Measurement of the speed of light

T.D. Schanzer (z5310829)
School of Physics, University of New South Wales, Sydney NSW 2052, Australia
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This report presents measurements of the speed of light and refractive index in air, acrylic glass and water using a simple teaching apparatus. The quality of the method and its limitations are assessed.

INTRODUCTION

Measurements of the speed of light have a long history, with discussion of whether or not it was finite traced back to the ancient Greeks. Rømer (1676) is usually credited with making the first accurate measurements for his observation of variations in Io's eclipse durations behind Jupiter as the Earth approached and receded in its annual orbit [1].

More sophisticated experiments later increased the precision with which the speed of light was known, to the point that it could be given an exact value in SI units in a 1983 resolution of the seventeenth General Conference on Weights and Measures. Presently, the metre "is defined by taking the fixed numerical value of the speed of light in vacuum, c, to be 299 792 458 when expressed in the unit $\mathrm{m\,s^{-1}}$" [2].

Refractometry, the measurement of refractive indices, has many applications in modern research, industry and medicine. These include concentration measurements of dissolved material in liquids (e.g., sugar in drinks), characterisation of glasses and plastics (e.g., for use in lenses) and analysis of biofluids (e.g., blood and urine) [3].

Modern Methods

Modern methods for measuring the refractive indices of other materials are numerous. Meeten (2014) [3] describes several, including:

Interferometry. An interferometer separates a laser into two beams, one of which travels through air and the other through the medium of interest, before recombining them to produce a pattern of interference fringes from which the phase difference and refractive index can be determined.

Deviation. A beam of light is passed at an angle through a flat sheet of the material with known thickness. The beam that emerges on the other side of the sheet is parallel to the incident beam but offset laterally due to refraction. The magnitude of the deviation can be related to the refractive index, thickness and angle of incidence using Snell's Law.

Critical angle (Abbe method). The incidence angle at which light, passing from the medium of interest into a medium of known lower refractive index, is refracted at 90° is found by measuring the reflectance or transmittance at the interface as a function of incidence angle. The refractive index is easily calculated using Snell's Law. This method is commonly used in commercial refractometers.

The Refractive Index of Air

Many experiments, including the one detailed here, are not conducted in vacuum and must therefore account for the refractive index of air, which is known to depend on wavelength, temperature, pressure and relative humidity. A commonly-used empirical formula was developed by Edlén in 1966 and later revised by Birch and Downs in 1994 to better match experimental results [4]. The revised formula calculates the refractive index n three steps: the first defines an intermediate quantity

$$\begin{aligned} n_\mathrm{s} = 1 + 10^{-8} \\ \times \Bigg(8342.54 + 2406147\left[130 - \left(\sigma/\mathrm{\mu m^{-1}}\right)^2\right]^{-1} \\ +15998\left[38.9 - \left(\sigma/\mathrm{\mu m^{-1}}\right)^2\right]^{-1}\Bigg) \end{aligned} \tag{1}$$

to account for dispersion (wavelength dependence); $\sigma = 1/\lambda$ is the wavenumber of the light in vacuum. The second step defines another intermediate quantity

$$\begin{aligned} n_\mathrm{tp} = 1 + \frac{(p/\mathrm{Pa})n_\mathrm{s}}{96095.43} \\ \times \frac{\left[1 + 10^{-8}(0.601 - 0.00972T/°\mathrm{C})p/\mathrm{Pa}\right]}{1 + 0.003661T/°\mathrm{C}} \end{aligned} \tag{2}$$

accounting for the air pressure p and temperature T. The final value is

$$\begin{aligned} n_\mathrm{a} = n_\mathrm{tp} - (f/\mathrm{Pa}) \\ \times \left[3.7345 - 0.0401\left(\sigma/\mathrm{\mu m^{-1}}\right)^2\right] \times 10^{-10}, \end{aligned} \tag{3}$$

where f is the partial pressure of water vapour.

The conditions measured in this experiment (using an inexpensive electronic weather station) were $T = (20.4 \pm 1.0)\,°\mathrm{C}$, $p = (1028 \pm 2)\,\mathrm{hPa}$ and relative humidity $\mathrm{RH} = (47 \pm 10)\,\%$ with an estimated wavelength $\lambda = 630\,\mathrm{nm}$, which when substituted into (1), (2) and (3) yield

$$n_\mathrm{a} = 1.000275. \tag{4}$$

By comparison to experimental results, Birch and Downs estimated the 1σ uncertainty of the formula to be 10^{-8} [4], but given the uncertainty in T, p, RH and λ this is more likely to be on the order of 10^{-6} [5], which is nonetheless negligible in comparison to the uncertainties in the results presented in this report.

AIM

This experiment seeks to determine the speed of light and refractive index in air, acrylic glass and water.

METHODS

The experiment used a combined laser source and detector unit (PHYWE 11226-00), containing a red laser source whose intensity was modulated at 50 MHz. The laser passed along an optical bench to a reflector mounted at a variable position x, was reflected back to the unit and directed onto a photodiode by a retro-reflector, as shown in Figure 1. The modulation of the laser intensity created pulses in both the source signal and the intensity measured by the photodiode, allowing the time difference between them to be measured on a digital oscilloscope (Tektronix TBS-1072B) connected to the source/detector unit. An illustration of this time measurement is shown in the top panel of Figure 1.

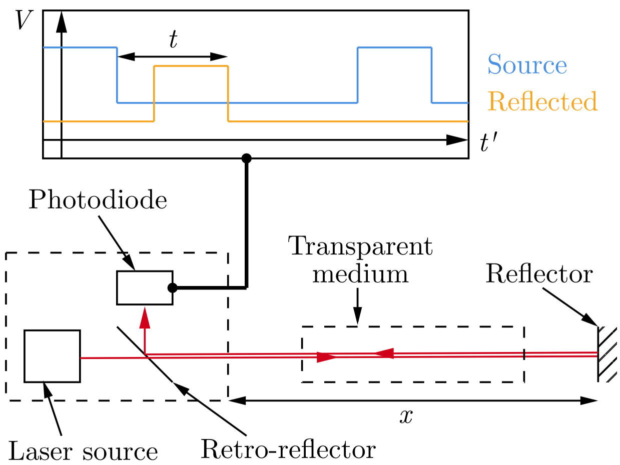

FIG. 1. Experimental setup, showing model oscilloscope trace (top).

The speed of light in air, c_a, was calculated by measuring the time difference t between the emitted and reflected beams as a function of the reflector's position x relative to a reference point and performing a linear regression using the simple relation

$$2x = c_a t. \tag{5}$$

Two different approaches were compared for measuring the speed of light in transparent media, namely acrylic and water.

Δx method. A cylinder of the transparent medium was placed in the path of the laser as shown in Figure 1 and the source/detector unit calibrated to show a zero time difference between the emitted and reflected beams. The cylinder was then removed, shortening the time taken by the light, and the reflector moved horizontally until the oscilloscope again showed a zero time difference, indicating that the initial and final optical path lengths were equal. Mathematically,

$$2n_a x' = 2n_a(x - l) + 2n_m l,$$

where x and x' are the initial and final reflector positions, l is the length of the cylinder and n_m is the refractive index of the medium. Consequently,

$$n_m = \frac{n_a(x' - x + l)}{l}.$$

Δt method. As before, the medium was placed in the optical path and the source/detector unit calibrated to show a zero time difference. The medium was removed and the resulting reduction in travel time Δt measured. Knowing that

$$\Delta t = 2l\left(\frac{1}{c_m} - \frac{1}{c_a}\right),$$

where c_m and c_a are the speeds of light in the medium and air respectively, it follows that

$$n_m = \frac{c}{c_m} = \frac{c\Delta t}{2l} + \frac{c}{c_a}.$$

RESULTS

The linear regression of distance travelled against time according to (5), shown in Figure 2, gives a speed of light in air of

$$c_a = (2.941 \pm 0.016) \times 10^8 \,\mathrm{m\,s^{-1}},$$

with the uncertainty (0.53%) accounting for uncertainties in $2x$ of 5 mm and in t of 80 ps. This value deviates from the accepted speed of light in air by 3.6σ or 1.9%, for reasons proposed in the Discussion.

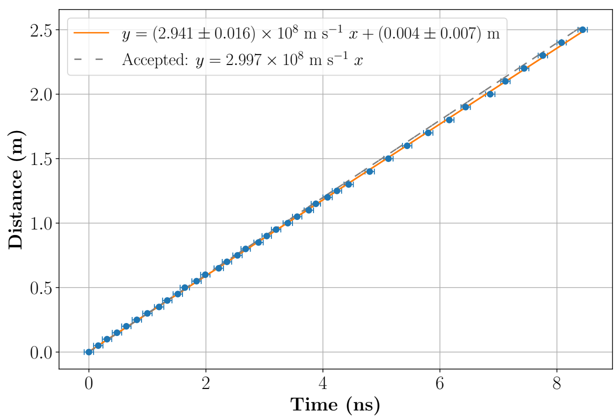

FIG. 2. Plot of $2x$ vs. t in air, with gradient c_a.

Figure 3 displays the results of the Δx (conducted on two occasions) and Δt methods in acrylic. The results of an alternative method of analysis for the Δx method are included for comparison, using the fact that

$$x' = x + l\left(\frac{n_m}{n_a} - 1\right)$$

to obtain n_m from the y-intercept of the linear regression line of x' and x. None of the methods agree with the accepted range of refractive indices, $1.48 - 1.52$, provided by the manufacturer [6].

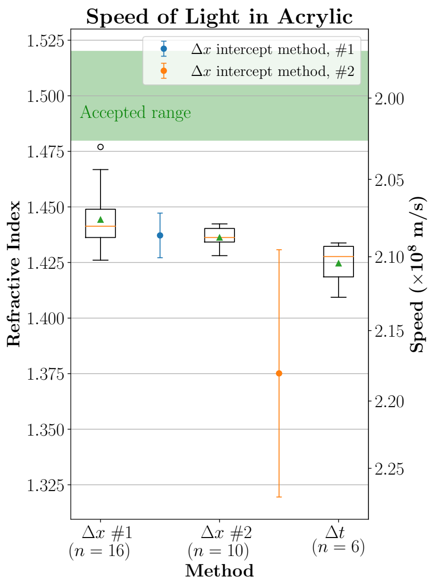

FIG. 3. Measured values of the speed of light in acrylic using the Δx and Δt methods. Green triangles are the mean values.

The same methods were applied for water, with the results shown in Figure 4. Only the intercept method for the second Δx measurement seems to be in agreement with the accepted refractive index of 1.33, but has a much larger uncertainty and deviates significantly from the other results.

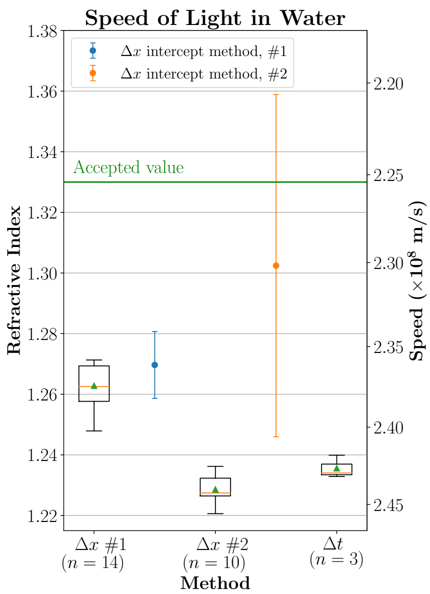

FIG. 4. Measured values of the speed of light in water using the Δx and Δt methods. Green triangles are the mean values.

Material	n	c ($\times 10^8\,\mathrm{m\,s^{-1}}$)
Air	1.019 ± 0.005	2.941 ± 0.016
Acrylic (Δx #2)	1.436 ± 0.004	2.087 ± 0.006
Acrylic (Δt)	1.425 ± 0.009	2.104 ± 0.014
Water (Δx #2)	1.229 ± 0.004	2.440 ± 0.009
Water (Δt)	1.236 ± 0.003	2.426 ± 0.006

TABLE I. Summary of results.

DISCUSSION

It is proposed that the disagreement between the measured and accepted values arose from the following factors that were very difficult to control:

Misalignment. There was a significant amount of play in the sliding mechanism of the reflector, making it necessary to support it with strips of paper to render the reflected beam horizontal. Furthermore, it was very difficult to ensure that the acrylic and water cylinders were exactly parallel to the beam with the equipment available. Both factors made it difficult to obtain an accurate measurement of the path length.

Imperfect cylinder shape. It is likely that the ends of the acrylic and water cylinders were not exactly parallel to each other, causing refraction and increasing the actual path length even with good alignment. The fact that the observed time difference between the emitted and reflected beams varied as the cylinders were rotated supports this hypothesis.

CONCLUSION

The speed of light and refractive index in air, acrylic glass and water were measured, but none were in agreement with accepted values, likely due to inevitable misalignment of the very simple apparatus and imperfection in the manufacturing of the acrylic and water cylinders.

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