Stat 324 Homework #2 Due Thursday Sept 27th 5:30pm

*Submit your homework to your TA's mailbox and Canvas if you wish before the due date/time. The mailboxes are to the left as you enter the Medical Science Center (1300 University Ave.) from the main University Ave. entrance.

*No late homework will be accepted for credit.

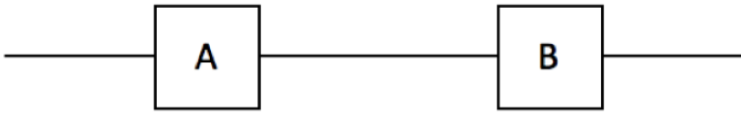
*If a problem asks you to use R, include a copy of the code and output. Please edit your code and output to be only the relevant portions.

*If a problem does not specify how to compute the answer, you may use any appropriate method but show the work that is necessary for the TA to follow and evluate your reasoning. I may ask you to use R or manual calculations on exam, so practice accordingly.

Total Points Available: 45 (35 pts for correctness; 10 points completeness)

10 points completion: (all problems attempted: 6 (scaled down if fewer); 2 (stapled); 2 (easy to read/follow))

1. A system contains two components, A and B, connected in series, as shown in the diagram. Assume that A and B function independently. For the system to function, both components must function.



   (a) If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions. ***2 points**: P(functions)=P(A functions and B functions)=0.95*0.97=0.9215*

   (b) If both A and B have probability of **p** of failing, what must the maximum value of p be so that the probability that the system functions is at least 0.90? ***2 points**:(1-p)(1-p)=.90, so* $1-p=\sqrt{.90}$*, 1-p=.9486833, so -p=.9486833-1, p=0.0513167*

2. A local taxi company owns 10 taxis. Three are randomly selected for compliance with emission standards (without replacement) and all are found to not be in compliance. The company claims all their other cars are in compliance.

   (a) Calcuate the probability of choosing the three taxis not in compliance, if in fact 7 out of the 10 are in compliance. ***2 points*** $\frac{3*2*1}{10*9*8}=\frac{6}{720}=0.008333333$

   (b) Comment on the believability of the company's claim. ***2 points*** *It is very unlikely for us to have seen three cars not incompliance if in fact only 3 out of the 10 were out of compliance.*

3. From the service records over the past year, the manager of an auto detailing shop has estimated the following partial probability distribution for X= the number of customers asking for service per day at his shop.

X	0	1	2	3	4	5
P(X=x)	0.04		0.34	0.20	0.11	0.06

(a) What does the probability of 1 customer asking for service $P(X = 1)$ have to be to make this a probability distribution? ***2 points:****So the probability values add up to 1* $P(X = 1)$ *has to be:0.25* $0.04 + 0.34 + 0.20 + 0.11 + 0.06 = 0.75$

(b) Suppose the shop currently has the capacity to serve up to 3 customers a day. What is the probability that one or more customer needs to be turned away? ***2 points:****0.11+0.06=0.17*

(c) What is the probability that the shop's capacity is not fully utilized on a day? ***2 points:****P*($X < 3) = 0.04 + 0.25 + 0.34 = 0.63$

(d) By how much must the capacity be increased so the probability of turning a customer away is no more than 0.10? ***2 points:****capacity increased by 1 to X=4 since* $P(X > 4) = 0.06$

4. A box contains five slips of paper. These slips are marked \$1, \$1, \$1, \$10, and \$25. The winner of a contest will select two slips of paper at random with replacement and will then get the larger of the dollar amounts on the two slips. Define a random variable W=the amount awarded.

(a) Determine the probability distribution of W (write out the pmf). **6 points, +2 for each probability**:

Max	Events	Probability
1	Both 1s	$(3/5)^2 = 0.36$
10	Both 10s or 1 10 and 1 1	$(1/5)^2 + 2 * (1/5)(3/5) = 0.28$
25	Both 25s or 1 10 and 1 25 or 1 25 and 1 1	$(1/5)^2 + 2 * (1/5) * (1/5) + 2(1/5)(3/5) = 0.36$

(b) What is the least probable outcome for the amount won? ***1 points; if got table wrong, grade based off of their table:****getting \$10.*

(c) What is the expected value for the amount won? ***2 points; if got table wrong, grade based off of their table:****EV*$(W) = .36 * 1 + .28 * 10 + .36 * 25 = 12.16$

(d) What is the variance and standard deviation for the amount won? ***2 points; if got table wrong, grade based off of their table:****VAR*$(W) = .36 * (1 - 12.16)^2 + .28 * (10 - 12.16)^2 + .36 * (25 - 12.16)^2 = 105.4944$,*SD*$(W) = \sqrt{105.4944} = 10.27105$

(e) What is the probability that the amount won is more than one standard deviation away from the expected value? ***2 points +1 range, +1 percent; if got table wrong, grade based off of their table:****12.15+10.27=22.42, 12.15-10.27=1.88 so 1s and 25s which is 2*.36=0.72*

5. For each of the following questions, say whether the random process is reasonably a binomial process or not, and explain your answer. As part of your explanation, you will want to comment on the potential validity of each of things that must be true for a process to be a binomial process. If it is a binomial process, identify n : the number of Bernoulli trials and π the probability of success.

(a) A fair die is rolled until a 1 appears, and X denotes the number of rolls. *Not binomial since not fixed number of trials*

(b) Ten different basketball players each attempt 1 free throw and X is the total number of successful attempts. *Not binomial since probability of sucess isn't same between players*

(c) It has been reported that nation-wide, one-third of all credit card users pay their bills in full each month. Let X be the number of people in a sample of 25 randomly chosen credit card users in Madison who pay their bill in full on a given month. *Binomial OK; n=25 fixed number of trials,* $\pi = .30$*, trials are independent*

(d) Let X be the number of months out of a randomly chosen year that one randomly chosen credit card user in Madison pays their bill in full. *probability not; trials do not seem independent if I don't pay this month, more likely to pay off in full next month?*

6. Exit polling has been a controversial practice in recent elections, since early release of the resulting information appears to affect whether or not those who have not yet voted do so. Suppose that 90% of all registered Wisconsin voters favor banning the release of information from exit polls in presidential elections until after the polls in Wisconsin close. A random sample of 25 Wisconsin voters is selected (You can assume that the responses of those surveyed are independent).

   (a) What is the probability that more than 23 favor the ban?

   Answer: ***2 points****:* $P(X > 23) = P(X = 24) + P(X = 25) = 0.1994161 + 0.0717898 = .2712059$

   (b) What is the probability that at least 23 favor the ban?

   Answer: ***2 points****:*$P(X \geq 23) = P(X = 23) + P(X = 24) + P(X = 25) = 0.2658881 + 0.1994161 + 0.0717898 = 0.537094$

   (c) What are the mean value and standard deviation of the number who favor the ban in a sample of size 25? Answer: ***2 points****:*$E(X) = 25 * .90 = 22.5$*,* $SD = \sqrt{25 * .90 * .10} = 1.5$

   (d) Is it probable that fewer than 23 in the sample favor the ban with the assertion that 90% of the populace favors the ban? (Hint: Consider $P(X < 23)$ when $\pi = 0.90$)

   Answer: *:*$P(X < 23) = 1 - P(X \geq 23) = 1 - 0.537094 = 0.462906$ *It is pretty likely we would see fewer than 23 people in our sample who favor the ban even if* 90% *of the populace favors the ban. It would happen in about* 46% *of the samples of size 25 we would take from this population.*

7. On the question of whether Interstate 90 should become a tollway, suppose the probability that a randomly selected person in Wisconsin favors tolling is 0.4, opposes it is 0.5, and has no opinion is 0.1. If six people are interviewed and they respond independently, find the probability that:

   (a) Five favor tolling:

   Answer: $\binom{6}{5}(.4)^5(.6)^1 = .0369$ *dbinom(5,6,.4)*

   (b) Two oppose tolling:

   Answer: $\binom{6}{2}(.5)^2(.5)^4 = 0.234375$ *dbinom(2,6,.5)*

   (c) More than 4 favor tolling:

   Answer: *X=number who favor tolling:* $P(X = 5) + P(X = 6) = \binom{6}{5}(.4)^5(.6)^1 + \binom{6}{6}(.4)^6(.6)^0 = .041$ *dbinom(5,6,.4)+dbinom(6,6,.4)*

   (d) At least one has no opinion on tolling:

   Answer: *Z: number that have no opinion:* $P(Z) \geq 1 = 1 - P(z = 0) = 1 - \binom{6}{0}(.1)^0(.9)^6 = 1 - 0.531441 = 0.468559$