

## Exercise Sheet 4

Handout: Sep 30 — Deadline: Oct 12, 4pm

**Question 4.1** (0.1 marks) Say whether the following array is a Max-Heap (justify your answer):

34	20	21	16	14	11	3	14	17	13
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**Question 4.2** (0.1 marks)

Consider the following input for HEAPSORT:

12	10	4	2	9	6	5	25	8
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Create a heap from the given array and sort it by executing HEAPSORT. Draw the heap (the tree) after BUILD-MAX-HEAP and after every execution of MAX-HEAPIFY in line 5 of HEAPSORT. You don't need to draw elements extracted from the heap, but you can if you wish.

**Question 4.3** (0.5 marks)

1. Provide the pseudo-code of a MAX-HEAPIFY( $A, i$ ) algorithm that uses a WHILE loop instead of the recursion used by the algorithm shown at lecture.
2. Prove correctness of the algorithm by loop invariant.

**Question 4.4** (1.25 marks)

1. Show that each child of the root of an  $n$ -node heap is the root of a sub-tree of at most  $(2/3)n$  nodes. (*HINT: consider that the maximum number of elements in a subtree happens when the left subtree has the last level full and the right tree has the last level empty. You might want to use the formula seen at lecture:  $\sum_{i=0}^{k-1} 2^i = 2^k - 1$ ).*)
2. As a consequence of (1) we can use the recurrence equation  $T(n) \leq T(2n/3) + \Theta(1)$  to describe the runtime of Max-Heapify( $A, n$ ). Prove the runtime of Max-Heapify using the Master Theorem.

**Question 4.5** (1 mark)

Argue that the runtime of HEAPSORT on an already sorted array of distinct numbers is  $\Omega(n \log n)$ .

**Question 4.6** (0.45 marks)

Implement HEAPSORT( $A, n$ ) and the two problems "Heap" and "Heap Operations" on the Judge system.