Disc_7_Soln

1. Specifications for a water pipe call for a mean breaking strength μ of more than 2000 lb per linear foot. Engineers will perform a test to decide whether or not to use a certain kind of pipe. A random sample of 1 ft sections of pipe is selected and their breaking strengths are measured. The pipe will not be used unless the engineers can conclude (statistically, not with certainty) that the mean breaking strength is greater than 2000. Prior literature suggests the breaking strengths of these pipes are roughly symmetric and unimodal and have a population standard deviation of 10.

   (a) Specify appropriate null and alternative hypotheses for this situation.

   ANSWER: $H_o : \mu = 2000$, $H_a : \mu > 2000$

   (b) What kind of evidence from the sample do you need to reject the null hypothesis?

   ANSWER: We would need to see averages in the sample higher than what is statistically likely if the null is true

   (c) Explain in non-statistical language what a Type I Error would be in this context.

   ANSWER: We'd say mean strength is higher than 2000 when in fact it is not.

   (d) Explain in non-statistical language what a Type II Error would be in this context.

   ANSWER: We'd say mean strength is not higher than 2000 when in fact it is.

   (e) Which type of Error, Type I or Type II, is worse in this situation? How does this knowledge affect the engineer's process?

   ANSWER: I'd say type 1 would be worse because we could end up using pipe that isn't strong enough. We should require stronger evidence before we reject the null. So we could set a lower $\alpha = 0.01$

   (f) If engineers plan to sample 40 sections of pipe, what power will they have to detect a true mean strength of 2004 lb per linear foot (assuming $\sigma = 10$ and $\alpha = 0.01$)? Draw a picture to help you calculate this. Comment on your findings. You can check your work by looking at: http://digitalfirst.bfwpub.com/stats_applet/stats_applet_9_power.html.

   ANSWER: First, I need to find my critical values from my null distribution. Note, since sample size is 40, the population of strength don't need to be normal, CLT tells us distribution of means will still be approx Normal. Since $\alpha = 0.01$ *and alternative is:* $H_a : \mu > 2000$*, values over* $z_.01 = 2.326348$ *will be in my rejection region. This translates to* $\bar{X} > 2.326348 * \frac{10}{\sqrt{40}} + 2000 = 2003.678$*. Now, power=P(Reject Null* H_0 */* $\mu_A = 2004, \sigma = 10$ *true)=P(*$\bar{X} > 2003.678|\mu_A = 2004, \sigma = 10 = P(Z > \frac{2003.678-2004}{10/\sqrt{40}}) = P(Z > -0.2036507) = 0.5806868$*. This is very low power - in only about* 60% *of samples we could choose from a population with true average of 2004, will we reject the null that* $H_0 : \mu = 2000$.

   (g) Suppose engineers are interested in increasing their power to detect a true mean strength of 2004 lb per linear foot (assuming $\sigma = 10$). What minimum sample size should they choose so their estimate of the mean is precise enough to have power of 0.85?

ANSWER: $n \approx (\frac{\sigma(z_\alpha + z_\beta)}{\mu_o - \mu_a})^2$. $\sigma = 10$, $z_\alpha = z_{.01} = 2.33$, $z_\beta = z_{.15} = 1.04$ *so* $n \approx (\frac{10(2.33+1.04)}{2000-2004})^2 = 70.98063$. *You can check n=71 on the simulation pushes power above 85.*