Data-driven subgrid-scale parametrisation for a simple dynamical system

Honours project update

Thomas Schanzer

School of Physics
Climate Change Research Centre and ARC Centre of Excellence for Climate Extremes

University of New South Wales, Sydney, Australia

Thursday 20 July 2023

The complexity of atmospheric models makes parametrisation testing difficult.

I aim to:

1. Use a simpler dynamical system as a test-bed
2. Build data-driven parametrisation schemes from scratch, with/without
   - i Stochasticity
   - ii Memory
3. Evaluate the schemes' performance

Rayleigh-Bénard convection in 2D

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} \pi + \left(\frac{\mathrm{Pr}}{\mathrm{Ra}}\right)^{1/2} \left(1 + \nu^* |\nabla^2 \boldsymbol{u}|\right) \nabla^2 \boldsymbol{u} + \theta \hat{\boldsymbol{z}}$$

$$\frac{\partial \theta}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \theta = (\mathrm{Ra}\,\mathrm{Pr})^{-1/2} \left(1 + \nu^* |\nabla^2 \theta|\right) \nabla^2 \theta$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

- Rayleigh number $\mathrm{Ra} = 10^9 \Rightarrow$ turbulent
- Prandtl number $\mathrm{Pr} = 1$
- Nonlinear hyperdiffusion for model stability at low resolution ($\nu^* = 10^{-3}$)
- No-slip, isothermal top/bottom boundaries; periodic lateral boundaries

Numerical methods

- Equations solved using `dedalus` v3: spectral PDE solver for Python
  - Highly flexible
  - Designed to run in parallel
- Horizontal Fourier series expansion
- Vertical Chebyshev polynomial expansion

Animation

Integration for 100 time units with 1024 horizontal and 128 vertical modes:

`https://youtu.be/SU8ColUYYlA`

Key properties:

- Turbulent flow
- Large and small scale features
- Only two prognostic variables
- Comparatively inexpensive (13 cpu-hr for the above run)

$\Rightarrow$ A simple dynamical system analogue of the atmosphere

I need:

1. A reasonably accurate high-resolution model to treat as "truth"
2. A coarse-resolution model to correct using parametrisation
3. Evidence that under-resolution introduces biases that are:
    i. *Systematic:* consistent over- or under-estimation, not mere numerical error
    ii. *Statistically significant*

Quantities of interest

Nusselt number (dimensionless rate of vertical heat transport):

$$\mathrm{Nu}(z,t) \equiv (\mathrm{Ra}\,\mathrm{Pr})^{1/2}\langle w\theta\rangle_x - \frac{\partial\langle\theta\rangle_x}{\partial z}$$

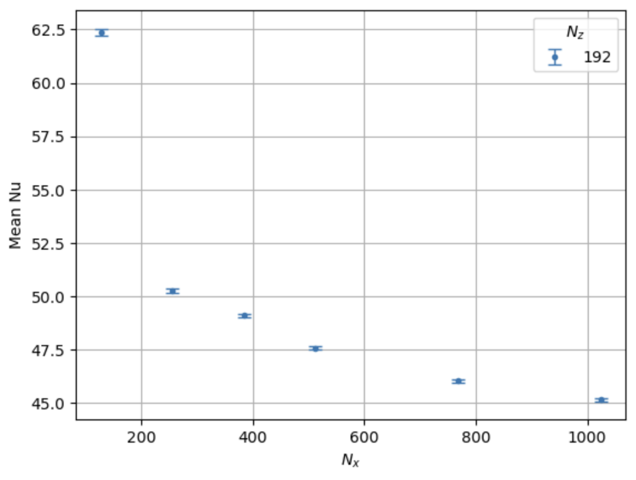
Root-mean-square velocity:

$$u_{\mathrm{rms}}(t) = \sqrt{\langle \boldsymbol{u}\cdot\boldsymbol{u}\rangle_{x,z}}$$

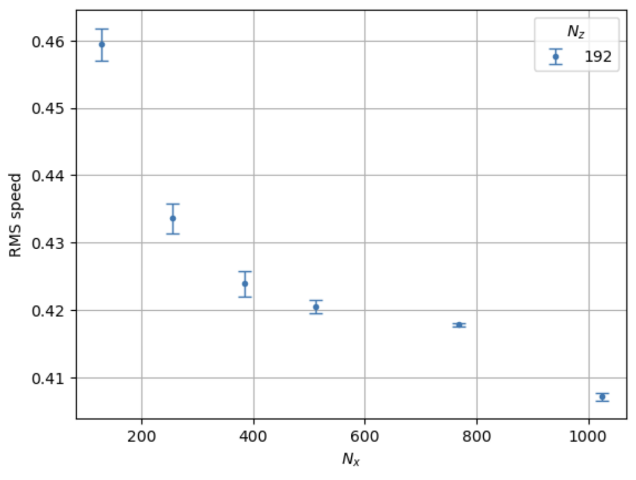
Mean values are obtained via:

1. 1000 time unit spin-up at coarse resolution
2. 300-time-unit integration at new resolution
3. Averaging over final 275 time units

Mean Nusselt number



Mean RMS velocity



Next steps: building the parametrisation

1. Given the state S_t of the fine model at time t,
   i Advance it by Δt using the fine model $\mathcal{F}_{\Delta t}$, giving $\mathcal{F}_{\Delta t} S_t$
   ii Coarse-grain it and advance it by Δt using the coarse model $\mathcal{C}_{\Delta t}$, giving $\mathcal{C}_{\Delta t}\langle S_t \rangle$
2. Determinine the *unresolved tendency*

$$T = \frac{\langle \mathcal{F}_{\Delta t} S_t \rangle - \mathcal{C}_{\Delta t}\langle S_t \rangle}{\Delta t}$$

3. Fit a model to the dataset of $(\langle S_t \rangle, T)$
4. Plug this into the coarse model as an external forcing and assess the results