Discussion 5: Sampling Distribution of X-bar and Confidence Intervals

1. Suppose the number of different computers used by students in the University computer labs last week has distribution:

Value	Probability
0	0.3
1	0.4
2	0.3

Suppose the distribution of computer use is still valid for this week and consider sampling two students at random. Let X_1 and X_2 be iid random draws from the population that represent the number of different computers used by the two students in your sample.

(a) Determine the missing elements in the table for the sampling distribution $\bar{X} = \frac{X_1 + X_2}{2}$.

$\bar{X}$	Probability
0.5	
1	0.34
	0.24
2.0	

(b) Calculate the expected value and standard error of $\bar{X}$ any way you want.

(c) Calculate the probability that the average number of different computers used by the two chosen students is atleast 1.5 using the pmf in part a.

(d) Calculate the probability that the average number of different computers used by the two chosen students is atleast 1.5 using a Normal approximation.

(e) Compare the the values in parts c and d. Explain the relationship between the two numbers.

(f) If the sample size is increased to 50, find the probability that the average number of different computers used by the 50 chosen students is atleast 1.5.

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2. A caffeine drink company sells a drink with a label that claims a caffeine content of 86 mg. Sixteen bottles of the drink are randomly selected and analyzed for caffeine content. The resulting observations are:

$$\left\|\begin{array}{cccccccc} 83.7 & 88.6 & 83.5 & 88.3 & 83.9 & 84.9 & 85.4 & 85.6 \\ 89.8 & 86.2 & 83.9 & 86.1 & 84.5 & 87.3 & 85.2 & 86.7 \end{array}\right\|$$

(a) Check that the assumptions for building a confidence interval are well met.

(b) Construct an appropriate 90% confidence interval.