CV II - Assignment 2

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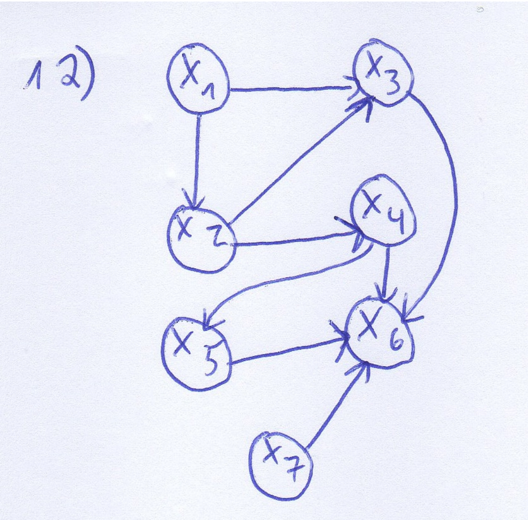
Group 40

Problem 1

1.1

A graphical model can display complex structures or problems in a compact, visual way. They are a very efficient way to handle complex problems.

1.2

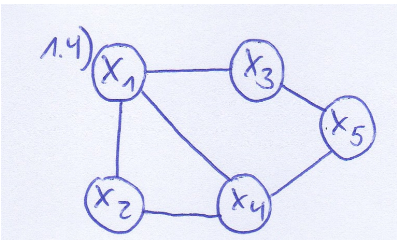


Markov blanket of x_3 : $\{x_1, x_2\}$

1.3

a): $p(x_1|x_3,x_4,x_5)\ p(x_2)\ p(x_3|x_7)\ p(x_4|x_7,x_8)\ p(x_5|x_4,x_8)\ p(x_6|x_5,x_9)\ p(x_7|x_8)\ p(x_8)\ p(x_9)$
b): $\frac{1}{Z}\phi_1(x_1,x_4,x_5)\ \phi_2(x_1,x_3)\ \phi_3(x_4,x_5,x_7,x_8)\ \phi_4(x_3,x_7)\ \phi_5(x_5,x_6)\ \phi_6(x_6,x_9)\ \phi_7(x_2)$

1.4



1.5

x_1	x_2	x_3	$\phi_1(x_1)$	$\phi_2(x_2,x_3)$	$\Pi\phi(\ldots)$	$p(x_1,x_2,x_3)$
0	0	0	0,5	0+0+1=1	0,5	0,024
0	0	1	0,5	0+3+1=4	2	0,095
0	1	0	0,5	2+0+1=3	1,5	0,071
0	1	1	0,5	2+3+1=6	3	0,143
1	0	0	1	0+0+1=1	1	0,048
1	0	1	1	0+3+1=4	4	0,190
1	1	0	1	2+0+1=3	3	0,143
1	1	1	1	2+3+1=6	6	0,286

$\Rightarrow Z = 21$

1.6

a): Fig.2 left:

$x_2 \perp x_1 | x_3$

$\Rightarrow$ x_3 blocks all routes from x_2 to x_1

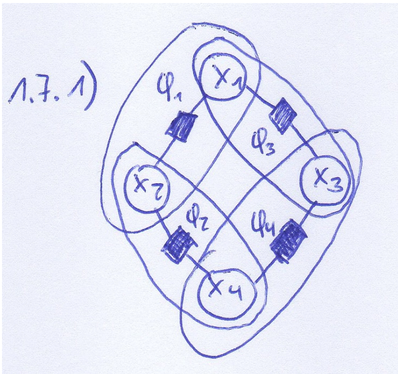
b): Fig.2 right:

$x_2 \perp x_1$

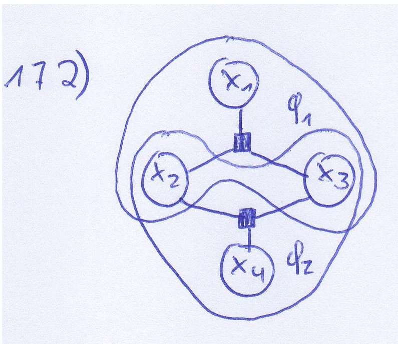
$\rightarrow$ Markov blanket of $x_2 = \{x_3\}$

$\Rightarrow$ independent of x_1

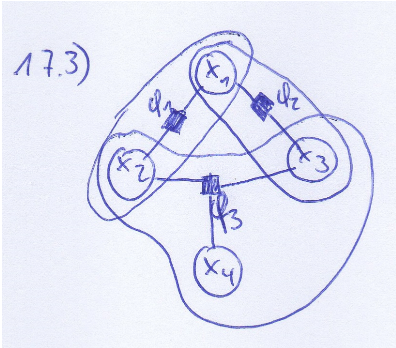
1.7



1.: $\frac{1}{Z}\phi_1(x_1,x_2)\ \phi_2(x_2,x_4)\ \phi_3(x_1,x_3)\ \phi_4(x_3,x_4)$



2.: $\frac{1}{Z}\phi_1(x_1,x_2,x_3)\ \phi_2(x_2,x_3,x_4)$



3.: $\frac{1}{Z}\phi_1(x_1,x_2)\ \phi_2(x_1,x_3)\ \phi_3(x_2,x_3,x_4)$

1.8

$\{w_2, w_4, w_6, w_6, x_5\}$

1.9

$\{w_2, w_3, w_5, x_4\}$

Problem2

2.1

$p(L=0|S=0)$

$$= \frac{p(L{=}1,S{=}0)}{p(S{=}0)} = \frac{\sum_R p(L{=}1,S{=}0,R)}{\sum_{R,L} p(S{=}0,R,L)} = \frac{p(L{=}1,S{=}0,R{=}0)+p(L{=}1,S{=}0,R{=}1)}{p(L{=}1,S{=}0,R{=}0)+p(L{=}1,S{=}0,R{=}1)+p(L{=}0,S{=}0,R{=}0)+p(L{=}0,S{=}0,R{=}1)}$$

$$= \frac{0{,}7\cdot 0{,}7\cdot 0{,}1+0{,}9\cdot 0{,}3\cdot 0{,}1}{0{,}7\cdot 0{,}7\cdot 0{,}1+0{,}9\cdot 0{,}3\cdot 0{,}1+0{,}3\cdot 0{,}7\cdot 0{,}9+0{,}8\cdot 0{,}3\cdot 0{,}9} = \frac{0{,}049+0{,}027}{0{,}049+0{,}027+0{,}189+0{,}216} = \frac{0{,}076}{0{,}481}$$

$\underline{\underline{\approx 0,158}}$

2.2

$p(L=1|S=0,R=0)$

$$= \frac{p(L{=}1,S{=}0,R{=}1)}{\sum_L p(L,S{=}0,R{=}1)} = \frac{p(L{=}1,S{=}0,R{=}1)}{p(L{=}1,S{=}0,R{=}1)+p(L{=}0,S{=}0,R{=}1)}$$

$$= \frac{p(S{=}0|R{=}1,L{=}1)\cdot p(R{=}1)\cdot p(L{=}1)}{p(S{=}0|R{=}1,L{=}1)\cdot p(R{=}1)\cdot p(L{=}1)+p(S{=}0|R{=}1,L{=}0)\cdot p(R{=}1)\cdot p(L{=}0)}$$

$$= \frac{0{,}9\cdot 0{,}3\cdot 0{,}1}{0{,}9\cdot 0{,}3\cdot 0{,}1+0{,}8\cdot 0{,}3\cdot 0{,}9} = \frac{0{,}027}{0{,}027+0{,}216} = \frac{0{,}027}{0{,}243}$$

$\underline{\underline{= 0,\overline{1}}}$

2.3

The observation that there are no free seats left creates an dependency between the two events L and R. Now, with the addition that we observe the raid outside, the probability that L=1 decreases. This event (R=1) provides an additional explanation for our S=0 and intuitively lowers our believing that L=1. So we can say that the observation of R=1 'explains away' the observation of S=0.