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Introduction to Image Processing:

Local Operations: Morphology

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Local Operators: Morphologocal Operators

Point Operators

- Binarisation
- Gray level reduction
- Contrast & Brightness manipulations
- Histogram Equalization
- Arithmetic Operations
- Logic Operations

Global Operators

- Discrete Fourier Transform
- Wavelet Transform
- Hough Transform
- Principal Component Transform

Local Operators

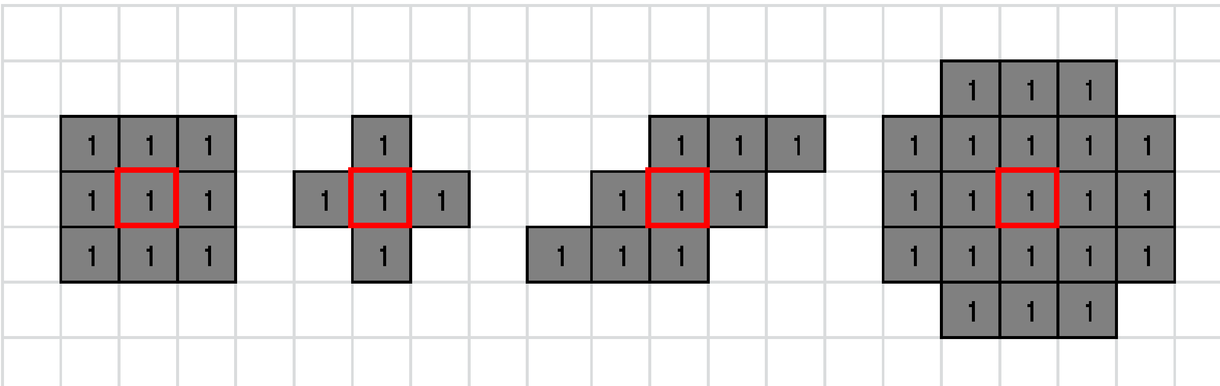
- Filters
  - Low Pass Filter
    - Gauss & Box Filter
  - High Pass Filter
    - Sobel Filter
    - Laplace Filter
- Morphological Operators
  - Erosion & Dilation
  - Opening & Closing
- Rank Order Operators
  - Min. & Max. Filter
  - Median Filter

Morphological Filters

- Morphology matters about **shapes** in different sciences.
- In image processing it was developed by **Frenchman Matteron & Serra** in the 1970's.
- We distinguish morphological operators in:
    - **binary images**
    - **grayscale images**

Morphological Filters

- We allways define a local area with a **Structured Element SE**:
  - It allways has a **anchor point**.
  - It is mostly **symmetric**, but it can have **any form**.
  - The value in a binary SE is either **1** or **0** but all are the same

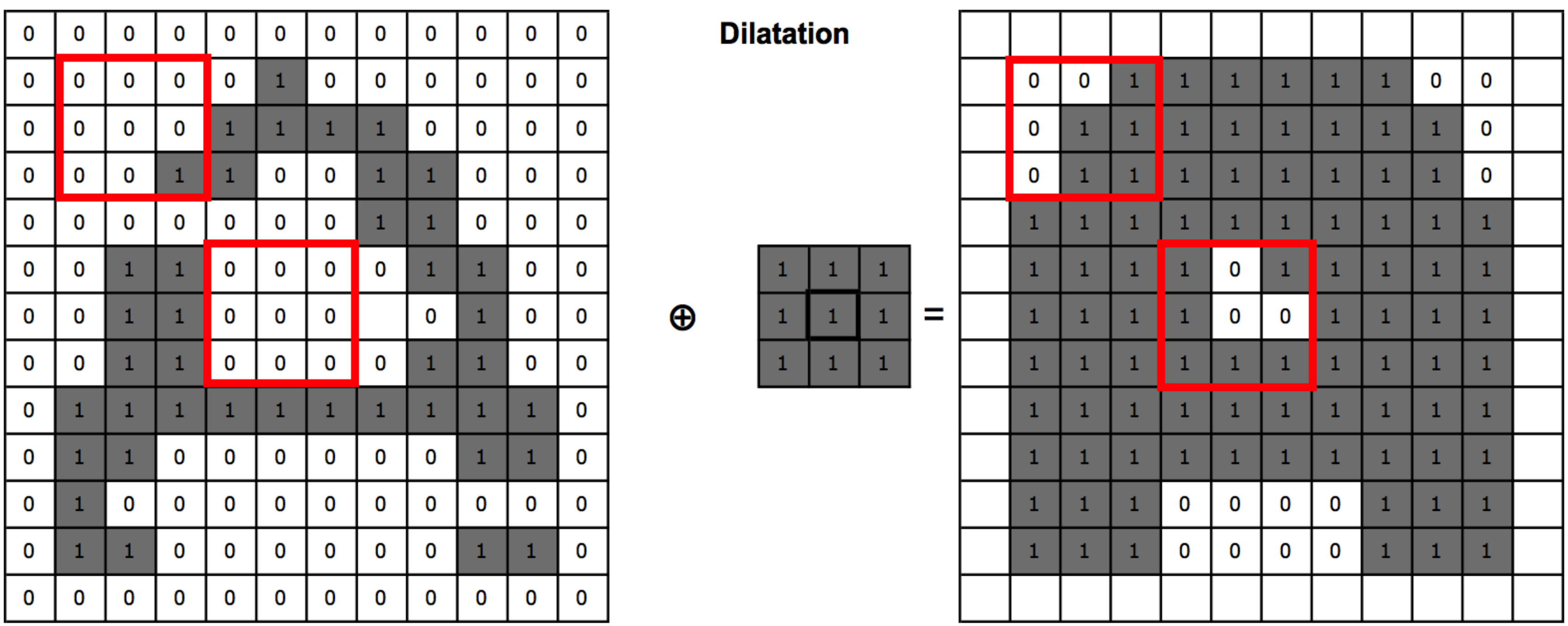


- The are two major morphological operations:
  - **Dilation** (in German Dilatation)
  - **Erosion**

Morphological Filters: Dilation

- Explanation with **set theory**:
  - B is the set of binary pixels in an image
  - S is the structure element
  - The dilation set constists of all pixels where the intersection of B & S is not nothing.

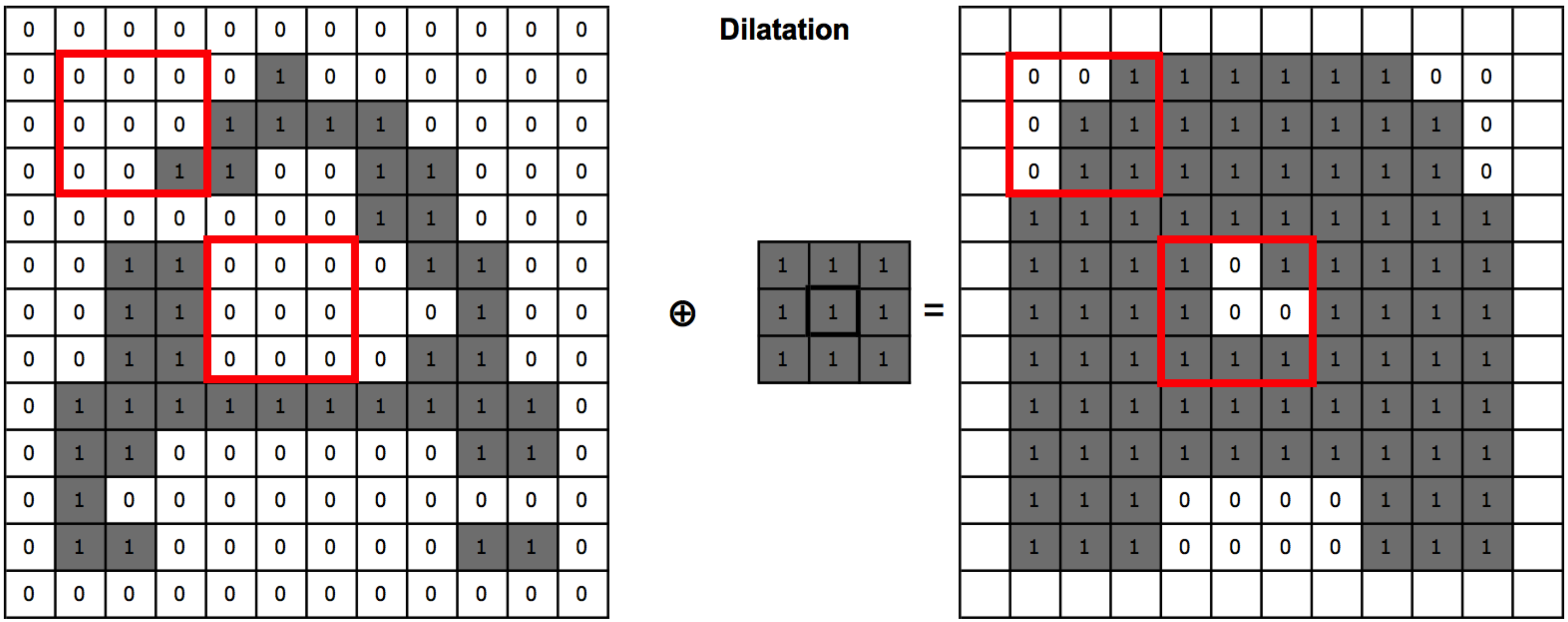
$$B_{dilation} = B \oplus S = \left\{ B \cap S \neq \emptyset \quad \text{für alle Pos. } S \text{ in } B \right\}$$



Morphological Filters: Dilation

- Explanation with **logic operation**:
  - The target pixel is the result of an **OR** of all pixels in the source image under the SE.

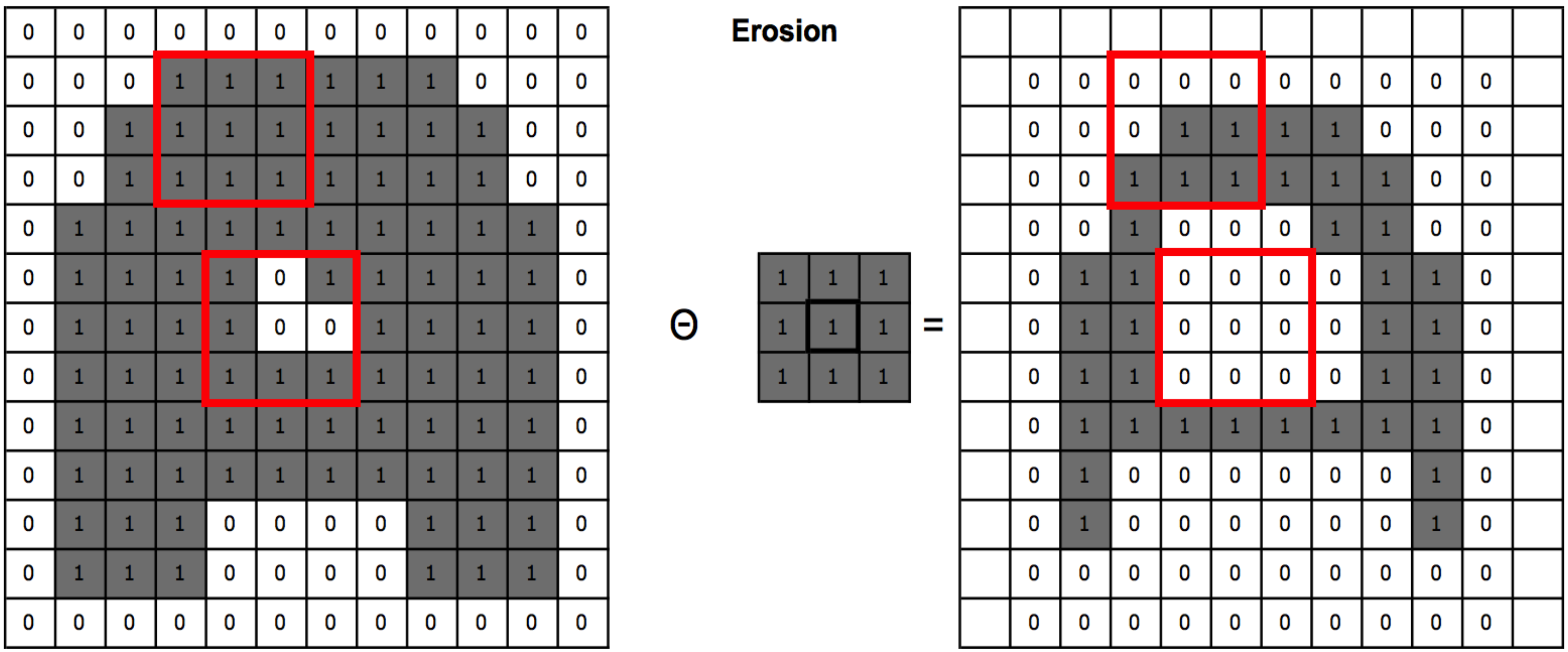
- Explanation with **rank order operation**:
  - The target pixel is the **maximum** of all source pixel under the SE.



Morphological Filters: Erosion

- Explanation with **set theory**:
  - B is the set of binary pixels in an image
  - S is the structure element
  - The erosion set constists of all pixels where the intersection of B & S is S.

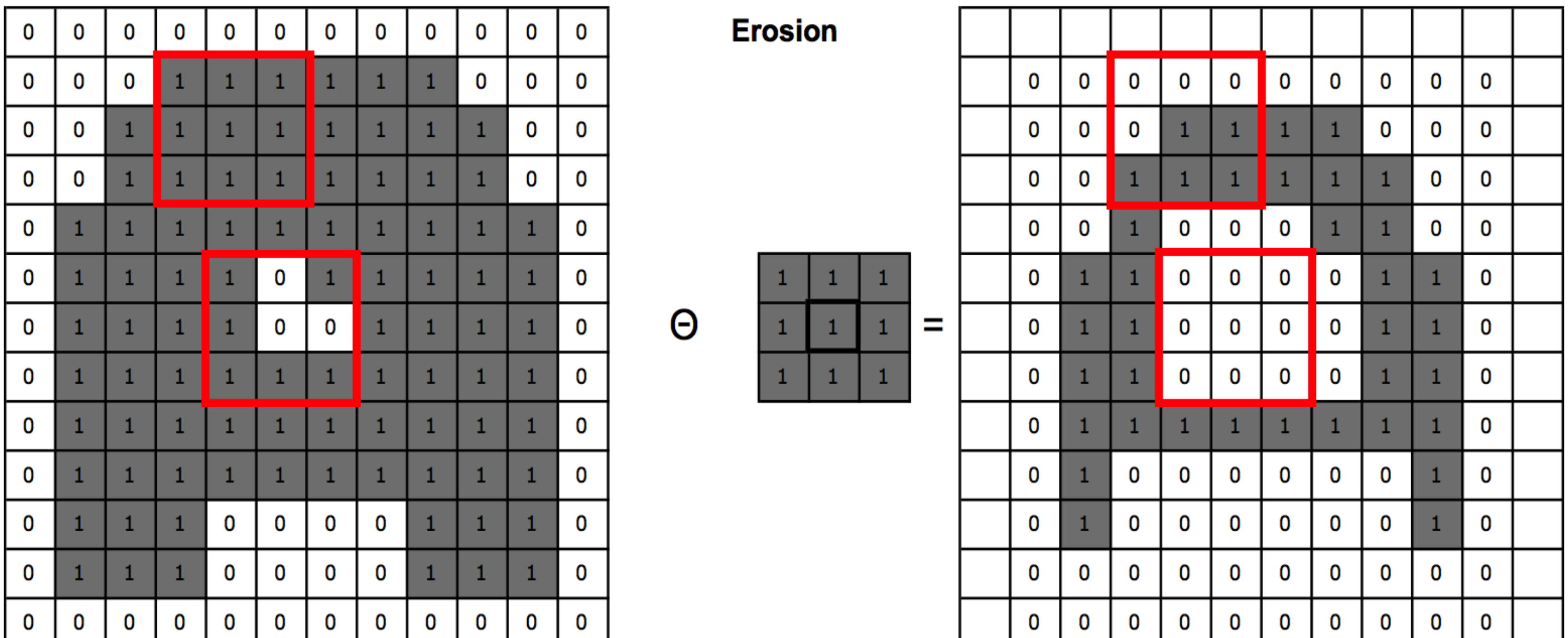
$$B_{erosion} = B \ominus S = \{ B \cap S = S \quad \text{für alle Pos.}\ S \text{ in } B \}$$



Morphological Filters: Erosion

- Explanation with **logic operation**:
  - The target pixel is the result of an **AND** of all pixels in the source image under the SE.

- Explanation with **rank order operation**:
  - The target pixel is the **minimum** of all source pixel under the SE.



Morphological Filters: Properties

- **Properties** of Dilation and Erosion:
  - **Dilation & Erosion are not invertable.**
  - **Dilation & Erosion are dual**: A dilation of the foreground can be done with an erosion on the background and vice versa:

$$\overline{B} \oplus S = \overline{\left(B \ominus S\right)} \qquad \overline{B} \ominus S = \overline{\left(B \oplus S\right)}$$

  - **Dilation is commutative**:

$$B \oplus S = S \oplus B$$

  - **Dilation is associative**:

$$\left(B \oplus S_1\right) \oplus S_2 = B \oplus \left(S_1 \oplus S_2\right)$$

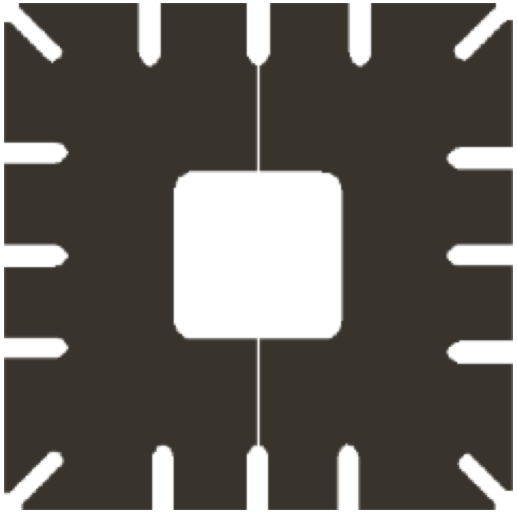
  - **Erosion is not commutative**:

$$B \ominus S \neq S \ominus B$$

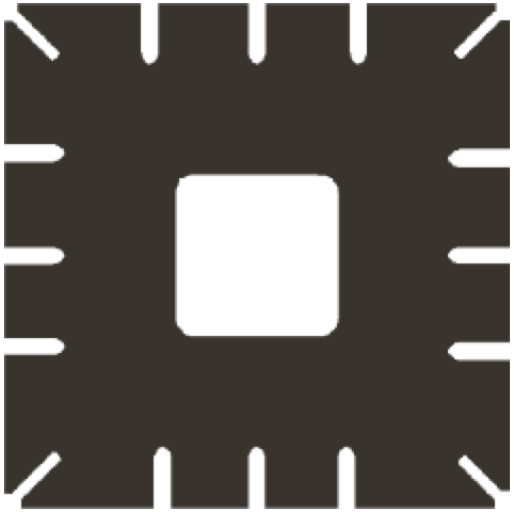
Morphological Filters: Example

Original 486x486

Erosion with 11x11



Erosion with 15x15



Erosion with 45x45

Image Source: Gonzales & Woods

Morphological Filters: Closing

- A **closing operation** is a dilation followed by an erosion with the mirrored SE:

$$B_{Closing} = B \bullet S = \left(B \oplus S \right) \ominus S'$$

- Holes and concave edges are filled up.
- Bridges between unconnected regions can result.

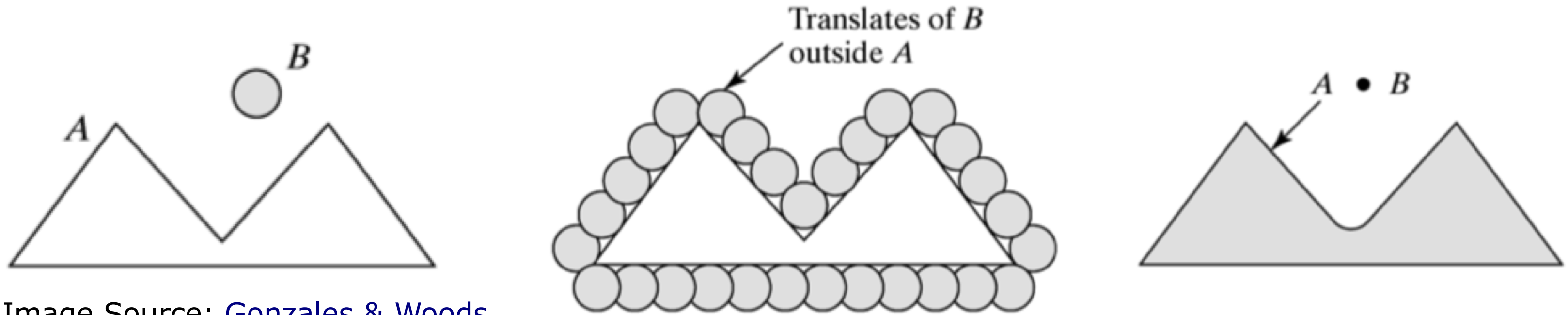


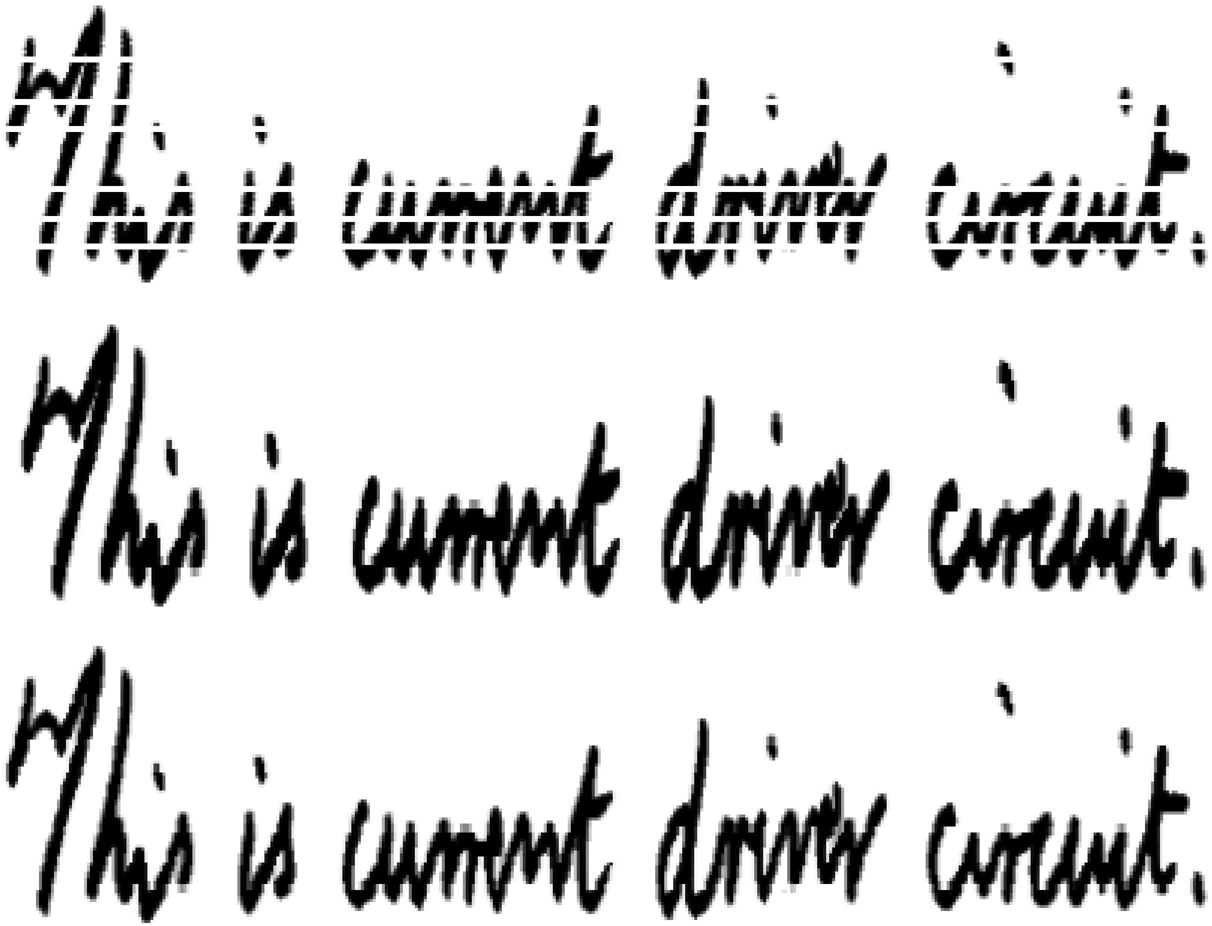
Image Source: Gonzales & Woods

Morphological Filters: Closing

- A **closing operation** is a dilation followed by an erosion with the mirrored SE:

$$B_{Closing} = B \bullet S = \left(B \oplus S \right) \ominus S'$$

- Example closing with 1x5 linear SE:



Morphological Filters: Opening

- A **opening operation** is an erosion followed by a dilation with the mirrored SE:

$$B_{Opening} = B \circ S = (B \ominus S) \oplus S'$$

- Small structures get deleted.
- Convex edges get flattened.

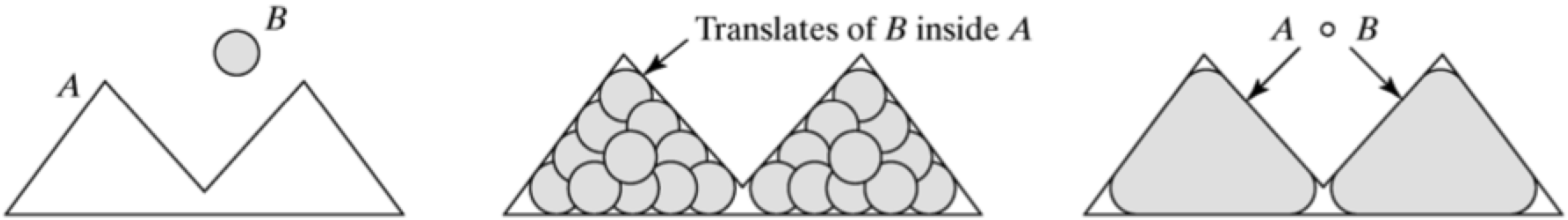


Image Source: Gonzales & Woods

Morphological Filters: Opening

- A **opening operation** is an erosion followed by a dilation with the mirrored SE:

$$B_{Opening} = B \circ S = \left(B \ominus S\right) \oplus S'$$

- Example of a closing followed by a opening by an 8-connecting SE:

$$B' = \left(\left(\left(B \ominus S_8\right) \oplus S_8\right) \oplus S_8\right) \ominus S_8 = \left(B \circ S_8\right) \bullet S_8$$

$\ominus S_8 =$

$\oplus S_8 =$

$\oplus S_8 =$

$\ominus S_8 =$

Image Source: Gonzales & Woods

Morphological Filters: Properties

- Properties of Opening & Closing:
  - Both are final: Another operation of the same doesn't change anything:

$$\left(B \circ S\right) \circ S = B \circ S$$

$$\left(B \bullet S\right) \bullet S = B \bullet S$$

  - Both are dual: An Opening on the foreground is equal to the Closing of the background:

$$B \circ S = \overline{\overline{B} \bullet S}$$

$$B \bullet S = \overline{\overline{B} \circ S}$$

Morphological Filters: Edge Extraction

- By differencing the source image with dilated or eroded image we can extract edges of the source image:
- Rule: Subtract the thinner from the thicker image:

$$B_{\text{edge intern}} = (B \ominus S) \setminus B = B - (B \ominus S)$$

$$B_{\text{edge extern}} = (B \oplus S) \setminus B = (B \oplus S) - B$$

$$B_{\text{edge thick}} = (B \oplus S) - (B \ominus S)$$

Morphological Filters: Edge Extraction

Internal edges with the 8-connection SE: $B_{\text{edge intern}} = (B \ominus S) \setminus B = B - (B \ominus S)$

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	0	1	1	1	1	0	0
0	0	1	1	1	1	0	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Erosion

$\ominus$

1	1	1
1	1	1
1	1	1

=

	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	1	0	0	0	0	0	
	0	0	0	1	1	1	1	1	0	0	0	
	0	0	1	1	0	0	0	0	0	0	0	
	0	0	1	1	0	0	0	0	0	0	0	
	0	0	1	1	0	0	0	0	0	0	0	
	0	0	1	1	0	0	0	1	1	0	0	
	0	0	1	1	0	0	0	1	1	0	0	
	0	1	1	1	1	1	1	1	1	1	0	
	0	1	0	0	0	0	0	0	0	1	0	
	0	0	0	0	0	0	0	0	0	0	0	

\ Original =

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	0	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0
0	0	1	0	0	1	1	1	1	1	1	0	0
0	0	1	0	0	1	0	0	0	0	0	0	0
0	0	1	0	0	1	0	1	1	1	1	0	0
0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	1	0	0	1	1	1	0	0	1	1	0
0	1	0	0	0	0	0	0	0	0	0	1	0
0	1	0	1	1	1	1	1	1	1	0	1	0
0	1	1	1	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Internal edges with the 4-connection SE:

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	0	1	1	1	1	0	0
0	0	1	1	1	1	0	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Erosion

$\ominus$

	1	
1	1	1
	1	

=

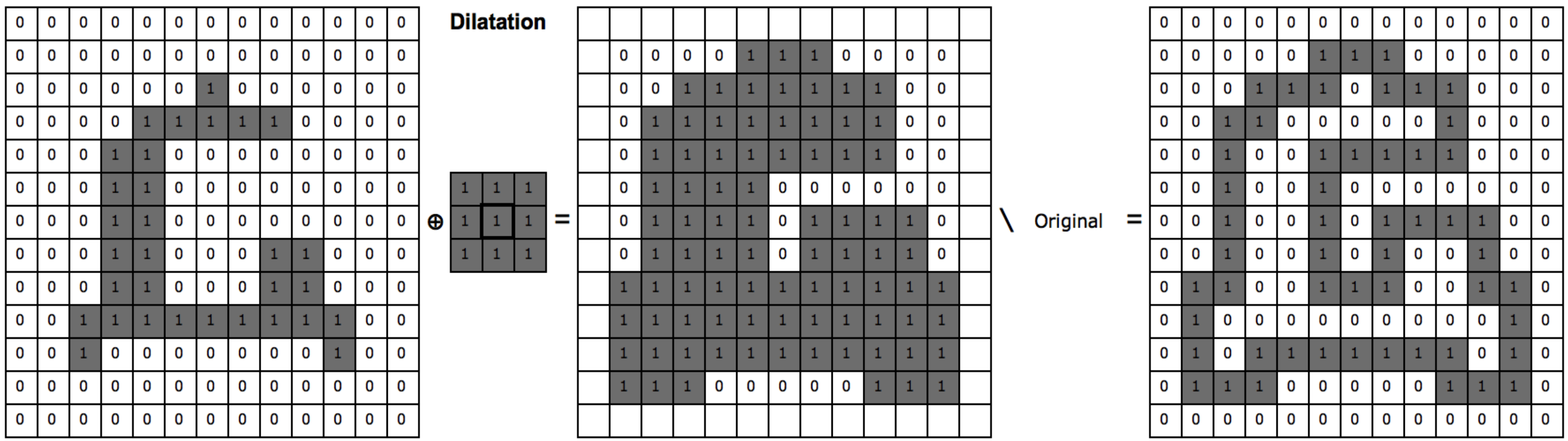
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	1	1	1	0	0	0	0	
	0	0	1	1	1	1	1	1	1	0	0	
	0	0	1	1	1	0	0	0	0	0	0	
	0	0	1	1	0	0	0	0	0	0	0	
	0	0	1	1	0	0	0	0	0	0	0	
	0	0	1	1	0	0	0	1	1	0	0	
	0	1	1	1	1	0	1	1	1	1	0	
	0	1	1	1	1	1	1	1	1	1	0	
	0	1	1	0	0	0	0	0	1	1	0	
	0	0	0	0	0	0	0	0	0	0	0	

\ Original =

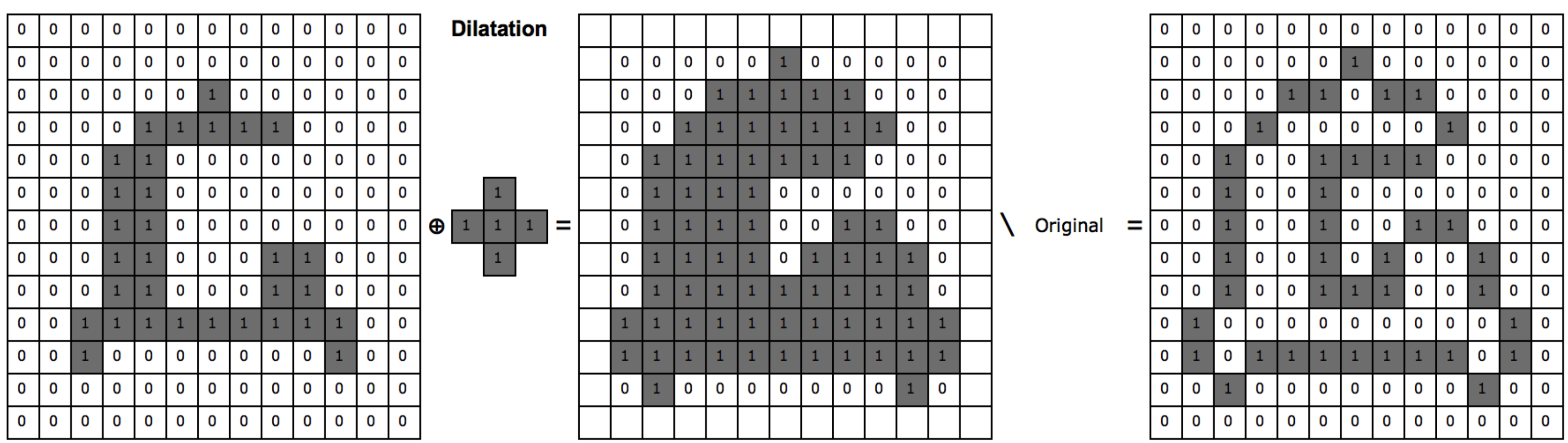
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	1	1	0	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	1	1	1	1	1	0	0
0	0	1	0	0	1	0	0	0	0	0	0	0
0	0	1	0	0	1	0	1	1	1	1	0	0
0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	0	0	0	1	0	0	0	0	1	0
0	1	0	0	0	0	0	0	0	0	0	1	0
0	1	0	0	1	1	1	1	1	0	0	1	0
0	1	1	1	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Morphological Filters: Edge Extraction

External edges with the 8-connection SE: $B_{\text{edge extern}} = \left(B \oplus S \right) \setminus B = \left(B \oplus S \right) - B$



External edges with the 4-connection SE:



Morphological Filters: Hit or Miss Operator

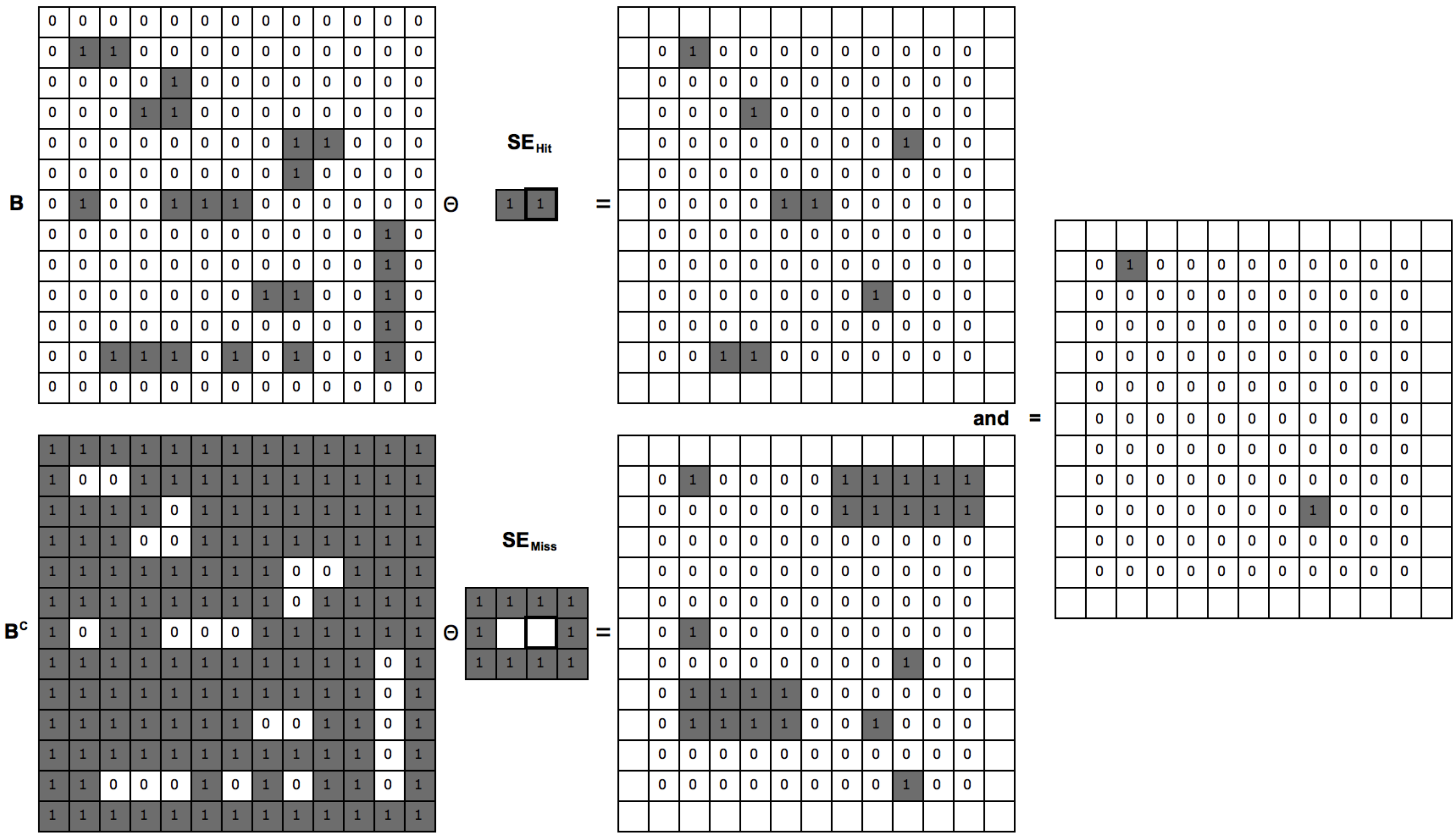
- With the **Hit or Miss operator** we can **detect patterns equal to the SE**.
- It can be done in two steps:
    - In the Hit step we get all patterns equal to the SE.
    - In the Miss step we get the same surrounding of the SE.
        - The Miss step is done on the inverted (complement) image & with the inverted SE.
    - Both steps are combined with an AND.

$$B \otimes E = \left(B \ominus S_{Hit} \right) \cap \left(B^C \ominus S_{Miss} \right) \quad \text{mit } B^C = \overline{B}$$

Morphological Filters: Hit or Miss Operator

- With the **Hit or Miss operator** we can **detect patterns equal to the SE**:

$$B \otimes E = \left(B \ominus S_{Hit} \right) \cap \left(B^{C} \ominus S_{Miss} \right) \quad \text{mit } B^{C} = \overline{B}$$



Morphological Filters: Hit or Miss Operator

- The **Hit or Miss operator** can be done in one step if we allow 0 in the SE:

$$B \otimes E = \left(B \ominus S_{Hit} \right) \cap \left(B^C \ominus S_{Miss} \right) \qquad \text{mit } B^C = \overline{B}$$

- Example: Detecting lower left corners:

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	1	1	1	1	1	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

$\ominus$

	1	
0	1	1
0	0	

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	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	1	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	1	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	

Morphological Filters: Fast Implementation

- If we **move the SE** as with alle local operators over the source image we get a **slow performance** (as usual with local operators).
- If we move the source image over the SE:
  - If we combine all shift with an OR we get a dilation.
  - If we combine all shift with an AND we get an erosion.

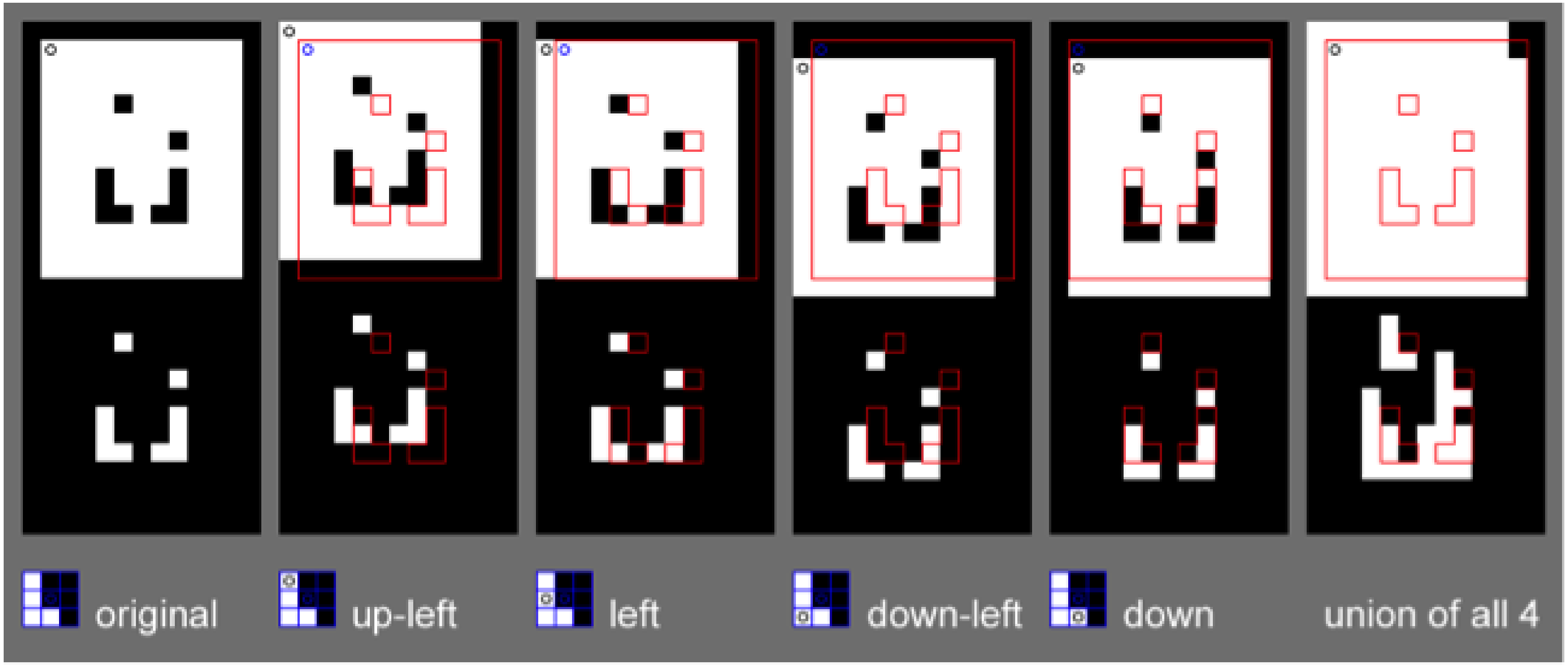
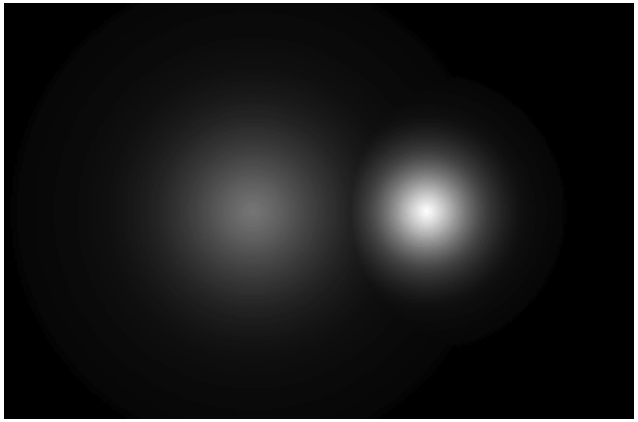
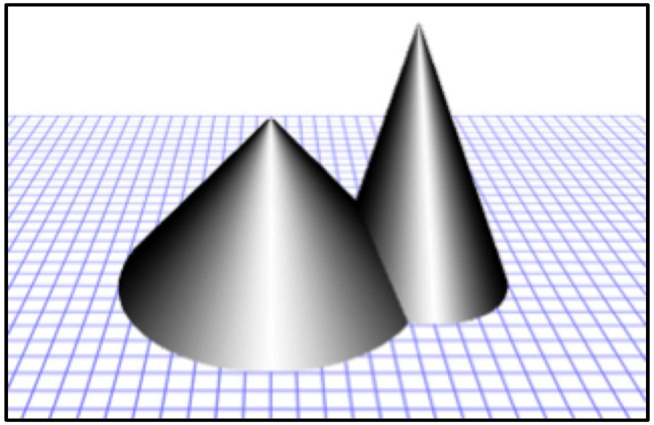


Image Source: R. A. Peters, Vanderbilt University

Morphological Filters: Grayscale Morphology

- Grayscale image can be interpreted as 3D structure:





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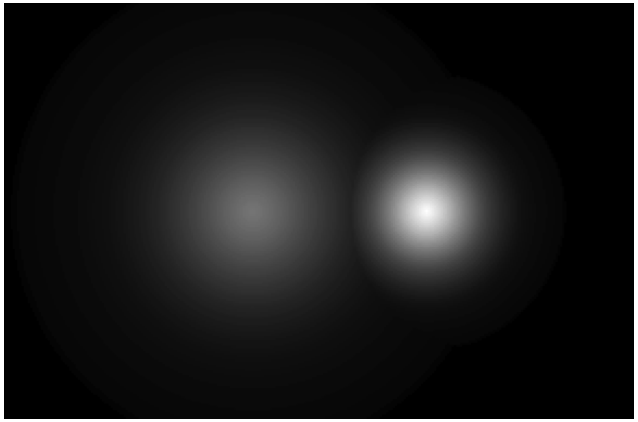
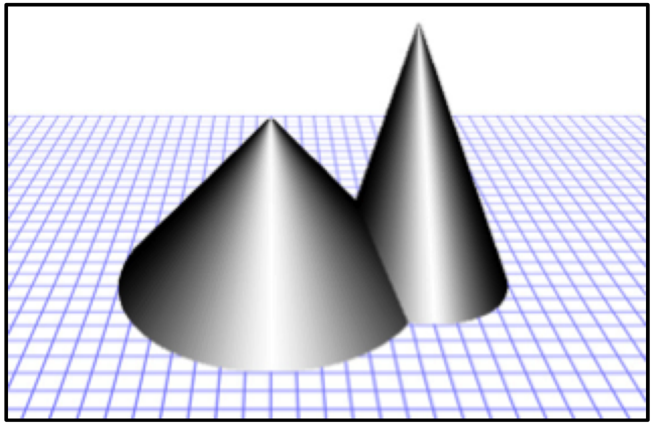




- Also morphological operations can be better explained with 3D set operations.
- If a set is a subset of another depends explicitelly on the hight of the sets:

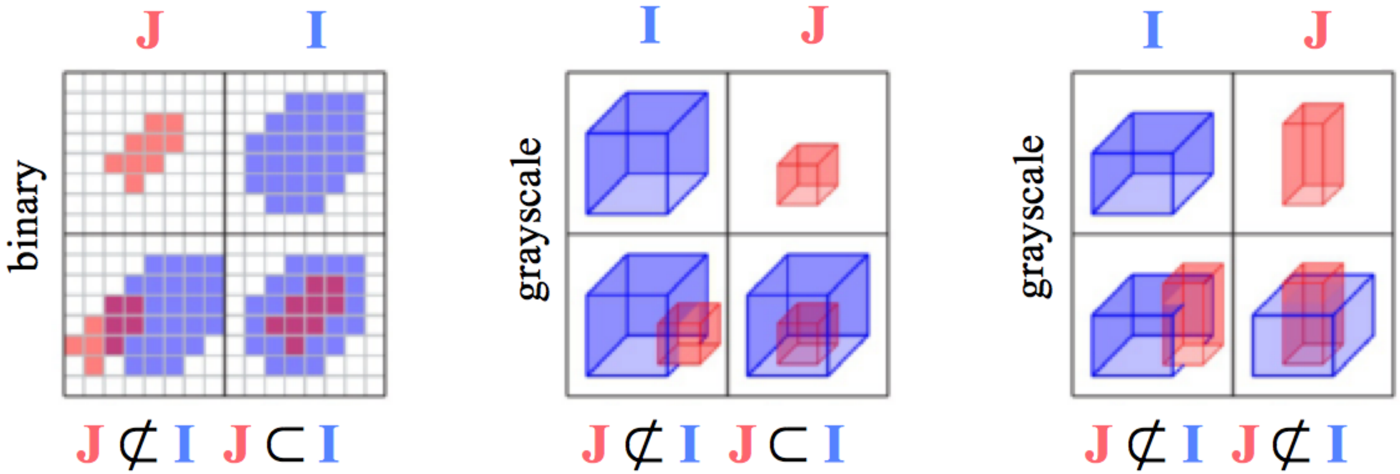
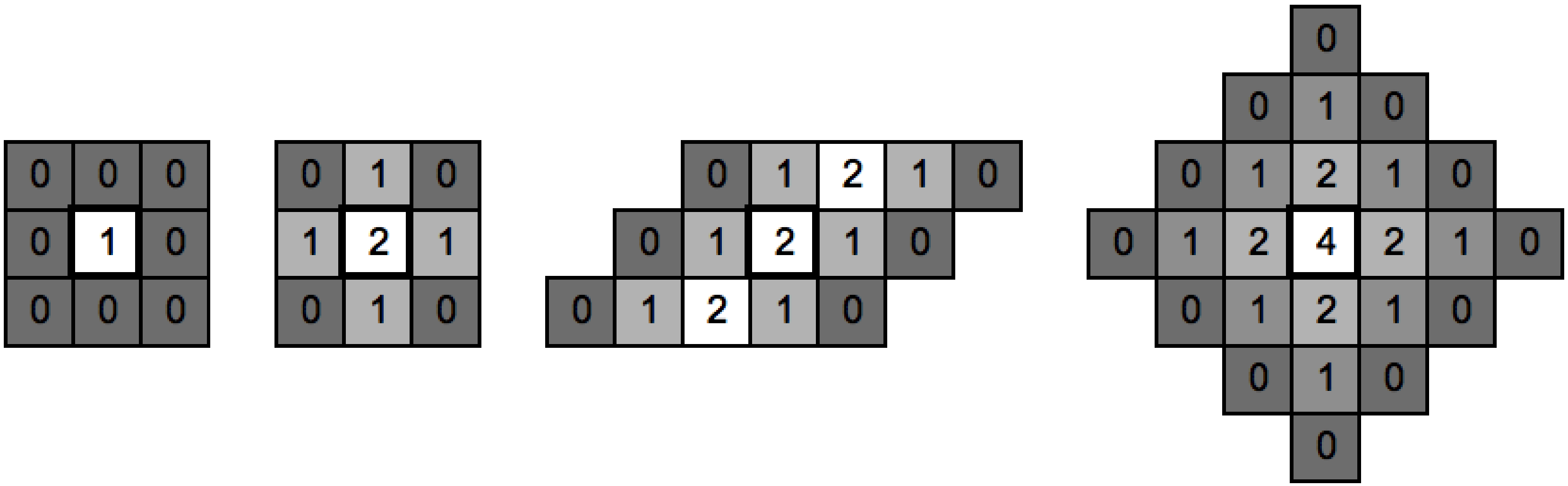


Image Source: R. A. Peters, Vanderbilt University

Morphological Filters: Grayscale Morphology

- For grayscale morphology the values in the SE range is 0...2^n
- In mathematical morphology the SE and the image contain real numbers.
- A 0 is not the same as outside of the SE.
- Some SE examples:



Morphological Filter: Grayscale Dilation & Erosion

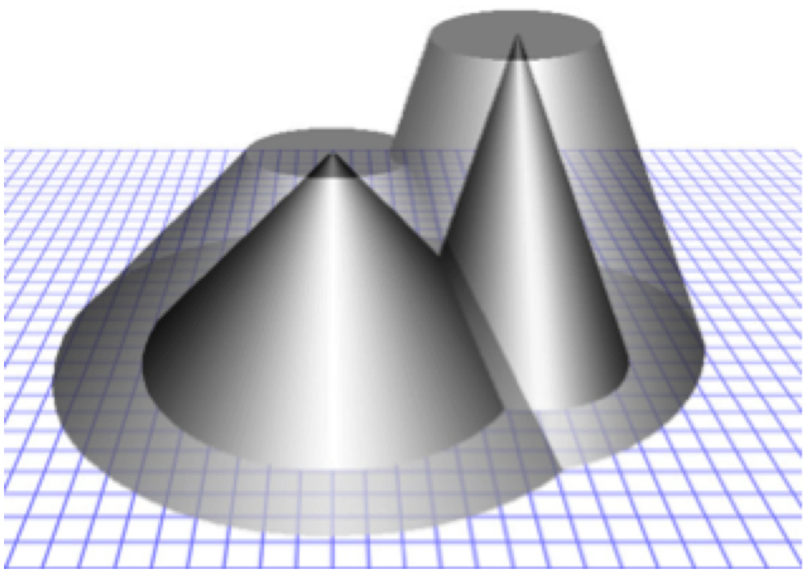
- The **grayscale dilation & erosion** are defined as **rank order operations**:

$$G'(x,y) = G(x,y) \oplus S(i,j) = max(G(x+i, y+j) + S(i,j))$$

$$G'(x,y) = G(x,y) \ominus S(i,j) = min(G(x+i, y+j) - S(i,j))$$

- If all elements in the SE are zero, this definitions correspond to the binary ones.

Dilation over original



Erosion under original

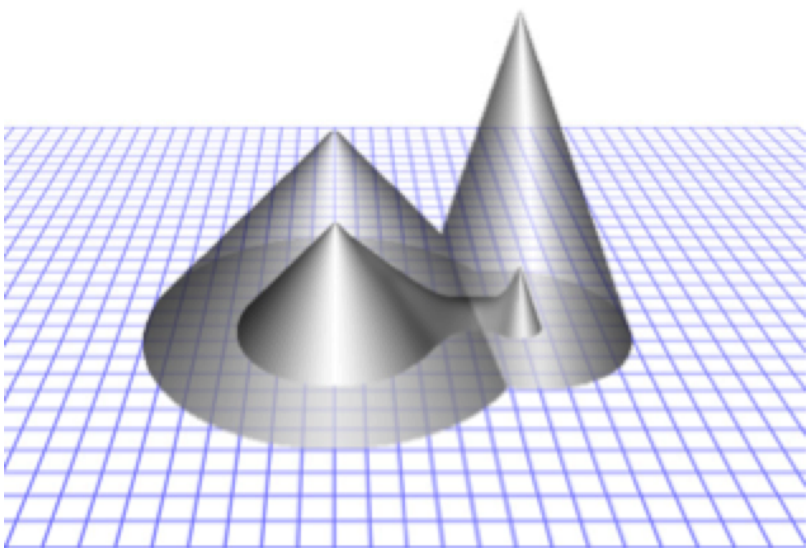


Image Source: R. A. Peters, Vanderbilt University

Morphological Filter: Grayscale Dilation & Erosion

- If all elements in the SE are zero, this definitions correspond to the binary ones.
- For all the following examples **the SE values are zero**!
- The dilation is a Maximum Filter:

$$G'(x,y) = G(x,y) \oplus S(i,j) = max(G(x+i, y+j))$$

- The erosion is a Minimum Filter:

$$G'(x,y) = G(x,y) \ominus S(i,j) = min(G(x+i, y+j))$$

Morphological Filter: Grayscale Dilation & Erosion

- **All SE values are zero**:

$$G'(x,y) = G(x,y) \oplus S(i,j) = max(G(x+i, y+j))$$

$$G'(x,y) = G(x,y) \ominus S(i,j) = min(G(x+i, y+j))$$

0	6	0	0	0	0	1	2	3	2	3	2	1
2	0	2	0	0	0	2	3	2	3	2	1	1
2	2	0	3	0	1	0	0	3	4	3	2	0
5	0	2	1	**0**	**2**	**0**	**5**	**0**	3	2	1	1
0	0	3	2	**5**	**3**	**4**	**0**	**2**	4	0	4	0
5	0	4	0	**3**	**68**	**2**	**1**	**3**	0	3	2	1
0	0	3	3	**0**	**0**	**3**	**2**	**4**	0	0	0	5
3	0	3	3	**3**	**2**	**5**	**3**	**2**	2	2	3	6
2	0	2	0	6	0	2	0	0	3	0	5	2
1	5	2	5	2	5	3	1	1	5	0	6	5
3	2	0	0	5	2	0	0	0	2	3	2	2
2	0	0	0	0	0	2	3	0	3	5	3	2
1	0	5	2	5	0	0	5	3	0	5	1	3
0	1	0	0	3	1	2	3	5	0	1	0	5

0	0	0	0	0	0	1	2	2	2	2	2	**3**	3	3	3	3	3	3	4	4	5	5	5	**68**

Minimum Median Maximum

Morphological Filter: Grayscale Dilation & Erosion

Original & Dilation with an 8-connection SE:

Morphological Filter: Grayscale Dilation & Erosion

Original & Erosion with an 8-connection SE:

Morphological Filters: Grayscale Median Filter

- A **very powerful denoising** filter is the **median filter**.
- It takes the **median value** from the sorted rank order:
- It is very good filtering out **salt & pepper noise**:

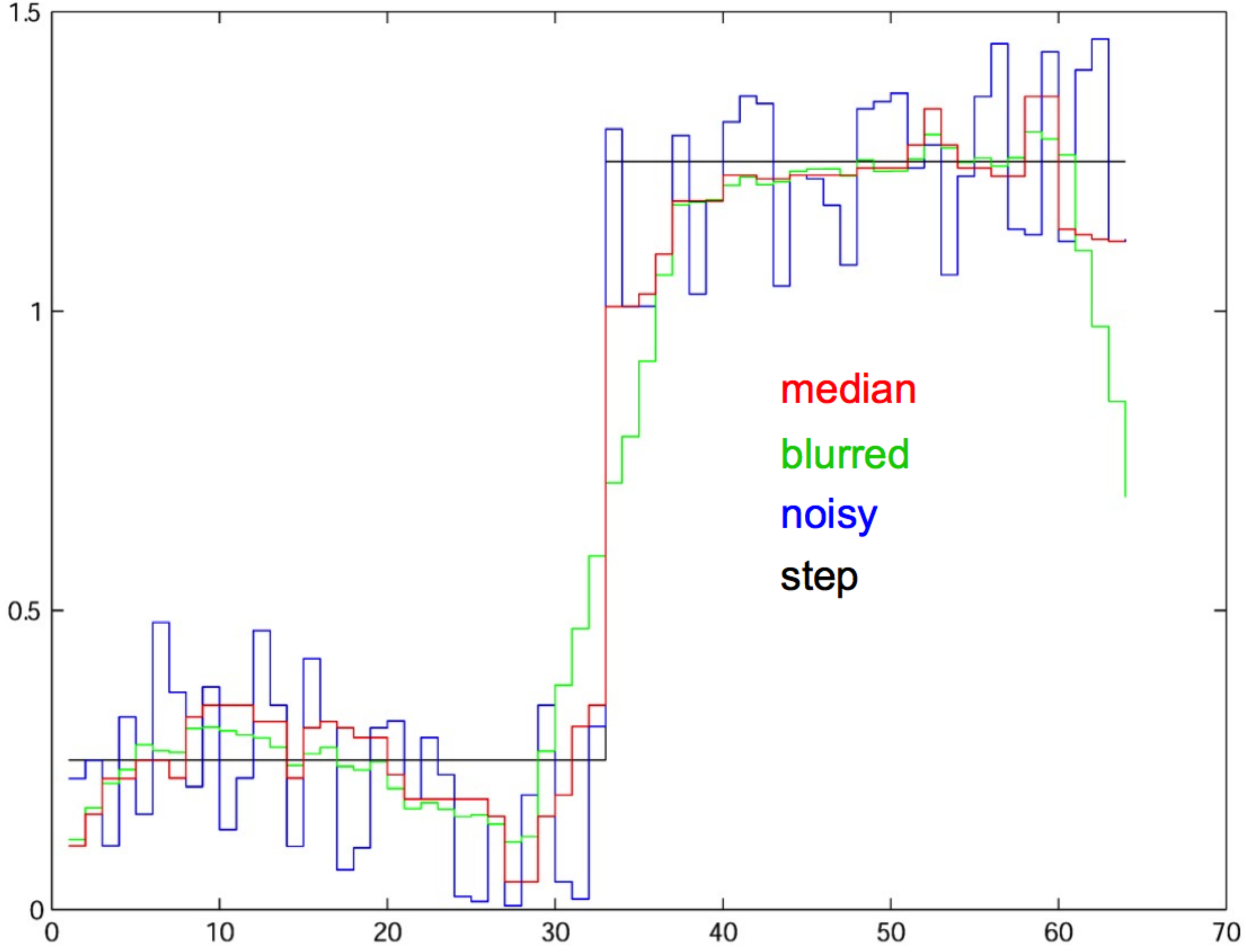
0	6	0	0	0	0	1	2	3	2	3	2	1
2	0	2	0	0	0	2	3	2	3	2	1	1
2	2	0	3	0	1	0	0	3	4	3	2	0
5	0	2	1	**0**	**2**	**0**	**5**	**0**	3	2	1	1
0	0	3	2	**5**	**3**	**4**	**0**	**2**	4	0	4	0
5	0	4	0	**3**	**68**	**2**	**1**	**3**	0	3	2	1
0	0	3	3	**0**	**0**	**3**	**2**	**4**	0	0	0	5
3	0	3	3	**3**	**2**	**5**	**3**	**2**	2	2	3	6
2	0	2	0	6	0	2	0	0	3	0	5	2
1	5	2	5	2	5	3	1	1	5	0	6	5
3	2	0	0	5	2	0	0	0	2	3	2	2
2	0	0	0	0	0	2	3	0	3	5	3	2
1	0	5	2	5	0	0	5	3	0	5	1	3
0	1	0	0	3	1	2	3	5	0	1	0	5

0	0	0	0	0	0	1	2	2	2	2	2	**3**	3	3	3	3	3	3	4	4	5	5	5	**68**

Minimum — Median — Maximum

Morphological Filters: Grayscale Median Filter

The median filter keeps sharp noise edges sharp:



Morphological Filters: Grayscale Median Filter

Original with salt & pepper noise and filtered with the 3x3 median filter:

Morphological Filters: Grayscale Median Filter

Original with salt & pepper noise and filtered with the 3x3 gaussian filter:

Morphological Filters: Grayscale Median Filter

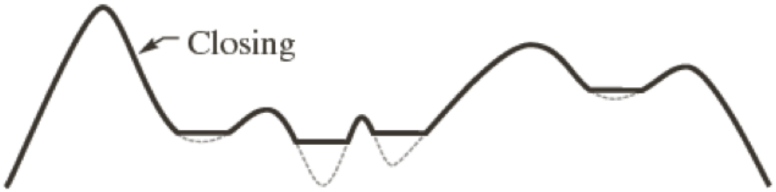
Original gaussian noise (sigma: 10.0) and twice filtered with 3x3 median filter:

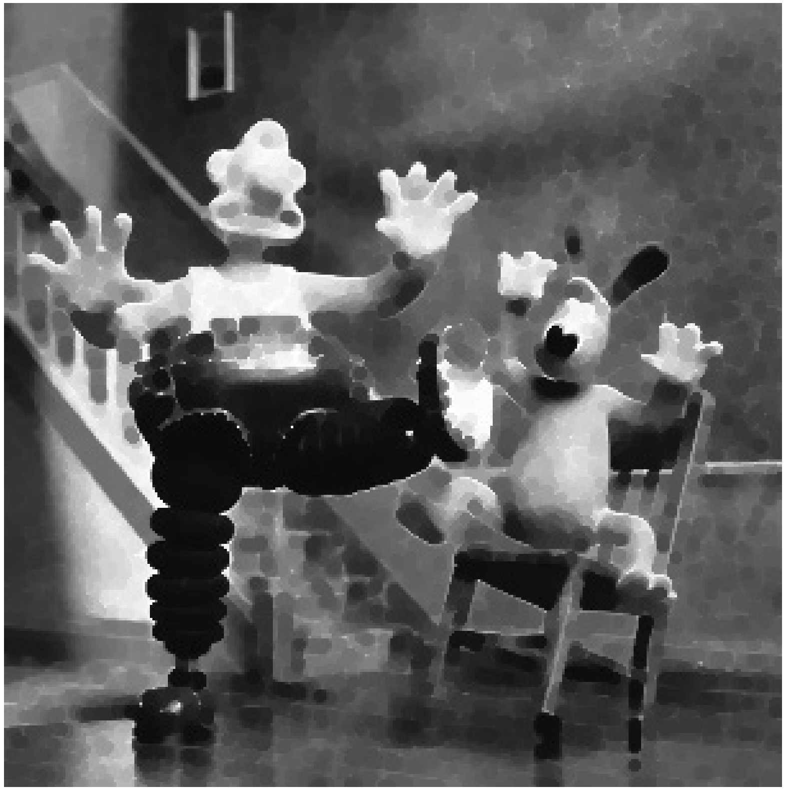
Original gaussian noise (sigma: 10.0) and twice filtered with 3x3 gaussian filter:

Morphological Filter: Grayscale Closing & Opening

- Closing & Opening are as well a combination dilation & erosion.
- A **closing keeps bright details** and **deletes dark details**:

$$G_{Closing} = G \bullet S = (G \oplus S) \ominus S'$$





Morphological Filter: Grayscale Closing & Opening

- Closing & Opening are as well a combination dilation & erosion.
- An **opening keeps dark details** and **deletes bright details**:

$$G_{Opening} = G \circ S = (G \ominus S) \oplus S'$$

Morphological Filters: Grayscale Edge Extraction

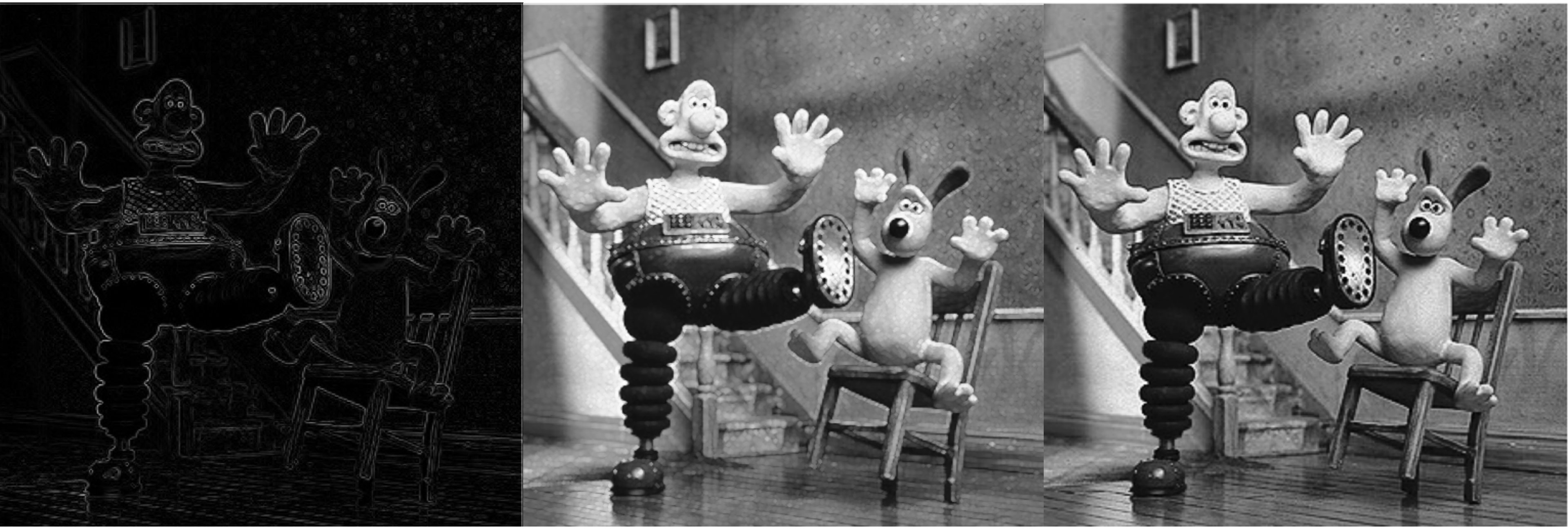
- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

$$G_{\text{edge intern}} = G - (G \ominus S)$$

Morphological Filters: Grayscale Edge Extraction

- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

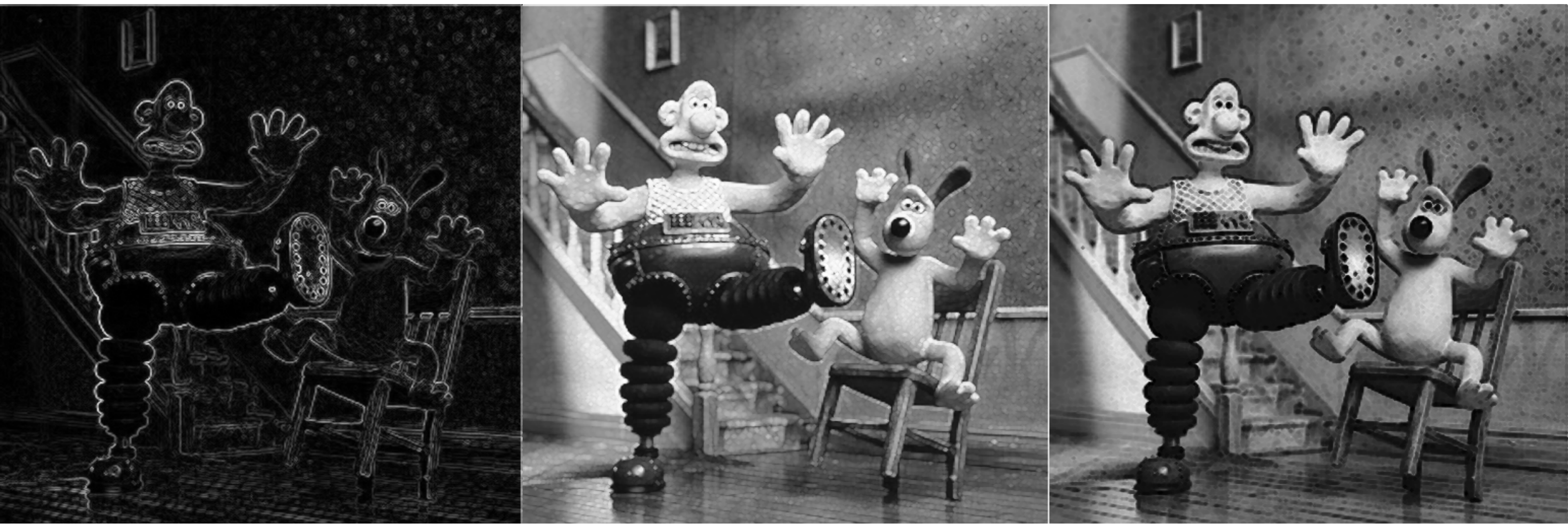
$$G_{\text{edge extern}} = \left(G \oplus S\right) - G$$



Morphological Filters: Grayscale Edge Extraction

- **Edge extraction can be achieved with differencing** the original and the eroded or dilated image.

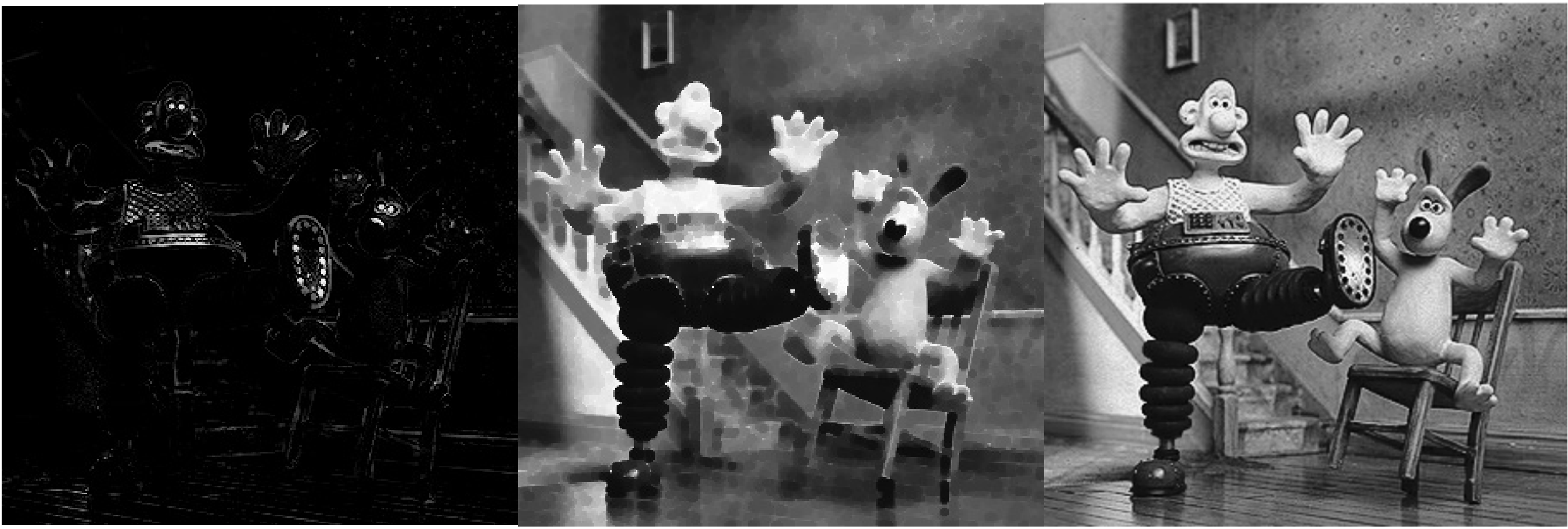
$$G_{\text{edge thick}} = (G \oplus S) - (G \ominus S)$$

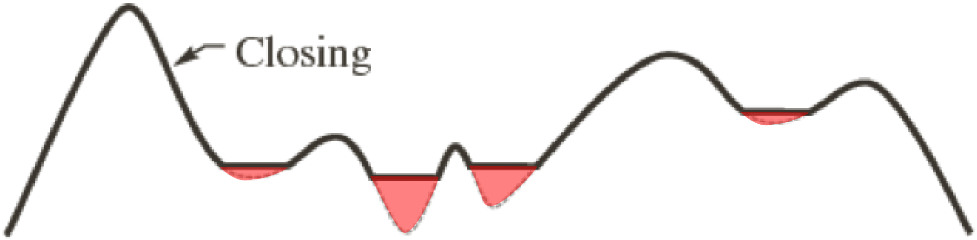


Morphological Filters: Bot Hat Operator

- The **Bot Hat** operator makes small details bright that are darker than the surrounding.

$$G_{Bothat} = G_{Closing} - G = (G \bullet S) - G$$

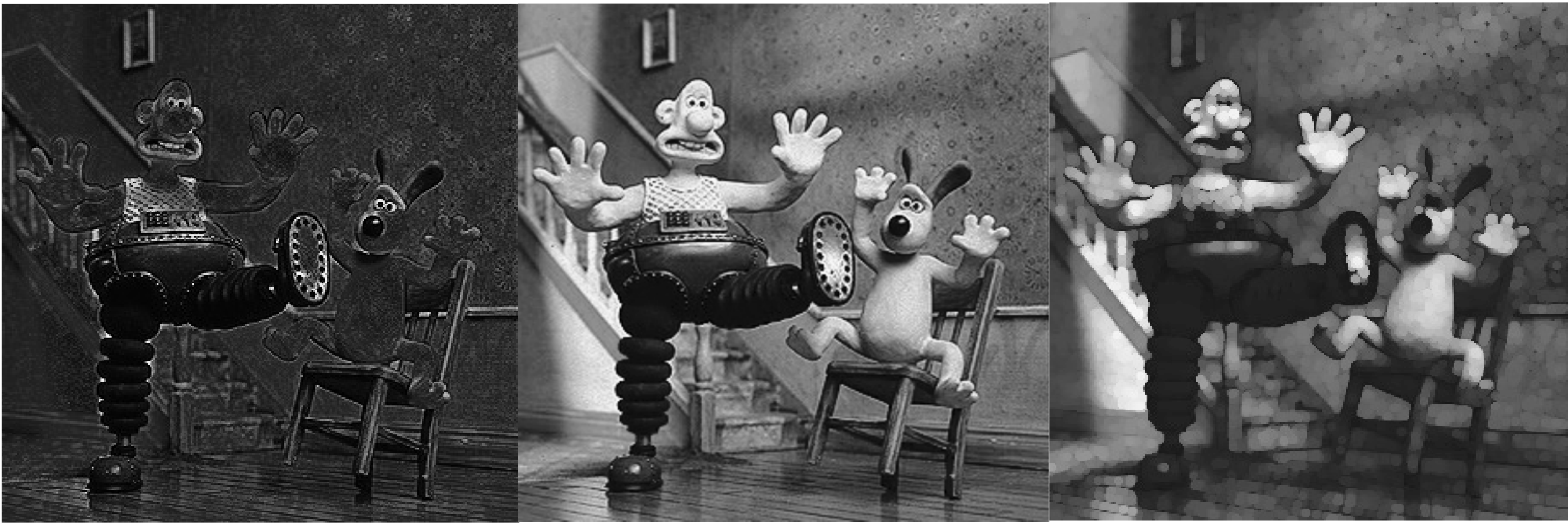


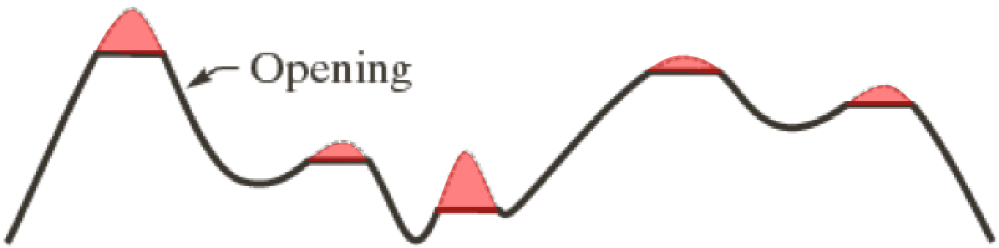


Morphological Filters: Top Hat Operator

- The **Top Hat** operator makes small details bright that are brighter than the surrounding.

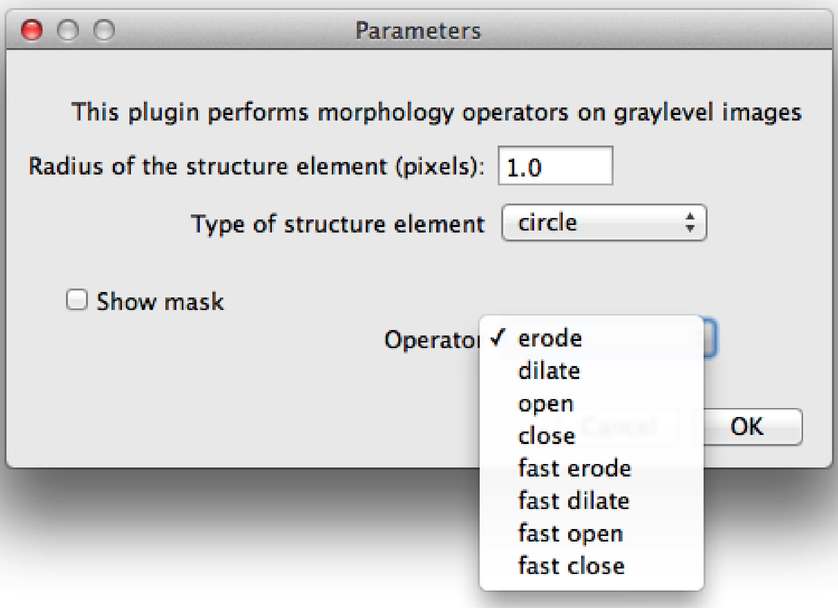
$$G_{Tothat} = G - G_{Opening} = G - (G \circ S)$$





Morphological Operators in ImageJ

- **ImageJ** offers in the menu *Process > Binary* the operators *Erode*, *Dilate*, *Open*, *Close* and *Outline.*
- All operators use the **8-connection SE**.
- **Fiji** offers in the menu *Process > Morphology > Gray Morphology* a dialog for grayscale morphology with various SE and operations:



Morphological Operators in ImageJ

- For programming the *ImageProcessor* class provides the essential operators *dilate()*, *erode()*, *open()* and *close()*.
- All methods can be applied on binary and grayscale images.
- The *ColorProcessor* provides the same for color images with the application on the color channels seperately.
- All operators use the 8-connection SE.
- There are additional ImageJ plugins with more detailed operators.

Morphological Operators in Matlab

- Matlab offer generic and specific morphological functions.
- A SE can be defined with the function **strel**:

```
I = imread('../images/circuit.png');      % Load image into matrix X
J = imcomplement(I);                      % Invert image
BW1 = im2bw(J,0.8);                       % make black & white
SE1 = strel('rectangle',[30 20]);         % 40x30 rectangulare SE
BW2 = imerode(BW1,SE1);                   % erode w. 40x30 rectangular SE
BW3 = imdilate(BW2,SE1);                  % dilate w. the same SE

imshow(I); title('Original');             % Show images
figure; imshow(BW2); title('After erosion');
figure; imshow(BW1); title('After binarisation');
figure; imshow(BW3); title('After dilation');
```

Morphological Operators: Exercise

- Implement the dilation & ersion in Excel or OpenOffice Calc.
- Add salt & pepper noise and gaussian noise to an image.
  - Denoise the image with the median and gaussian filter.
  - Try various sizes of the SE.
  - Try multiple subsequent filterings.
- Optional: Implement the same operations in ImageJ as in the Matlab example.