

**Algorithm 2** URef-cVote**Function**  $\bar{\mathbf{U}} = \text{URef-cVote}(\mathcal{U})$ 

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- 1: Randomly select a partition from  $\mathcal{U}$  as a reference  $\mathbf{U}^0$
  - 2:  $\dot{\mathbf{U}} = 0, \bar{\mathbf{U}} = 0$  {Initialize two  $k_0 \times n$  matrices  $\dot{\mathbf{U}}$  and  $\bar{\mathbf{U}}$ }.
  - 3: **for**  $i = 1$  to  $b$  **do**
  - 4:    $\mathbf{W}^i = \mathbf{U}^{iT} \mathbf{U}^0$  .
  - 5:    $\dot{\mathbf{V}}^i = \mathbf{U}^i \mathbf{W}^i$
  - 6: **end for**
  - 7:  $N = \sum_{i=1}^b \sum_{j=1}^n \sum_{q=1}^{k_0} \dot{v}_{jq}^i$
  - 8:  $\dot{\mathbf{U}} = \frac{1}{N} \sum_{i=1}^b \dot{\mathbf{V}}^i$  { $\dot{\mathbf{U}}$  represents the empirical joint distribution  $p(c^0, x)$ }
  - 9: **for**  $j = 1$  to  $n$  **do**
  - 10:    $P_{x_j} = \sum_{q=1}^{k_0} \dot{u}_{qj}$  .
  - 11:    $\bar{\mathbf{u}}_j = \dot{\mathbf{u}}_j / P_{x_j}$  {each row vector  $\bar{\mathbf{u}}_j$  of  $\bar{\mathbf{U}}$  represents the distribution  $p(c^0|x_j)$ }
  - 12: **end for**
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the thesis is focused on the normalized scheme. In [39], empirical results for the un-normalized cumulative voting scheme are presented.

### 4.1.3 Preserving Class Distribution

Consider the random variables  $\{C^i\}_{i=1}^b$ , defined over the cluster labels of each ensemble partition  $\{\mathbf{U}^i\}_{i=1}^b$ , with probability distribution  $p(c_l^i) = n_l^i/n$  (assuming hard ensemble partitions). The optimally relabeled partition  $\mathbf{V}^i$  computed with respect to a given reference partition  $\mathbf{U}^0$ , using the cumulative voting scheme described in Ch. 3, is a soft partition that is viewed as representing a conditional probability distribution  $p^i(c^0|x)$ , where the random variable  $C^0$  is defined over the initial reference clusters  $\{c_q^0\}_{q=1}^{k_0}$ , and  $X$  is defined over the objects  $x \in \mathcal{X}$ . Suppose that the simplifying assumption that the marginal probabilities  $p(x_i) = \frac{1}{n}$ ,  $\forall j$  is made. The joint distribution  $p^i(c^0, x)$ , based on  $p^i(c^0|x)$ , can be computed using Bayes rule. Let  $p(c^0)$  represent