Assignment IX - DSAA(H)

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Question 9.1 (1 mark)

1. Prove by induction that every complete binary tree of height h has $2^h - 1$ internal nodes.
2. Prove by induction that in every full nonempty binary tree the number of leaves is one more than the number of internal nodes.
3. Prove by induction that every nonempty binary tree satisfies $|V| = |E| + 1$.

PF:

1. Base $h = 0$: internal $0 = 2^0 - 1$.

   Step: two complete subtrees of height $h - 1$, then $I = 1 + 2\left(2^{h-1} - 1\right) = 2^h - 1$.

   `Q.E.D.`.

2. Base: single node $L = 1, I = 0$.

   Step: subtrees T_L, T_R and $L(T_L) = I(T_L) + 1$, $L(T_R) = I(T_R) + 1$, then

   $L = L(T_L) + L(T_R) = I(T_L) + I(T_R) + 2 = I + 1$.

   `Q.E.D.`.

3. Base $1 = 0 + 1$.

   Step: Remove a leaf, then $|V'| = |V| - 1$ and $|E'| = |E| - 1$. By $|V'| = |E'| + 1$, thus

   $|V| = |E| + 1$.

   `Q.E.D.`.

Question 9.2 (0.25 marks)

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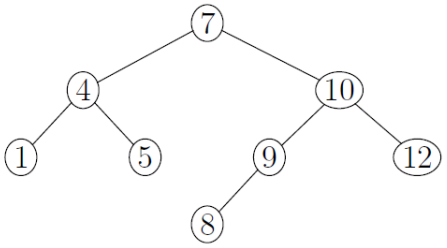
1. Insert a node with key 11 into the following binary search tree. Give a step-by-step explanation.



2. Delete the node with key 10 into the resulting binary search tree. Give a step-by-step explanation.
3. Insert a node with key 10 into the resulting binary search tree. Give a step-by-step explanation.
4. Delete the node with key 8 from the resulting binary search tree. Give a step-by-step explanation.
5. Delete the node with key 7 from the resulting binary search tree. Give a step-by-step explanation.

Sol:

1.

- Compare with 7 → go right to 10.
- 11 > 10 → go right to 12.
- 11 < 12 and 12's left is NIL → insert as left child of 12.

```
        7
      /   \
     4     10
    / \   /  \
   1   5 9   12
        /   /
       8   11
```

2.

- Node 10 has two children. The min of right subtree is 11.
- Splice 11 out of under 12.
- Replace 10 by 11; set 11.left ← 9, 11.right ← 12.

```
          7
        /   \
       4     11
      / \   /  \
     1   5 9   12
           /
          8
```

3.

- 10 > 7 → go right to 11.
- 10 < 11 → go left to 9.
- 10 > 9 and 9.right is NIL → insert as right child of 9.

```
          7
        /   \
       4     11
      / \   /  \
     1   5 9   12
           / \
          8  10
```

4.

- 8 is a leaf → remove directly.

```
        7
      /   \
     4     11
    / \   /  \
   1   5 9   12
          \
          10
```

5.

- Root 7 has two children. Min of right subtree: from 11 go left to 9 (now left is NIL), then it's 9.
- Splice 9 out by linking its right child 10 up to 11.left.
- Replace 7 by 9, set 9.left ← 4, 9.right ← 11.

```
        9
      /   \
     4     11
    / \   /  \
   1   5 10  12
```

Question 9.3 (0.25 marks)

Delete two different nodes in different order from a binary search tree (e.g. first node x and then node y, or alternatively first node y and then node x). Can the resulting trees be different? Explain your answer.

Sol: That will cause different shapes.

Reason: Deleting a two-child node replaces it by its successor or predecessor and performs transplants along that path, if the other key lies on or near that path, the first deletion changes which successor or predecessor the second uses, so the final structure is differ.

For instance, when y is the successor of x, and y has right child.

Question 9.4 (0.25 marks)

Write the TREE-PREDECESSOR(x) procedure.

Sol:

Algorithm 1: Tree-Maximum(x)

```
1 while x.right ≠ NIL do
2 |   x ← x.right
3 end
4 return x
```

Algorithm 2: Tree-Predecessor(x)

```
1 if x.left ≠ NIL then
2 |   return TREE-MAXIMUM(x.left)
3 end
4 y ← x.parent
5 while y ≠ NIL and x = y.left do
6 |   x ← y
7 |   y ← y.parent
8 end
9 return y
```

Tips: Implement Tree-Predecessor from root, we can find the predecessor without parent pointer:

Algorithm 2: Tree-Predecessor($root, x$)

```
1  if root = NIL then
2  |  return NIL
3  end
4  if x ≤ root.key then
5  |  return PREDECESSOR-BY-KEY(root.left, x)
6  end
7  best ← PREDECESSOR-BY-KEY(root.right, x)
8  if best ≠ NIL then
9  |  return best
10 end
11 return root
```

Question 9.5 (0.25 marks)

Question 9.5 (0.25 marks)

You can sort a set of n numbers by the following procedure:

1. Build a binary search tree by inserting each element using TREE-INSERT (n times)
2. Print the numbers in sorted order by an INORDER tree walk.

What are the worst case and best case runtimes of this sorting algorithm?

Sol:

- Worst case: $\Theta(n^2)$.

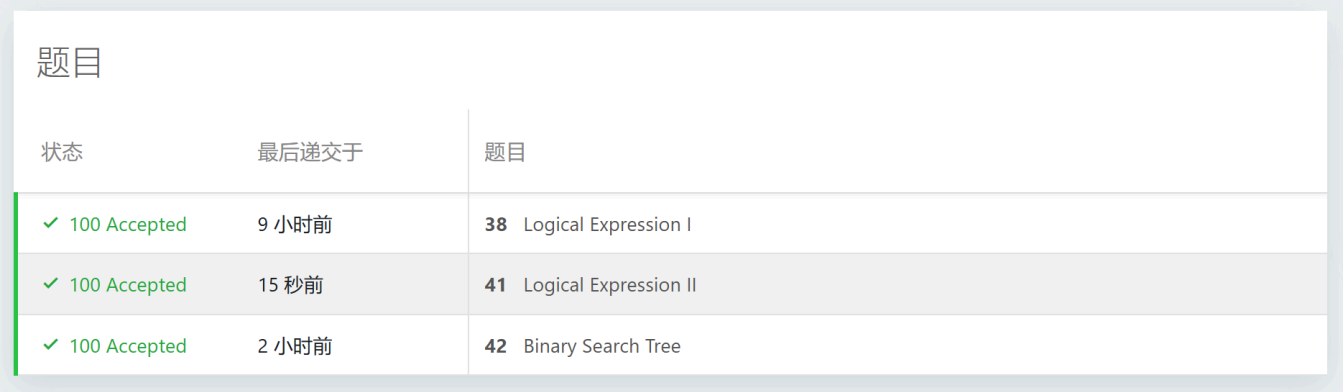
  Let chain height $h_t = t$, build $\sum_{t=0}^{n-1}(t+1) = \Theta(n^2)$.

- Best case: $\Theta(n \log n)$.

  With balanced heights $h_t = \Theta(\log t)$, build $\sum \Theta(\log t) = \Theta(n \log n)$.

Question 9.6 (1 mark)

Solve the three problems on Binary Search Trees on Online Judge.



题目

状态	最后递交于	题目
✓ 100 Accepted	9 小时前	38 Logical Expression I
✓ 100 Accepted	15 秒前	41 Logical Expression II
✓ 100 Accepted	2 小时前	42 Binary Search Tree

```
int main(){

    int T = read();
    while(T--){
        string S; cin >> S;
        stack < char > cur;
        for(int i = 0; i < S.size(); ++i){
            char c = S.at(i);
            if(c == '-')++i, cur.push('>');
            else if(c != ')')cur.push(c);
            else{
                char v2 = cur.top(); cur.pop();

                if(!cur.empty() && cur.top() == '!'){
                    cur.pop();
                    if (!cur.empty() && cur.top() == '(')cur.pop();
                    cur.push(v2 == '1' ? '0' : '1');
                }else{
                    char op = cur.top(); cur.pop();
                    char v1 = cur.top(); cur.pop();
                    if (!cur.empty() && cur.top() == '(') cur.pop();
                    bool a = v1 == '1', b = v2 == '1', r(false);
                    switch(op){
                        case '&': r = a & b; break;
                        case '|': r = a | b; break;
```

```
                        case '^': r = a ^ b; break;
                        case '>': r = (!a) | b; break;
                    }
                    cur.push(r ? '1' : '0');
                }

            }
        }printf("%c\n", cur.top());
    }

    // fprintf(stderr, "Time: %.6lf\n", (double)clock() /
CLOCKS_PER_SEC);
    return 0;
}
```

```
struct Edge{
    Edge* nxt;
    int to;
};

const int LIM = 1100000;

int main(){
    vector < int > A(LIM, 0);
    vector < int > label(LIM, 0);
    vector < int > res(LIM, 0);
    vector < Edge* > head(LIM, nullptr);
    vector < int > L(LIM, 0), R(LIM, 0);
    vector < int > flag(LIM, 0), flag2(LIM, 0), F(LIM, 0);

    string S;
    stack < int > cur;
    int N(0), cnt(0);
    while(cin >> S){
        if(!S.empty() && isdigit(S[0])){N = stoi(S); break;}
        if(S == "!"){
            if(!cur.empty()){
                int tp = cur.top();
                if(tp > 0) flag2[tp] ^= 1;
```

```
                else flag[-tp] ^= 1;
            }
        }else if(S[0] == 'x'){
            int val(0);
            for(int i = 1; i < (int)S.length(); ++i)
                val = val * 10 + int(S[i] - '0');
            cur.push(val);
        }else if(S == "&" || S == "|"){
            int y = cur.top(); cur.pop();
            int x = cur.top(); cur.pop();
            label[++cnt] = (S == "&" ? 2 : 3);
            L[cnt] = x; R[cnt] = y;
            cur.push(-cnt);
        }
    }

    for(int i = 1; i <= N; ++i)A[i] = read();

    auto mapID = [&](int t)->int{return t > 0 ? t : (N + (-t));};

    int root = cur.empty() ? N : mapID(cur.top());
    int Q = read();

    for(int i = 1; i <= N; ++i)F[i] = flag2[i];
    for(int i = 1; i <= cnt; ++i){
        int p = N + i;
        F[p] = flag[i];
        A[p] = label[i];
        head[p] = new Edge{head[p], mapID(L[i])};
        head[p] = new Edge{head[p], mapID(R[i])};
    }

    vector < char > vis(LIM, 0), vis2(LIM, 0);
    vector < int > val(LIM, 0);

    auto dfs = [&](auto&& self, int p, int g)->int{
        if(p <= N)return A[p] ^ g;
        if(vis[p])return val[p];
        vis[p] = 1;

```

```
        int c1(-1), c2(-1), k(0);
        for(auto i = head[p]; i && k < 2; i = i->nxt)
            (k++) == 0 ? c1 = i->to : c2 = i->to;
        int rs = c1, ls = c2;
        int x = self(self, ls, g ^ F[ls]);
        int y = self(self, rs, g ^ F[rs]);

        int opt = (A[p] ^ g);
        if(opt == 2){
            if(!x)res[rs] = 1;
            if(!y)res[ls] = 1;
            val[p] = (x & y);
        }else{
            if(x == 1)res[rs] = 1;
            if(y == 1)res[ls] = 1;
            val[p] = (x | y);
        }
        return val[p];
    };

    auto dfs2 = [&](auto&& self, int p)->void{
        if(p <= N)return;
        if(vis2[p])return;
        vis2[p] = 1;
        for(auto i = head[p]; i; i = i->nxt){
            res[i->to] |= res[p];
            self(self, i->to);
        }
    };

    int ans = dfs(dfs, root, F[root]);
    dfs2(dfs2, root);

    while(Q--)printf("%d\n", res[read()] ? ans : !ans);

    return 0;
}
```

```
class Node{
```

```
public:
    Node *ls, *rs;
    int val, siz, cnt;
};

Node* root;

#define siz(p) ((p) ? (p)->siz : 0)
class Tree{
private:
public:
    void Pushup(Node* p){
        if(!p)return;
        p->siz = siz(p->ls) + siz(p->rs) + p->cnt;
    }
    Node* QueryMx(Node* p = root){
        if(!p)return p;
        if(p->rs)return QueryMx(p->rs);
        return p;
    }
    Node* QueryMn(Node* p = root){
        if(!p)return p;
        if(p->ls)return QueryMn(p->ls);
        return p;
    }
    Node* Insert(int val, Node* p = root){
        if(!p)return new Node{nullptr, nullptr, val, 1, 1};
        if(val < p->val)p->ls = Insert(val, p->ls);
        else if(val > p->val)p->rs = Insert(val, p->rs);
        else ++p->cnt;
        Pushup(p);
        return p;
    };
    Node* Delete(int val, Node* p = root){
        if(!p)return p;
        if(val < p->val)p->ls = Delete(val, p->ls);
        else if(val > p->val)p->rs = Delete(val, p->rs);
        else{
            if(p->cnt > 1)--p->cnt;
            else{
```

```
                if(!p->ls)delete exchange(p, p->rs);
                else if(!p->rs)delete exchange(p, p->ls);
                else{
                    auto succ = QueryMn(p->rs);
                    p->val = succ->val, p->cnt = succ->cnt;
                    succ->cnt = 1;
                    p->rs = Delete(succ->val, p->rs);
                }
            }
        }Pushup(p); return p;
    }
    int QueryRnk(int val, Node* p = root){
        if(!p)return 0;
        if(val == p->val)return siz(p->ls);
        if(val < p->val)return QueryRnk(val, p->ls);
        return siz(p->ls) + p->cnt + QueryRnk(val, p->rs);
    }
    Node* QueryByRnk(int rnk, Node* p = root){
        if(!p)return p;
        // printf("l siz = %d, rnk = %d, cnt = %d\n", siz(p->ls), rnk, p->cnt); fflush(stdout);
        if(siz(p->ls) + 1 <= rnk && rnk <= siz(p->ls) + p->cnt)return p;
        if(rnk <= siz(p->ls))return QueryByRnk(rnk, p->ls);
        return QueryByRnk(rnk - siz(p->ls) - p->cnt, p->rs);
    }
    Node* QuerySuc(int val, Node* p = root){
        if(!p)return p;
        if(val >= p->val)return QuerySuc(val, p->rs);
        auto res = QuerySuc(val, p->ls);
        return res ? res : p;
    }
    Node* QueryPre(int val, Node* p = root){
        if(!p)return p;
        if(val <= p->val)return QueryPre(val, p->ls);
        auto res = QueryPre(val, p->rs);
        return res ? res : p;
    }
}tr;

```

```cpp
int main(){
    int T = read();
    while(T--){
        int opt = read(), val = read();
        switch(opt){
            case 1: root = tr.Insert(val); break;
            case 2: root = tr.Delete(val); break;
            case 3: printf("%d\n", tr.QueryRnk(val) + 1); break;
            case 4: printf("%d\n", tr.QueryByRnk(val)->val); break;
            case 5: printf("%d\n", tr.QueryPre(val)->val); break;
            case 6: printf("%d\n", tr.QuerySuc(val)->val); break;
        }
    }

    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
}

```