BFH - TI
Computer Sciences

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CPVR Module 7281
Exercises

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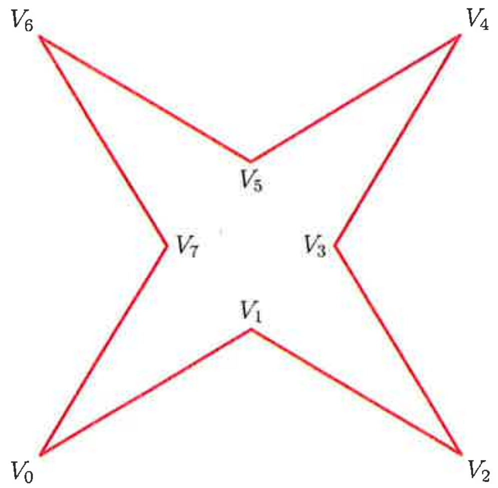
1 Computingthenormaltoaplane

Exercise1 :Computingthenormalusingthecrossproduct

A classical problem of computer graphic is to compute the normal to a plane or the normal to a planary face. The face is a polygon defined by the coordinates of its vertices.

Consider the following polygon :

$V_0 = (1;1;1)$
$V_1 = (6;6-K;1)$
$V_2 = (11;1;1)$
$V_3 = (6+K;6;1)$
$V_4 = (11;11;1)$
$V_5 = (6;6+K;1)$
$V_6 = (1;11;1)$
$V_7 = (6-K;6;1)$



The parameter K may take any value between 1 and 4. Using the normal vector of this planary polygon using the cross product between 3 consecutive vertices, for example V_0, V_1 and V_2. What are the values for the different values of K?

Solution : Consider the vectors defined by $\vec{a} = \overrightarrow{V_0V_1}$ and $\vec{b} = \overrightarrow{V_1V_2}$. One should has :

$$\begin{aligned}\vec{a} &= V_1 - V_0\\ &= (6;6-K;1)-(1;1;1)\\ &= (5;5-K;0)\end{aligned} \quad \text{and} \quad \begin{aligned}\vec{b} &= V_2 - V_1\\ &= (11;1;1)-(6;6-K;1)\\ &= (5;-5-K;0)\end{aligned}$$

The cross product of these two vectors is (with the parameter K) :

$$\vec{a}\times\vec{b} = \begin{bmatrix}5\\5-K\\0\end{bmatrix} \times \begin{bmatrix}5\\-5-K\\0\end{bmatrix} = \begin{bmatrix}0\\0\\-50\end{bmatrix}$$

Since the parameter K is positive, one can see that this normal vector points in the negative

Z direction.

What happens of one use the vertices V_1, V_2 and V_3 to compute this normal vector?

Solution : Now one can define the vector $\vec{a}$ as $\overrightarrow{V_1V_2}$ and the vector $\vec{b}$ as $\overrightarrow{V_2V_3}$. This gives :

$$\begin{aligned}\vec{a} &= V_2 - V_1 \\ &= (11;1;1) - (6;6-K;1) \\ &= (5;-5-K;0)\end{aligned} \quad \text{and} \quad \begin{aligned}\vec{b} &= V_3 - V_2 \\ &= (6+k;6;1) - (11;1;1) \\ &= (-5+K;5;0)\end{aligned}$$

If one compute the cross product of these two vectors, one has :

$$\vec{a} \times \vec{b} = \begin{bmatrix} 5 \\ -5-K \\ 0 \end{bmatrix} \times \begin{bmatrix} -5+K \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ K^2 \end{bmatrix}$$

One can see that the normal computed at vertice V_1 points into the negative Z direction and the normal computed at vertice V_2 points into the positive Z direction. This does not seems very coherent.

One cannot rely on the denomination of the vertices since the start point is randomly choosed

Another idea solution to compute the normal vector is to compute an *averaged* vector as a mean value of all normal vectors at all vertices of the polygon. One may for example use the following formula :

$$\vec{n} = \frac{1}{n}\sum_{i=0}^{n} \vec{n}_i = \frac{1}{n}\sum_{i=0}^{n} (V_{i+1} - V_i) \times (V_i - V_{i-1})$$

Solution : If one compute the normal to this polygon using this formula, for the different values of K, one can find :

K	$\vec{n}$
1	(0 ; 0 ; -124)
2	(0 ; 0 ; -56)
3	(0 ; 0 ; 4)
4	(0 ; 0 ; 56)

As one can see in the table, the normal changes direction between $K = 2$ and $K = 3$. For $K = 2.928932$ the normal vector is almost a zero vector and this method fails catastrophically!

Exercise2 :ComputingthenormalusingtheformulaofNewell

There is a much better approach to compute the normal of a polygon. This is the *Newell's method.* This method states that the normal's components are :

$$n_x = \sum_{i=0}^{n} (V_{i_y} - V_{i+1_y}) \cdot ((V_{i_z} - V_{i+1_z})$$
$$n_y = \sum_{i=0}^{n} (V_{i_z} - V_{i+1_z}) \cdot ((V_{i_x} - V_{i+1_x})$$
$$n_z = \sum_{i=0}^{n} (V_{i_x} - V_{i+1_x}) \cdot ((V_{i_y} - V_{i+1_y})$$

Using this method, computes the normal of the polygon for the different values of K.

Solution : If one write a little program to do that, one can see that the normal takes the values :

K	$\vec{n}$
1	(0; 0; 40)
2	(0; 0; 80)
3	(0; 0; 120)
4	(0; 0; 160)