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Introduction to Computer Vision:

Local Operations: Morphology

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Local Operators: Morphologocal Operators

Point Operators

- Binarisation
- Gray level reduction
- Contrast & Brightness manipulations
- Histogram Equalization
- Arithmetic Operations
- Logic Operations

Global Operators

- Discrete Fourier Transform
- Wavelet Transform
- Hough Transform
- Principal Component Transform

Local Operators

- Filters
 - Low Pass Filter
 - Gauss & Box Filter
- High Pass Filter
- Sobel Filter
- Laplace Filter

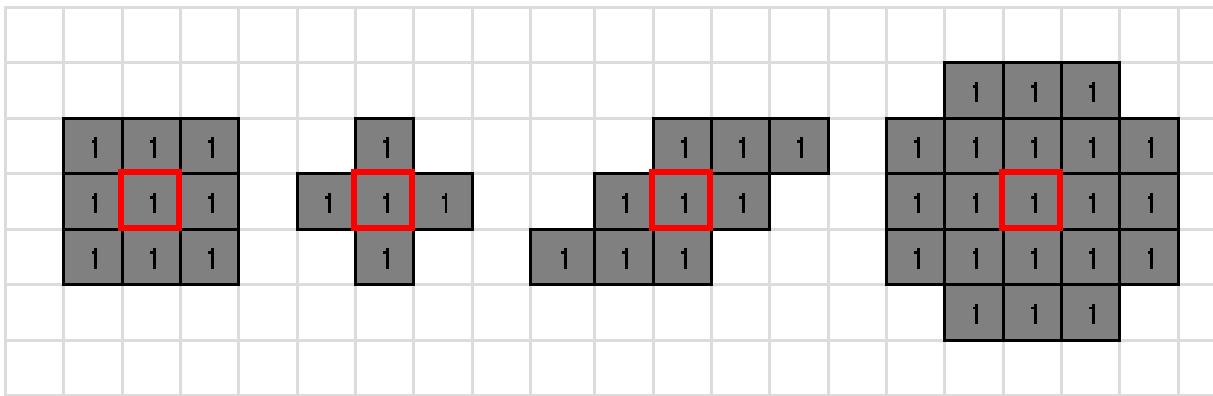
- Morphological Operators
 - Erosion & Dilation
 - Opening & Closing
- Rank Order Operators
 - Min. & Max. Filter
 - Median Filter

Morphological Filters

- Morphology matters about **shapes** in different sciences.
- In image processing it was developed by **Frenchman Matheron & Serra** in the 1970's.
- We distinguish morphological operators in:
 - **binary images**
 - **grayscale images**

Morphological Filters

- We always define a local area with a **Structured Element SE**:
 - It always has a **anchor point**.
 - It is mostly **symmetric**, but it can have **any form**.
 - The value in a binary SE is either **1** or **0** but all are the same



- There are two major morphological operations:
 - **Dilation** (in German Dilatation)
 - **Erosion**

Morphological Filters: Dilation

- Explanation with **set theory**:
 - B is the set of binary pixels in an image
 - S is the structure element
 - The dilation set consists of all pixels where the intersection of B & S is not nothing.

$$B_{dilation} = B \oplus S = \{ B \cap S \neq \emptyset \text{ für alle Pos. } S \text{ in } B \}$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0
0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Dilatation

\oplus

1	1	1
1	1	1
1	1	1

=

0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Morphological Filters: Dilation

- Explanation with **logic operation**:
 - The target pixel is the result of an **OR** of all pixels in the source image under the SE.
 - Explanation with **rank order operation**:
 - The target pixel is the **maximum** of all source pixel under the SE.

Dilatation

+

1

0	0	1	1	1	1	1	1	0	0	
0	1	1	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	1	1	0	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	
1	1	1	1	0	0	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	0	0	0	0	1	1	1	
1	1	1	0	0	0	0	1	1	1	

Morphological Filters: Erosion

- Explanation with **set theory**:
 - B is the set of binary pixels in an image
 - S is the structure element
 - The erosion set consists of all pixels where the intersection of B & S is S.

$$B_{erosion} = B \ominus S = \{ B \cap S = S \text{ für alle Pos. } S \text{ in } B \}$$

Erosion

-

2

Morphological Filters: Erosion

- Explanation with **logic operation**:
 - The target pixel is the result of an **AND** of all pixels in the source image under the SE.
 - Explanation with **rank order operation**:
 - The target pixel is the **minimum** of all source pixel under the SE.

Erosion

Θ

1	1	1
1	1	1
1	1	1

=

Morphological Filters: Properties

- **Properties** of Dilation and Erosion:
 - **Dilation & Erosion are not invertable.**
 - **Dilation & Erosion are dual:** A dilation of the foreground can be done with an erosion on the background and vice versa:

$$\overline{B} \oplus S = \overline{(B \ominus S)} \quad \overline{B} \ominus S = \overline{(B \oplus S)}$$

- **Dilation is commutative:**

$$B \oplus S = S \oplus B$$

- **Dilation is associative:**

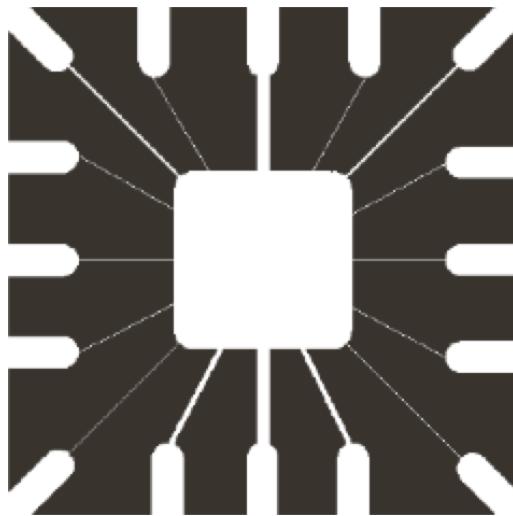
$$(B \oplus S_1) \oplus S_2 = B \oplus (S_1 \oplus S_2)$$

- **Erosion is not commutative:**

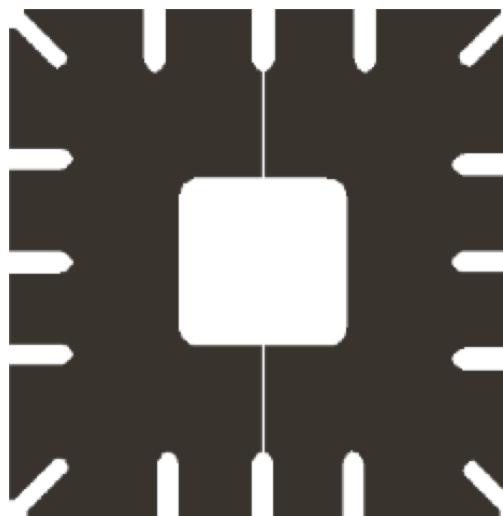
$$B \ominus S \neq S \ominus B$$

Morphological Filters: Example

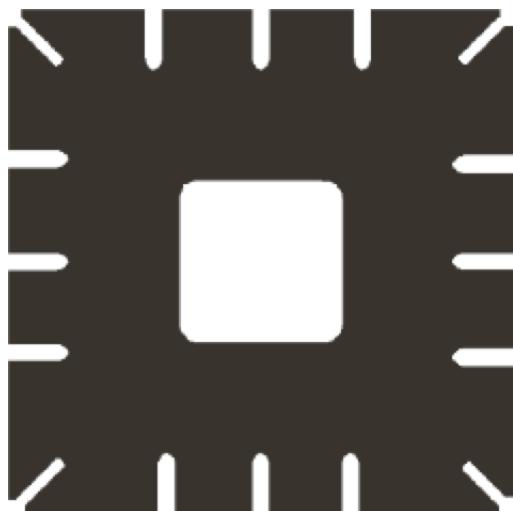
Original 486x486



Erosion with 11x11



Erosion with 15x15



Erosion with 45x45



Image Source: Gonzales & Woods

Morphological Filters: Closing

- A **closing operation** is a dilation followed by an erosion with the mirrored SE:

$$B_{Closing} = B \bullet S = (B \oplus S) \ominus S'$$

- Holes and concave edges are filled up.
- Bridges between unconnected regions can result.

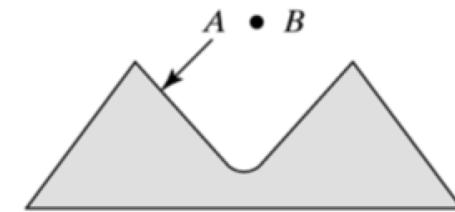
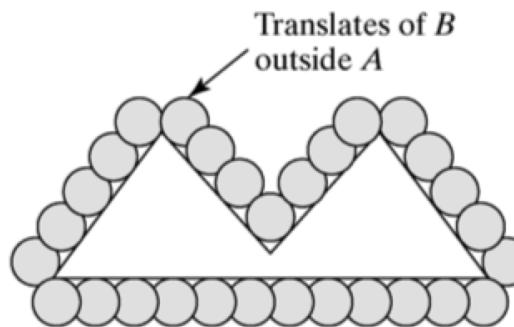
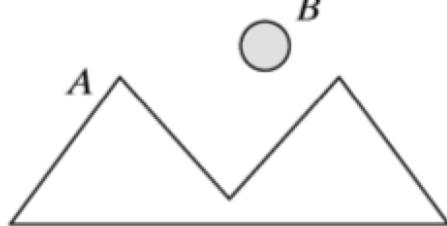


Image Source: Gonzales & Woods

Morphological Filters: Closing

- A **closing operation** is a dilation followed by an erosion with the mirrored SE:

$$B_{Closing} = B \bullet S = (B \oplus S) \ominus S'$$

- Example closing with 1x5 linear SE:

This is current driver circuit.

This is current driver circuit.

This is current driver circuit.

Morphological Filters: Opening

- A **opening operation** is an erosion followed by a dilation with the mirrored SE:

$$B_{Opening} = B \circ S = (B \ominus S) \oplus S'$$

- Small structures get deleted.
- Convex edges get flattened.

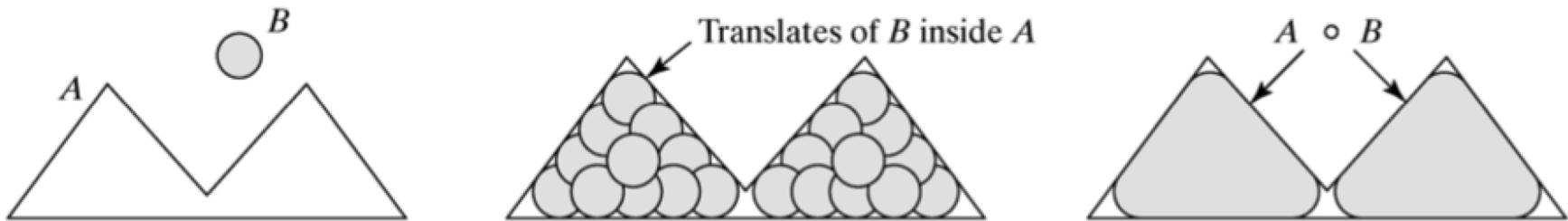


Image Source: Gonzales & Woods

Morphological Filters: Opening

- A **opening operation** is an erosion followed by a dilation with the mirrored SE:

$$B_{Opening} = B \circ S = (B \ominus S) \oplus S'$$

- Example of a closing followed by a opening by an 8-connecting SE:

$$B' = (((B \ominus S_8) \oplus S_8) \oplus S_8) \ominus S_8 = (B \circ S_8) \bullet S_8$$



$$\ominus S_8 =$$



$$\oplus S_8 =$$



$$\oplus S_8 =$$



$$\ominus S_8 =$$



Image Source: [Gonzales & Woods](#)

Morphological Filters: Properties

- Properties of Opening & Closing:

- Both are final: Another operation of the same doesn't change anything:

$$(B \circ S) \circ S = B \circ S$$

$$(B \bullet S) \bullet S = B \bullet S$$

- Both are dual: An Opening on the foreground is equal to the Closing of the background:

$$B \circ S = \overline{\overline{B} \bullet S}$$

$$B \bullet S = \overline{\overline{B} \circ S}$$

Morphological Filters: Edge Extraction

- By differencing the source image with dilated or eroded image we can extract edges of the source image:
- Rule: Subtract the thinner from the thicker image:

$$B_{\text{edge intern}} = (B \ominus S) \setminus B = B - (B \ominus S)$$

$$B_{\text{edge extern}} = (B \oplus S) \setminus B = (B \oplus S) - B$$

$$B_{\text{edge thick}} = (B \oplus S) - (B \ominus S)$$

Morphological Filters: Edge Extraction

Internal edges with the 8-connection SE: $B_{\text{edge intern}} = (B \ominus S) \setminus B = B - (B \ominus S)$

Erosion

Erosion													
Θ		=											
1	1	1											
1	1	1											
1	1	1											
0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0

\Original

Internal edges with the 4-connection SE:

Erosion

\ Original

Morphological Filters: Edge Extraction

External edges with the 8-connection SE: $B_{\text{edge extem}} = (B \oplus S) \setminus B = (B \oplus S) - B$

External edges with the 4-connection SE:

Morphological Filters: Hit or Miss Operator

- With the **Hit or Miss operator** we can **detect patterns equal to the SE**.
- It can be done in two steps:
 - In the Hit step we get all patterns equal to the SE.
 - In the Miss step we get the same surrounding of the SE.
 - The Miss step is done on the inverted (complement) image & with the inverted SE.
 - Both steps are combined with an AND.

$$B \otimes E = (B \ominus S_{Hit}) \cap (B^C \ominus S_{Miss}) \quad \text{mit } B^C = \overline{B}$$

Morphological Filters: Hit or Miss Operator

- With the **Hit or Miss operator** we can **detect patterns equal to the SE**:

$$B \otimes E = (B \ominus S_{\text{Hit}}) \cap (B^C \ominus S_{\text{Miss}}) \quad \text{mit } B^C = \overline{B}$$

$$\Theta = \text{SE}_{\text{Hit}} \times \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$$

		0	1	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	1	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	1	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	1	1	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	1	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	1	1	0	0	0	0	0	0	0

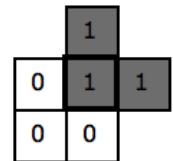
Morphological Filters: Hit or Miss Operator

- The **Hit or Miss operator** can be done in one step if we allow 0 in the SE:

$$B \otimes E = (B \ominus S_{Hit}) \cap (B^C \oplus S_{Miss}) \quad \text{mit } B^C = \overline{B}$$

- Example: Detecting lower left corners:

8



Morphological Filters: Fast Implementation

- If we **move the SE** as with all local operators over the source image we get a **slow performance** (as usual with local operators).
- If we move the source image over the SE:
 - If we combine all shift with an OR we get a dilation.
 - If we combine all shift with an AND we get an erosion.

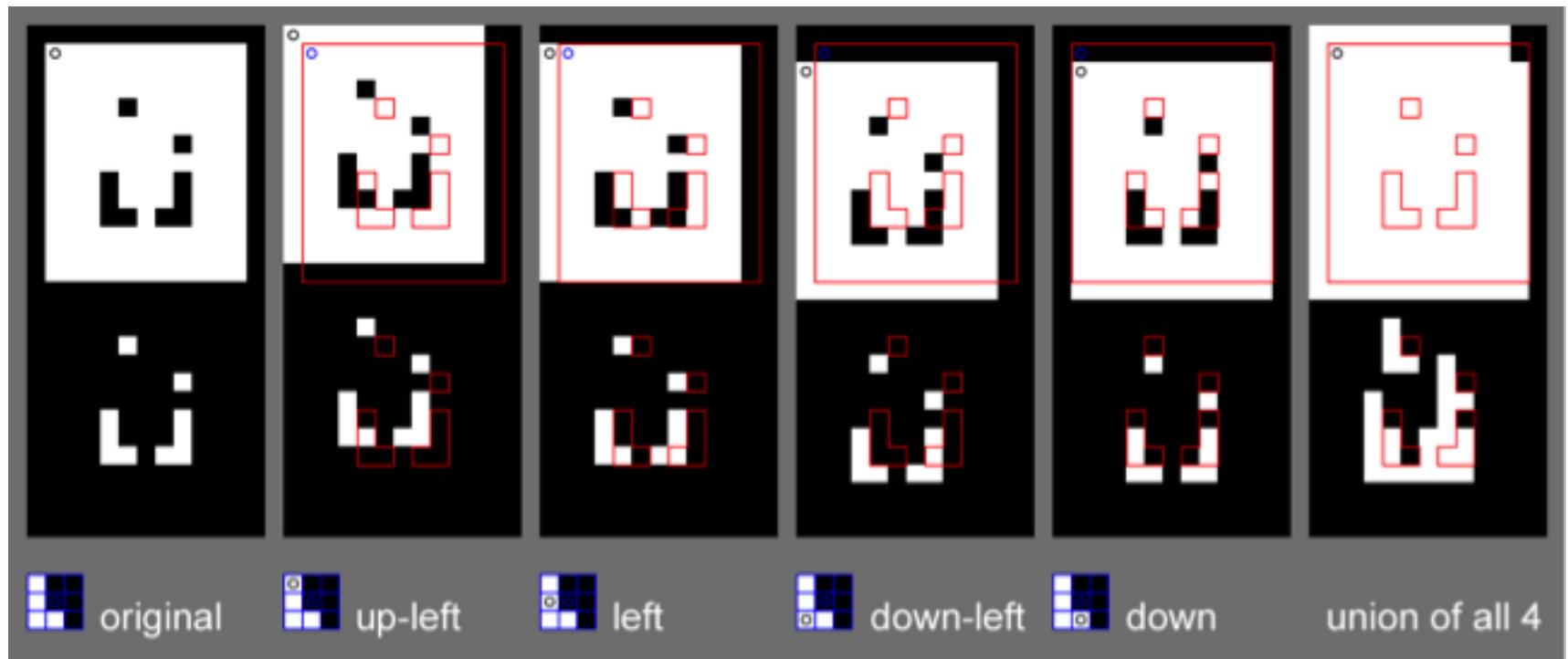


Image Source: R. A. Peters, Vanderbilt University

Morphological Filters: Grayscale Morphology

- Grayscale image can be interpreted as 3D structure:

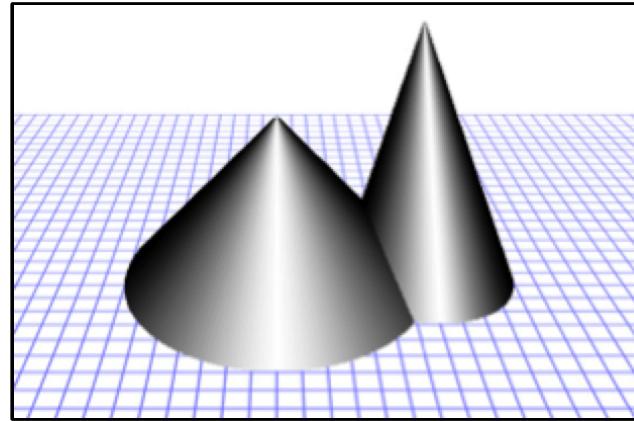
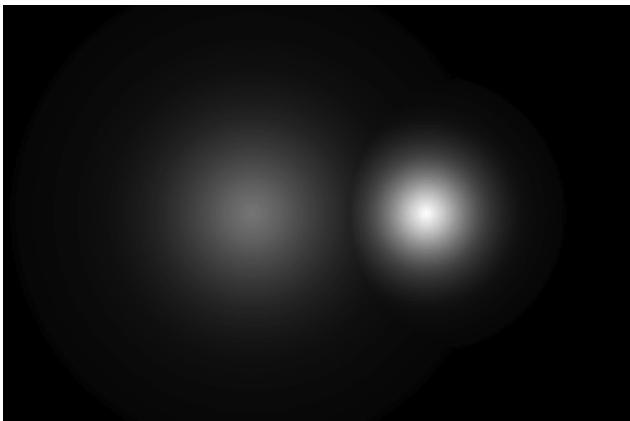
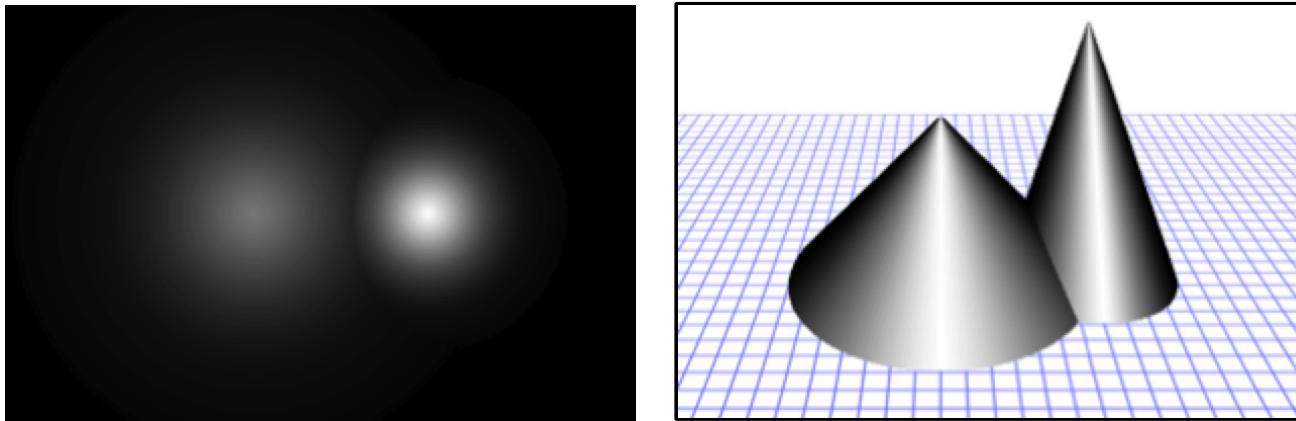


Image Source: R. A. Peters, Vanderbilt University

Morphological Filters: Grayscale Morphology

- Grayscale image can be interpreted as 3D structure:



- Also morphological operations can be better explained with 3D set operations.
- If a set is a subset of another depends explicitly on the height of the sets:

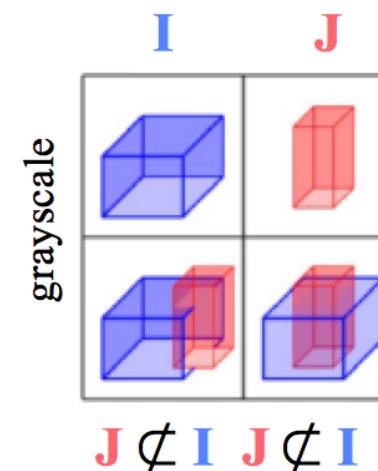
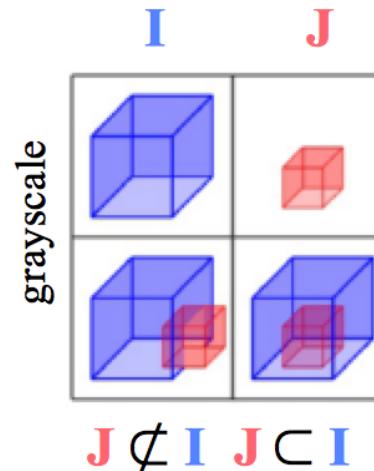
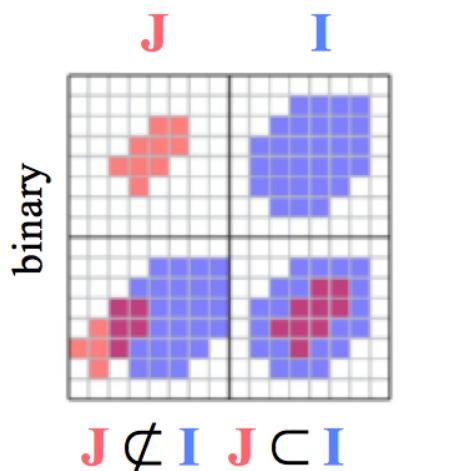


Image Source: R. A. Peters, Vanderbilt University

Morphological Filters: Grayscale Morphology

- For grayscale morphology the values in the SE range is 0... 2^n
- In mathematical morphology the SE and the image contain real numbers.
- A 0 is not the same as outside of the SE.
- Some SE examples:

0	0	0
0	1	0
0	0	0

0	1	0
1	2	1
0	1	0

0	1	2	1	0
0	1	2	1	0
0	1	2	1	0

0	1	2	4	2	1	0
0	1	2	1	0		
0	1	0				
0						

Morphological Filter: Grayscale Dilatation & Erosion

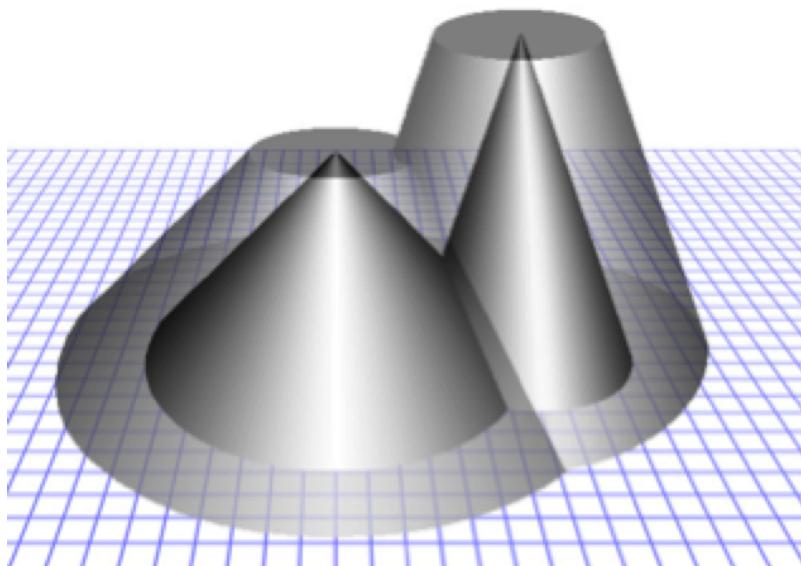
- The **grayscale dilation & erosion** are defined as **rank order operations**:

$$G'(x, y) = G(x, y) \oplus S(i, j) = \max(G(x+i, y+j) + S(i, j))$$

$$G'(x, y) = G(x, y) \ominus S(i, j) = \min(G(x+i, y+j) - S(i, j))$$

- If all elements in the SE are zero, this definitions correspond to the binary ones.

Dilation over original



Erosion under original

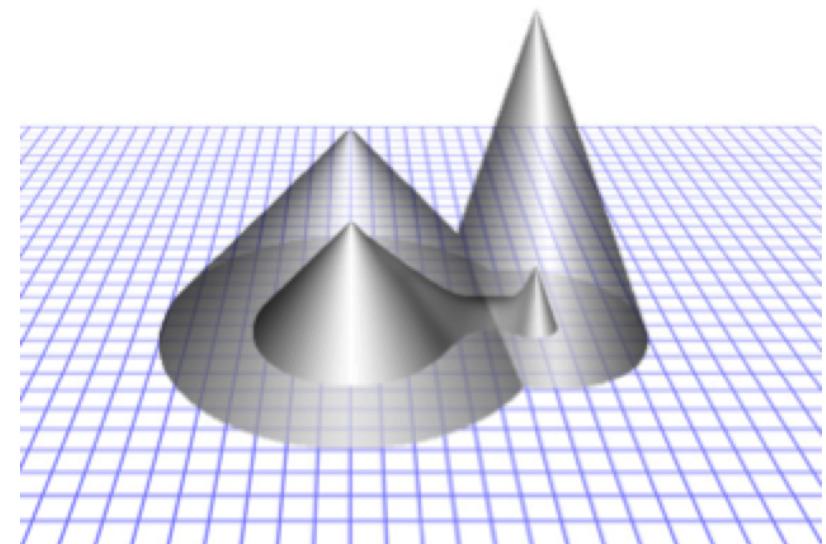


Image Source: R. A. Peters, Vanderbilt University

Morphological Filter: Grayscale Dilation & Erosion

- If all elements in the SE are zero, these definitions correspond to the binary ones.
- For all the following examples **the SE values are zero!**
- The dilation is a Maximum Filter:

$$G'(x, y) = G(x, y) \oplus S(i, j) = \max(G(x+i, y+j))$$

- The erosion is a Minimum Filter:

$$G'(x, y) = G(x, y) \ominus S(i, j) = \min(G(x+i, y+j))$$

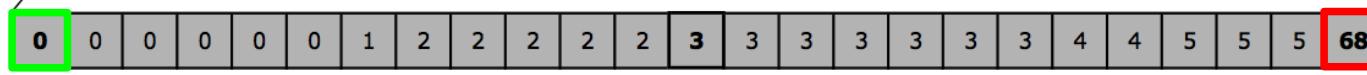
Morphological Filter: Grayscale Dilation & Erosion

- All SE values are zero:

$$G'(x, y) = G(x, y) \oplus S(i, j) = \max(G(x+i, y+j))$$

$$G'(x, y) = G(x, y) \ominus S(i, j) = \min(G(x+i, y+j))$$

0	6	0	0	0	0	0	1	2	3	2	3	2	1
2	0	2	0	0	0	0	2	3	2	3	2	1	1
2	2	0	3	0	1	0	0	3	4	3	2	0	
5	0	2	1	0	2	0	5	0	3	2	1	1	
0	0	3	2	5	3	4	0	2	4	0	4	0	
5	0	4	0	3	68	2	1	3	0	3	2	1	
0	0	3	3	0	0	3	2	4	0	0	0	5	
3	0	3	3	3	2	5	3	2	2	2	3	6	
2	0	2	0	6	0	2	0	0	3	0	5	2	
1	5	2	5	2	5	3	1	1	5	0	6	5	
3	2	0	0	5	2	0	0	0	2	3	2	2	
2	0	0	0	0	0	2	3	0	3	5	3	2	
1	0	5	2	5	0	0	5	3	0	5	1	3	
0	1	0	0	3	1	2	3	5	0	1	0	5	



Morphological Filter: Grayscale Dilation & Erosion

Original & Dilation with an 8-connection SE:



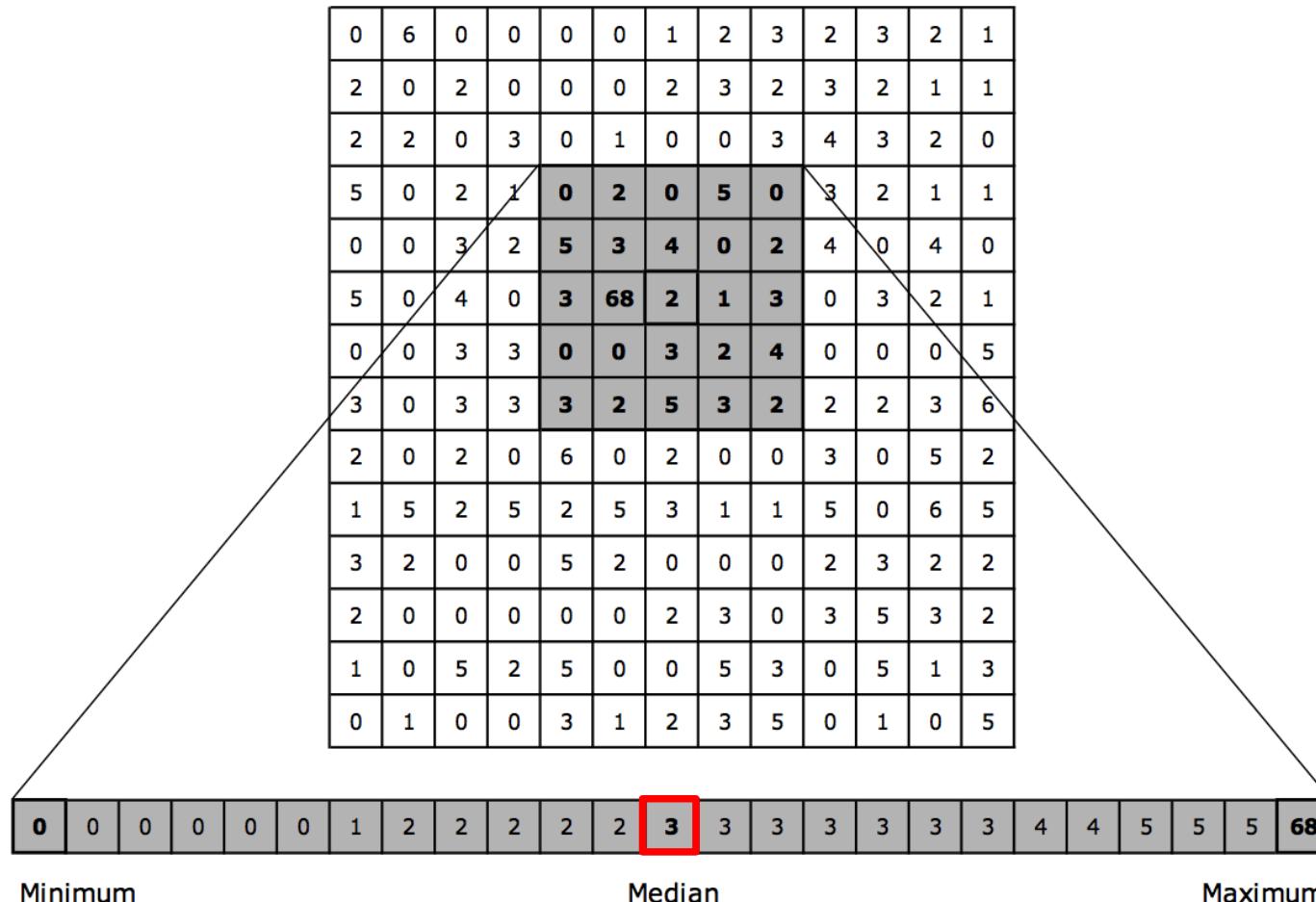
Morphological Filter: Grayscale Dilation & Erosion

Original & Erosion with an 8-connection SE:



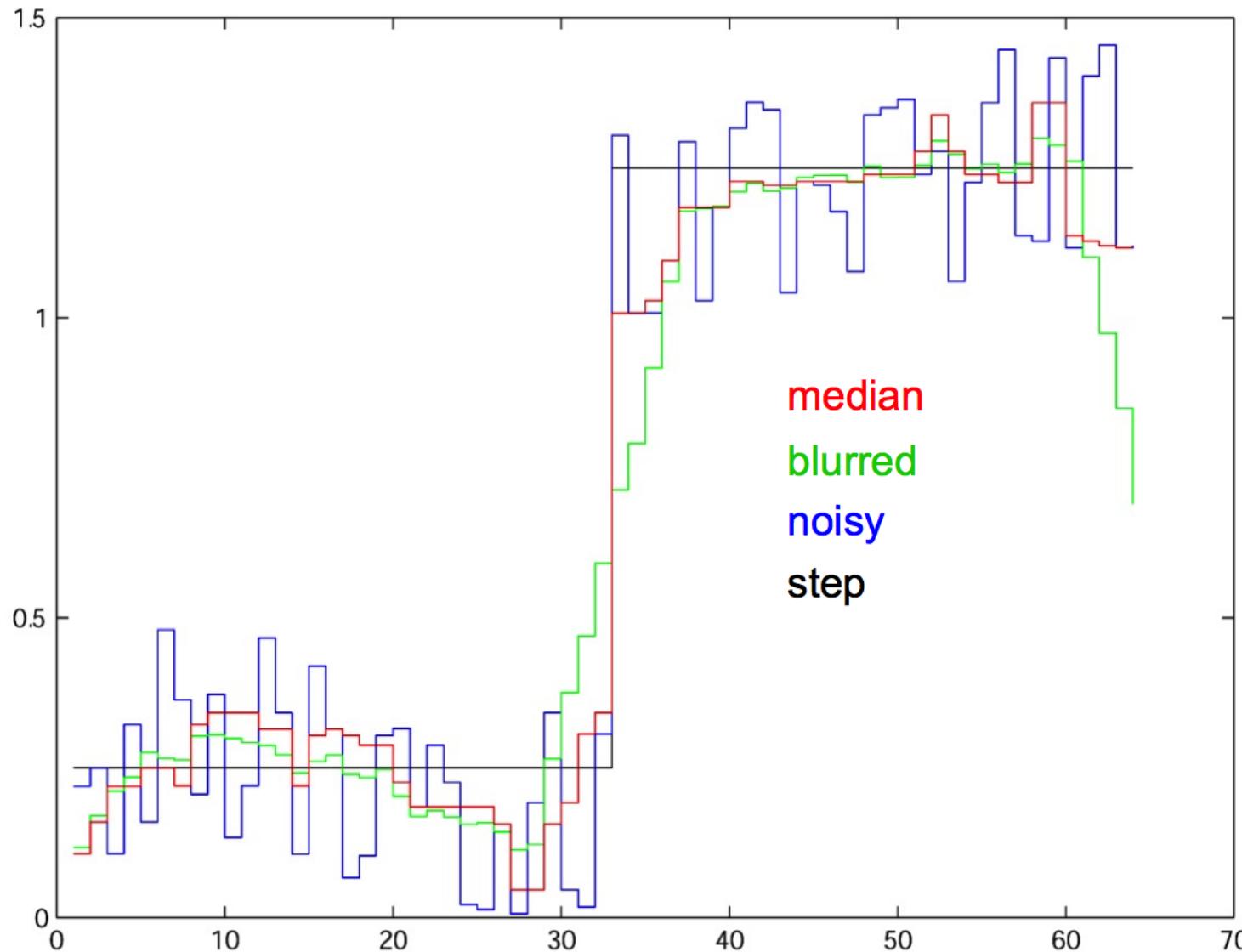
Morphological Filters: Grayscale Median Filter

- A **very powerful denoising** filter is the **median filter**.
- It takes the **median value** from the sorted rank order:
- It is very good filtering out **salt & pepper noise**:



Morphological Filters: Grayscale Median Filter

The median filter keeps sharp noise edges sharp:



Morphological Filters: Grayscale Median Filter

Original with salt & pepper noise and filtered with the 3x3 median filter:



Morphological Filters: Grayscale Median Filter

Original with salt & pepper noise and filtered with the 3x3 gaussian filter:



Morphological Filters: Grayscale Median Filter

Original gaussian noise (sigma: 10.0) and twice filtered with 3x3 median filter:



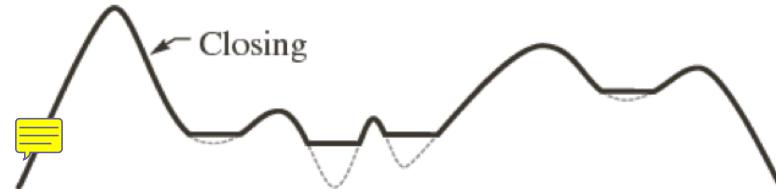
Original gaussian noise (sigma: 10.0) and twice filtered with 3x3 gaussian filter:



Morphological Filter: Grayscale Closing & Opening

- Closing & Opening are as well a combination dilation & erosion.
- A **closing keeps bright details** and **deletes dark details**:

$$G_{Closing} = G \bullet S = (G \oplus S) \ominus S'$$



Morphological Filter: Grayscale Closing & Opening

- Closing & Opening are as well a combination dilation & erosion.
- An **opening keeps dark details** and **deletes bright details**:

$$G_{Opening} = G \circ S = (G \ominus S) \oplus S'$$



Morphological Filters: Grayscale Edge Extraction

- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

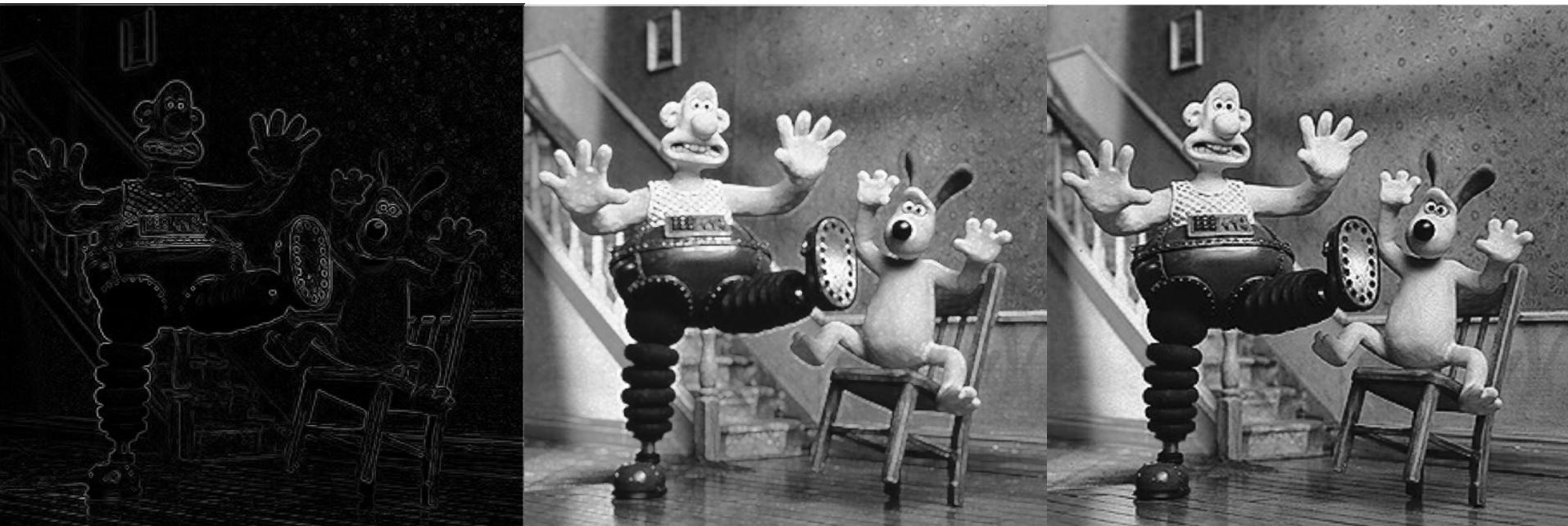
$$G_{\text{edge intern}} = G - (G \ominus S)$$



Morphological Filters: Grayscale Edge Extraction

- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

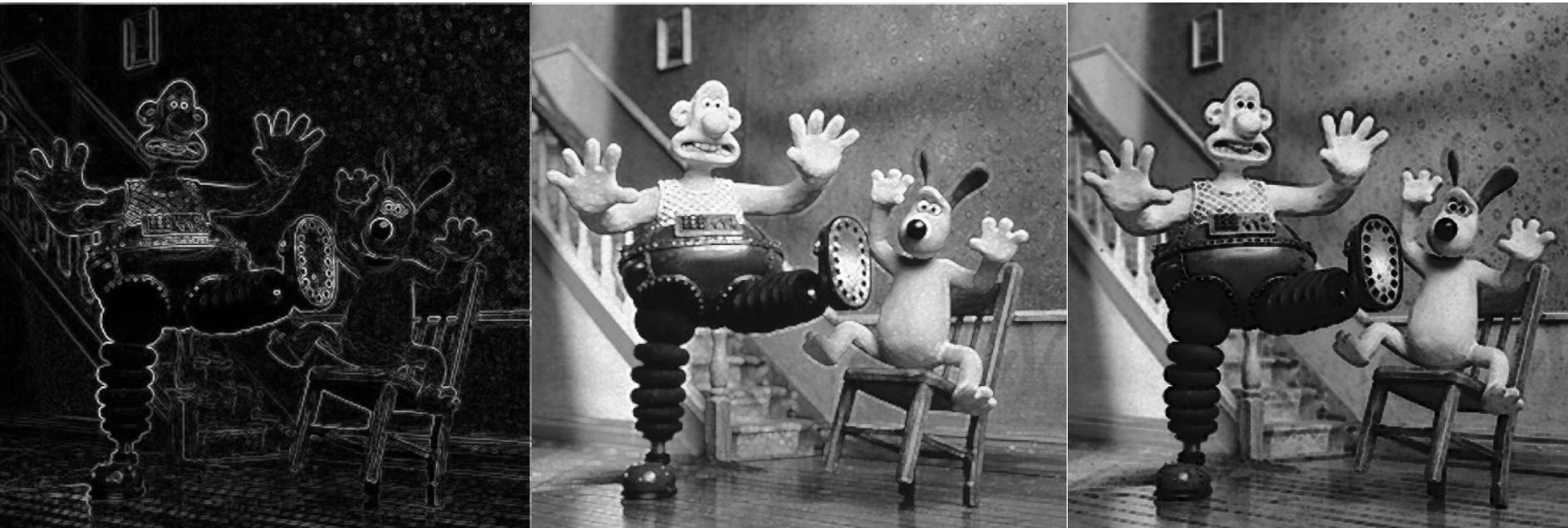
$$G_{\text{edge ext}} = (G \oplus S) - G$$



Morphological Filters: Grayscale Edge Extraction

- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

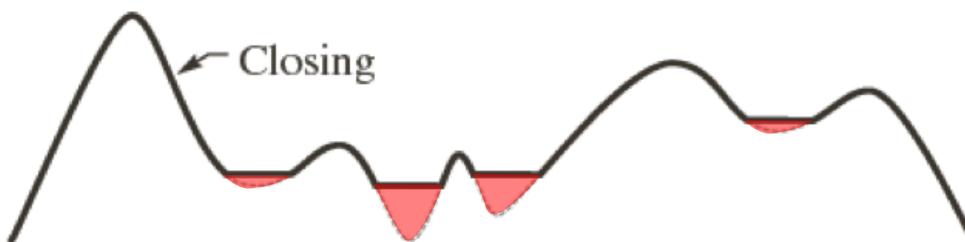
$$G_{\text{edge thick}} = (G \oplus S) - (G \ominus S)$$



Morphological Filters: Bot Hat Operator

- The **Bot Hat** operator makes small details bright that are darker than the surrounding.

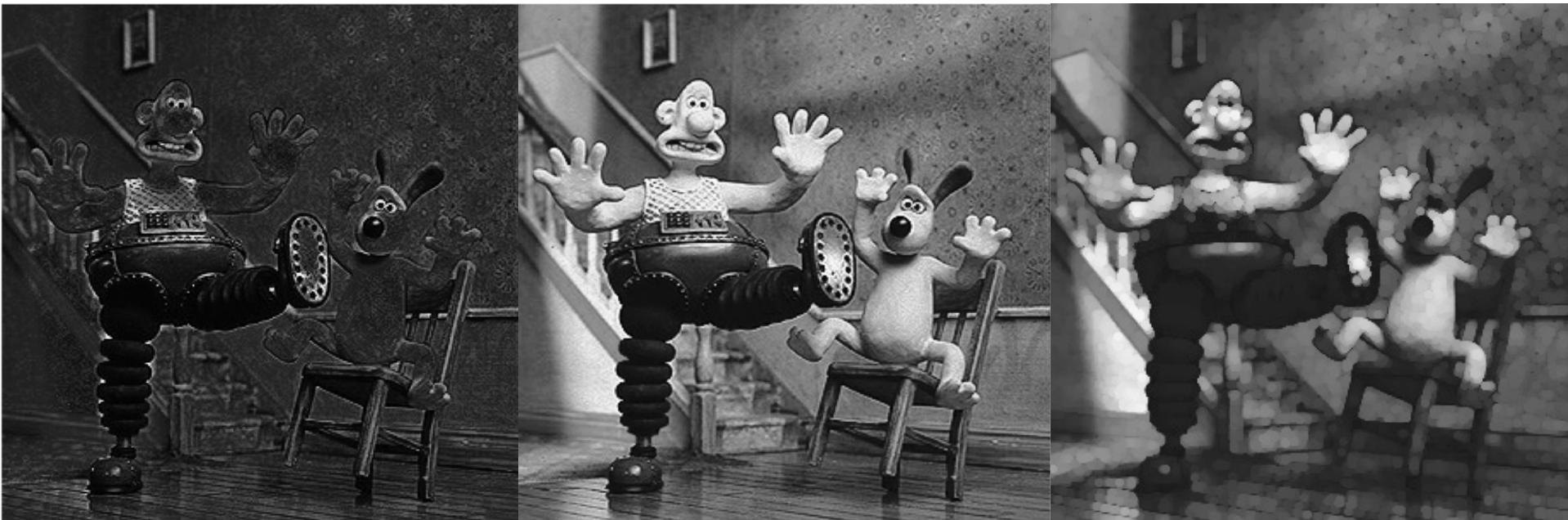
$$G_{Bothat} = G_{Closing} - G = (G \bullet S) - G$$



Morphological Filters: Top Hat Operator

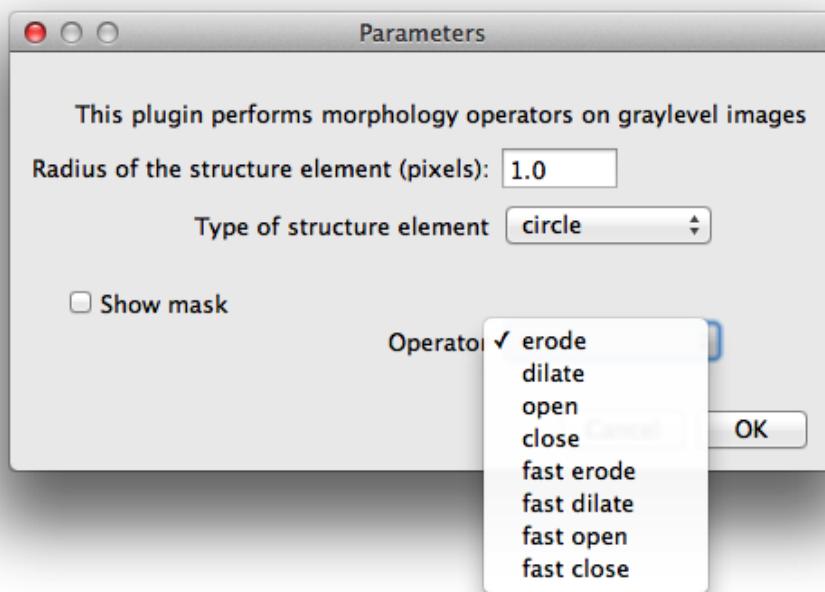
- The **Top Hat** operator makes small details bright that are brighter than the surrounding.

$$G_{\text{Tothat}} = G - G_{\text{Opening}} = G - (G \circ S)$$



Morphological Operators in ImageJ

- **ImageJ** offers in the menu *Process > Binary* the operators *Erode*, *Dilate*, *Open*, *Close* and *Outline*.
- All operators use the **8-connection SE**.
- **Fiji** offers in the menu *Process > Morphology > Gray Morphology* a dialog for grayscale morphology with various SE and operations:



Morphological Operators in ImageJ

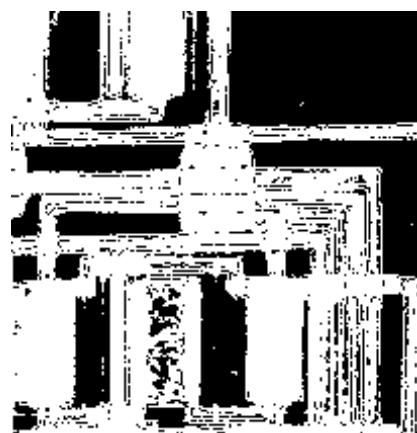
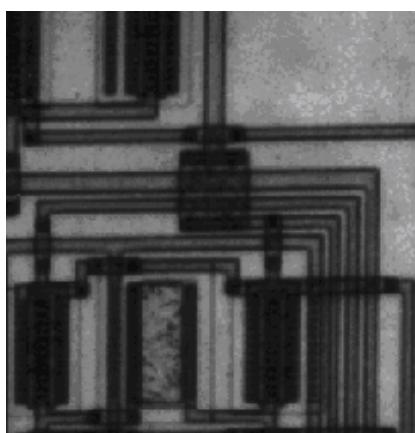
- For programming the *ImageProcessor* class provides the essential operators *dilate()*, *erode()*, *open()* and *close()*.
- All methods can be applied on binary and grayscale images.
- The *ColorProcessor* provides the same for color images with the application on the color channels separately.
- All operators use the 8-connection SE.
- There are additional ImageJ plugins with more detailed operators.

Morphological Operators in Matlab

- Matlab offer generic and specific morphological functions.
- A SE can be defined with the function **strel**:

```
I = imread('..../images/circuit.png');           % Load image into matrix X
J = imcomplement(I);                          % Invert image
BW1 = im2bw(J,0.8);                         % make black & white
SE1 = strel('rectangle',[30 20]);            % 40x30 rectangular SE
BW2 = imerode(BW1,SE1);                      % erode w. 40x30 rectangular SE
BW3 = imdilate(BW2,SE1);                     % dilate w. the same SE

imshow(I); title('Original');                 % Show images
figure; imshow(BW2); title('After erosion');
figure; imshow(BW1); title('After binarisation');
figure; imshow(BW3); title('After dilation');
```



Morphological Operators: Exercise

- Implement the dilation & erosion in Excel or OpenOffice Calc.
- Add salt & pepper noise and gaussian noise to an image.
 - Denois the image with the median and gaussian filter.
 - Try various sizes of the SE.
 - Try multiple subsequent filterings.
- Optional: Implement the same operations in ImageJ as in the Matlab example.