# Heat Transfer Final Project

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May 1 2023

**Abstract**

The objective of this study is to design a heat sink in order to enhance the heat transfer from a logic chip. The use of this heat sink will allow for superior heat transfer from the chip to the surrounding fluid, allowing for more voltage to be applied to the processor before any damage occurs. Through a parametric study, which explored the effects of the number of fins, their width, and their spacing, it was found that the most efficient heat sink overall was the one which contained the maximal number of fins with minimum thickness. This design also was the most efficient when considering the density of heat dissipation, or the heat dissipation divided by the mass of the heat sink, but only when the base plate was considered as well. If base plate thickness was neglected, the most efficient design (that contained fins) was a single fin that was as thin as possible.

## 1 Design Criteria

The logic chip for which the cooling is to be designed is a square that is $W_c = 25mm$ wide. The heat sink is immersed into a dielectric liquid with $h = 1500\frac{W}{m^2K}$ and consists of copper with $k = 400\frac{W}{mK}$. The base of the heat sink is considered to have negligible heat resistance, with zero contact resistance to the chip. The maximal height of the fins $L$ is $6mm$ and the maximal chip temperature $T_b$ is $85°C$. The ambient temperature of the liquid $T_\infty$ is $25°C$.

The manufacturing constraints on the chip are as follows: The minimal spacing between fins is to be $1mm$ and the minimal thickness of the fins is also 1mm.

## 2 Theory

The fins in this heat sink have the following properties, which allow for some simplifications to be made:

1. Constant Conductivity $k$
2. Uniform Cross Sectional Area $A_c$
3. Constant Perimeter $P$

4. negligible generation $\dot{q} = 0$

5. negligible radiation $\ddot{q}_{rad} = 0$

Thus the fin equation is as follows:

$$\frac{d^2T}{dx^2} - \frac{hP}{kAc}(T - T_\infty) = 0 \tag{1}$$

Solving this for the boundary condition that states that convection occurs at the tip of the fin as well yields the fin heat rate as:

$$q_f = M\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL} \tag{2}$$

with:

$$m = \sqrt{\frac{hP}{kA_c}} \tag{3}$$

and:

$$M = \sqrt{hPkA_c}\theta_b \tag{4}$$

Additionally, the fin efficiency is defined as:

$$\eta_f = \frac{q_f}{hA_f\theta_b} \tag{5}$$

For a heat sink, which is an array of fins, the total heat rate is defined as:

$$q_t = \eta_0 hA_t\theta_b \tag{6}$$

where $A_t$ is the total area or $NA_f + A_b$ with $N$ being the number of fins and $A_f$ and $Ab$ the respective areas of each fin. The overall surface efficiency for this assembly is:

$$\eta_0 = 1 - \frac{NA - f}{A_t}(1 - \eta_f) \tag{7}$$

# 3 Example Calculation

Using the above equations for an array of 13 fins 1mm thick, with 12 1mm gaps between them for the heat sink design being evaluated would yield the following:

Initially, geometric constants of the anatomy of the heat sink need to be calculated. These are:

- $P_fin = 0.052m$
- $A_c = 2.5\dot{1}0^{-5}m^2$
- $A_f = 0.0003m^2$
- $A_b = 0.0003m^2$

- $A_t = 0.0042m^2$

From the problem statement we also know $\theta_b = 65°C$ and from the equations above we can calculate:

$$m = \sqrt{\frac{hP}{kA_c}} = 88.3 \tag{8}$$

and:

$$M = \sqrt{hPkA_c}\theta_b = 52.99 \tag{9}$$

Thus for a single fin:

$$q_f = M\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL} = 27.4W \tag{10}$$

and:

$$\eta_f = \frac{q_f}{hA_f\theta_b} = 1.0 \tag{11}$$

This yields:

$$\eta_0 = 1 - \frac{NA - f}{A_t}(1 - \eta_f) = 1.0 \tag{12}$$

and finally:

$$q_t = \eta_0 hA_t\theta_b = 383.227W \tag{13}$$

The heat dissipation density was calculated by calculating the volume of the fins, as well as adding the volume of a base plate for the fins to be mounted to (thickness 2mm). This was then multiplied by the density of copper ($8.9g.cm^3$) which yielded the results shown in the graphs below.

# 4 Computational Results

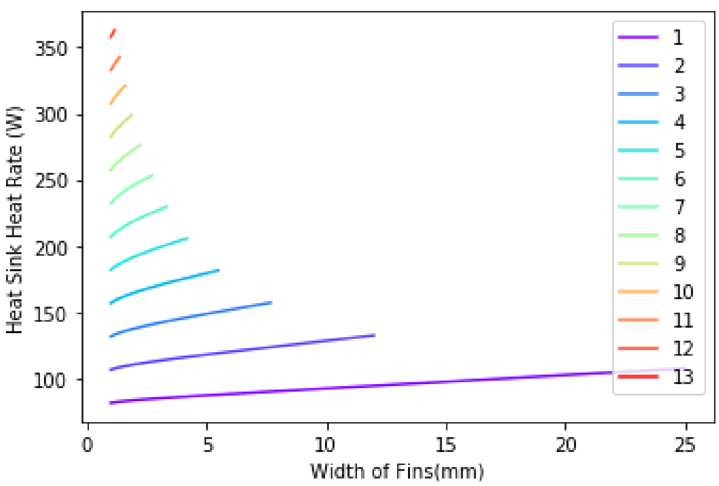


Figure 1: Heat Rate Vs Fin Width for Different Numbers of Evenly Spaced Fins

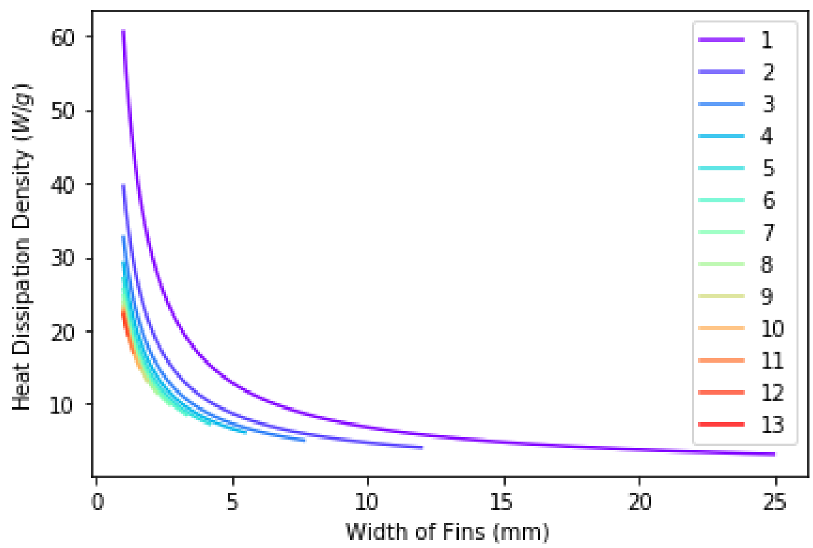


Figure 2: Heat Rate Density Vs Fin Width for Different Numbers of Evenly Spaced Fins with no base plate

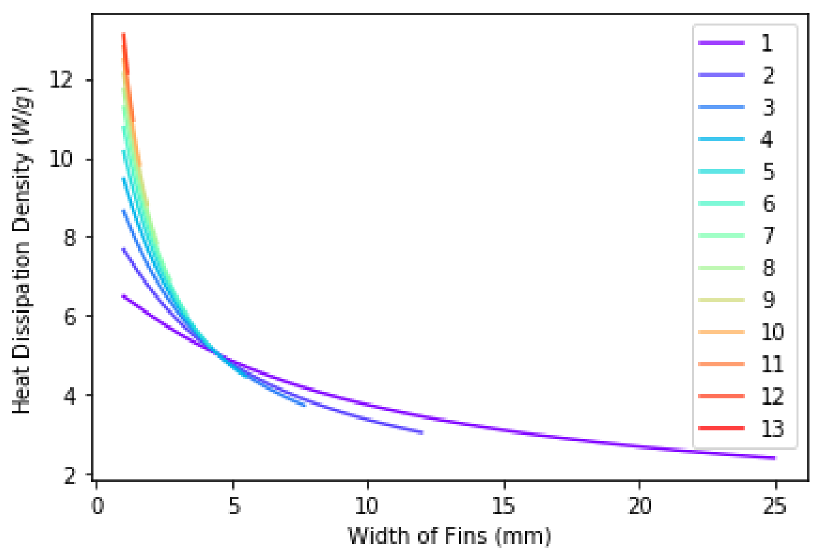


Figure 3: Heat Rate Density Vs Fin Width for Different Numbers of Evenly Spaced Fins with a 2mm base plate

# 5 Conclusion

As seen in the figures above, as more fins were added, the heat dissipated through the heat sink increased. Additionally, given that $h$ remained constant, the thicker the fins became, the better the heat sink was at shedding heat. Considering material usage as well, however, yielded entirely different results. If the base plate material was neglected, the most successful heat sinks were those that used the fewest fins. The heat sink that only contained 1 fin was by far the most efficient, but only when the fin thickness was minimized. When adding some mass to the base plate, however, as would be the case in any real manufacturing of this heat sink, it was once found that the greater number of fins was once again more successful. Thus, the most efficient heat sink for realistic manufacture would be one that contained 13 fins of 1 mm thickness each spaced 1mm apart. The total efficiency for this sink is $13.36w/g$.

# 6 Code

This project was done in Jupyter Notebook which is a python variant.

```
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from mpl_toolkits import mplot3d
from matplotlib.pyplot import cm
Wc=.025 #m
Tsur=25 #C
h=1500 # W/m^2 K
k=400 #W/mk
Tb=85 #C
L=0.006
def P(Wc,w):
    return 2*Wc+2*w
def Ac(Wc,w):
    return(Wc*w)
def Af(Wc,L):
    return 2*Wc*L
def Ab(N,Ac,Wc):
    return (Wc*Wc)-(N*Ac)
def At(Af,N,Ab):
    return N*Af+Ab
def ThetaB(Tb,Tsur):
    return(Tb-Tsur)
def m(h,P,k,Ac):
    return np.sqrt(h*P/(k*Ac))
def M(h,P,k,Ac,ThetaB):
    return (np.sqrt(h*P*k*Ac))*ThetaB
```

```
def qf(M,m,L,h,k):
    return M*(np.sinh(m*L)+(h/(m*k))*np.cosh(m*L))/(np.cosh(m*L)+(h/(m*k))*np.sinh(m*L))
def qt(nu0,h,At,ThetaB):
    return nu0*h*At*ThetaB
def nu0(N,Af,nuf,At):
    return 1-((N*Af)*(1-nuf)/(At))
def nuf(qf,h,Af,ThetaB):
    return qf/(h*Af*ThetaB)
def qtotal(N,w,Wc,Tsur,h,k,Tb,L):
    Pr=P(Wc,w)
    Acr=Ac(Wc,w)
    Afr=Af(Wc,L)
    Abr=Ab(N,Acr,Wc)
    Atr=At(Afr,N,Abr)
    ThetaBr=ThetaB(Tb,Tsur)
    mr=m(h,Pr,k,Acr)
    Mr=M(h,Pr,k,Acr,ThetaBr)
    qfr=qf(Mr,mr,L,h,k)
    nufr=nuf(qfr,h,Afr,ThetaBr)
    nu0r=nu0(N,Afr,nufr,Atr)
    return qt(nu0r,h,Atr,ThetaBr)
def maxwidth(N):
    return (26-N)/N
color = iter(cm.rainbow(np.linspace(0, 1, 13)))
for N in np.arange(1,14,1):
    c = next(color)
    w=((np.arange(1,maxwidth(N),.01))/1000)
    qoverall=qtotal(N,w,Wc,Tsur,h,k,Tb,L)
    plt.plot(w*1000,qoverall,label=N,c=c)
plt.xlabel('Width of Fins(mm)')
plt.ylabel('Heat Sink Heat Rate (W)')
plt.legend()
def mass (N,Acr,L,Wc):
    return(N*Acr*L*8960)+Wc*Wc*.00*8960
color = iter(cm.rainbow(np.linspace(0, 1, 13)))
for N in np.arange(1,14,1):
    c = next(color)
    w=np.arange(1,maxwidth(N),.01)/1000
    qoverall=qtotal(N,w,Wc,Tsur,h,k,Tb,L)
    Acr=Ac(Wc,w)
    Mar=mass(N,Acr,L,Wc)
    heateff=qoverall/Mar
    plt.plot(w*1000,heateff/1000,label=N,c=c)
plt.xlabel('Width of Fins (mm)')
plt.ylabel('Heat Dissipation Density ($W/g$)')
plt.legend()
```