Berner Fachhochschule
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Introduction to Image Processing:

Image Statistics

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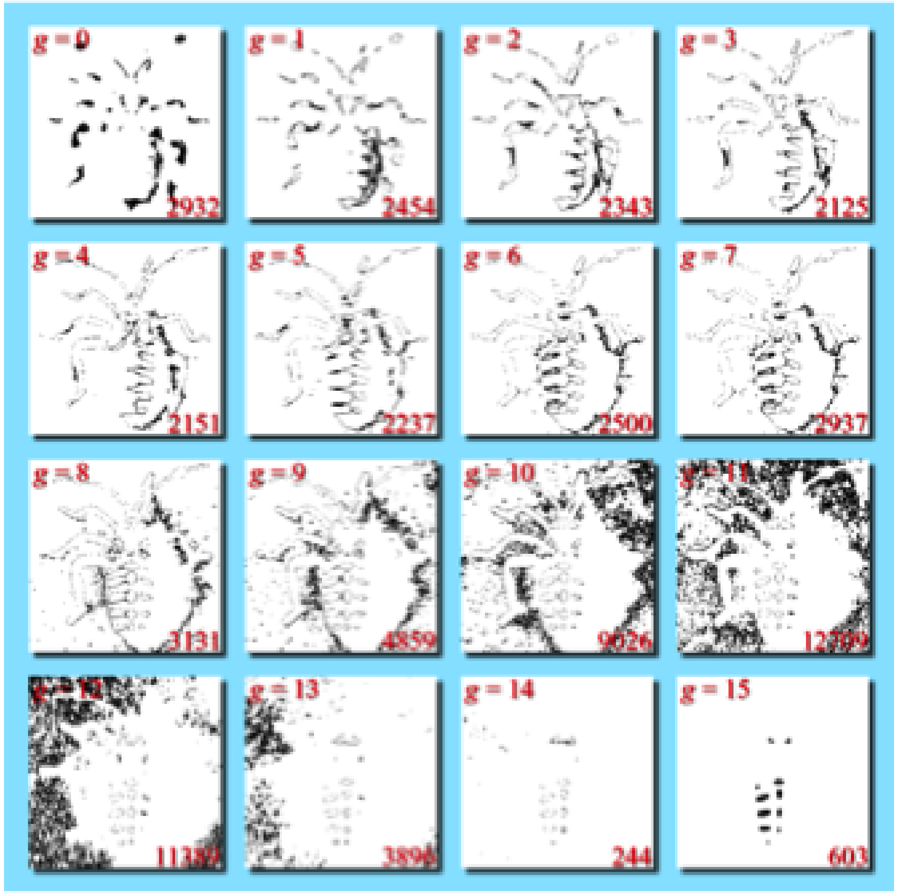
Image Statistics: Histogram

- Image Acquisition is a measuring process.
- It can be therefore **statistically analyzed**.
- The most common statistic is the NO. of pixels (n_g) in each gray level:

$$h(g)=n_g$$



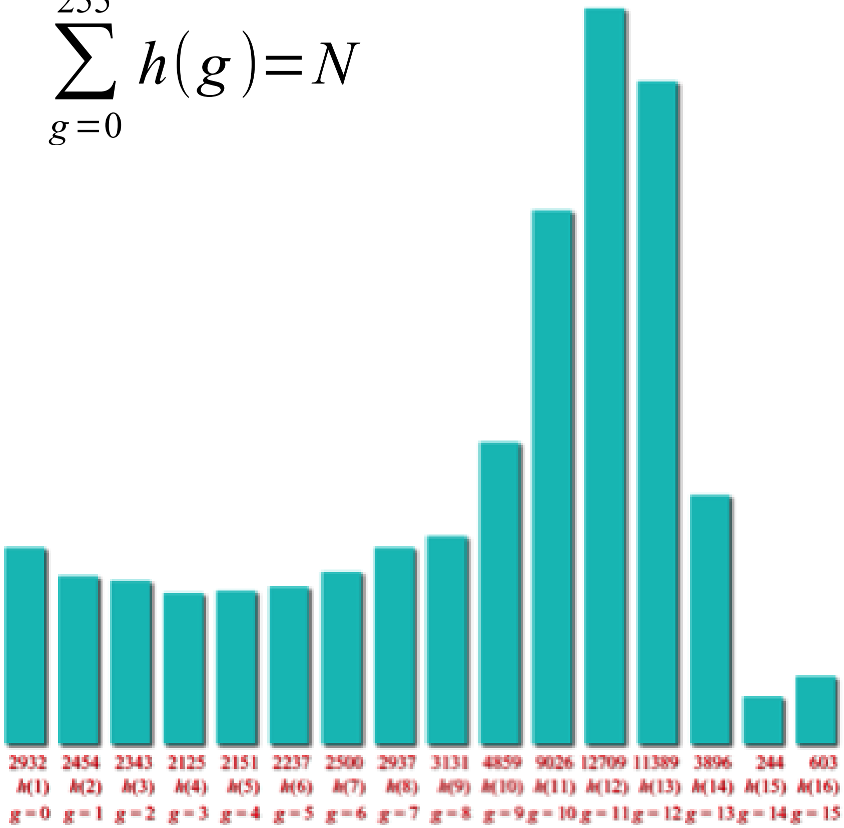
Images by R.A. Peters, Vanderbilt University

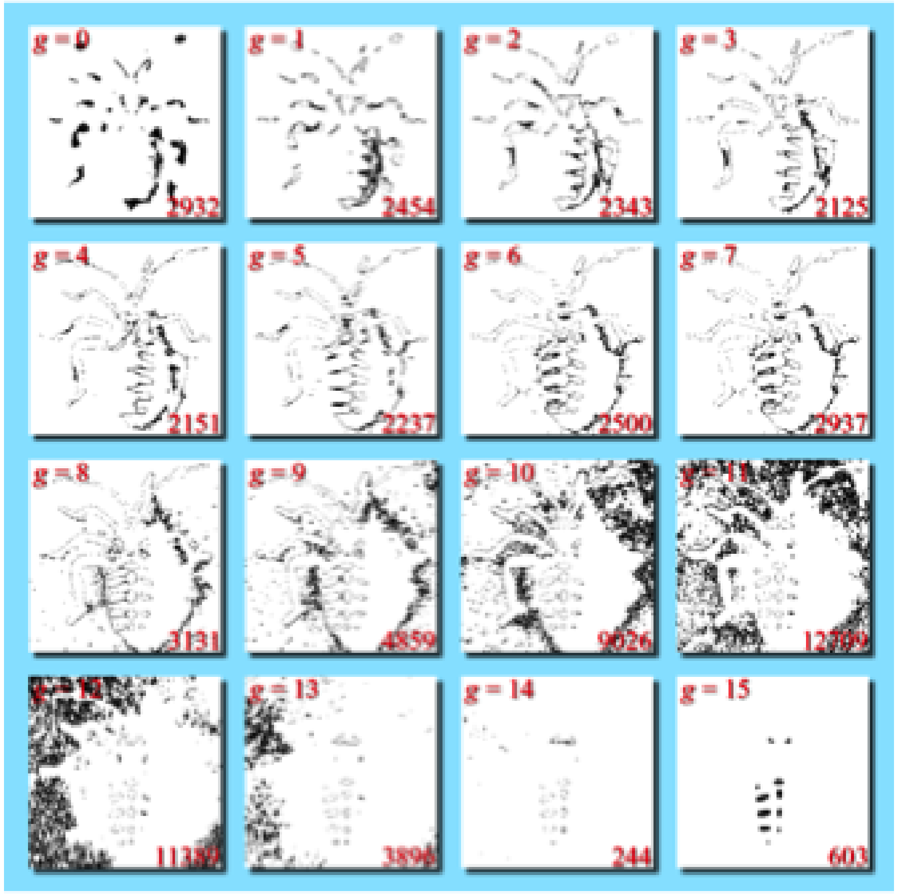
Image Statistics: Histogram

- The **bar chart** of 16 gray level count is called **histogram.**
- The sum of all levels is equal to the sum of all pixels:

$$h(g) = n_g$$

$$\sum_{g=0}^{255} h(g) = N$$





Images by R.A. Peters, Vanderbilt University

Image Statistics: Histogram

- The visualize small occurencies the histogram can be **logarithmic scaled**:

Logarithmic scale

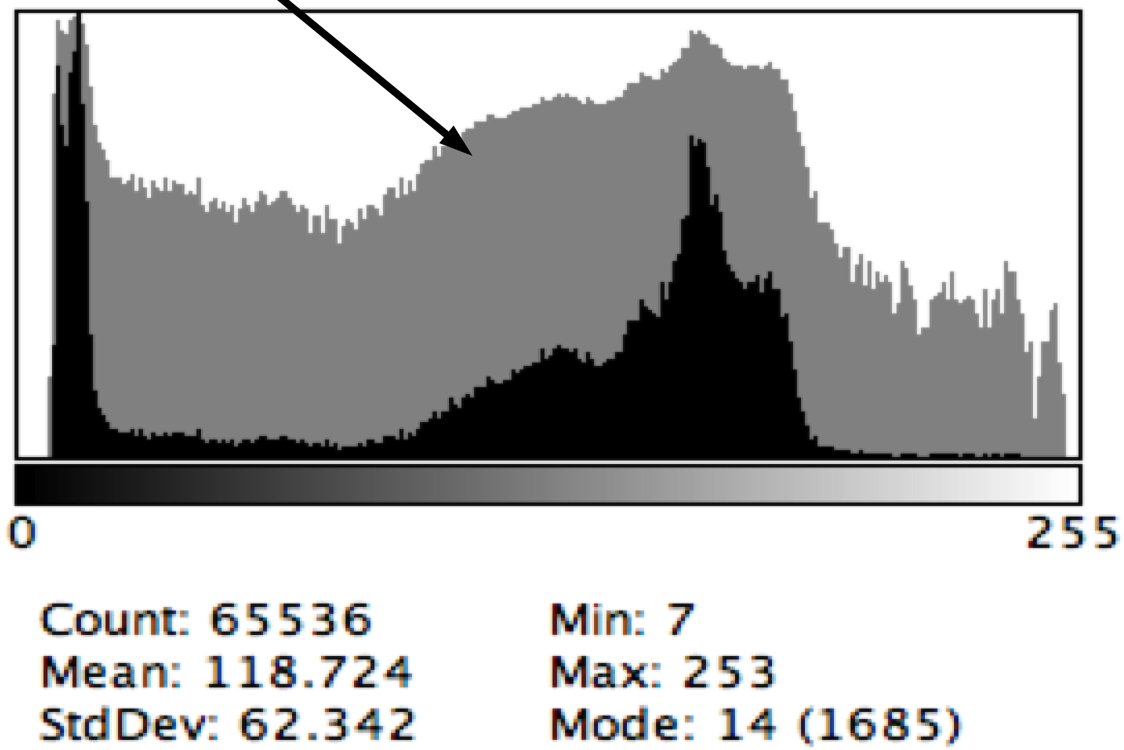


Image Statistics: Color Histogram

Images by R.A. Peters, Vanderbilt University

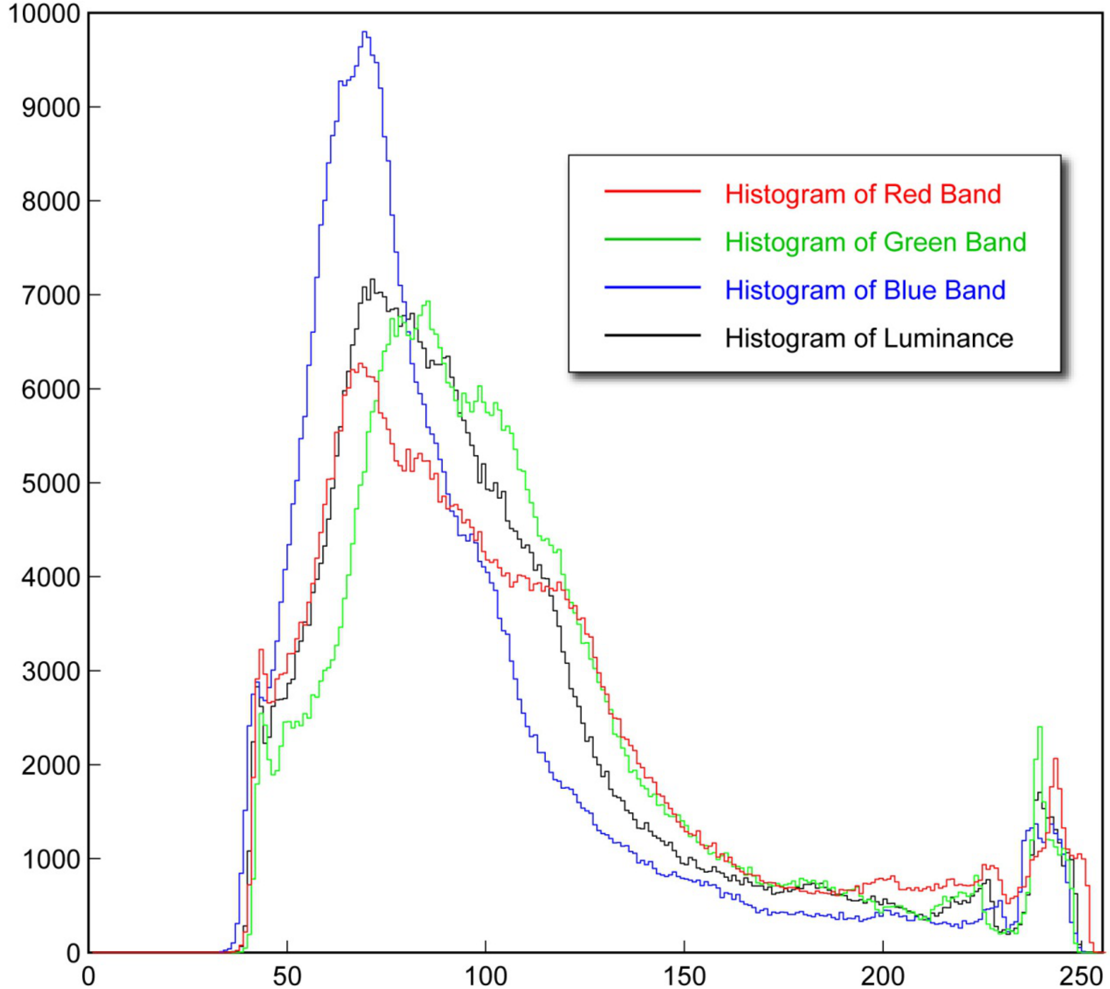
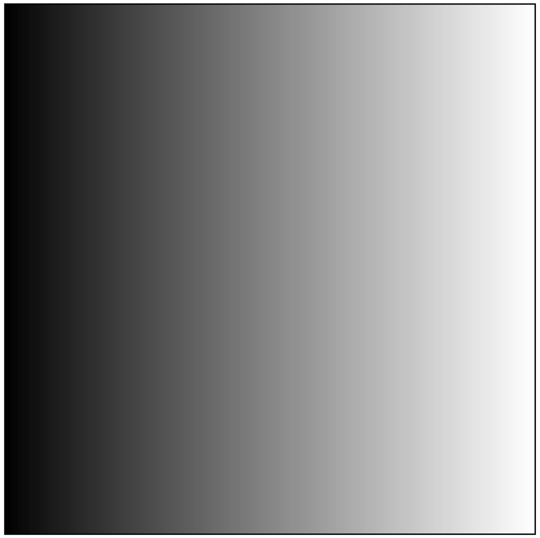
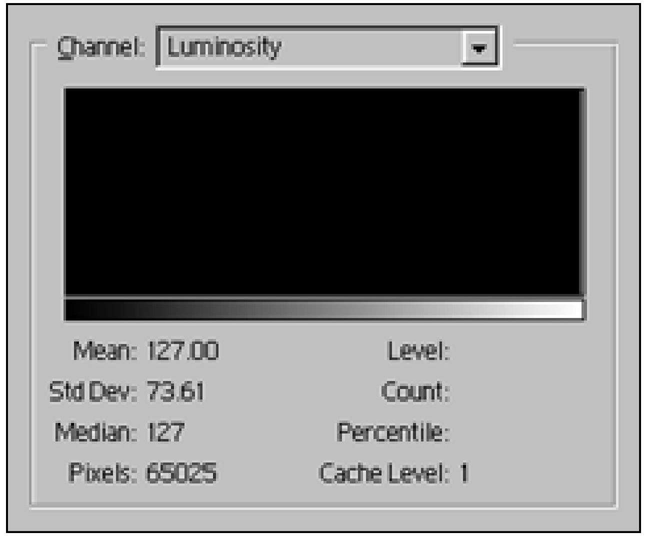
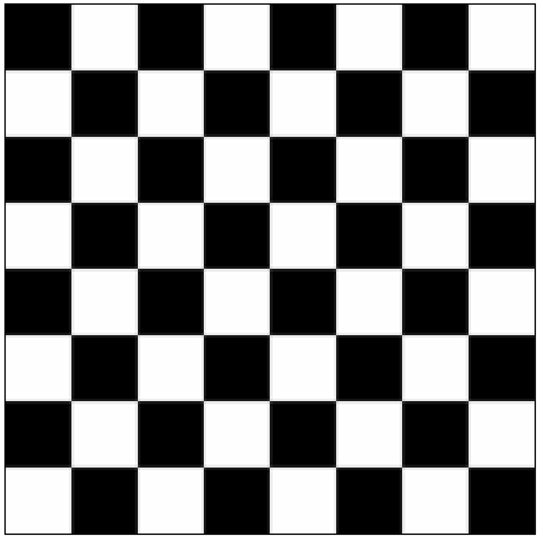


Image Statistics: Histogram

How does the histogram look like from the followin images:







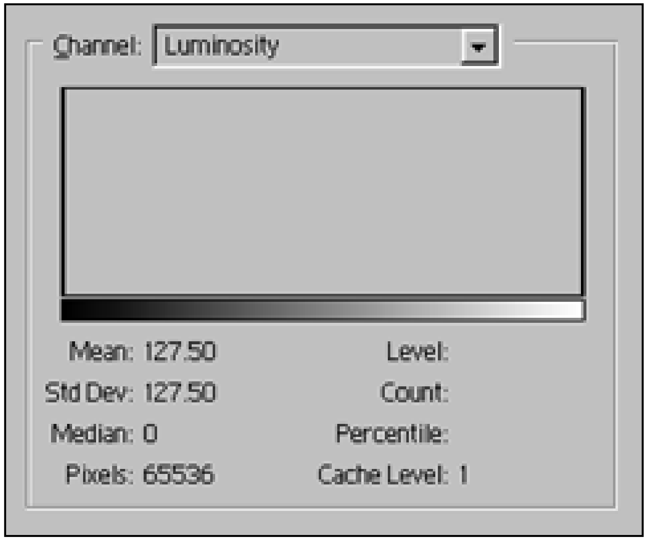


Image Statistics: Probability Density Function

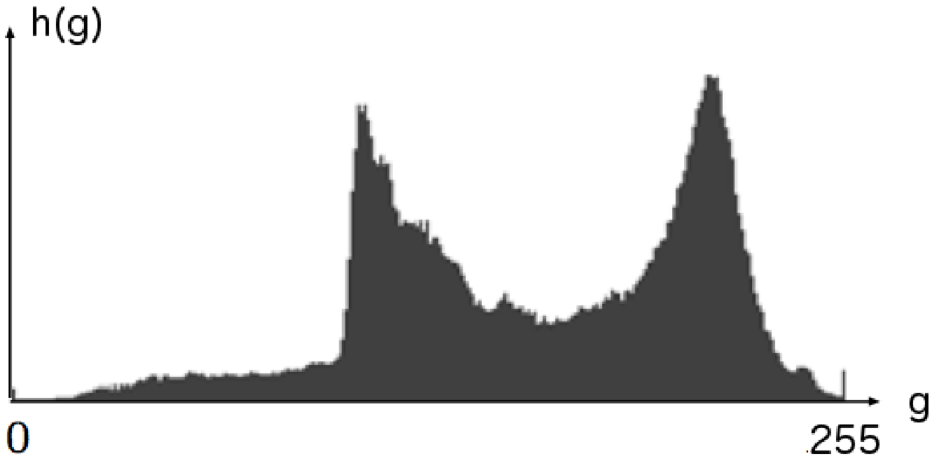
- If the histogram is normalized we get the **Probability Density Function (PDF)**.
- To normalize it we divide all frequencies h(g) by the sum of all pixels.
- p(g) sais how probable a pixel has a gray value = g.
- The sum of all probabilities is 1 (100%).

$$p(g)=\frac{n_g}{N}$$

$$\sum_{g=0}^{255} p(g)=1$$

Image Statistics: Cumulative Histogram

- The **cumulative histogram** sums up all level frequencies.
- It is used in algorithms like the histogram equalization.



$$H(g) = \sum_{j=0}^{g} h(j)$$

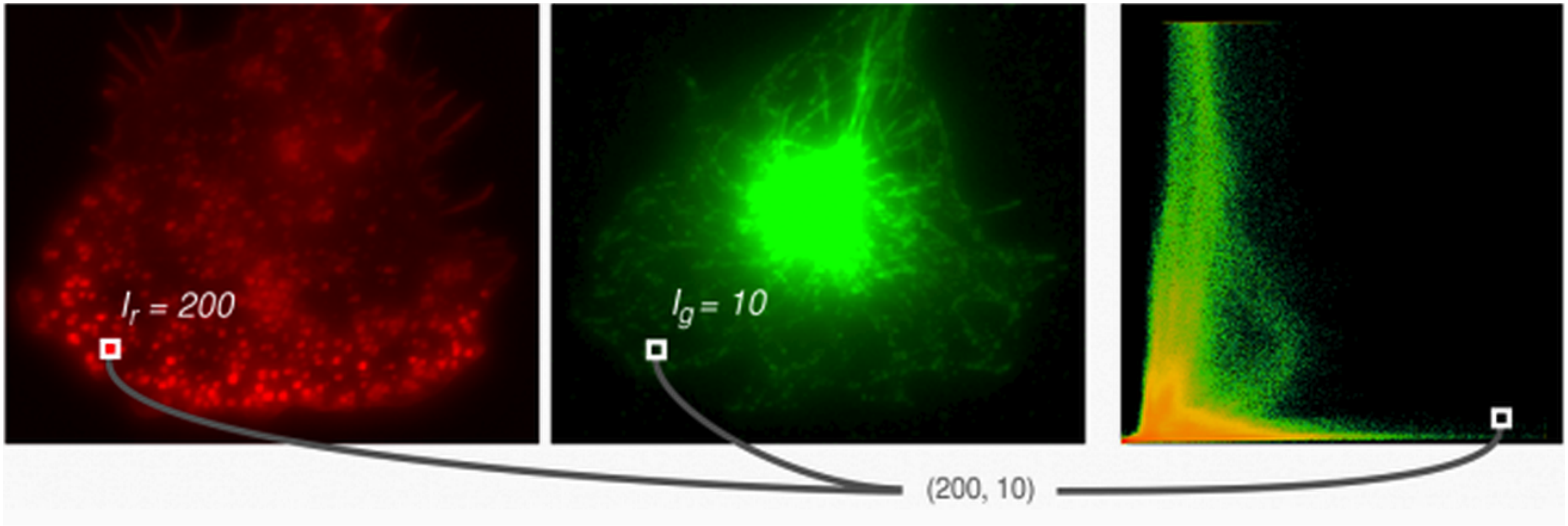
Image Statistics: Cumulative Density Function

- If the cumulativ histogram is normalized we get the **Cumulative Probability Density Function (cdf)**.
- To normalize it we divide all cumulative sums H(G) by the sum of all pixels.
- P(g) sais how probable a pixel has a gray value <= g.

$$P(g)=\frac{H(g)}{N}$$

Image Statistics: Multidimensional Histogram

- A histogram can be built from multiple images.
- E.g. from 2 color channels of the same image:
- The gray value in each pixel is used as the coordiante of the counter in 256 x 256 histogram image:



- The frequencies of occurence are scaled and displayed with color palettes:

$$h(g_0, h_1, \ldots h_{N-1}) = \frac{a_{g_0 g_1 \ldots g_{N-1}}}{N} \qquad s(x, y) = s(g_0, g_1) = C \cdot h(g_0, h_1)$$

Image Statistics: Multidimensional Histogram

- A **2D-Histogram** can show:
  - **Over-exposure** of an image:

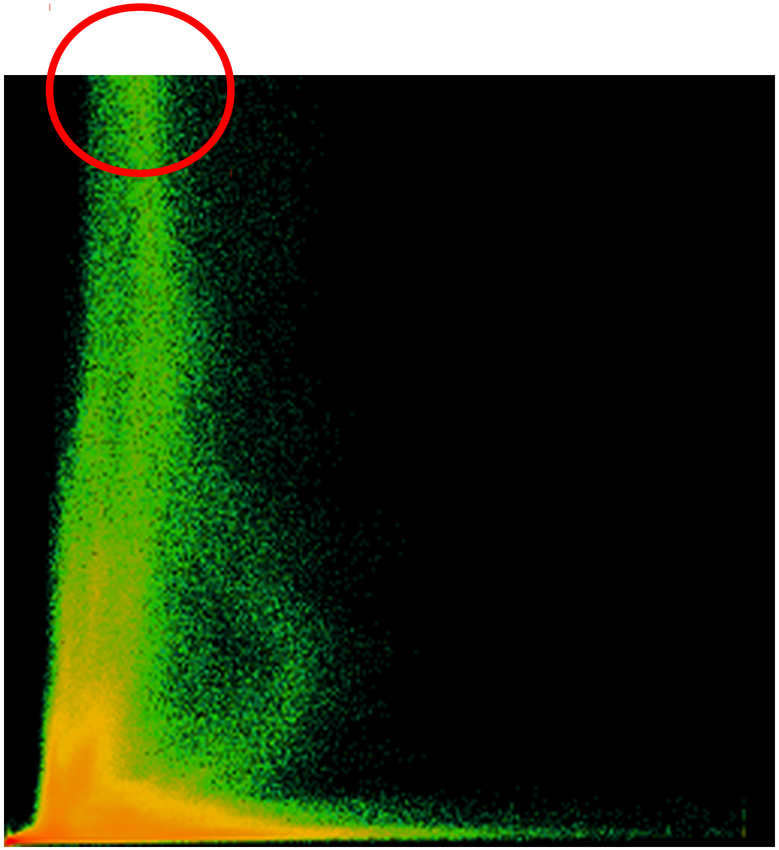


Image Statistics: Multidimensional Histogram

- A **2D-Histogram** can show:
  - Over-exposure of an image
  - **Correlation** between 2 images show in concentration along the diagonal:

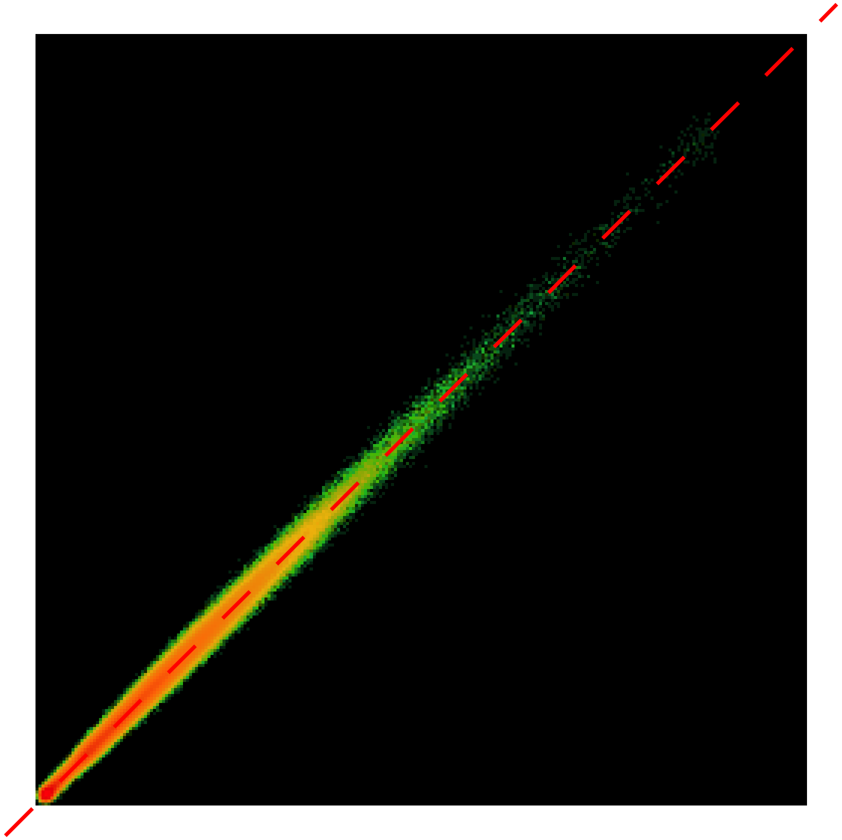


Image Statistics: Multidimensional Histogram

- A **2D-Histogram** can show:
  - Over-exposure of an image
  - Correlation between 2 images
  - **Distribution of Dynamic** in one color channel:

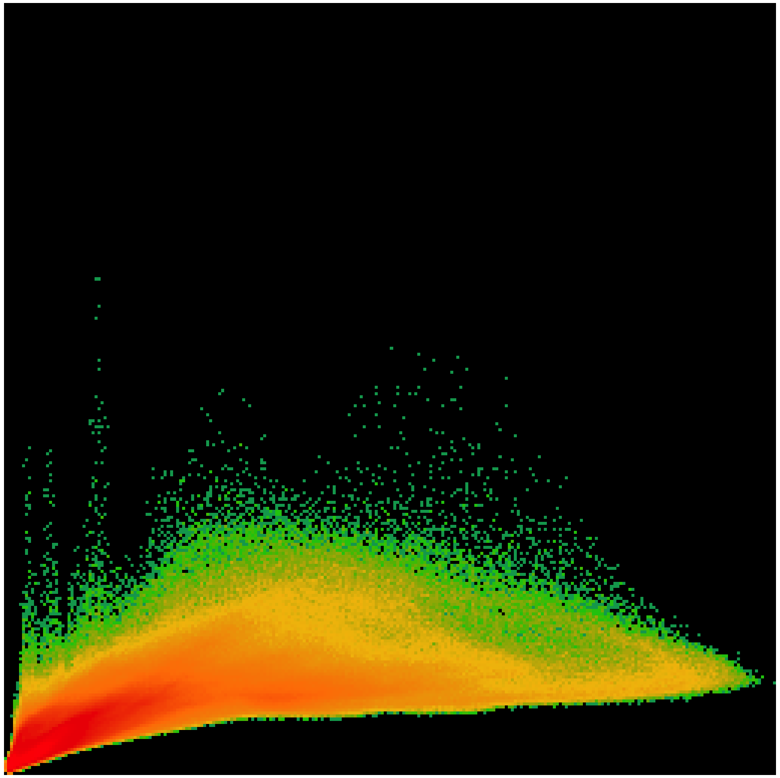


Image Statistics: Multidimensional Histogram

- A **2D-Histogram** can show:
  - Over-exposure of an image
  - Correlation between 2 images
  - Distribution of Dynamic
  - **Hot pixels** are single defect pixels that saturate without light:

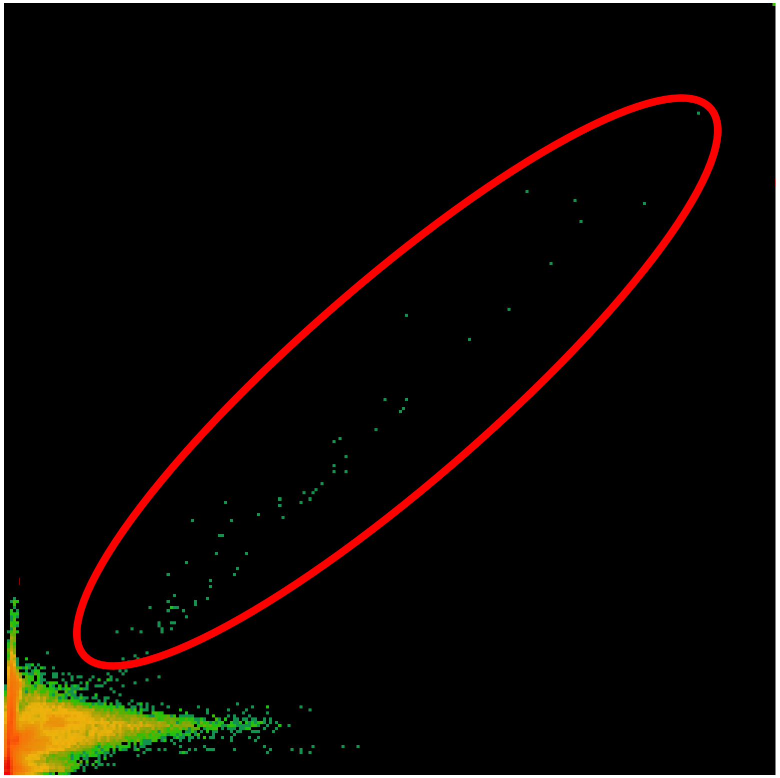


Image Statistics: Multidimensional Histogram

- Fiji can display also 3D histograms in the color inspector:

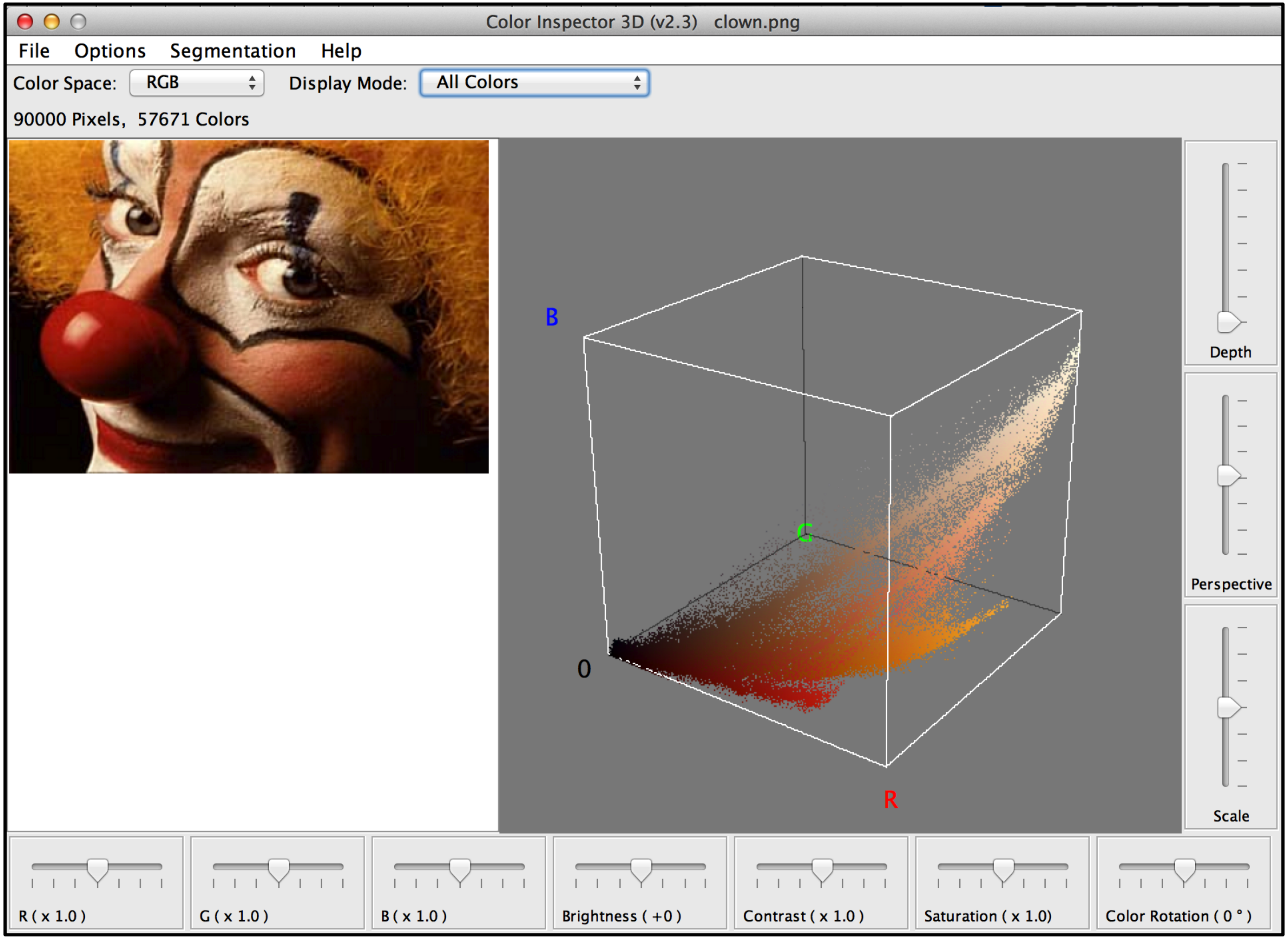


Image Statistics Values:

- **Min. & max.:** $$D = g_{max} - g_{min}$$
- **Contrast:** $$C = \frac{g_{max} - g_{min}}{g_{max} + g_{min}}$$
- **Mean (Mittelwert):** $$\bar{g} = \frac{1}{N} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} g(x, y)$$
- **Mode (Modalwert):** $$\mathring{g} = max(h(g))$$

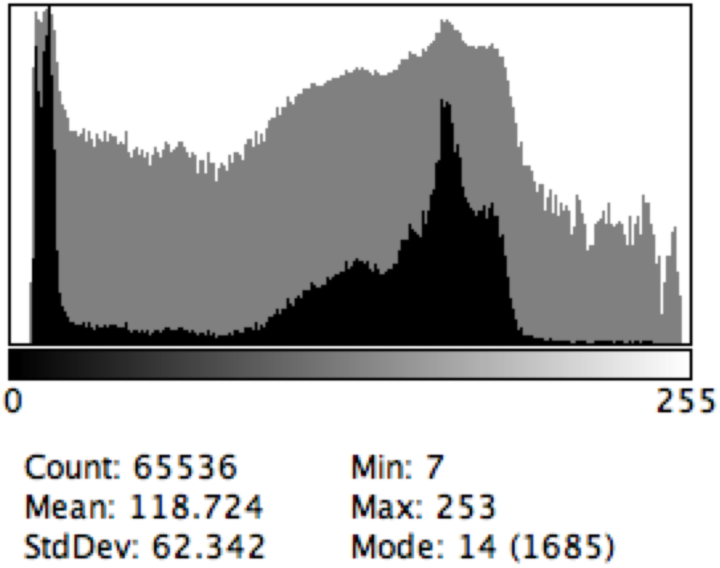


Image Statistics Values:

- **Variance (Varianz):** $$var = \frac{1}{N} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} \left(g(x,y) - \bar{g} \right)^2$$

- **Stadard Deviation:** $$s = \sigma = \sqrt{var}$$

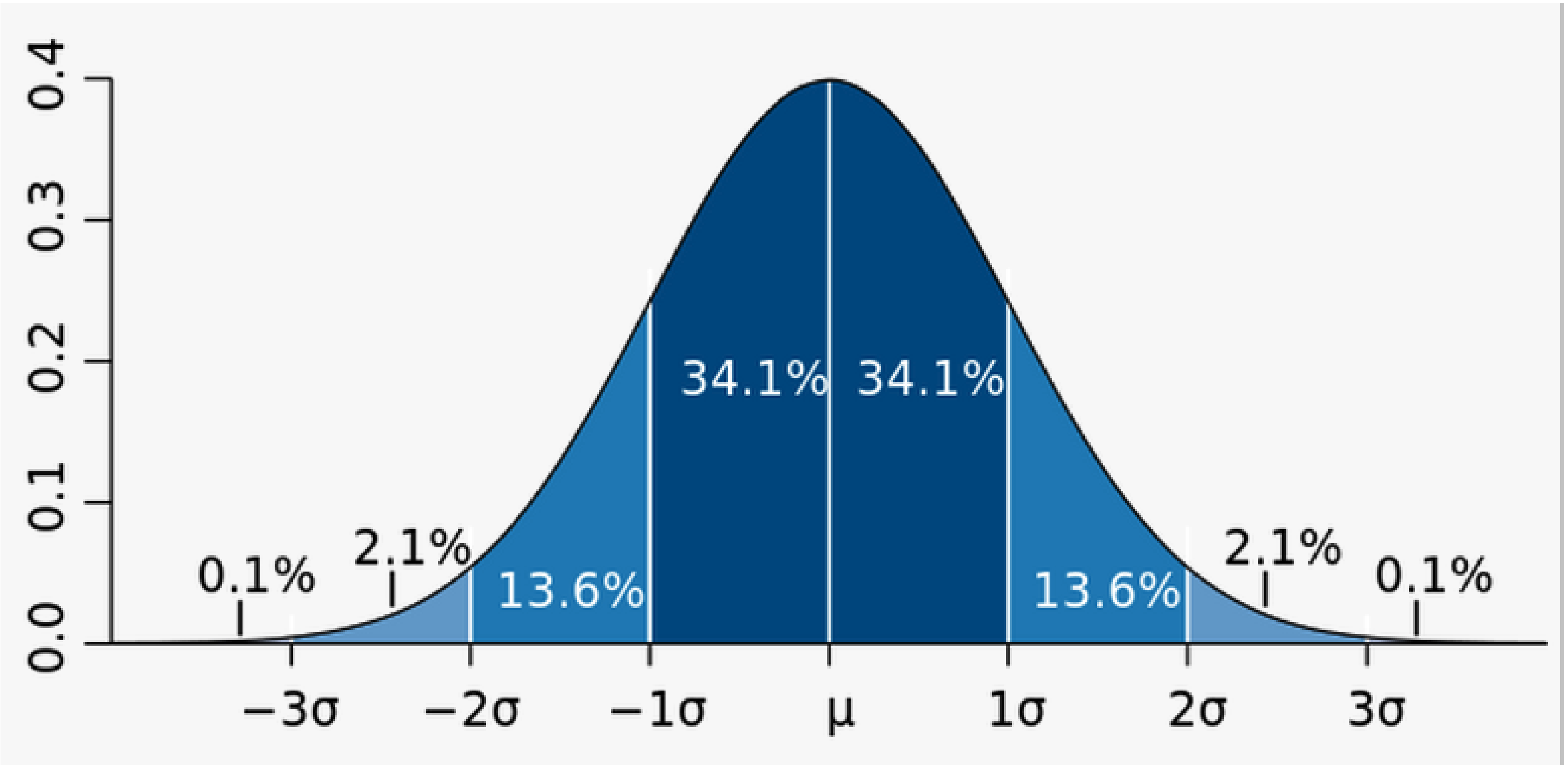


Image Statistics Values:

- **Skewness (Schiefe):** $$v=\frac{1}{N}\sum_{x=0}^{m-1}\sum_{y=0}^{n-1}\left(\frac{g(x,y)-\bar{g}}{s}\right)^3$$

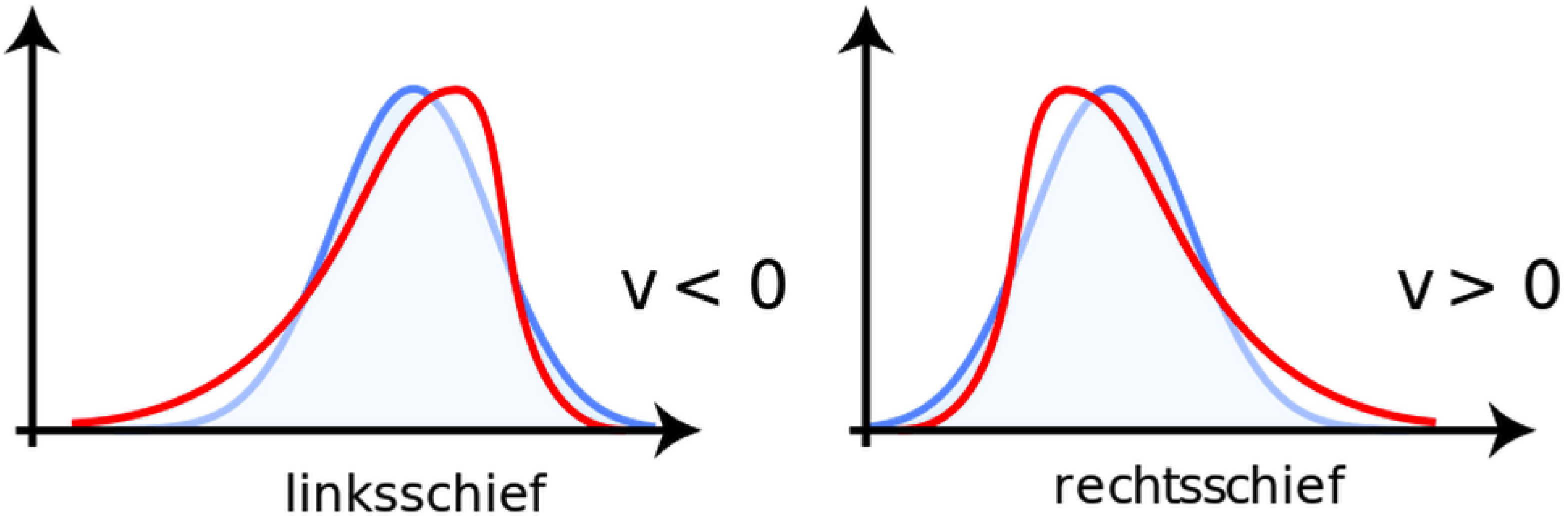
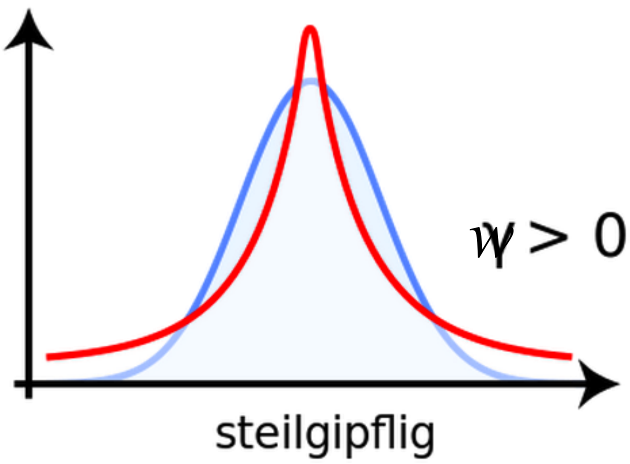


Image Statistics Values:

- **Kurtosis (Wölbung):** $w = \frac{1}{N} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} \left(\frac{g(x,y) - \bar{g}}{s} \right)^4$



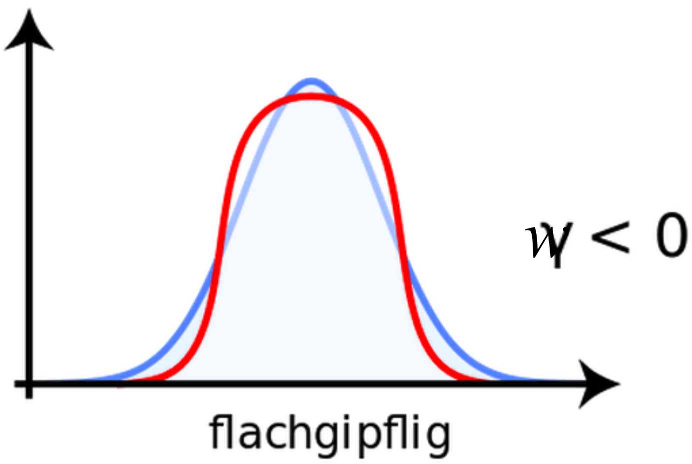


Image Statistics Values:

- **Entropy (Entropie):** $H = -\sum_{0}^{255} h(g)\log_2(h(g))$

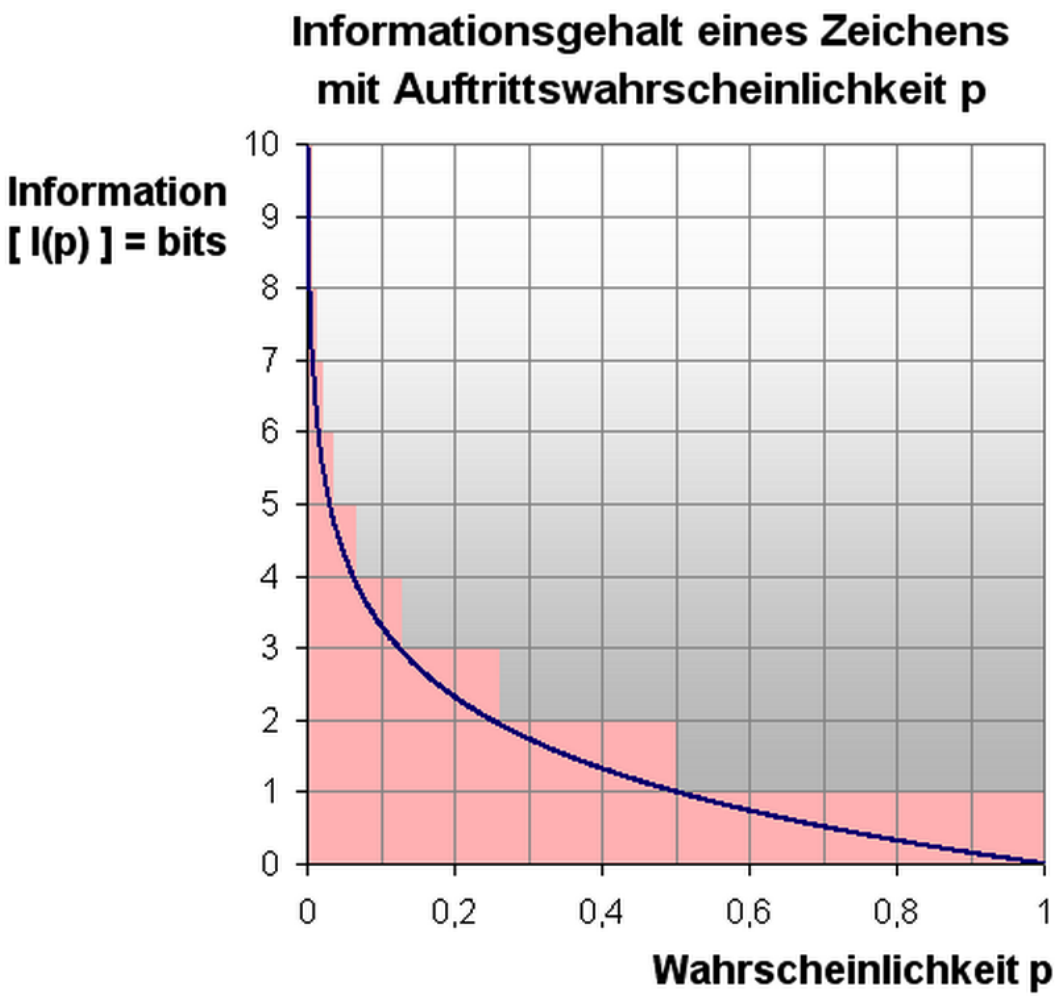
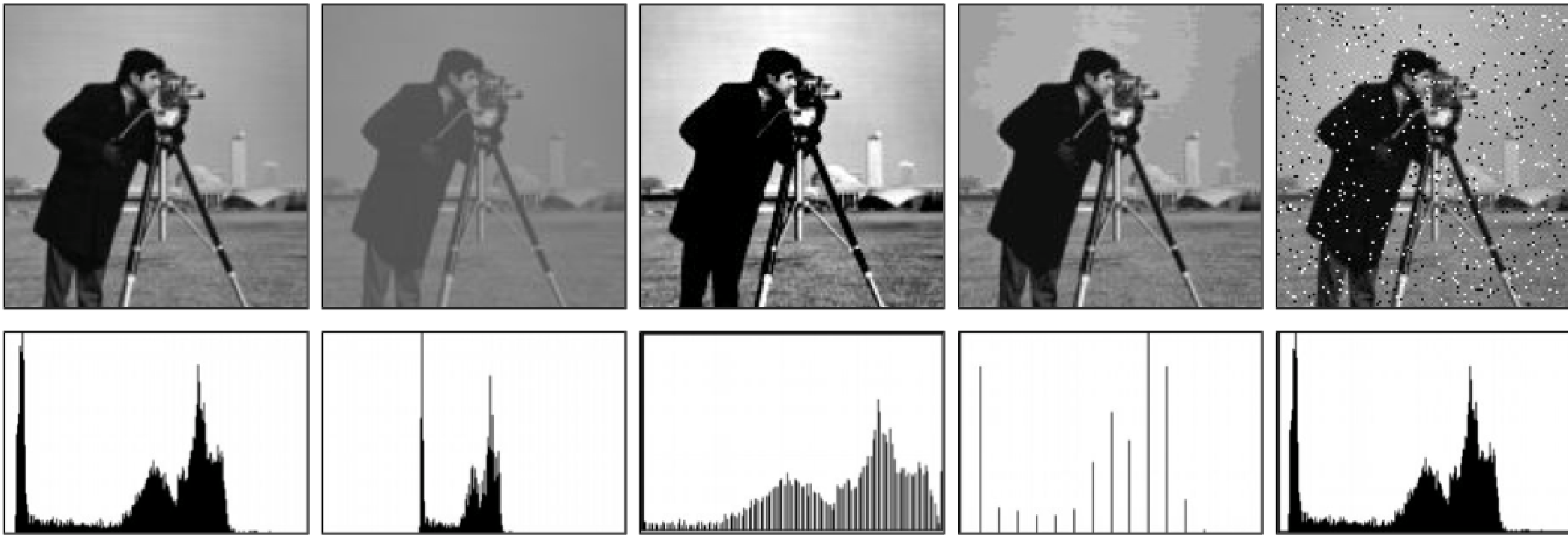


Image Statistics Examples:

Originalbild	Kontrast reduziert	Kontrast erhöht	Graustufen reduziert	Mit Salt & Pepper



Min:	8	81	0	16	0
Max:	246	172	255	239	255
Dynamic:	238	91	255	223	255
Contrast:	0.94	0.36	1.00	0.87	1.00
Mean:	118.7	123.6	130.7	118.9	119.0
StdDev:	59.6	22.9	87.3	58.5	64.6
Mode:	14	83	0	159	14
Median:	138	131	150	143	138
Skewness:	-0.79	-0.79	-0.43	-0.77	-0.53
Kurtosis:	-0.81	-0.81	-1.29	-0.83	-0.73