BFH - TI
Computer Sciences 17 avril 2015

CPVR Module 7281
Exercises

Table des matières

1 Linear Algebra

Exercise 1: Unit vector

Let $\vec{v} = (8,6)$. Compute the length of $\vec{v}$. Give the coordinates of a *unit* vector $\vec{u}$ **normal** to $\vec{v}$

Exercise 2: Barycentric coordinates

For two points $P = (1;2)$ and $Q = (4;8)$ compute the intermediate points that are $1/3$, $1/2$ and $3/4$ the way between P and Q.

Exercise 3: Angle between two vectors

Let $\vec{v} = (8;6)$ and $\vec{u} = (0;5)$, and θ the angle between $\vec{v}$ and $\vec{u}$. Using the dot and perp products, compute $\cos\theta$ and $\sin\theta$.

Exercise 4: Cross product

Let $\vec{v} = (v_1; v_2; v_3)$ and $w = (w_1; w_2; w_3)$ be any two 3D vectors. Prove that the equations $(\vec{v} \times \vec{w}) \cdot \vec{v} = 0$ and $(\vec{v} \times \vec{w}) \cdot \vec{w} = 0$ are always true, and thus $\vec{v} \times \vec{w}$ is always perpendicular to $\vec{v}$ and $\vec{w}$.

Exercise 5: Triple product

Let $\vec{u} = (u_1; u_2; u_3)$, $\vec{v} = (v_1; v_2; v_3)$ and $\vec{w} = (w_1; w_2; w_3)$ be any three 3D vectors. Prove that the equation $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$ is alway true.

Exercise 6: Line geometry

Let a 2D (infinite) line pass through two points P_0 and P_1. Given any arbitrary point P in the plane, show that the perp product $P - P_0) \perp (P_1 - P_0)$ will be positive for points P on one side of the line and negative on the other side of the line.

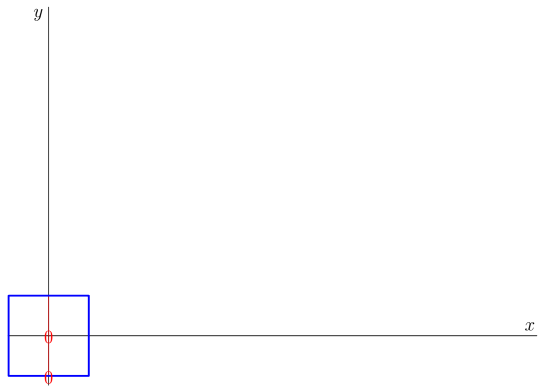
Exercise 7: Segment geometry

Using the previous exercise, develop a test for wether a **finite segment** between points Q_0 and Q_1 crosses (i.e.intersects) the (infinite) line throught P_0 and P_1. Use this to develop another test for wether the **finite segment** Q_0Q_1 intersects with the **finite segment** P_0P_1 without actually computing the point of intersection.

2 Transformation in plane and space

Exercise 8: Transformation is plane 1

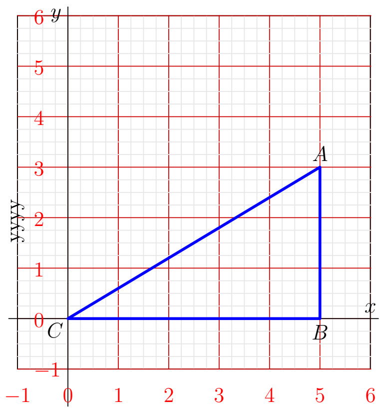
Let S be a square with the vertices $v_1 = (-0.5;-0.5)$, $v_2 = (0.5;-0.5)$, $v_3 = (0.5;0.5)$ and $v_4 = (-0.5;0.5)$

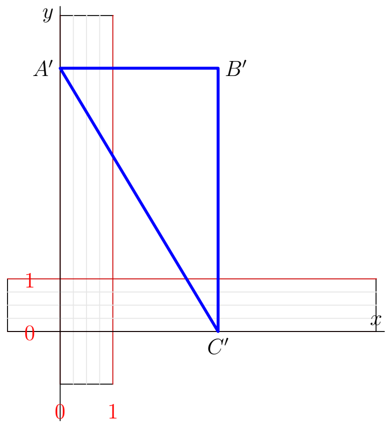


1. Apply to that square a rotation of $\pi/4$ and a translation along vector $x = (5.0;0.0)$. Compute the new coordinates of the vertices of the square.
2. Apply to that square a translation along vector $x = (5.0;0.0)$ and then a rotation of $\pi/4$. Compute the new coordinates of the vertices of the square.

Exercise 9: Transformation in plane 2

Let T be a triangle with the following vertices : $A = (5;3)$, $B = (5,0)$ and $C = (0;0)$. Which tranformation should be applied to this triangle to have the figure on the right, i.e. $A' = (0;5)$, $B' = (3;5)$ and $C' = (3;0)$.





Exercise 10: Reflection around a line in plane

- Given a 2D point given by its homogeneous coordinates $\tilde{P} = (\tilde{x}; \tilde{y}; 1)$. Compute the coordinates of the point $\tilde{P}'$ which is obtained by a reflexion of $\tilde{P}$ around an axis inclined by $\pi/6$ with the x-axis.
- Same exercise with an axis inclined by $\pi/4$ with the x-axis

Exercise 11: Quaternions

Using the quaternion algebra, rotate the vector $\vec{v} = (1;0;0)$ around the z-axis with an angle of π.

Exercise 12: Scaling preserve some parallelism

Let be two segment $\sigma_1 = (x_{11}; y_{11}; 1) - (x_{12}; y_{12}; 1)$ and $\sigma_2 = (x_{21}; y_{21}; 1) - (x_{22}; y_{22}; 1)$. The two segments are defined as parallel. If one apply a scaling transformations to these two segments, their images σ_1^* and σ_2^* remain parallel. Prove this affirmation.

3 Geometry of simple elements

Exercise 13: Equation of a plane

Let Π be a plane given by three points $P_1 = (0;1;3)$, $P_2 = (2;3;5)$ and $P_3 = (0;0;5)$. Compute the cartesian equation of this plane.

Exercise 14: Intersection between a line and a sphere

Let d a line given by two points $P_1 = (-2;2;2)$ and $P_2 = (0;3;5)$. Where does this line intersect the sphere of radius 5, with center at $(0;0;0)$.

Exercise 15: Distance between a point and a line

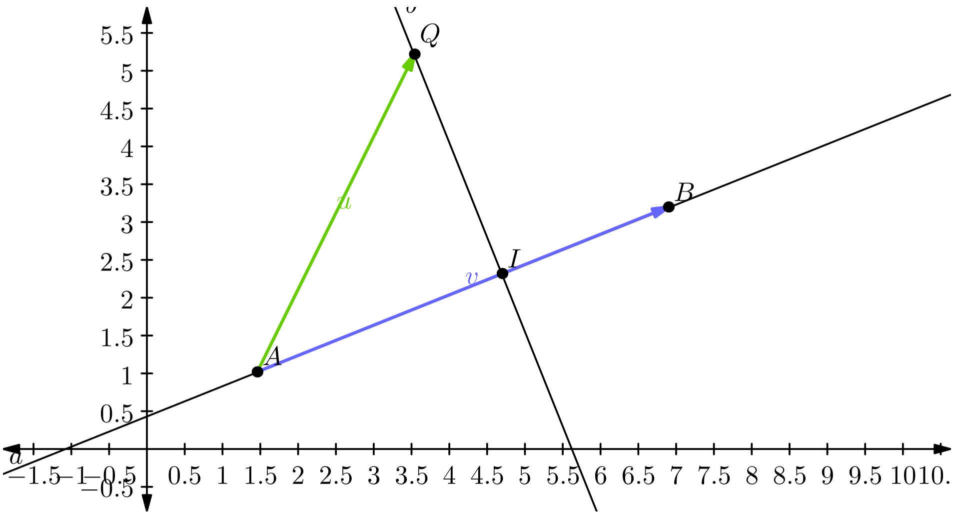
Let be $\mathscr{L}$ a 3D line given by its parametric equation, that is :

$$\mathscr{L} = \mathbf{P} + t \cdot \vec{d}$$

and a point **Q** not on the line.

The distance between point **Q** is defined as the shortest distance between **Q** and a point **X** $\in \mathscr{L}$.

Give a formula to compute the distance between **Q** and $\mathscr{L}$.



Exercise 16: Distance between a point and a half-line

Let be ℓ a half-line given by :

$$\ell = \mathbf{P} + t \cdot \vec{d} \qquad \text{with } t \geq 0$$

and a point $\mathbf{Q}$ not on the line.

Using the result of the previous exercise, find a formula to compute the distance between ℓ and $\mathbf{Q}$.

Exercise 17: Distance between a point and a line segment

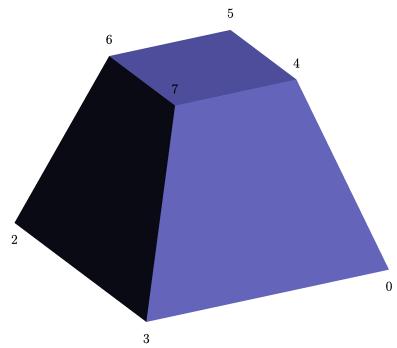
Let be S a line segment given by two points $\mathbf{P_0}$ and $\mathbf{P_1}$, and a point $\mathbf{Q}$ not on the segment.

Using the results previously seen, find a formula to compute the distance between S and $\mathbf{Q}$.

4 Volume modeling

Exercise 18: Volume modeling

Consider the truncated pyramid below :



The coordinates of the vertices are :

Vertex ID	x	y	z
0	1.0	0.0	0.0
1	0.0	1.0	0.0
2	-1.0	0.0	0.0
3	0.0	-1.0	0.0
4	0.5	0.0	1.0
5	0.0	0.5	1.0
6	-0.5	0.0	1.0
7	0.0	-0.5	1.0

1. Describe this pyramid using the *Indexed Face Set* model.
2. Describe this pyramid using the *Winged edge* model.

Exercise 19: Computing normal vectors

A robust algorithm to compute the normal of a plane polygon is given by the Newell algorithm.

Let P be a polygon given by N vertices $(x_i; y_i; z_i)$. The normal $\vec{n} = (n_x; n_y; n_z)$ is given by :

$$n_x = \sum_{i=0}^{N-1} (y_i - y_{succ(i)})(z_i + z_{succ(i)}) \tag{1}$$

$$n_y = \sum_{i=0}^{N-1} (z_i - z_{succ(i)})(x_i + x_{succ(i)}) \tag{2}$$

$$n_z = \sum_{i=0}^{N-1} (x_i - x_{succ(i)})(y_i + y_{succ(i)}) \tag{3}$$

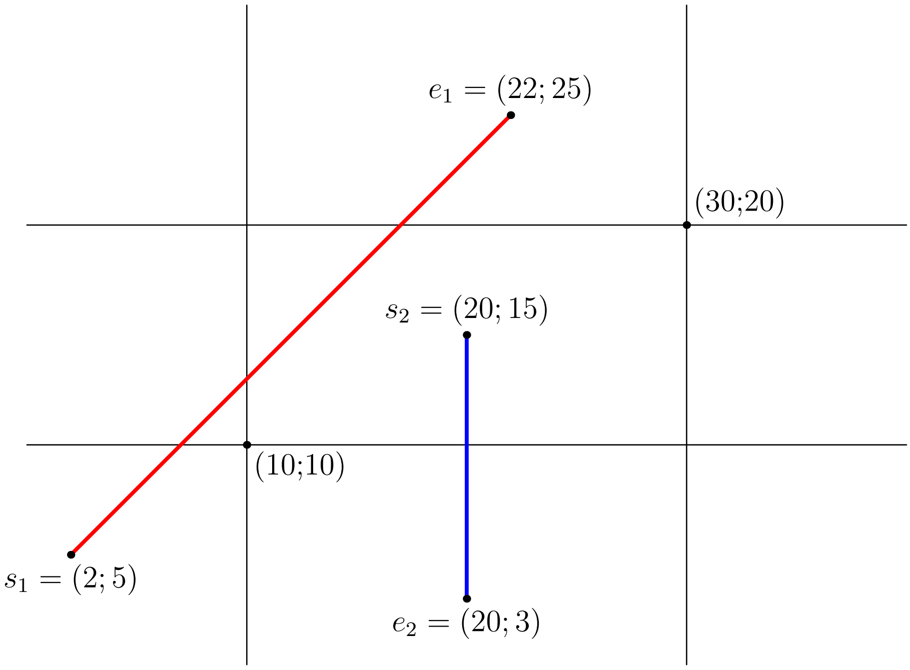
Where the function $succ(i)$ is $(i+1) \bmod N$. The normal vector computer by this algorithm is not normalized (i.e. $\|\vec{n}\| \neq 1$) and therefore, most of the time this operation should be added at the end of the computing.

Using these equations, compute the normal of the lateral faces of the pyramid.

5 Clipping

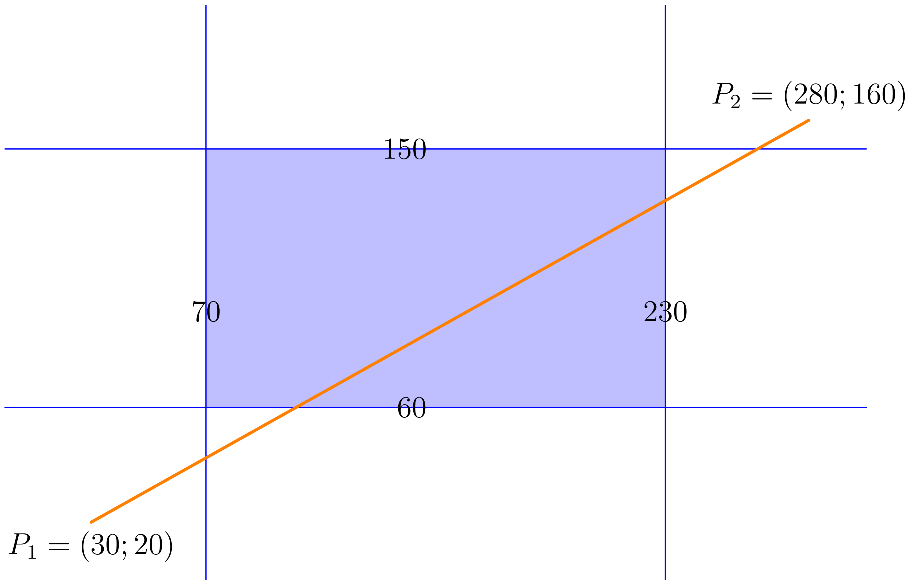
Exercise 20: Cohen-Sutherland Clipping

Using the Cohen-Sutherland algorithm, clip the segment $(s_1; e_1)$ and $(s_2; e_2)$. The clipping phase must be processed in order : `BOTTOM`,`TOP`, `LEFT` and `RIGHT`.



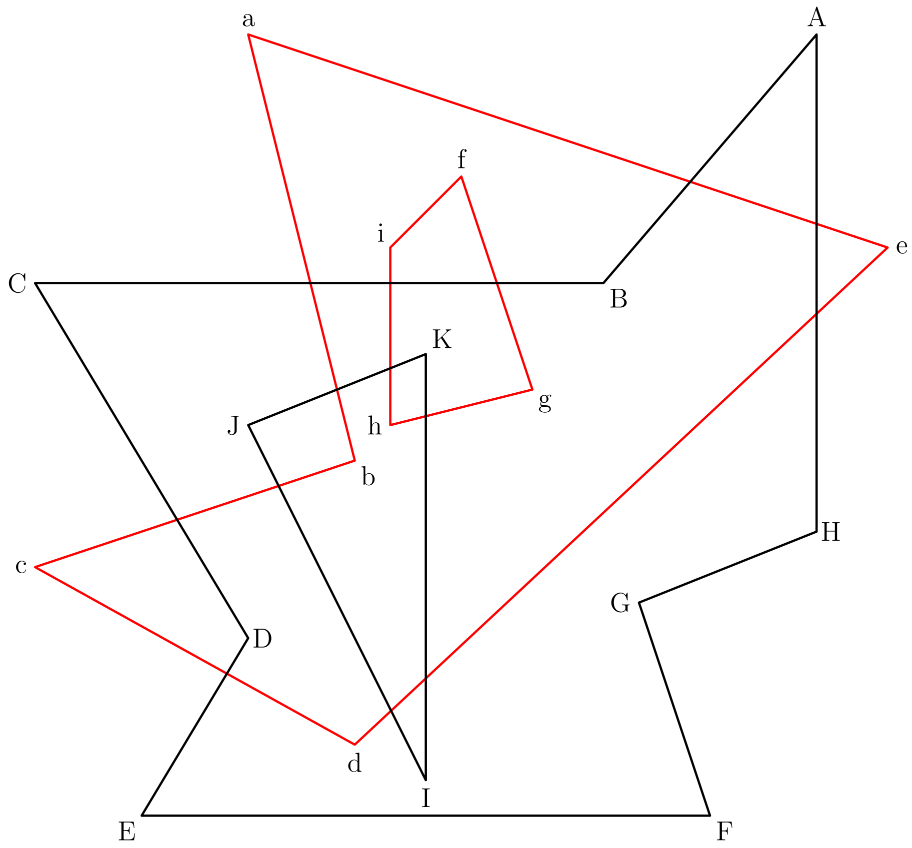
Exercise 21: Liang-Barsky Clipping

Using the Liang-Barsky, clip the following segment.



Exercise 22: Weiler-Atherton Clipping

Using the Weiler-Atherton algorithm, clip the following subject polygon (red) using the "window" in black. Explain an algorithm that can be used.



6 Back and hidden faces

Let be a 2D scene, with 2 objects. Theses two object are given by the following coordinates.

Edges : A = (5 ;12), B = (9 ;12), C = (9 ;16), D = (5 ;16), E = (7 ;5), F = (11 ;5), G = (11 ;9), H = (7 ;9), cam = (7 ;2)

The faces are defined by : A–B, B–C, C–D, D–A, E–F, F–G, G–H, H–E.

The camera is positionned at point (7 ;2), the lookat vector is (0 ;1) and the field of view is 90°. As the scene is only 2D, up vector is not needed.

Questions :

- For each of theses faces, compute which are front faces and wich are back faces.
- Compute the z-buffer in the direction of the view of the camera.
- Adapt the Warnock algorithm for the one-D case and compute how much subdivsion are needed to have only one object for the scene described above.