

## Exercise Sheet 9

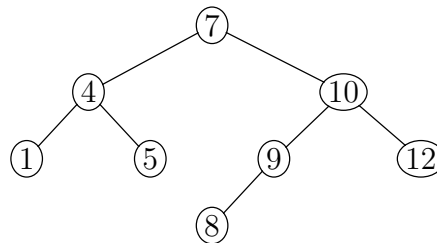
Handout: Nov 4th — Deadline: Nov 11th, 4pm

### Question 9.1 (1 mark)

1. Prove by induction that every complete binary tree of height  $h$  has  $2^h - 1$  internal nodes.
2. Prove by induction that in every full nonempty binary tree the number of leaves is one more than the number of internal nodes.
3. Prove by induction that every nonempty binary tree satisfies  $|V| = |E| + 1$ .

### Question 9.2 (0.25 marks)

1. Insert a node with key 11 into the following binary search tree. Give a step-by-step explanation.



2. Delete the node with key 10 into the resulting binary search tree. Give a step-by-step explanation.
3. Insert a node with key 10 into the resulting binary search tree. Give a step-by-step explanation.
4. Delete the node with key 8 from the resulting binary search tree. Give a step-by-step explanation.
5. Delete the node with key 7 from the resulting binary search tree. Give a step-by-step explanation.

### Question 9.3 (0.25 marks)

Delete two different nodes in different order from a binary search tree (e.g. first node  $x$  and then node  $y$ , or alternatively first node  $y$  and then node  $x$ ). Can the resulting trees be different? Explain your answer.

### Question 9.4 (0.25 marks)

Write the TREE-PREDECESSOR( $x$ ) procedure.

### Question 9.5 (0.25 marks)

You can sort a set of  $n$  numbers by the following procedure:

1. Build a binary search tree by inserting each element using TREE-INSERT ( $n$  times)
2. Print the numbers in sorted order by an INORDER tree walk.

What are the worst case and best case runtimes of this sorting algorithm?

**Question 9.6** (1 mark)

Solve the three problems on Binary Search Trees on Online Judge.