot \lg^2 n)$ for the base of log only changes the constants, satisfy case 2, thus $T(n) = \Theta(n^{\frac{1}{2}} \log^3 n)$.
4. We have $a = 2, b = 4, f(n) = n$, the watershed: $W(n) = n^{\log_b^a} = n^{\frac{1}{2}}$. Then, let $\epsilon = \frac{1}{2}$, $f(n) = n = \Omega(n^{\log_b(\frac{1}{2}+\epsilon)})$. And for the regularity, let $c = \frac{1}{2}$, we obtain $af(\frac{n}{b}) = 2f(\frac{n}{4}) = \frac{1}{2}n \leq cf(n) = \frac{1}{2}n$, satisfy case 3, thus $T(n) = \Theta(n)$.

Question 3.4 (0.45 marks)

Question 3.4 (0.45 marks) Write the pseudo-code of the *recursive* BINARYSEARCH$(A, x, \text{low}, \text{high})$ algorithm discussed during the lecture to find whether a number x is present in an increasingly sorted array of length n. Write down its recurrence equation and prove that its runtime is $\Theta(\log n)$ using the Master Theorem.

Sol: Pseudo-code:

Algorithm 1: BinarySearch$(A, x, low, high)$

```
if low > high then
    return false
end
mid ← ⌊(low + high) / 2⌋
if A[mid] = x then
    return true
end
else if x < A[mid] then
    return BinarySearch(A, x, low, mid − 1)
end
else
    return BinarySearch(A, x, mid + 1, high)
end
```

Recurrence equation:

$$T(n) = T(\frac{n}{2}) + 1$$

We have $a = 1, b = 2$, $f(n) = 1$. Watershed: $n^{\log_b^a} = 1$. Let $k = 0$, $f(n) = 1 = \Theta(1 \cdot \lg^0 n)$, satisfy case 2, thus $T(n) = \Theta(\log n)$.

Question 3.5 (0.6 marks)

Question 3.5 (0.6 marks) Solve programming problems "Heybale Feast", "A good problem", "Swiss" and "Bubble Sort II" provided on the Judge system.

题目

状态	最后递交于	题目
✓ 100 Accepted	5 天前	**22** Haybale Feast
✓ 100 Accepted	3 天前	**23** A Good Problem
✓ 100 Accepted	3 天前	**24** Swiss
✓ 100 Accepted	5 天前	**25** Bubble Sort II

```cpp
int N; ll M;

class SegTree{
private:
    int tr[110000 << 2];
    #define LS (p << 1)
    #define RS (LS | 1)
    #define MID ((gl + gr) >> 1)
public:
    void Pushup(int p){
        tr[p] = max(tr[LS], tr[RS]);
    }
    void Build(const vector < int > &A, int p = 1, int gl = 1, int gr = N){
        if(gl == gr)return tr[p] = A[gl = gr], void();
        Build(A, LS, gl, MID), Build(A, RS, MID + 1, gr);
        Pushup(p);
    }
    int Query(int l, int r, int p = 1, int gl = 1, int gr = N){
        if(l <= gl && gr <= r)return tr[p];
        if(gr < l || r < gl)return -1;
        return max(Query(l, r, LS, gl, MID), Query(l, r, RS, MID + 1, gr));
    }
}st;

int main(){
    N = read(); M = read < ll >();
    vector < int > F(N + 10, 0), S(N + 10, 0);
    for(int i = 1; i <= N; ++i)F[i] = read(), S[i] = read();
    st.Build(S);
    vector < ll > sumF(N + 10, 0);
    for(int i = 1; i <= N; ++i)sumF[i] = sumF[i - 1] + F[i];
    auto Check = [](ll sum)->bool{return sum >= M;};
    int res(INT_MAX);
    for(int beg = 1; beg <= N; ++beg){
        int l = beg, r = N, ans = -1;
        while(l <= r){
            int mid = (l + r) >> 1;
```

```
            if(Check(sumF[mid] - sumF[beg - 1]))ans = mid, r = mid - 1;
            else l = mid + 1;
        }
        if(ans != -1)res = min(res, st.Query(beg, ans));
    }
    printf("%d\n", res);
    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
}
```

```
int main(){
    int N = read();
    vector < int > A(N + 10, 0);
    for(int i = 1; i <= N; ++i)A[i] = read();

    vector < pair < int, int > > res;

    auto Solve = [&](auto &&self, int l, int r)->void{
        if(l == r)return;
        int mid = (l + r) >> 1;
        for(int i = 1; i <= N; ++i)
            if(mid + 1 <= A[i] && A[i] <= r)res.push_back({2, i});
        int cur(l + 1);
        while(cur < mid + 1)res.push_back({1, cur++});
        self(self, l, mid), self(self, mid + 1, r);
    }; Solve(Solve, 0, N);

    printf("%d\n", res.size());
    for(auto [a, b] : res)printf("%d %d\n", a, b);
    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
}
```

```
int main(){
    int N = read(), R = read(), Q = read();
    N <<= 1;
```

```
    vector < tuple < int, int, int > > players;
    for(int i = 1; i <= N; ++i)players.push_back({read(), 0, i});
    for(int i = 1; i <= N; ++i)get < 1 >(players[i - 1]) = read();
    sort(players.begin(), players.end(), [](const tuple < int, int, int > &a, const tuple < int, int, int > &b)->bool{
        return get < 0 >(a) == get < 0 >(b) ? get < 2 >(a) < get < 2 >(b) : get < 0 >(a) > get < 0 >(b);
    });
    while(R--){
        vector < tuple < int, int, int > > winner, loser;
        for(auto it = players.begin(); it < prev(players.end()); advance(it, 2))
            get < 1 >(*it) > get < 1 >(*next(it)) ? (++get < 0 >(*it), winner.push_back(*it), loser.push_back(*next(it))) : (++get < 0 >(*next(it)), winner.push_back(*next(it)), loser.push_back(*it));
        if(players.size() & 1)loser.push_back(*prev(players.end()));

        players.clear();
        for(auto it1 = winner.begin(), it2 = loser.begin(); it1 != winner.end() || it2 != loser.end();){
            if(it1 == winner.end())players.push_back(*it2), ++it2;
            else if(it2 == loser.end())players.push_back(*it1), ++it1;
            else
                get < 0 >(*it1) == get < 0 >(*it2) ? (get < 2 >(*it1) < get < 2 >(*it2) ? (players.push_back(*it1), ++it1) : (players.push_back(*it2), ++it2)) : get < 0 >(*it1) > get < 0 >(*it2) ? (players.push_back(*it1), ++it1) : (players.push_back(*it2), ++it2);
        }
    }
    // sort(players.begin(), players.end(), [](const tuple < int, int, int > &a, const tuple < int, int, int > &b)->bool{
    //     return get < 0 >(a) == get < 0 >(b) ? get < 2 >(a) < get < 2 >(b) : get < 0 >(a) > get < 0 >(b);
    // });
    printf("%d\n", get < 2 >(players[Q - 1]));


    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
```

```
}
```

```
int main(){
    int N = read();
    vector < int > A(N + 10, 0);
    for(int i = 1; i <= N; ++i)A[i] = read();
    ll res(0);
    vector < int > cur;
    for(int i = 1; i <= N; ++i){
        if(cur.empty())cur.push_back(A[i]);
        else{
            auto it = upper_bound(cur.begin(), cur.end(), A[i]);
            res += distance(it, cur.end());
            cur.insert(it, A[i]);
        }
    }printf("%lld\n", res);
    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
}
```