Assignment XIV - DSAA(H)

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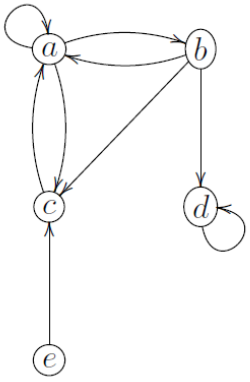
Question 14.1 (0.25 marks)

Perform a depth-first search on the following graph visiting nodes in alphabetical order. Assume that all adjacency lists are sorted alphabetically. Write down the timestamps and the π-value of each node.



Sol:

1. The traversal $a \to b \to c$ (finish c) $\to d$ (finish d) (finish b) (finish a), then e (finish e).

---

2. The timestamps and parents are: $a : (d = 1, f = 8, \pi = \text{NIL})$, $b : (d = 2, f = 7, \pi = a)$, $c : (d = 3, f = 4, \pi = b)$, $d : (d = 5, f = 6, \pi = b)$, $e : (d = 9, f = 10, \pi = \text{NIL})$.

Question 14.2 (0.5 marks)

Prove or refute the following claim: if some depth-first search on a directed graph yields precisely one back edge, then all depth-first searches on this graph yield precisely one back edge.

PF:

The claim is **false**.

Counterexample: Consider the directed graph with $V = \{1, 2, 3\}$ and $E = \{(1, 2), (2, 3), (3, 1), (3, 2)\}$. An edge (u, v) is a back edge iff v is an ancestor of u in the DFS tree.

Run DFS with vertex order $1, 2, 3$. The DFS tree can be $1 \to 2 \to 3$, and from 3 both edges $(3, 1)$ and $(3, 2)$ go to gray ancestors, so there are 2 back edges.

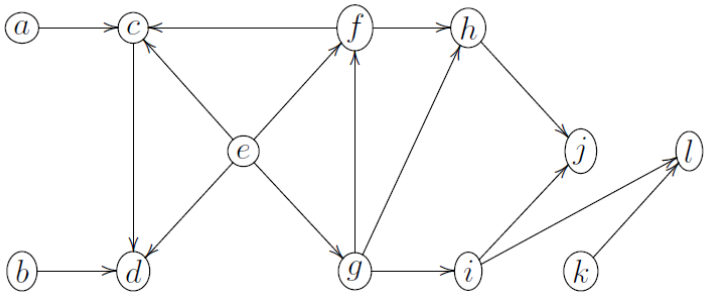
Then run DFS with vertex order $3, 1, 2$ (start at 3). One possible DFS tree is $3 \to 1 \to 2$, so $(2, 3)$ is a back edge, and when $(3, 2)$ is examined, 2 is already black, so it is not a back edge.

Hence one DFS has exactly 1 back edge while another has 2, refuting the claim.

`Q.E.D.`

Question 14.3 (0.25 marks)

Run TOPOLOGICAL-SORT on the following directed acyclic graph. Assume that depth-first search visits nodes in alphabetical order and that adjacency lists are sorted alphabetically.



Sol:

Using Topological-Sort, the finish times are

$f[k] = 24, f[e] = 22, f[g] = 21, f[i] = 20, f[l] = 19, f[f] = 15, f[h] = 14, f[j] = 13, f[b] = 8, f[a] = 6, f[c] = 5, f[d] = 4$

.

Hence the topological ordering is $k,\ e,\ g,\ i,\ l,\ f,\ h,\ j,\ b,\ a,\ c,\ d$.

Question 14.4 (0.5 marks)

Recall from the lecture that DFS can be used to check whether a directed graph $G = (V, E)$ is acyclic or not, and that DFS runs in time $\Theta(|V| + |E|)$.

Give an algorithm that checks whether or not an *undirected* graph $G = (V, E)$ is acyclic and that *runs in time only* $O(|V|)$.

Sol:

Run DFS on the undirected graph while storing $parent[u]$.

In DFS-Visit(u, p), mark u visited and scan each neighbor $v \in Adj[u]$: if v is unvisited, set $parent[v] = u$ and recurse; otherwise, if $v \neq p$, report "cyclic". If all DFS trees finish without triggering $v \neq p$, report "acyclic". The DFS work is $O(|V| + |E|)$, and for an undirected acyclic graph (a forest) we have $|E| \leq |V| - 1$, so the full check runs in $O(|V|)$.

Algorithm 1: Undirected-Acyclic(G)

```
1  foreach u ∈ V do
2  |   visited[u] ← false
3  |   parent[u] ← NIL
4  end
5  foreach u ∈ V do
6  |   if not visited[u] then
7  |   |   if DFS-VISIT(u, NIL) = true then
8  |   |   |   return false
9  |   |   end
10 |   end
11 end
12 return true
```

Algorithm 2: DFS-Visit(u, p)

```
1  visited[u] ← true
2  parent[u] ← p
3  foreach v ∈ Adj[u] do
4  |   if not visited[v] then
5  |   |   if DFS-VISIT(v, u) = true then
6  |   |   |   return true
7  |   |   end
8  |   end
9  |   else if v ≠ p then
10 |   |   return true
11 |   end
12 end
13 return false
```

Question 14.5 (1 mark)

Implement the problems "Drainage System", "Logical Chain" and "2-SAT" on the online Judge system.

Sol:

题目

状态	最后递交于	题目
✓ 100 Accepted	16 小时前	**59** Drainage System
✓ 100 Accepted	15 小时前	**60** Logical Chain
✓ 100 Accepted	17 秒前	**61** 2-SAT

```cpp
struct Fraction{//non-negative
    __int128_t a, b;
    Fraction Shrink(void){
        __int128_t div = __gcd(a, b);
        a /= div, b /= div;
        return *this;
    }
    friend const Fraction operator + (const Fraction &x, const Fraction &y){
        __int128_t below = x.b * y.b / __gcd(x.b, y.b);
        return Fraction{below / x.b * x.a + below / y.b * y.a, below}.Shrink();
    }
    friend const Fraction operator / (const Fraction &x, const int &v){
        return Fraction{x.a, x.b * v}.Shrink();
    }
    friend const Fraction operator / (const Fraction &x, const Fraction &y){
        return Fraction{x.a * y.b, x.b * y.a}.Shrink();
    }
    friend const Fraction operator * (const Fraction &x, const Fraction &y){
        return Fraction{x.a * y.a, x.b * y.b}.Shrink();
    }
    friend const bool operator <= (const Fraction &x, const Fraction &y){
        return x.a * y.b <= y.a * x.b;
    }
    friend const bool operator >= (const Fraction &x, const Fraction &y){
        return x.a * y.b > y.a * x.b;
    }
    friend const bool operator < (const Fraction &x, const Fraction &y){
        return x.a * y.b < y.a * x.b;
    }
    friend const bool operator > (const Fraction &x, const Fraction &y){
        return x.a * y.b > y.a * x.b;
    }
    void Desc(void){
```

```
        this->Shrink();
        printf("%lld/%lld\n", (ll)this->a, (ll)this->b);
    }
};


void print(__int128_t x) {
    if(x > 9)print(x / 10);
    putchar(x % 10 + '0');
}

int main(){
    int N = read(), M = read();
    vector < vector<int> > adj(N + 10);
    vector < int > d(N + 10, 0);
    vector < int > inDeg(N + 10, 0);
    vector < Fraction > water(N + 10, {0, 1});

    for(int i = 1; i <= N; ++i){
        d[i] = read();
        if(d[i] == 0)continue;
        for(int j = 1; j <= d[i]; ++j){
            int to = read();
            adj[i].push_back(to);
            ++inDeg[to];
        }
    }

    queue < int > Q;
    for(int i = 1; i <= M; ++i){
        water[i] = {1, 1};
        if(inDeg[i] == 0)Q.push(i);
    }
    while(!Q.empty()){
        int u = Q.front();
        Q.pop();
        if(d[u] == 0)continue;
        Fraction flowOut = water[u] / d[u];
        for(int v : adj[u]){
            water[v] = water[v] + flowOut;
            --inDeg[v];
            if(inDeg[v] == 0)Q.push(v);
        }
    }
```

```
    for(int i = 1; i <= N; ++i){
        if(d[i] == 0){
            if(water[i].a == 0)printf("0 1\n");
            else {
                print(water[i].a); putchar(' ');
                print(water[i].b); putchar('\n');
            }
        }
    }
    return 0;
}

```

```
int main(){
    int N = read(), M = read();

    vector < bitset < 260 > > adj(N + 10), rev(N + 10);

    for(int i = 0; i < N; ++i){
        char c = getchar();
        while(!isdigit(c))c = getchar();
        if(c - '0')adj[i][0] = 1, rev[0][i] = 1;
        for(int j = 1; j < N; ++j){
            c = getchar();
            if(c - '0')adj[i][j] = 1, rev[j][i] = 1;
        }
    }

    bitset < 260 > vis;
    vector < int > ord;
    int cnt(0);

    auto dfs1 = [&](auto &&self, int p)->void{
        vis[p] = 1;
        while(true){
            int q = (adj[p] & ~vis)._Find_first();
            if(q >= N)break;
            self(self, q);
        }
        ord.push_back(p);
    };

    auto dfs2 = [&](auto &&self, int p)->void{

```

```
        vis[p] = 1;
        ++cnt;
        while(true){
            int q = (rev[p] & ~vis)._Find_first();
            if(q >= N)break;
            self(self, q);
        }
    };

    for(int i = 1; i <= M; ++i){
        int K = read();
        while(K--){
            int p(read() - 1), q(read() - 1);
            adj[p].flip(q), rev[q].flip(p);
        }

        vis.reset();
        ord.clear();
        for(int p = 0; p < N; ++p)
            if(!vis[p])dfs1(dfs1, p);

        vis.reset();
        ll ans(0);
        for(int j = N - 1; j >= 0; --j){
            int p(ord[j]);
            if(!vis[p]){
                cnt = 0;
                dfs2(dfs2, p);
                ans += (1ll * cnt * (cnt - 1)) >> 1;
            }
        }
        printf("%lld\n", ans);
    }

    // fprintf(stderr, "Time: %.6lf\n", (double)clock() / CLOCKS_PER_SEC);
    return 0;
}

```

```
struct Edge{
    Edge* nxt;
    int to;
};
```

```
5
6   int main(){
7       int N(0), M(0);
8
9       bool bol(true);
10      int ch(0);
11      while((ch = getchar()) != EOF){
12          if(bol && ch == 'c'){
13              while(ch != '\n' && ch != EOF)ch = getchar();
14              bol = true;
15              continue;
16          }
17          if(bol && ch == 'p')break;
18          bol = (ch == '\n');
19      }
20
21      N = read(), M = read();
22      int V(N << 1);
23
24      vector < Edge* > head(V + 10, nullptr), rHead(V + 10, nullptr);
25      vector < Edge > ed((M << 1) + 10), red((M << 1) + 10);
26      int ec(0), rc(0);
27
28      auto Id = [&](int lit)->int{
29          int p((abs(lit) - 1) << 1);
30          if(lit < 0)p ^= 1;
31          return p;
32      };
33
34      auto AddEdge = [&](int s, int t)->void{
35          ed[ec] = Edge{head[s], t}, head[s] = &ed[ec++];
36          red[rc] = Edge{rHead[t], s}, rHead[t] = &red[rc++];
37      };
38
39      auto AddClause = [&](int a, int b)->void{
40          int p(Id(a)), q(Id(b));
41          AddEdge(p ^ 1, q), AddEdge(q ^ 1, p);
42      };
43
44      for(int i = 1; i <= M; ++i){
45          int a = read(), b = read();
46          int z = read();
47          while(z != 0)z = read();
48          AddClause(a, b);
```

```
    }

    vector < char > vis(V + 10, 0);
    vector < Edge* > it(V + 10, nullptr);
    vector < int > ord, stk;

    for(int p = 0; p < V; ++p){
        if(vis[p])continue;

        stk.clear();
        stk.push_back(p);
        vis[p] = 1, it[p] = head[p];

        while((int)stk.size()){
            int u(stk.back());
            auto &e = it[u];

            while(e && vis[e->to])e = e->nxt;

            if(!e){
                ord.push_back(u);
                stk.pop_back();
            }else{
                int v(e->to);
                e = e->nxt;
                if(!vis[v])vis[v] = 1, it[v] = head[v], stk.push_back(v);
            }
        }
    }

    for(int p = 0; p < V; ++p)vis[p] = 0;

    vector < int > comp(V + 10, -1);
    int scc(0);

    for(int idx = (int)ord.size() - 1; idx >= 0; --idx){
        int p(ord[idx]);
        if(vis[p])continue;

        stk.clear();
        stk.push_back(p);
        vis[p] = 1, comp[p] = scc;

        while((int)stk.size()){
```

```
            int u(stk.back());
            stk.pop_back();
            for(auto i = rHead[u]; i; i = i->nxt)
                if(!vis[i->to])vis[i->to] = 1, comp[i->to] = scc,
stk.push_back(i->to);
        }

        ++scc;
    }

    for(int i = 0; i < N; ++i){
        int p(i << 1);
        if(comp[p] == comp[p ^ 1]){
            printf("s UNSATISFIABLE\n");
            return 0;
        }
    }

    printf("s SATISFIABLE\n");
    printf("v ");
    for(int i = 0; i < N; ++i){
        int p(i << 1);
        int val = comp[p] > comp[p ^ 1] ? (i + 1) : -(i + 1);
        printf("%d ", val);
    }
    printf("0\n");

    return 0;
}

```