Compressible Flow

Alex Tschinkel

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These lecture notes were taken in Fall of 2023 at NYU Tandon in the Lectures of Prof. Dizinno in AE-UY 4603 Compressible Flow. All mistakes belong to the author. If you find a correction please contact me at at4227@nyu.edu

Contents

Figure 1: Professor Dizinno Demonstrating the Movement of a Shock Wave

0.1 Useful Constants

Gas Constant R	8.314 $m^3 Pa/(Kmol)$
R_{air}	287 J/(kg K)
R_{air}	$8.315\,\mathrm{J \cdot mol^{-1}} \cdot K^{-1}$
γ_{air}	1.4
$C_{P_{air}}$	1.005 kJ/(kg K)
$C_{V_{air}}$	0.7171 kJ/(kgK)
Re_{crit}	Between 2500 and 4000
$\rho_{air,SL}$	1.23 kg/m^3

0.2 Administration

- 2 Midterms 10/4, 11/13
- 2 Assignments 9/25, 11/1
- Final (Optional)

All exams closed book closed notes, but he will give us an equation sheet.

0.3 Reading list

- When we left earth, the NASA Missions
- Apollo 8
- First Man
- The Rocketeer
- From the earth to the moon
- October sky
- Contact
- Truth Lies & O rings
- A Man on the moon
- The Wright Bros, (McCullough)
- Into the Black
- The Mercury 13
- Space Cowboys
- The Space Shuttle Decision
- The Challenger disaster
- Through the glass ceiling to the stars

There are two EC assignments on brightspace. Do em

1 Lecture 1

Substances have elastic properties. Applying a force to a material will result in a deformation. This yields:

$$Modulus = \frac{Stress}{Strain} \tag{1}$$

One example of this is Young's Modulus:

$$E = \frac{\sigma}{\epsilon} \tag{2}$$

Shear Loading yields:

$$G = \frac{\tau}{\gamma} \tag{3}$$

In a fluid you get viscosity:

$$\mu = \frac{\tau}{\dot{\gamma}} \tag{4}$$

If you have compressive loading in a solid you have the bulk modulus:

$$\beta = -\frac{\Delta p}{\frac{\Delta V}{V}} \tag{5}$$

Or in the calculus limit:

$$\beta = -\frac{dp}{\frac{dV}{V}} \tag{6}$$

Compressibility: The reciprocal of the bulk modulus, is the change in unit volume per unit applied pressure.

$$k = -\frac{\frac{dV}{V}}{dp} \tag{7}$$

Takeaways:

- All materials are compressible
- The nature of the response is the difference as is the degree
- Compressibility measures the ability of a material to withstand changes in volume when under uniform compression

The Process matters: Isothermal compressibility implies a constant temperature, Isentropic compressibility implies constant entropy, reversibility, no heat transfer.
Assuming the fluid has mass the result is that:

$$k = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \tag{8}$$

Thus pressure changes yield density changes.
Mach number:

$$M = \frac{V}{a} \tag{9}$$

Assumptions: if $M > 0.3$ the flow is compressible. Otherwise it is incompressible.
As you approach Mach 1 C_D gets very big all of a sudden. Additionally shock waves are caused by supersonic flow.
So What actually is Mach 1?

2 Physics of Wave Propagation

How Does a fluid know that there's an obstacle in its path? How does the flow adjust to the presence of an obstacle?

2.1 Compression Waves

When you push on a fluid it creates some momentum. This creates a region of increased density that propagates through the flow. An oscillating source creates a wave that propagates at:

$$c = \lambda \nu \tag{10}$$

The speed at which that wave propagates is the speed of sound in that medium.

$$\Delta V = c \frac{\Delta \rho}{\rho} \tag{11}$$

applying the momentum equation yields:

$$\Delta V = \frac{\Delta p}{\rho c} \tag{12}$$

or:

$$c \frac{\Delta \rho}{\rho} = \frac{\Delta p}{\rho c} \tag{13}$$

which yields:

$$c^2 = \frac{\Delta p}{\Delta \rho} \tag{14}$$

$$a = \sqrt{\frac{v}{k_s}} \rightarrow \frac{\text{Specific Volume}}{\text{Compressibility}} = \text{Speed of Sound} \tag{15}$$

Thus if you have an ideal gas and an isentropic process you have:

$$a = \sqrt{\gamma R T} \tag{16}$$

This is $340m/s$ or $1117ft/s$.

2.2 Example 3.1

Issac Newton assumed (incorrectly) that sound waves travelled iso-thermally. By how much was his estimate of the speed of sound in error? Iso Thermal:

$$a_T^2 = \frac{\partial p}{\partial \rho} \| = \frac{\partial}{\partial \rho} (\rho R T)_T = RT \tag{17}$$

isentropic:

$$a^2 = \gamma R T \tag{18}$$

or

$$\frac{a_T - a_s}{a_s} = \frac{1}{\sqrt{\gamma}} - 1 \tag{19}$$

Revisiting the Mach Number:

$$\frac{KE}{U} = \frac{V^2}{2U} = \frac{V^2}{2 C_v T} = \frac{\gamma(\gamma - 1)}{2} M^2 \tag{20}$$

2.3 Disturbances

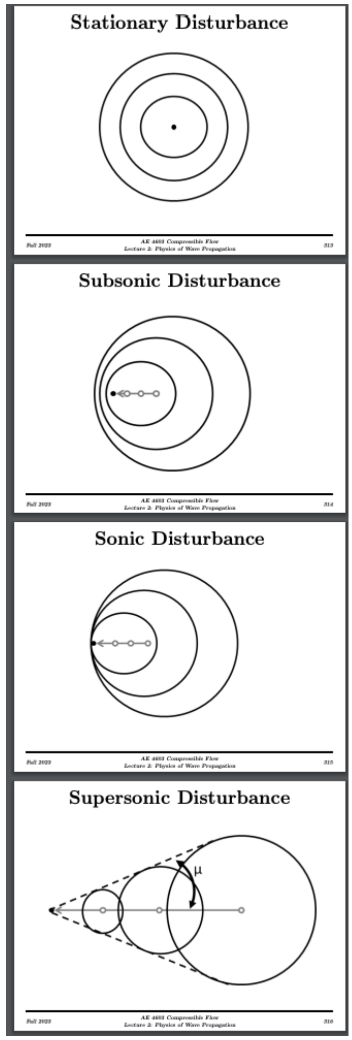


Figure 2: The Behavior of waves emitted by a disturbance

2.4 Shock Waves

If a source is traveling at or above mach 1, there is a shock wave that is emitted in a conical form.

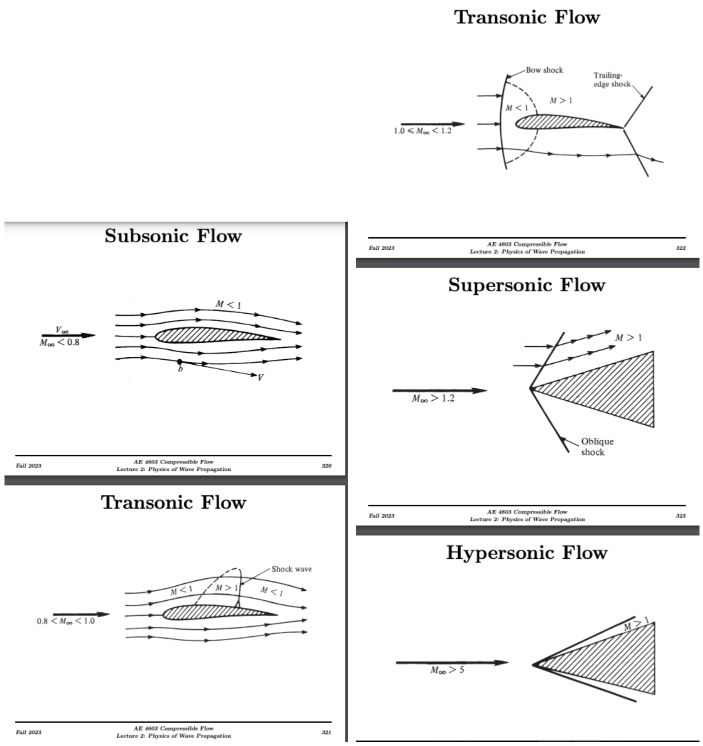


Figure 3: Flow Regimes over an Airfoil

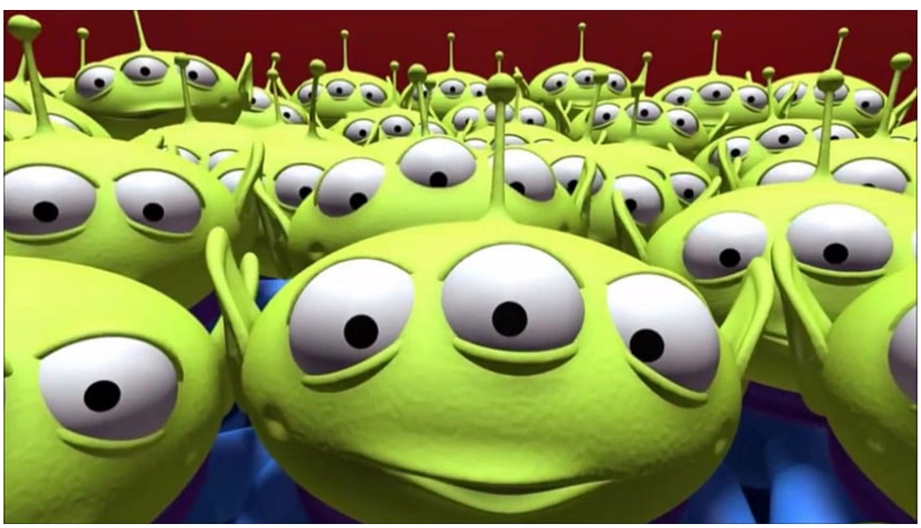


Figure 4: Us When Dizinno shows us the tool that can calculate the speed of sound at different altitudes and pressures

3 Isentropic Flows

Start with a stream tube, means no mass transfer, apply the conservation of energy to the stream tube:

$$\frac{\partial}{\partial t}\int_{\forall}\rho E\,\forall + \int_{S}\left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} = q + \dot{W} \tag{21}$$

by applying conservartion of mass:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{22}$$

3.1 Stagnation Enthalpy

$$h_0 = h + \frac{V^2}{2} = \text{Const} \tag{23}$$

Anything that is a stagnation quantity is also a total quantity. Stagnation enthalpy and total enthalpy are the same. $h_0 = h_t$

Definition: The enthalpy that a flow achieves when it is adiabatically brought to rest.

The flow does not actually have to be brought to rest (nor done so adiabatically). h_0 is associated with a fluid element moving at speed V.

3.2 Stagnation Temperature

For a calorically perfedct gas $\Delta h = C_p \Delta T$ apply this between the stagnation state and the actual state:

$$h_0 - h = C_p(T_0 - T) \tag{24}$$

Whicb yields:

$$T_0 = T + \frac{v^2}{2C_p} \tag{25}$$

The stagnation temperature is the maximum temperature that a flow can achieve without adding any extra energy

As you take $\lim V \to 0$ all the KE of the gas is converted to thermal. The result is the maximum T of the gas.

As $T \to 0$ the thermal energy converts to kinetic

3.3 Mach Number Dependence

$$\frac{T_0}{T} = 1 + \frac{V^2}{2C_pT} \tag{26}$$

or:

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma - 1}{2}\right)\frac{V^2}{\gamma RT} \tag{27}$$

or finally:

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma - 1}{2}\right)M^2 \tag{28}$$

- For a 747 at $M = 0.85$ this yields: 249K

- For the concorde at $M = 2$ this is: 391K
- For the SR-71 at $M = 3.2$ you get: 661K

3.4 Stagnation Pressure

In all earlier calculations the flow was stopped Adiabatitically. Now we assume its reversible as well. Thus these are Isentropic flows.
Recall:

$$\frac{p_0}{p} = \frac{T_0}{T}^{\gamma/\gamma-1} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} \tag{29}$$

Definition: The stagnation pressure is the pressure that the flow would reach if the flow at mach number M was brought to rest by an isentropic process.

3.4.1 Example 4.2

Calculate the stagnation pressure at the nose of a Bugatti Veyron at 120 m/s

Figure 5: Nyoom

- p=101kPa,T=298K
- $a = \sqrt{\gamma RT} = 346 m/s$
- $M \frac{V}{a}$

$$\frac{p_0}{p} = \frac{T_0}{T}^{\gamma/\gamma-1} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} \tag{30}$$

Yields

$$p_0 = 110 kPa \tag{31}$$

3.5 Stagnation speed of Sound

$$\frac{a_0}{a} = \sqrt{\frac{T_0}{T}} = \sqrt{1 + \frac{\gamma-1}{2}M^2} \tag{32}$$

3.6 Stagnation Properties

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{33}$$

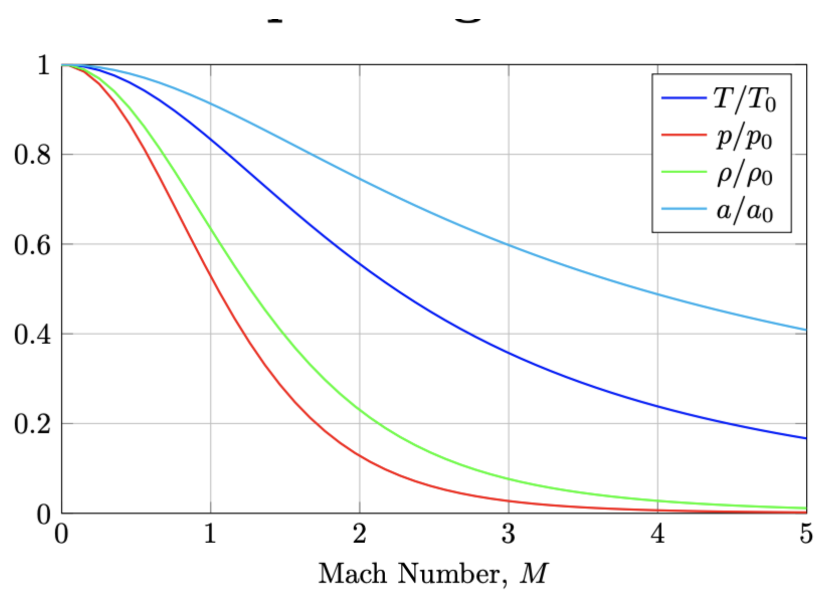
$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1} \tag{34}$$

$$\frac{a_0}{a} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/2} \tag{35}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma-1} \tag{36}$$

Stagnation temperature requires adiabatic conditions while p_0, ρ_0 require isentropic conditions. The actual flow field does not have to be adiabatic or isentropic from one point to the next. If the flow is isentropic the T_0, p_0,ρ_0 are constant throughout the flow field. If the flow is not isentropic than we cannot assume this.
Wow there's an app that will do this for us too! Also tables in books (ugh)

3.7 Expanding Flows



3.8 The Critical State

Consider a fluid that is not moving at Mach 1. Now imagine that it is sped up or slowed down to Mach 1. The state of the fluid when $M = 1$ is known as the critical state.

Critical properties are obtained by substituting $M = 1$ into the isentropic relations.

$$\frac{p^*}{p_0} = 0.528 \tag{37}$$

$$\frac{T^*}{T_0} = 0.833 \tag{38}$$

$$V^* = a^* \tag{39}$$

$$\frac{\rho^*}{\rho_0} = 0.6339 \tag{40}$$

3.8.1 Assumptions

Since the values for the properties at the critical state were derived usign isentropic relations the same assumptions hold true. The flow does not actually have to be at $M = 1$.

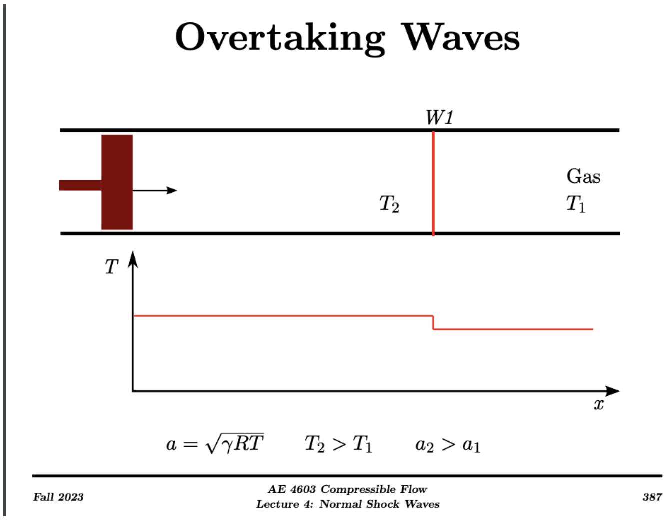
4 an Apollo Interlude

Dizinno talks about rockets for a sec.

Figure 6: Dizinno when the fuel starts to slosh

5 Normal Shock Waves

As flows move around an object the are redirected around said object. This acoustic wave is a weak wave of particles rebounding off of an object.



Because of this phenomenon, waves that are following move slightly faster than a leading wave. Given enough time and space waves will catch up to one another and combine.
Now Supersconic the molecules that hit the object and rebound cannot propagate upstream. Thus a shock wave forms.
There are discontinuities across the thickness of the shock. The thickness of the shock is actually quite small and is simplified away in a macroscopic perspective.

5.1 Math Time ☺

5.1.1 Normal Shock Relations

Drawing a control volume across a shock wave we have these governing equations for a steady flow:

$$\rho_1 v_1 = \rho_2 v_2 \tag{41}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{42}$$

$$h_1 \frac{u_1^2}{2} = h_2 \frac{u_2^2}{2} \tag{43}$$

Big Assumption: The flow field is adiabatic: $T_{01} = T_{02}$
Recalling:

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 \tag{44}$$

applying this across both sides of the wave:

$$T_1 \left(1 + \left(\frac{\gamma - 1}{2}\right) M_1^2\right) = T_2 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right] \tag{45}$$

which yields the following temperature ratio:

$$\frac{T_1}{T_2} = \frac{\left(1 + \left(\frac{\gamma-1}{2}\right) M_2^2\right)}{\left(1 + \left(\frac{\gamma-1}{2}\right) M_1^2\right)} \tag{46}$$

but unfortunately we don't know M_2 so this approach doesn't work.

$$p_1 \left(1 + \gamma M_1^2\right) = p_2 \left(1 + \gamma M_2^2\right) \tag{47}$$

which gives us the pressure ratio of:

$$\frac{p_1}{p_2} = \frac{\left(1 + \gamma M_2^2\right)}{\left(1 + \gamma M_1^2\right)} \tag{48}$$

Once again we need to know M_2 for this approach to yield results.
Doing a whole bunch of algebra that I cant type out we get an equation with $M_2 = f(M_1)$ which can vaguely be represented as a quadform:

$$M_2^2 = \frac{\left(-\gamma M_1^4 - 1\right) \pm \left(M_1^2 + \gamma M_1^4 - 1 - \gamma M_1^2\right)}{\gamma\left(1 - 2M_1^2\right) - 1} \tag{49}$$

Evaluating in the − case yields a trivial solution: $M_2^2 = M_1^2$. There are no changes in properties and this corresponds to isentropic flows.
Conversely evaluating at the + case yields:

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \tag{50}$$

5.2 The Normal Shock Relations

Plugging this into the prior equations yields the following normal shock relations:

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \tag{51}$$

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma-1}{2} M_1^2\right)\left(\frac{2\gamma}{\gamma-1} M_1^2 - 1\right)}{\left[\frac{(\gamma+1)^2}{2(\gamma-1)}\right] M_1^2} \tag{52}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1} \tag{53}$$

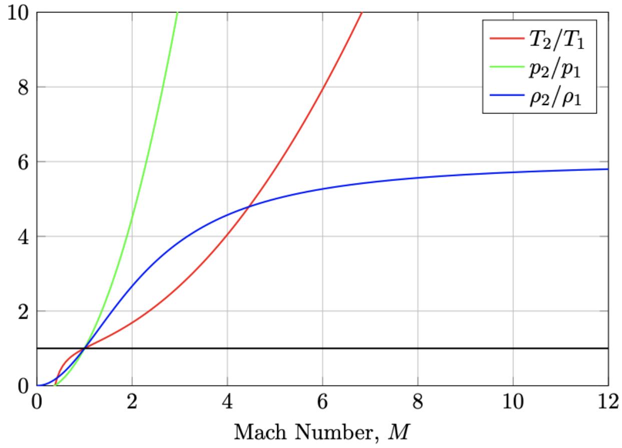
$$\frac{\rho_2}{\rho_1} = \frac{M_1^2(\gamma + 1)}{(\gamma - 1)M_1^2 + 2} \tag{54}$$

$$T_{01} = T_{02} \tag{55}$$

phew that was brutal to type.
Now Some statements:

- For $M_1 = 1$, $\Delta s = 0$
- For $M < 1$, $\Delta s < 0$ **WHICH VIOLATES 2nd LAW and IS NOT POSSIBLE**
- For $M > 1$, $\Delta s > 0$ which is completely plausible

This entropy change is caused by the manner in which the flow slows significantly across the shock. This yields the following trends across the shock: The stagnation pressure decreases across the flow



as:

$$\frac{p_{02}}{p_{01}} = \exp\left(-\frac{s_2 - s_1}{R}\right) \tag{56}$$

As $M_1 \to \infty$:

- $M_2 = 0.378$
- $\frac{\rho_2}{\rho 1} = 6$
- $\frac{p_2}{p_1} = \infty$
- $\frac{T_2}{T_1} = \infty$

And there exist online tools to calculate this as well.

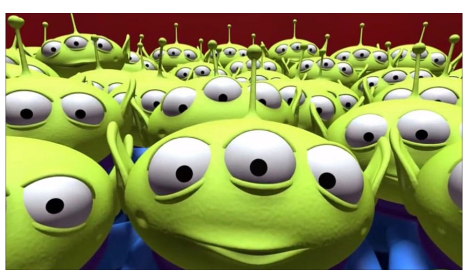


Figure 7: Us When Dizinno shows us the tool that can calculate the Normal Shock Relations

5.3 Example 4.1

A normal shock wave is standing in the test section of a wind tunnel. Upstream $M_1 = 3$, $p_1 = 0.5$atm, and $T_1 = 200K$. Find M_2, p_2, T_2, and v_2 downstream of the shock.
Going to trusty tables we have:

- $M_2 = 0.475$
- $p_2 = 5.17$atm
- $T_2 = 536K$
- Remembering that $a = \sqrt{\gamma R T_2}$, we find that $a = 464m/s$,$v_2 = 200m/s$

5.4 Example 4.2

A blunt nosed missile is flying Mach 2 at sea level conditions. What are the Temperature and Pressure at the nose of the missile?
Sea Level Conditions:

- $T = 519°R$
- $p_1 = 2116$ Psi

Isentropic Table at $M_1 = 2$ to determine values at the stagnation state yields:

$$\frac{T_{01}}{T_1} = 1.8 \tag{57}$$

and

$$\frac{p_{01}}{p_1} = 7.824 \tag{58}$$

Thus getting values for $M_1 = 2$ from the Normal Shock Tables

$$T_{02} = \left(\frac{T_{02}}{T_{01}}\right)\left(\frac{T_{01}}{T_1}\right)T_1 = 1 \cdot 1.8 \cdot 519 = 934R° \tag{59}$$

Doing the same for p_{02} yields from the table $\frac{p_{02}}{p_{01}} = 0.7209$ and:

$$p_{02} = \left(\frac{p_{02}}{p_{01}}\right)\left(\frac{p_{01}}{p_1}\right)p_1 = 0.7209 \cdot 7.824 \cdot 2116 = 11935\text{Psi} \tag{60}$$

5.5 Velocity Measurement

For incompressible flow the velocity is measured by comparing the static and stagnation pressures using bernouli yielding:

$$V = \sqrt{\frac{2(p_0 - p)}{\rho g}} \tag{61}$$

For subsconic compressible flow we modify this equation:

$$V_1 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{62}$$

For supersconic flow we need to first compute the flow across the other side of the shock wave, and from there we can use the same equation.

5.6 Example 4.3

For a pitot tube in a supersconic flow where the static pressure is 0.4 atm, the tube measures 3 atm. What is the Mach number?

$$\frac{p_{02}}{p_1} = \frac{3}{0.4}\text{atm} = 7.5 \tag{63}$$

From the Normal Shock Table at: $p_{02}/p_1 = 7.5$ we get $M_1 = 2.34$
The key assumption in this problem is that the pitot tube is not measuring the static pressure, we simply know it.

5.7 Example 4.4

A Pitot tube is inserted into an airflow where the static pressure of 1 atm. Calculate the mach number when the pitot tube measures:

- 1.276 atm
- 2.7414 atm
- 12.06 atm

To solve this we first have to determine what region we are in. From the isentropic table, at $M = 1$ $p/P_0 = 5.28$. Thus if sconic conditions:

$$p_0 = \frac{p}{0.528} = 1.89p \tag{64}$$

Thus if $p_0 \leq 1.89p$ the flow is subsconic. Otherwise it is supersconic.

- Thus for 1.276 atm we can reference the isentropic table and $M = 0.6$
- For 2.714 atm the flow is supersconic. we go to the normal shock tables at $\frac{p_{02}}{p_1} = 2.174$ which yields $M = 1.3$
- and 12.06 atm yields $M = 3$

END OF MATERIAL FOR EXAM 1

6 The Hugoniot

Figure 8: Hugoniot

Can we describe flow across a shock wave without using mach number? Because the static pressure increases across the wave the shock can be visualized as a thermodynamic device that compresses the gas.
We begin with the continuity equation

$$\rho_1 v_1 = \rho_2 v_2 \tag{65}$$

Which can be used to solve for the speed on either side as a function of the ratio of densities. This yields the following Energy equation:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{66}$$

$$u_{T1} + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = u_{T2} + \frac{p_2}{\rho_2} + \frac{u_2^2}{2} \tag{67}$$

This then yields the Hugoinoit Relationship:

$$u_{T2} - u_{T1} = \left(\frac{p_1 + p_2}{2}\right)(v_1 - v_2) \tag{68}$$

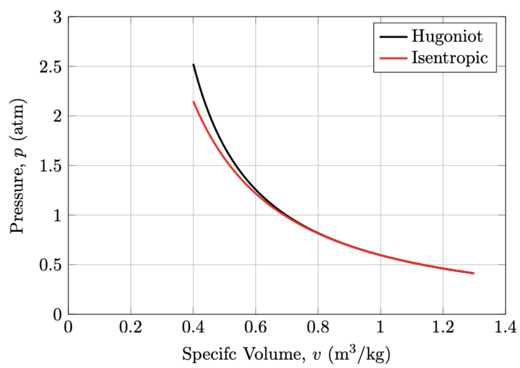


Figure 9: Hugoniot

If we **THEN** assume a calorically perfect gas this all yields:

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{v_1}{v_2} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{v_1}{v_2}} \tag{69}$$

v **is SPECIFIC VOLUME HERE NOT VELOCITY**
u_T **is the internal energy not the velocity**

6.1 Example 5.1

A Normal Shock is standing in the test section of a supersonic tunnel. Upstream of the wave $M_1 = 3$, $p_1 = 0.5atm$, $T_1 = 200K$ Show that this shock wave satisfies the Hugoniot for a calorically perfect gas.

So why do we care to use the Hugoniot? Because this doesn't deal with the speed of sound, we no longer have to care about the frame of reference. The hugoniot deals with thermal quantities which are scalars, not dynamic quantities which are vectors.
Normal Shock Tables $M = 3$ $p_2/p_1 = 10.33$, $\rho_2/\rho_1 = 3.86$

$$\rho_1 = \frac{p_1}{RT_1} = 0.88kg/m^3 \tag{70}$$

$$\rho_2 = \frac{\rho_2}{\rho_1}\rho_1 = 3.4kg/m^3 \tag{71}$$

Applying the hugoniot:

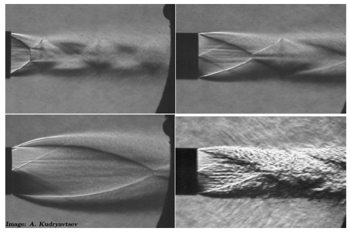
$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{v_1}{v_2} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{v_1}{v_2}} \tag{72}$$

yields 10.36 which is close enough to the 10.33 from the tables.

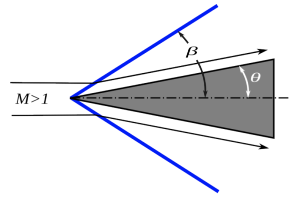
Figure 10: Dizinno When James

7 Oblique Shock Waves

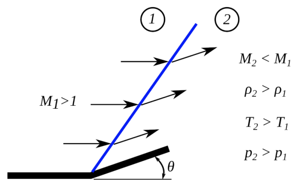
Why are jet intakes shaped the way they are? What is the function of an inlet? How do they achieve their goal?
Oblique shockwaves are not normal to the flow (shocker). They form at changes in geometry, or where the flow needs to be redirected. When you have an exhaust out of a jet, the shock wave bounces across the exhaust several times in a diamond formation:
The boundary conditions for the figure 15b imply that the fluid must be parallel to the surface since the surface is impermeable. Thus the velocity normal to the surface must be zero. The oblique shock forms when the flow needs to be turned in order to satisfy this boundary condition. The wave angle β reflects the angle of the oblique wave. θ is the wedge angle.



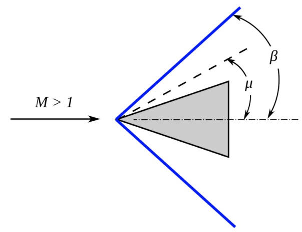
(a) Oblique waves in an exhaust plume



(b) Supersconic flow creating an oblique wave



(a) Relations of Upstream and Downstream properties



(b) As the flow accelerates, the wave angle approaches the mach angle

The velocity decreases across the shock wave ie $M_2 < M_1$ but is not necessarily below 1. Going back to disturbance propagation, if a source is moving supersconically, when a source emits a wave. The mach angle μ is defined as

$$\sin\mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M} \tag{73}$$

As you approach hypersconic speeds, not only does the boundary layer approach the wave, an entropy layer begins to form as well. In this class we wont deal with absurdly crazy mach numbers. *Into* the wave you have V_1, M_1, which can be decomposed into components normal and tangential to the wave. These are u_1 and w_1 respectively with M_{n1} and M_{t1}. On the back end of the shock wave exists V_2 at M_2 which are deflected relative to V_1. Which yield w_2, and u_2 which are parallel and perpendicular to the wave as well as M_{t2} and M_{n2}
Now we make a control volume out of a stream tube across the shock wave. We begin our analysis with the conservation of mass, which we simplify to:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \tag{74}$$

$$\rho_1 u_1 = \rho_2 u_2 \tag{75}$$

We specifically only chose the u component because the u component carries the velocity across the wave but the w tangential velocities dont carry any volume across the wave (nicely).
Tangential Momentum:

$$\frac{\partial}{\partial t}\int_{\forall} w\rho d\forall + \int_S w\rho\vec{V}\cdot d\vec{A} = \sum F_t \tag{76}$$

which considering steady state means that:

$$w_1 = w_2 \tag{77}$$

Normal Momentum:

$$\frac{\partial}{\partial t}\int_{\forall} w\rho d\forall + \int_S w\rho\vec{V}\cdot\vec{dA} = \sum F_n \tag{78}$$

$$u_1(-\rho_1 u_1 A_1) + u_2(-\rho_2 u_2 A_2) = p_1 A_1 - p_2 A_2 \tag{79}$$

which yields:

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_1 u_2^2 \tag{80}$$

Energy Equation:

$$\frac{\partial}{\partial t}\int_{\forall} e\rho d\forall + \int_S \left(u_T + \frac{p}{\rho} + \frac{v^2}{2} + gz\right)\rho\vec{V}\cdot\vec{dA} = q - \dot{w} \tag{81}$$

which yields:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{82}$$

7.1 Recall

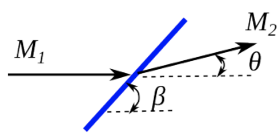


Figure 13: Oblique shock wave

which yields the following three equations for oblique shock waves:

- $\rho_1 u_1 = \rho_2 u_2$
- $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$
- $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$

7.2 Oblique Shock Relationships

$$M_{n1} = M_1 \sin\beta \tag{83}$$

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} \tag{84}$$

$$M_{n2}^2 = \frac{M_{n1}^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_{n1}^2 - 1} \tag{85}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n1}^2 - 1) \tag{86}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2 + 2} \tag{87}$$

$$\frac{T_2}{T_1} = \frac{p_2\rho_1}{p_1\rho_2} \tag{88}$$

For a normal shock all equations are a function of γ and M_1, whereas for an oblique shock they are a function of γ M_1, β and θ.

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma\cos 2\beta) + 2}\right] \tag{89}$$

That's nasty but there's a graph for that: For every wedge and wedge angle there are two possible

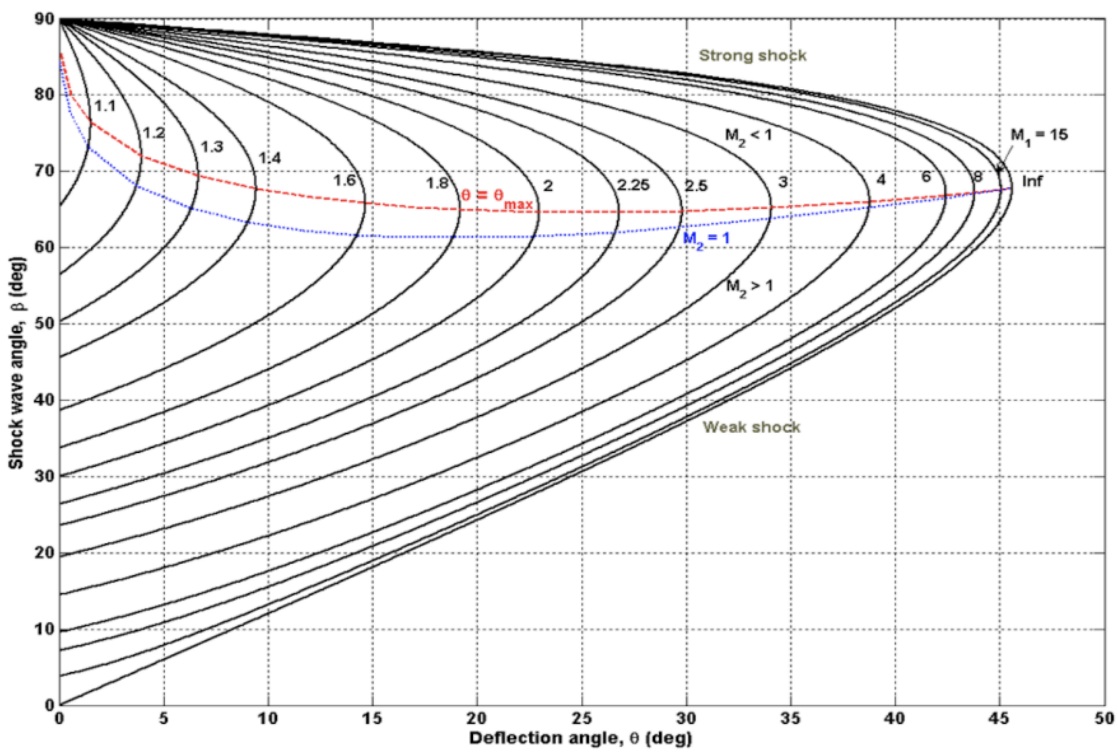
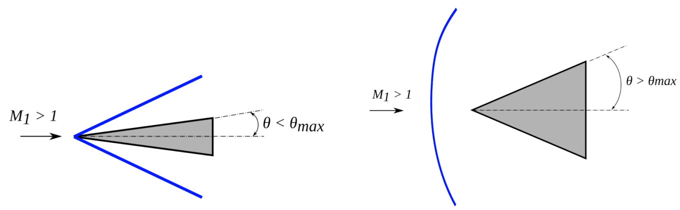


Figure 14: θ-β-M Graph (in air so $\gamma = 1.4$)

shock waves that could form. There is a weak shock solution and a strong shock solution, with the strong solution having a larger wave angle. *Most* of the time we see the weak solution, but sometimes we see the strong solution, depending on what the pressute wants to or has to be downstream of the shock.

The strong shock solution will have a M_2 less than 1 and the weak solution will have $M_2 > 1$. Above θ_{max} You have a detached wave. The distance between the nose and the shock is the detachment



(a) Attached Wave (b) Detached wave

distance, and we really haven't fully calculated it. If you have a corner flow it will look as follows:

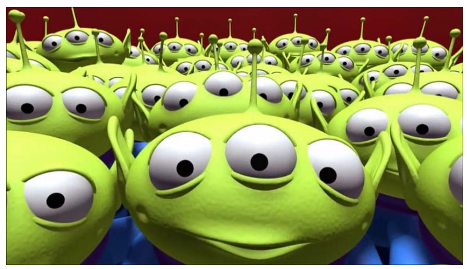


Figure 16: Us When Dizinno shows us the online tool that can calculate the Oblique Shock Relations

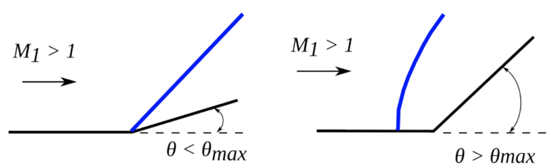


Figure 17: Corner Flow

7.3 Example 6.1

A uniform subsconic stream with $M_1 = 3$ $p_1 = 1$ atm and $T_1 = 288$K encounters a compression corner that deflects the stream bu 20°. Calculate the shock angle and p_2, T_2, M_2, p_{02}, and T_{02} behind the shock.
From the θ -β-M graph we find that at 20° and $M_1 = 3$ we get that $\beta = 37.8°$.

$$M_{n1} = M_1 \sin \beta = 1.84 \tag{90}$$

and the normal shock table at $M_{n1} = 1.84$ yields:

- $\frac{p_2}{p_1} = 3.77$
- $\frac{T_2}{T_1} = 1.56$
- $M_2 \frac{M_{n2}}{\sin\beta - \theta} = 1.99$
- $p_{02} = \frac{p_{02}}{p_{01}} \frac{p_{01}}{p_1} p_1 = 29.26 atm$
- $T_{02} = T_{01} = \frac{T_{01}}{T_1} T_1 = 806.5K$

7.4 Example 6.2

In example 6.1 the compression corner is increased to 30°. Now find p_2 and M_2 and compare them to the 20° case.
From the chart we get $\beta = 52°$ and $M_{n1} = 2.36$ which yields fro the normal shock table $M_{n2} = 0.527$ and $\frac{p_2}{p_1} = 6.35$ which yields $p_2 = 6.35atm$ and $M_2 = 1.4$.
Increasing the wedge angle gives a higher pressure and lower trailing mach number, increasing the strength of the wave.

7.5 Example 6.3

Same as example 6.1 but with $M_1 = 5$
From θ-β-M chart $\beta = 29.8°$ and $M_{n1} = 2.48$. This yields $p_2 = 7.037$ and $M_2 = 3.0$
By increasing the mach number of the incoming flow we have a substantial pressure rise and a bigger flow deceleration.

7.6 Conclusions:

Increasing the normal component of the upstream mach number increases the shock strength: ie increasing β by increasing θ or by increasing M_1
As θ increases for fixed M_1 the shock becomes stronger and β increases. As M_1 increases for fixed θ the shock becomes stronger and β decreases.

7.6.1 A Challenge:

Code your own θ-β-M chart in Python. Extra credit.

7.7 Example 6.4

A 10° half angle wedge is placed into a mystery flow of unknown Mach number. Using a Schlieren system the shock wave angle is 44°. What is the Mach number?
Going to the θ-β-M chart we see that the mach number is $M_1 = 1.8$

7.8 Example 6.5

Consider a 15° half angle wedge at zero angle of attack. Calculate the pressure coefficient on the wedge surface in a Mach 3 flow of air.
The pressure coefficient is:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \tag{91}$$

So we know that:

- $\theta = 15°$
- $M_1 = 3$

We can also rearrange C_p as:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{p_2 - p_1}{\frac{1}{2}\frac{p_1}{RT_1}V_1^2\frac{\gamma}{\gamma}} = \frac{p_2 - p_1}{\frac{\gamma}{2}p_1 M_1^2} \tag{92}$$

Which we simplify as:

$$C_p = \frac{1}{\gamma M_1^2}\left(\frac{p_2}{p_1} - 1\right) \tag{93}$$

From the Normal Shock online calculator we find that $\frac{p_2}{p_1} = 2.82$ and thus that $C_p = 0.29$. If we don't have the online tool, we use the Normal shock properties table after finding M_{n1} from M_1 and β which we find from the θ-β-M chart.

7.9 3D effects

Because of the curvature of a 3D surface, the shock effects are weaker. For example:

M_1	Wedge	Cone
θ	20°	20°
β	53°	37°
p_2/p_1	2.843	1.588
M_2	1.21	1.55

Table 1: 2D to 3D comparison

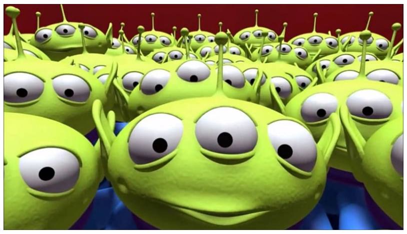


Figure 18: Us When Dizinno shows us the online tool that can calculate the Conical Shock Relations

7.9.1 The Shock Polar

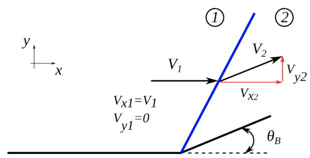


Figure 19: 2D Oblique shock wave plane

But *NOW* we get to plot it in terms of velocity: The different points are the velocity vectors

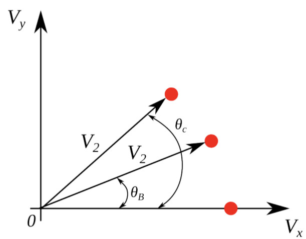


Figure 20: Shock Polar

after deflections of different values of θ. By connecting these dots we end up with a parabola. Comparing to the speed of sound yields: Point E represents a normal shock, and point A represents

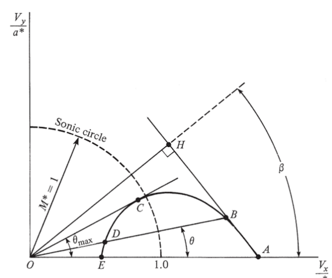


Figure 21: Shock Polar with respect to a

no redirection. Points on the parabola outside the sonic circle are weak shocks, while points inside the sonic circle are stronger shocks.
I MISSED A CLASS HERE GET NOTES
There's a new computer assignment here

7.10 Example 6.8

We now have reflected shocks, imma need to catch up here
The stagnation pressure is higher when we decelerate the flow by two shocks as opposed to one.

7.11 Reflections on Reflection

The idea of stagnation pressure: Lets say I have a supersonic flow at $M_1 = 3$. If I want to decelerate this flow lets say I have a single normal shock: Upstream the stagnation pressure is P_{01} and downstream it is P_{02}. I can then go to a table and I see that $\frac{P_{02}}{P_{01}} = .3283$. I we instead slowed the flow through an oblique shock whose wave angle is $\beta = 40°$, and then hit it with a following normal shock: we compute $\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}}\frac{P_{02}}{P_{01}}$. We then find that $\frac{P_{03}}{P_{01}} = 0.58$. Thus we see that two shocks are better than one shock, with a 70% increase in efficiency.

8 Expansion Waves

We now get to address the opposite of shock waves. Shock waves compress the flow by decreasing the static pressure and decelerate the flow. Now we move onto an expansion wave, which accelerates fluid elements. All the prior shocks we have seen are dissipating. These waves accelerate a flow. Expansion waves are isotropic. As a flow goes around a corner it experience a gradual change,

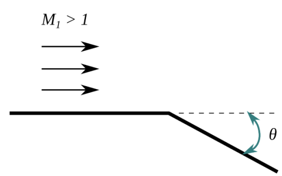


Figure 22: Expansion Wave

which is different from the normal and oblique shocks, which present a sudden change.
Mach Lines at an angle of $\mu_1 = \sin^{-1}(\frac{1}{M_1})$ and $\mu_2 = \sin^{-1}(\frac{1}{M_2})$. Expansion waves have the following features:

- The flow is accelerated $M_2 > M_1$
- Static properties decrease. $p_2 < p_1$, $T_2 < T_1$, $\rho_2 < \rho_1$
- Streamlines are smooth curves
- The expansion is Isentropic

For a given M_1, p_1, T_1 and θ_1 how do we find M_2, p_2 and T_2?
Along a mach line at angle μ from $\vec{V}$ the flow is deflected by θ and the new velocity is $V + dV$. Applying the law of sines to a triangle constructed from these lines we find that:

$$\frac{V}{\sin(\frac{\pi}{2} - \mu - d\theta)} = \frac{V + dV}{\sin(\frac{\pi}{2} + \mu)} \tag{94}$$

Which we eventually find leads to:

$$d\theta = \frac{dV/V}{\tan\mu} \tag{95}$$

and:

$$\sin\mu = \frac{at}{Vt} = \frac{1}{M} \tag{96}$$

which yields:

$$d\theta = \sqrt{M^2 - 1}\frac{dV}{V} \tag{97}$$

which is approximate for finite $d\theta$ and becomes more accurate as $d\theta$ gets smaller.

8.1 Prandl-Meyer Function

$$\nu(M) = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2}M^2}\frac{dM}{M} \tag{98}$$

Which yields:

$$\theta_2 = \nu(M_2) - \nu(M_1) \tag{99}$$

Using this function goes as follows:

- Obtain $\nu(M_1)$ from the tables at given M_1
- Calculate $\nu(M_2)$ from the equation above and the given θ_2
- From the tables at $\nu(M_2)$ find M_2
- Since the wave is isentropic T_0 and p_0 remain constant throughout the wave.

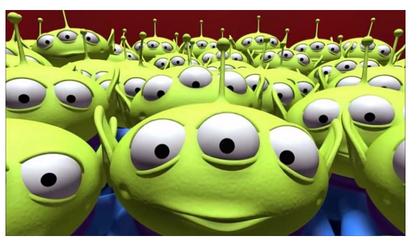


Figure 23: Us When Dizinno shows us the online tool that can sorta calculate the Prandl-Meyer Function

8.2 Example 7.1

A uniform supersconic stream with $M_1 = 1.5$, $p_1 = 1700psf$ and $T_1 = 460°R$ encounters an expansion corner which deflects the flow by 20°. Calculate M_2, p_2, T_2, p_{02}, and T_{02} and the angles of the forward and reversed mach lines make with respect to the upstream flow.
Starting from the tables at $M_1 = 1/5$ we get $\nu_1 = 11.91°$ and $\mu_1 = 41.81°$. Since θ_2 is $\nu(M_2) - \nu(M_1)$ we get $\nu(M_2) = 31.91°$. Going back to the table at this value yields:$M_2 = 2.2$ and $\mu_2 = 27°$.
Going back to the isentropic tables we can then solve for the other flow properties as we have done in the past. Yielding $T_2 = 339°R$ and $p_2 = 584psf$

8.3 Example 7.2

A 15° hald angle diamond wedge airfloi is in a supersconic flow at zero α. A pitot probe is inserted into the flow on the upper backface of the airfoil. The probe reads 2.586 atm. and the pressure just ahead of the probe is 0.1 atm. Find the free stream mach number.
This problem is juicy You have an oblique shock, an expansion shock, a normal shock, and a pitot probe. At the pitot probe you can solve:

$$\frac{p_{04}}{p_3} = \frac{2.596}{0.1} = 25.96 \tag{100}$$

From the normal shock tables we then can find that $M_3 = 4.45$. The Prandtl-Meyer table at $M_3 = 4.45$ yields $\nu_3 = 71.27°$. Given that $\theta_3 = 30°$ we find that $\nu_2 = 41.27°$. Plugging this back ingo the table we find that $M_2 = 2.6$. Using the Oblique shock table at $\theta = 15°$ we find that $M_2 = 2.6$ Which then yields $M_1 = 2.5$

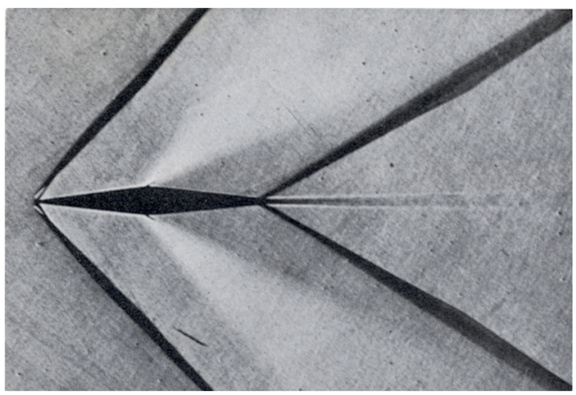


Figure 24: Schlieren Photo of the Shock Waves in Example 7.2

9 Converging and Diverging Nozzles

How can we accelerate a fluid element from subsconic speeds to superscionic speeds?

Figure 25: Dizinno telling us about Diverging and Converging Nozzles

We begin with Quasi 1-D flow:

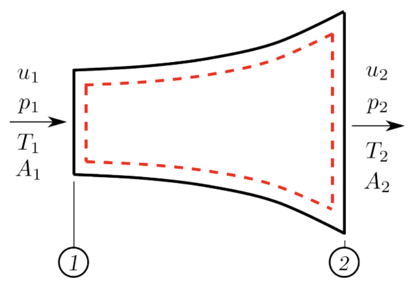


Figure 26: Quasi 1-D Flow with a control volume

There is an extremely gradual taper such that we can say that at one particular point the cross sectional area is constant, however over the scale of the whole we have different XC areas.
We Begin with the Continuity Equation:

$$\frac{\partial}{\partial t}\int_{V\!\!\!/}\rho dV\!\!\!/ + \int_S \rho\vec{V}\cdot\vec{dA} = 0 \tag{101}$$

which we reduce down to:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \tag{102}$$

and the Momentum equation:

$$\frac{\partial}{\partial t}\int_{V\!\!\!/}\vec{V}\rho dV\!\!\!/ + \int_S \vec{V}\rho\vec{V}\cdot\vec{dA} = \sum\vec{F} \tag{103}$$

Which we reduce down to:

$$p_1 A_1 + \rho_1 u_1^2 A_1 + \int_{A_1}^{A_2} p d\vec{A} = p_2 A_2 + \rho_2 u_2^2 A_2 \tag{104}$$

And the Energy Equation:

$$\frac{\partial}{\partial t}\int_{\forall} e\rho d\forall + \int_S \left(u_T + \frac{p}{\rho} + \frac{v^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} = q - \dot{w} \tag{105}$$

which is simplified to:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{106}$$

performing a differential analysis on a thin section of that control volume yields:

$$d(\rho u A) = 0 \tag{107}$$

$$dp = -\rho u du \tag{108}$$

$$dh + udu = 0 \tag{109}$$

We can then derive an area-velocity relationship:

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u} \tag{110}$$

This yields some interesting results:

- as $M \to 0$ (and flow is incompressible) $\frac{dA}{A} = -\frac{du}{u}$
- $0 \leq M \leq 1$, ie subsconic flow: an increase in velocity is caused by a decrease in area
- $M \geq 1$ yields, an increase in velocity is casued by an *increase* in area.

Figure 27: That moment when for Supersconic Flow Bernoulli is backwards all of a sudden

When you are at precisely equal to $M = 1$ you see that:

$$\frac{dA}{A} = 0 \tag{111}$$

which corresponds to a minimum in the distribution.

We can thus accelerate a gas isentropically to supersconic speeds through a convergent divergent duct. The location of the minimum area is known as the throat.
The critical state occurs where $M = 1$. At this location we get:

$$\frac{A}{A^*} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho}\frac{a^*}{u} \tag{112}$$

and:

$$\frac{\rho_0}{\rho^*} = \left(\frac{\gamma+1}{2}\right)^{1/\gamma-1} \tag{113}$$

9.1 Area-Mach Number Relationship

The Area-Mach No. Relationship is:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{1}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\gamma+1/\gamma-1} \tag{114}$$

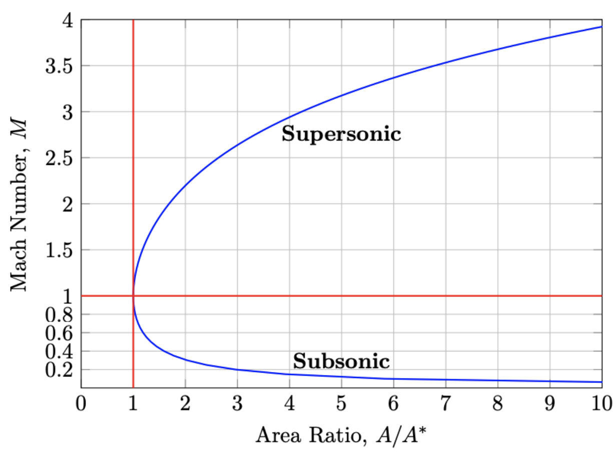


Figure 28: Area Mach Number Relationship

9.2 Terminology

- p_0 *Stagnation pressure:* The pressure in the upstream chamber where the velocity is essentially 0.
- p_e *Exit Plane Pressure:*the pressure at the exit of the nozzle
- p_b *The Back Pressure:* The pressure int he large downstream receiver
- p_t *The Throat Pressure:* The pressure at the minimum area, for a converging nozzle this is also the exit plane pressure.

- p^* *Critical Pressure:* The pressure when the mach numebr is 1.

if: $p_e = p_b = p_0$ we have no flow. If $p_e = p_b$ but p_0 is greater than either of those two, we know that $M < 1$. We can take this all the way to $M = 1$. This is still working nicely.
Choked Flow: A limiting condition where mass flow rate will not increase with further decreases in the exit pressure (for a fixed geometry condition). The parameter that becomes choked is the fluid velocity. This condition sets in when $p_e = 0.528p_0$.

9.3 Example 8.1

Air is allowed to flow from a large reservoir through a converging nozzle with an exit area of $50cm^2$. The reservoir pressure and temperature are 500kPa and 400K respectively. Determine the mass flow rate for the back pressures of 0, 125,250 and 375 kPa.
If the nozzle is choked:

$$\frac{p_b}{p_0} \leq 0.528. \tag{115}$$

Thus for any pressure lower than $p_b = 264.2$ kPa the nozzle will be choked, and the answer will be the same.
We know that the mass flow rate is:

$$\dot{m} = \rho_e V_e A_e = \frac{p_e}{RT_e}(M_e\sqrt{\gamma RT_e})A_e \tag{116}$$

For the three cases where the flow is choked, $M_e = 1$ and we find that:

$$\dot{m} = 5.05\text{kg/s} \tag{117}$$

For the back pressure of 375 kPa we get:

$$\frac{p_b}{p_0} = 0.75 \tag{118}$$

Which from the isentropic flow calculator yields $M_e = -.654$ and $T_e/T_0 = 0.921$. We calculate the mass flow rate using the same equation as above to find that:

$$\dot{m} = 4.46\text{kg/s} \tag{119}$$

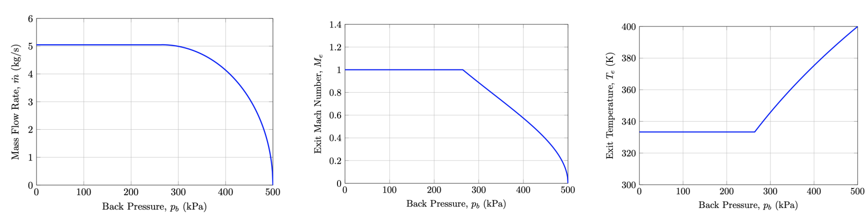


Figure 29: Experimental Data from Example 8.1

9.4 Midterm 2 Subjects

- Normal Shocks
- Oblique Shocks
- Expansion Waves
- Converging Nozzles

9.5 Extra Credit?

- 1 Page paper on whats your favorite plane and why
- name all four planes in dizinnos office
- Reports on the watchlist movies/books

9.6 Diverging Nozzles

9.6.1 Design Conditions for C-D Nozzles

9.7 Example

- 2 sections where $\frac{A}{A^*} = 6$
- P_0=10 atm, $T_0 = 300K$
- Find (M,P,T) at each of these spots

From the Isentropic calculator at $\frac{A}{A^*} = 6$ we find that:

- 0.097
- P/P_0=0.9934
- T/T_0=0.9881

Yield for the subsonic portion:

9.8 Post Covid Dizinno

If we are attempting to determine the location of a shock in a converging-diverging nozzle knowing the geometry, the reservoir conditions, and the back pressure.
First we find the Mach number in the exit plane:

$$M_e^2 = \frac{-1}{\gamma - 1} + \sqrt{(\frac{1}{\gamma - 1})^2 + (\frac{2}{\gamma - 1})(\frac{2}{\gamma + 1})^{\frac{\gamma+1}{\gamma-1}}(\frac{P_{01}}{P_b})^2(\frac{A_t}{A_e})^2} \tag{120}$$

Next we use M_e to find $\frac{P_e}{P_{02}}$

$$\frac{P_{02}}{P_e} = (1 + \frac{\gamma - 1}{2} M_e)^{\frac{\gamma}{\gamma-1}} \tag{121}$$

Figure 30: Dizinno fighting through a COVID to give us a Zoom lecture

Figure 31: James Pulling Up to Class 15 Minutes Late Every Day

or use tables Then $P_e = P_b$

$$\frac{P_b}{P_{01}}\frac{P_{02}}{P_e} = \frac{P_{02}}{P_{01}} \tag{122}$$

noting that $P_{01}A_1^* = P_{02}A_2^*$
This then allows us to solve iteratevly for M_1

$$\frac{P_02}{P_{01}} = (1 + \frac{\gamma - 1}{2}M_e)^{\frac{\gamma}{\gamma-1}}(\frac{1}{\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}}^{\frac{1}{\gamma-1}} \tag{123}$$

And we solve for:

$$(\frac{A_1}{A^*})^2 = \frac{1}{M_1^2}((\frac{2}{\gamma+1})(1 + \frac{\gamma-1}{2}M_1^2))^{\frac{\gamma+1}{\gamma-1}} \tag{124}$$

which gives $\frac{A_1}{A^*}$ which is the area just in front of the shock wave

9.9 Example

The backpressure to resevoir ratio in a C-D Nozzle is 0.7. The exit to throat area ratio is 2. Where is the normal shock?
Dizinno solved this in excel. What he found is that $\frac{A}{A^*} = 1.507$

10 Moving Shock Waves

When a wave begins to propagate it induces motion. W the wave propagation speed induces the fluid behind it to move in the same direction with an induced (mass) motion u_p.
This flow is unsteady! Time matters....
Recalling that for stationary shocks:

$$\rho_1 v_1 = \rho_2 v_2 \tag{125}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{126}$$

$$h_1 \frac{u_1^2}{2} = h_2 \frac{u_2^2}{2} \tag{127}$$

These relations are still true, however you need the proper interpretation of the velocity.

- u_1 The velocity of the gas ahead of the wave relative to the wave, w
- u_2 The velocity of the gas behind the wave relative to the wave $w - u_p$.

Yielding:

$$\rho_1 W = \rho_2 (w - u_p) \tag{128}$$

$$p_1 + \rho_1 W^2 = p_2 + \rho_2 (w - u_p)^2 \tag{129}$$

$$h_1 \frac{W^2}{2} = h_2 \frac{(w - u_p)^2}{2} \tag{130}$$

This also means that the Hugoniot relationship is the same because Thermodynamic relationships don't care about your reference frame.
It can be shown: That the pressure rise across the shock is:

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_s^2 - 1) \tag{131}$$

or that:

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma}(\frac{P_2}{P_1} - 1) + 1} \tag{132}$$

and that:

$$w = a_1 \sqrt{\frac{\gamma + 1}{2\gamma}(\frac{P_2}{P_1} - 1) + 1} \tag{133}$$

and:

$$u_p = w(1 - \frac{\rho_1}{\rho_2}) \tag{134}$$

$$u_p = \frac{a_1}{\gamma}(\frac{\rho_1}{\rho_2} - 1)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{\frac{1}{2}} \tag{135}$$

10.1 Example

A normal shock moves into still air at 300K. The pressure ratio across the shock is 10. Calculate the shock velocity and the induced velocity behind the wave.

10.1.1 Method 1: Equations

Start with the speed of sound that we are moving into:

$$a_1 = \sqrt{\gamma RT} = 347.2\text{m/s} \tag{136}$$

And then we can solve directly for the speed:

$$w = a_1\sqrt{\frac{\gamma+1}{2\gamma}(\frac{P_2}{P_1} - 1) + 1} = 1025\text{m/s} \tag{137}$$

Allowing us to solve for u_p which is 756 m/s using this equation:

$$u_p = \frac{a_1}{\gamma}(\frac{\rho_1}{\rho_2} - 1)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{\frac{1}{2}} \tag{138}$$

10.1.2 Method 2: Tables

Table A2 (normal shock) at $\frac{P_2}{P_1} = 10$ yields $M_s = 2.95$ relative to the wave.
Since the wave is moving into still air, this is what a stationary observer would see.

$$M_s = \frac{w}{a_1} \text{ yields } w = M_s a_1 = 1024\text{m/s} \tag{139}$$

At the same entry M_2=0.4782, $T_2 = 2.621T_1$. Which lets us find that $T_2 = 786$K which we can subsequently use to calculate a_2.
Knowing that $u_p = u_2 = m_2 a_2$ we find that $u_2 = 269$m/s. Subtracting that from 1024 to account for the reference frame change yields 756 m/s, which is the same as in the first method.

10.2 Example 2

$T_1 = 15°C$, $P_1 = 100kPa$, $\frac{P_2}{P_1} = 1.25$, find V_2, P_2, T_2.
Using the normal shock table at $P_2/P_1 = 1.25$ we find that $M_1 = 1.1$, $M_2 = 0.9118$, and $T_2/T_1 = 1.065$
All relative to the wave. This lets us solve for all the prior quantities relative to the wave.

Figure 32: James Pulling Up to Class 47 Minutes Late Today

10.3 Shock Tubes

11 Rayleigh Flow

Trends for Rayleigh Flow:

- When heat is added to a:
  - Supersconic Flow $M > 1$
    - $M_2 < M_1$, $P_2 > P_1$, $T_2 > T_1$
    - $u_2 < u_1$, $T_{02} > T_{01}$, $P_{02} < P_{01}$
  - Subsconic flow $M < 1$
    - $M_2 > M_1$, $P_2 < P_1$, $T_2 < T_1$ if $M_1 > \gamma^{-1/2}$
    - $T_2 > T_1$ if $M_1 < \gamma^{-1/2}$
    - $u_2 > u_1$, $T_{02} > T_{01}$, $P_{02} < P_{01}$

Thus we can conclude that deat addition always drives the flow to $M = 1$

12 Blunt Body Problem

13 Fanno Flow

Flow with friction.
Lets take a tube of length L diameter d. Taking a control volume thickness dx, thats a small slice of the tube, with shear stress acting on the wall τ_w.
Applying continuity, momentum, and the adiabatic energy equation we find that:

Figure 33: Only half an hour today