CS217 - Data Structures & Algorithm Analysis (DSAA)

Lecture #8

Elementary Data Structures

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Reading: Part III Introduction & Chapter 10

Aims of this lecture

- To introduce **data structures** and their typical operations.
- **Stacks**, **queues**, **priority queues** and **linked lists**.
- To work out the **running time** for operations on these data structures.
- To identify pros and cons for data structures in terms of efficiency.

Data Structures

- **Dynamic sets** that can store and retrieve elements.
- Data structures are techniques for representing finite dynamic sets of elements
- Each element can contain:
  - a **key,** used to identify the element
  - **Satellite data,** carried around but unused by the data structure
  - **Attributes,** that are manipulated by the data structure eg., pointers to other objects
- Often keys stem from a **totally ordered set** (e. g. numbers)
  - Allows to define the minimum, successor and predecessor

Data Structure Operations

- Operations on a dynamic sets S can be grouped into **queries** and **modifying operations**:
- Typical operations:
  - **Search(S, k):** returns a pointer to element $x \in S$ with **key k**, $(x.key = k)$ or NIL
  - **Insert(S, x):** adds element pointed to by x to S
  - **Delete(S, x):** given a pointer x to an element in S removes it from S
  - **Minimum(S), Maximum(S):** return a pointer x to element resp. with smallest or largest key
  - **Successor(S, x), Predecessor(S, x):** next larger (smaller) than $x.key$
- **Time** often measured using n as the number of elements in S.

Data Structure Operations

- What's the runtime of each operation on an **array**?
  - **Search(S, k):** returns a pointer to element $x \in S$ with **key k**, $(x.key = k)$ or NIL — $\Theta(n)$
  - **Insert(S, x):** adds element pointed to by x to S — $\Theta(1)$
  - **Delete(S, x):** given a pointer x to an element in S removes it from S — $\Theta(1)$
  - **Minimum(S), Maximum(S):** return a pointer x to element resp. with smallest or largest key — $\Theta(n)$
  - **Successor(S, x), Predecessor(S, x):** next larger (smaller) than $x.key$ — $\Theta(n)$

Data Structure Operations

- What's the runtime of each operation on a **sorted array**?
  - **Search(S, k):** returns a pointer to element $x \in S$ with **key k**, $(x.key = k)$ or NIL — $\Theta(\log n)$
  - **Insert(S, x):** adds element pointed to by x to S — $\Theta(n)$
  - **Delete(S, x):** given a pointer x to an element in S removes it from S — $\Theta(n)$
  - **Minimum(S), Maximum(S):** return a pointer x to element resp. with smallest or largest key — $\Theta(1)$
  - **Successor(S, x), Predecessor(S, x):** next larger (smaller) than $x.key$ — $\Theta(1)$

We'll now see some data structures that improve on the array implementation for many of the dynamic-set operations.

Roadmap for the next lectures

- Simple data structures
  - Stacks
  - Queues
  - Linked lists
  - Binary search trees
  - Graphs
- Advanced data structures
  - Balanced trees
  - Priority queues

Stacks

3
6
8

- Only the **top element** is accessible in a stack.
  - Last-in, first-out policy (LIFO)
- Insert is usually called **Push**, and Delete is called **Pop**.

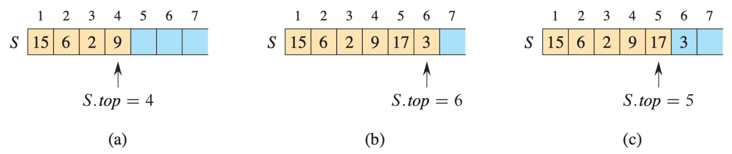
Stacks implemented using arrays

- Stacks can be implemented as an array S with attribute S.top.

```
PUSH(S, x)
if S.top == S.size
    error "overflow"
else S.top = S.top + 1
    S[S.top] = x
```

```
STACK-EMPTY(S)
if S.top == 0
    return TRUE
else return FALSE
```

```
POP(S)
if STACK-EMPTY(S)
    error "underflow"
else S.top = S.top - 1
    return S[S.top + 1]
```



- All stack operations take time $O(1)$.

Stacks Application (1): Bracket Balance Checking

- $1 + \{2 * [x + (4y - z)] * [5x - (5y + z)] - 5t\}$
- {[()][()]}
- Are the brackets correctly balanced or not?
- Read the expression: **Push** each opening bracket and **pop** for each closing bracket
- If the type of popped bracket always matches return **true,** else return **false**
- What's the runtime of the algorithm?

Stacks Application (2): Postfix expression

- $5 * ((9 + 3) * (4 * 2) + 7)$ (infix expression)
- 5 9 3 + 4 2 * * 7 + * (postfix expression)
- Parsing postfix expressions is somewhat easier than infix expressions. Why?
- Read the tokens one at a time:
  - If it is an operand, **push** it on the stack
  - If it is a binary operator **pop** twice, apply the operator, and **push** the result back on the stack
- What is the runtime of the algorithm?

Stacks Application (2): Postfix expression

- $5 * ((9 + 3) * (4 * 2) + 7)$ (infix expression)
- 5 9 3 + 4 2 * * 7 + * (postfix expression)

Stack operations	Stack elements
push(5)	5
push(9)	5 9
push(3)	5 9 3
push(pop() + pop())	5 12
push(4)	5 12 4
push(2)	5 12 4 2
push(pop() * pop())	5 12 8
push(pop() * pop())	5 96
push(7)	5 96 7
push(pop() + pop())	5 103
push(pop() * pop())	515

Queues

head | 3 | 6 | 8 | tail

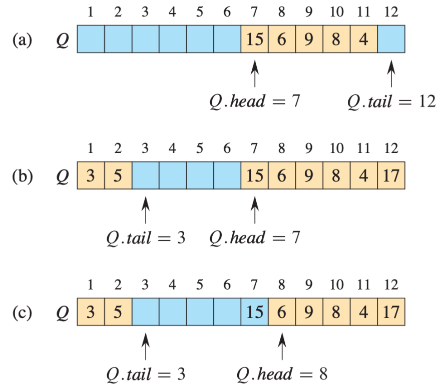
- The British love them ☺
- The first element in a queue is accessible.
  - First-in, first-out policy (FIFO)
- Insert is called **Enqueue**, Delete is called **Dequeue**.
- Queues have a **head** and a **tail**, like in a supermarket
  - Elements are added to the tail
  - Elements are extracted from the head

Queues implemented using arrays

- Queues can be stored in an array “**wrapped around**”.

ENQUEUE(Q, x)
$Q[Q.tail] = x$
if $Q.tail == Q.size$
  $Q.tail = 1$
else $Q.tail = Q.tail + 1$

DEQUEUE(Q)
$x = Q[Q.head]$
if $Q.head == Q.size$
  $Q.head = 1$
else $Q.head = Q.head + 1$
return x



- All queue operations take time $O(1)$.

Queues: Applications

- Playlists (eg., iTunes)
- Dispensing requests on a shared resource (eg., a printer, a server, a processor etc.,)
- Data buffers (eg., streaming services)
- What if I have priorities on the use of the resource?

Priority Queues: Motivation

- Schedule jobs on a computer shared among multiple users
- A max-priority queue keeps track of the jobs to be performed and their relative priorities
- When a job is finished the scheduler selects the job with highest priority from those pending
- Jobs can be added to the scheduler at any time

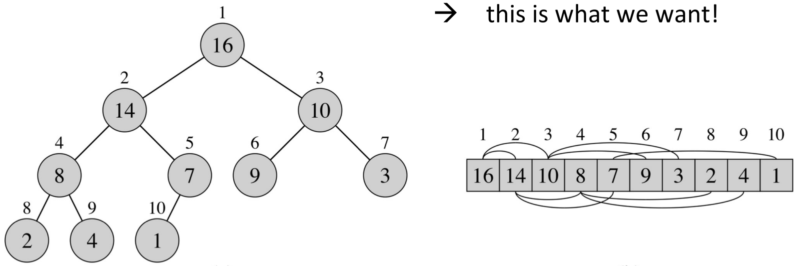
Job	Owner	Priority (key)
Job 1	Tang Ke	35
Job 12	Oliveto Pietro	2
Job 24	Hao Qi	22
Job 25	Yu Shiqi	18
Job 72	Tang Ke	30

- **Use a heap!**

Heap Properties

- **Max-heap property**: for every node other than the root, the parent is no smaller than the node, $A[\mathit{Parent}(i)] \geq A[i]$.

- In a max-heap, the **root** always stores a **largest** element.
  → this is what we want!



(a) (b)

- **Min-heap property**: for every node other than the root, the parent is no larger than the node, $A[\mathit{Parent}(i)] \leq A[i]$.

Priority Queue based on max-heap

- A data structure for maintaining a set S of elements with an associated element called key (the priority).

Operation	Time
Insert(S, x, k) – inserts *x* with key *k* into *S*	$O(\log n)$
Maximum (S) – returns the element in *S* with the largest key	$O(1)$
Extract-Max(S) – removes and returns element in *S* with the largest key	$O(\log n)$
Increase-Key(S, x, k) – increases they key of *x* to a larger value *k* (element may float up in the heap)	$O(\log n)$

Min-priority queue based on min-heap also exist: we will use them in graph algorithms (eg., Djikstra, Prim)

Priority Queues: handles

Operation	Time
Insert(S, x, k) – inserts *x* with key *k* into *S*	$O(\log n)$
Maximum (S) – returns the element in *S* with the largest key	$O(1)$
Extract-Max(S) – removes and returns element in *S* with the largest key	$O(\log n)$
Increase-Key(S, x, k) – increases they key of *x* to a larger value *k* (element may float up in the heap)	$O(\log n)$

- Elements in the priority queue correspond to objects (eg., jobs)
- For an operation such as Increase-Key we need a way to **map** objects to and from their position in the heap (and update it as it moves in the heap)
- One way is to use **handles**: extra information stored in the object that allows to do the mapping
- A Job x: **x.key** (priority) **x.heap_index** (handle) x.satellite_data;
- The heap needs to contain for each element **a pointer to the object**

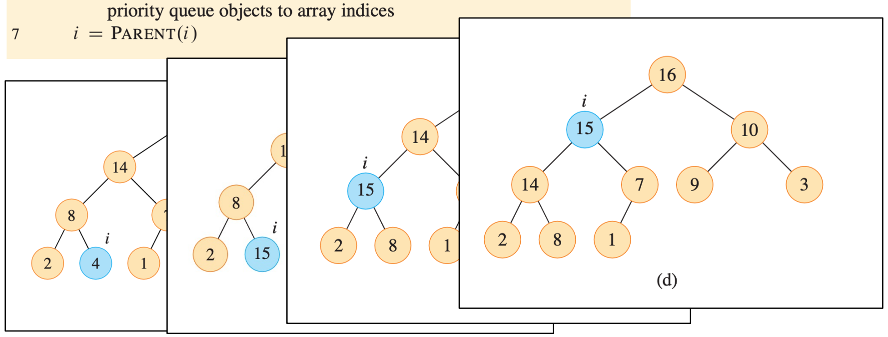
Find and extract next job

```
MAX-HEAP-MAXIMUM(A)
  if A.heap-size < 1
      error "heap underflow"
  return A[1]

MAX-HEAP-EXTRACT-MAX(A)
  max = MAX-HEAP-MAXIMUM(A)
  A[1] = A[A.heap-size]
  A.heap-size = A.heap-size − 1
  MAX-HEAPIFY(A, 1)
  return max
```

Increase job priority

```
MAX-HEAP-INCREASE-KEY(A, x, k)
if k < x.key
    error "new key is smaller than current key"
x.key = k
find the index i in array A where object x occurs
while i > 1 and A[PARENT(i)].key < A[i].key
    exchange A[i] with A[PARENT(i)], updating the information that maps
        priority queue objects to array indices
    i = PARENT(i)
```



Insert new job

```
MAX-HEAP-INSERT(A, x, n)
if A.heap-size == n
    error "heap overflow"
A.heap-size = A.heap-size + 1
k = x.key
x.key = −∞
A[A.heap-size] = x
map x to index heap-size in the array
MAX-HEAP-INCREASE-KEY(A, x, k)
```


Linked Lists: Array Disadvantages

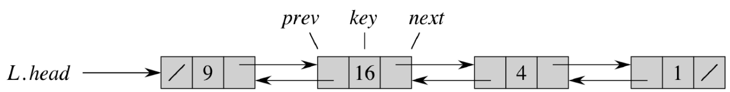
- You need to specify an initial size
- Changing the size of an array is troublesome
- Inserting and deleting elements in specific positions is difficult
- Let's say we want to delete 10 and keep the order of the rest:

A	5	8	10	13	16	19	27	46	51	86
A	5	8		13	16	19	27	46	51	86
A	5	8	13	16	19	27	46	51	86	

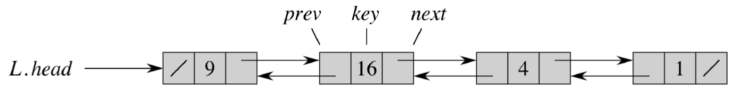
- What's the time complexity?

Linked Lists

- Objects are linked using **pointers to the next element**.
- Linked lists can be **singly linked** or **doubly linked**: pointers to next and previous elements.
- Each element x has attributes
  - x.key – the key used to identify the element
  - x.next – a pointer to the next element
  - x.prev – a pointer to the previous element
  - Optional: further satellite data



Linked Lists: Searching

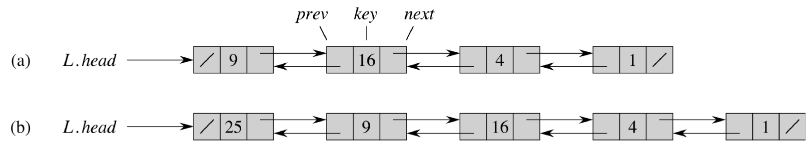


- Search inspects all elements in sequence and stops when the key has been found or the end of the list is reached.

```
LIST-SEARCH(L, k)
x = L.head
while x ≠ NIL and x.key ≠ k do
    x = x.next
return x
```

- The worst-case time is $\Theta(n)$, since it may have to search the entire list.

Linked Lists: Inserting at the front

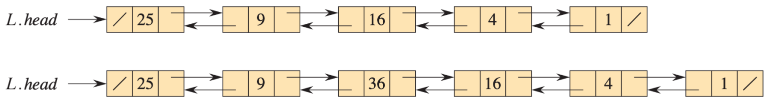


- New elements are added to the front of the list.

```
LIST-PREPEND(L, x)
x.next = L.head
x.prev = NIL
if L.head ≠ NIL
    L.head.prev = x
L.head = x
```

- The time for an insertion is $O(1)$.

Linked Lists: Inserting after element x

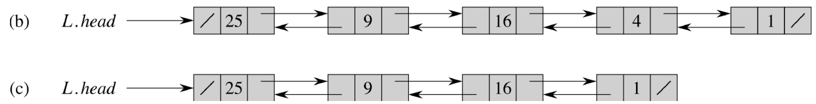


- New element added after element y.

```
LIST-INSERT(x, y)
x.next = y.next
x.prev = y
if y.next ≠ NIL
    y.next.prev = x
y.next = x
```

- The time for an insertion is $O(1)$ if you know the pointer to y

Linked Lists: Deleting



- If element x is known, update pointers to take it out.

```
LIST-DELETE(L, x)
if x.prev ≠ NIL then
    x.prev.next = x.next
else
    L.head = x.next
if x.next ≠ NIL then
    x.next.prev = x.prev
```

- The time for a deletion is $O(1)$.
  But if we only have the key and need to search the element x, it's time $\Theta(n)$ in the worst case.

Summary

- **Stacks** and **Queues** are simple data structures that can
  - be implemented efficiently in arrays (modulo space issues)
  - Have a restricted set of operations, but these run in time $O(1)$.
- **Priority Queues**: all operations in at most $O(\log n)$ time
- Linked lists form an **unordered list** of elements
  - **Insertion** is fast if not important where it occurs: time $O(1)$.
  - **Searching** takes worst-case time $\Theta(n)$.
  - **Deletion** runs in time $O(1)$ if the element is known, otherwise we need to run a search beforehand and incur time $\Theta(n)$.