Assignment on Markov Chains

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1 Markov Chains for Politics (1 Point)

Consider the following *opinion shift table* that cross-classifies the attitude of 493 randomly selected Swiss people towards a referendum in July 2013 (rows) by their attitude towards the same referendum in October 2013 (columns).

	yes	no	undecided	total
yes	167	36	15	218
no	19	131	10	160
undecided	45	50	20	115
total	231	217	45	493

1. Derive a transition matrix from this opinion shift table.

$$\begin{bmatrix} \frac{167}{218} & \frac{36}{218} & \frac{15}{218} \\ \frac{19}{160} & \frac{131}{160} & \frac{10}{160} \\ \frac{45}{115} & \frac{50}{115} & \frac{20}{115} \end{bmatrix} = \begin{bmatrix} 0.766 & 0.165 & 0.069 \\ 0.119 & 0.819 & 0.062 \\ 0.391 & 0.435 & 0.174 \end{bmatrix}$$

2. Compute the stationary distribution for the transition matrix.

$$[p_1 \quad p_2 \quad p_3]\begin{bmatrix} 0.766 & 0.165 & 0.069 \\ 0.119 & 0.819 & 0.062 \\ 0.391 & 0.435 & 0.174 \end{bmatrix} = [p_1 \quad p_2 \quad p_3]$$

$$\text{I)}\ 0.766p_1 + 0.119p_2 + 0.391p_3 = p_1 \Rightarrow -0.234p_1 + 0.119p_2 + 0.391p_3 = 0$$

$$\text{II)}\ 0.165p_1 + 0.819p_2 + 0.435p_3 = p_2 \Rightarrow 0.165p_1 - 0.181p_2 + 0.435p_3 = 0$$

$$\text{III)}\ p_1 + p_2 + p_3 = 1$$

$$\text{I} + 0.234 \cdot \text{III}: \quad -0.234p_1 + 0.119p_2 + 0.391p_3 = 0$$
$$0.234p_1 + 0.234p_2 + 0.234p_3 = 0.234$$
$$0.353p_2 + 0.625p_3 = 0.234 \Rightarrow 0.353p_2 = -0.625p_3 + 0.234$$
$$\Rightarrow p_2 = -1.771p_3 + 0.663$$

$$\text{II} - 0.165 \cdot \text{III}: \quad 0.165p_1 - 0.181p_2 + 0.435p_3 = 0$$
$$-0.165p_1 - 0.165p_2 - 0.165p_3 = -0.165$$
$$-0.346p_2 + 0.270p_3 = -0.165 \Rightarrow 0.346p_2 = 0.270p_3 + 0.165$$
$$p_2 = 0.780p_3 + 0.477$$

$$\Rightarrow -1.771p_2 + 0.663 = 0.780p_3 + 0.477$$
$$2.551p_3 = 0.186$$
$$p_3 = 0.073$$

$$\Rightarrow p_2 = 0.780p_3 + 0.477 = 0.057 + 0.477 = 0.534$$

$$\Rightarrow p_1 = 1 - p_2 - p_3 = 1 - 0.534 - 0.073 = 0.393$$

$$\Rightarrow [p_1 \quad p_2 \quad p_3] = [0.393 \quad 0.534 \quad 0.073]$$

3. The dynamics of opinion shift is 3 months (October minus July). If everybody was initially in favor of the initiative what is the proportion of people remaining in the YES state after 3 months, after 6 months and in the long run? Can this information be useful for the person in charge of the referendum campaign?

Initial results: $[1 \quad 0 \quad 0]$

after 3 months: $[1 \quad 0 \quad 0]\begin{bmatrix} 0.766 & 0.165 & 0.069 \\ 0.119 & 0.819 & 0.062 \\ 0.391 & 0.435 & 0.174 \end{bmatrix} = [0.766 \quad 0.165 \quad 0.069]$

after 6 months: $[0.766 \quad 0.165 \quad 0.069]\begin{bmatrix} 0.766 & 0.165 & 0.069 \\ 0.119 & 0.819 & 0.062 \\ 0.391 & 0.435 & 0.174 \end{bmatrix} = [0.633 \quad 0.292 \quad 0.075]$

The opinion is trending towards the stationary distribution, i.e. if enough time passes, the referendum will not get enough votes. People in charge of the referendum campaign should therefore delay the vote as long as possible.