

Boolsche_Algebra

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1 Boolsche Algebra

- https://de.wikipedia.org/wiki/Boolesche_Algebra
- <https://kogler.wordpress.com/2008/03/21/latex-use-of-math-symbols-formulas-and-equations/>

1.1 Operators

| | Operator | Latex Symbol | Latex Code |
|--------|-------------------------|--------------|----------------------------------------|
| NEGATE | $\neg \bar{x} \bar{x}$ | | <code>\neg \overline{x} \bar{x}</code> |
| AND | $\bigwedge \wedge * \&$ | | <code>\bigwedge \wedge * \&</code> |
| OR | $\bigvee \vee + $ | | <code>\bigvee \vee + </code> |
| XOR | \oplus | | <code>\oplus</code> |

1.2 Algebra rules

1.2.1 Duality Rules

$$\bar{0} = 1$$

$$\bar{1} = 0$$

1.2.2 Neutrality Rules

$$a * 1 = a$$

$$a + 0 = a$$

1.2.3 Extremity Rules

$$a * 0 = 0$$

$$a + 1 = 1$$

1.2.4 Dual Negation Rule (Involution)

$$\overline{(\bar{a})} = a$$

1.2.5 Idempotence Rules

Idempotenz ist ein Begriff aus der Mathematik und Informatik. In der Mathematik bezeichnet man ein Objekt a *a, das mit einer Verknüpfung $\{\circ\}$ die Eigenschaft $a \circ a = a$ erfüllt*

$$a * a = a$$

$$a + a = a$$

1.2.6 Complementary Rules

$$a * \bar{a} = 0$$

$$a + \bar{a} = 1$$

1.2.7 Commutativity Rules

Vertauschungsgesetz, ist eine Regel aus der Mathematik. Wenn sie gilt, können die Argumente einer Operation vertauscht werden.

$$a * b = b * a$$

$$a + b = b + a$$

1.2.8 Associativity Rules

Verknüpfungsgesetz oder auch Verbindungsgesetz, ist eine Regel aus der Mathematik. Eine (zweistellige) Verknüpfung ist assoziativ, wenn die Reihenfolge der Ausführung keine Rolle spielt.

$$(a * b) * c = a * (b * c) = a * b * c$$

$$(a + b) + c = a + (b + c) = a + b + c$$

1.2.9 Distributivity Rules

$$a * (b + c) = (a * b) + (a * c)$$

$$a + (b * c) = (a + b) * (a + c)$$

1.2.10 De Morgansche Rules

$$\overline{(a * b)} = \bar{a} + \bar{b}$$

$$\overline{(a + b)} = \bar{a} * \bar{b}$$

1.2.11 Absorption Rules

$$a + (a * b) = a$$

$$a * (a + b) = a$$

$$a + (\bar{a} * b) = a + b$$

$$a * (\bar{a} + b) = a * b$$

1.3 Some Proofs

| Name | Rule | Solution |
|-------------------|-----------------------------|-----------------------------------------------------------------------------------------|
| Absorption Rule 1 | $a * (a + b) = a$ | $a * (a + b)$ $(a * a) + (a * b)$ $a + a * b$ $a * 1 + a * b$ $a * (1 + b)$ $a * a$ a |
| Absorption Rule 2 | $a + (a * b) = a$ | $a + (a * b)$ $(a + a) * (a + b)$ $a * a + b$ $a + 0 * a + b$ $a + (0 * b)$ $a + 0$ a |
| Absorption Rule 3 | $a + (\bar{a} * b) = a + b$ | $a + (\bar{a} * b)$ $a + \bar{a} * a + b$ $1 * (a + b)$ $a + b$ |
| Absorption Rule 4 | $a(\bar{a} + b) = ab$ | $a * (\bar{a} + b)$ $a * \bar{a} + a * b$ $0 + a * b$ $a * b$ |

| Name | Rule | Solution |
|-----------------------|--------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Xor 4 Nand Solution 1 | $a \oplus b = \overline{a\overline{b}} + \overline{\overline{a}b}$ | $a \oplus b$ $\overline{a\overline{b}} + \overline{\overline{a}b}$ $\overline{(\overline{a} + \overline{\overline{b}})} +$ $\overline{(\overline{\overline{a}} + \overline{b})} +$ $\overline{(\overline{a} + b)} +$ $\overline{(a + \overline{b})}$ <hr/> $\overline{(\overline{a} + b)} * \overline{(a + \overline{b})}$ $\overline{(\overline{a} + b)} * \overline{(a + \overline{b})}$ $\overline{\overline{a}a + \overline{a}\overline{b} + ab + b\overline{b}}$ $\overline{0 + \overline{a}\overline{b} + ab + 0}$ $\overline{\overline{a}\overline{b} + ab}$ $\overline{\overline{a}\overline{b} * \overline{ab}}$ $\overline{ab * \overline{a}\overline{b}}$ $\overline{ab * \overline{a}\overline{b}}$ $\overline{ab * \overline{a} + \overline{b}}$ $\overline{ab * (a + b)}$ $\overline{\overline{a}ba + \overline{a}bb}$ $\overline{\overline{a}ba + \overline{a}bb}$ $\overline{\overline{\overline{a}ba} * \overline{\overline{a}bb}}$ $\overline{\overline{a}ba * \overline{abb}}$ |
| Xor 4 Nand Solution 2 | $a \oplus b = \overline{a\overline{b}} + \overline{\overline{a}b}$ | $a \oplus b$ $\overline{a\overline{b}} + \overline{\overline{a}b}$ $\overline{a\overline{a}} + \overline{a\overline{b}} +$ $\overline{b\overline{b}} + \overline{\overline{a}b}$ $\overline{a(\overline{a} + \overline{b})} +$ $\overline{b(\overline{b} + \overline{a})}$ $\overline{\overline{a}\overline{a}\overline{b}} + \overline{\overline{b}\overline{a}\overline{b}}$ $\overline{\overline{a}ba} * \overline{\overline{a}bb}$ |

1.4 Summary

| Topic | Rules |
|-------|-----------------------------------------------------------------------------------------------------------|
| Misc | $\overline{0} = 1$ $\overline{1} = 0$ $\overline{(\overline{a})} = a$ |
| AND | $a0 = 0 \setminus a1 = a \setminus aa = a \setminus$ $a\overline{a} = 0 \setminus (ab)c = a(bc) = abc$ |

| Topic | Rules |
|----------------|----------------------------------------------------------------------------------------------|
| OR | $a+0 = a \ \ a+1 = 1 \ \ a+a = a \ \$ $a+\bar{a} = 1 \ \ (a+b)+c = a+(b+c) = a+b+c$ |
| Commutativity | $ab = ba \ \ a+b = b+a$ |
| Associativity | $(a*b)*c = a*(b*c) = a*b*c$ $(a+b)+c = a+(b+c) = a+b+c$ |
| Distributivity | $a*(b+c) = (a*b)+(a*c)$ $a+(b*c) = (a+b)*(a+c)$ |
| De Morgan | $\overline{(a*b)} = \bar{a} + \bar{b}$ $\overline{(a+b)} = \bar{a} * \bar{b}$ |
| Absorption | $a + (a * b) = a$ $a*(a+b) = a$ $a+(\bar{a} * b) = a + b$ $a*(\bar{a} + b) = a * b$ |