Discussion 10 Solutions

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1. A three-month course in marketing advertises that it increase sales for the average participant. A company wants to get some evidence that the program is truely effective before sending all of its sales people, so they start by sending only 17. The sales, before and after the course in Marketing (in Thousands of Dollars) is given in the table below.

Salesperson	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Sales Before	3	8	7	10	6	7	12	7	6	8	9	7	10	7	4	5	7
Sales After	7	12	14	8	5	7	8	6	13	9	6	8	10	10	11	12	12

(a) What kind of tests can we consider conducting to answer company's question based on the information we have? What is an assumption that is common to all of these tests?

We have matched pair data, since we have Before and After measurements on a set of 17 individuals. With matched - pair data, we typically consider a 1 sample z test, 1 sample t test, bootstrap test for mean, sign test, Wilcoxon signed-rank, or 1 sample proportion test depending on whether we are interested in the value of or just sign of the difference. All of these tests are assuming the differences are all independent (so we would want to make sure that all of these individuals didn't attend the same 3 week session if we wanted to talk about the efficacy of the 3 week sessions in general (ok if we want to talk about the specific 3 week session.))

(b) Perform the relevant t test and bootstrap test after specifying the hypotheses and assumptions the tests are making. Also comment on how well these assumptions are met. Compare the conclusions of the tests.

T TEST ANSWER: $H_o : \mu_D = 0$ *and* $H_A : \mu_D > 0$ *where D=The amount of sales after the training - amount of sales before the training. A The matched-pair t test is additionally assuming the population of differences is normal. This assumption does not seem to be well met (our sample histogram and qqplot are showing very heavy tails). Going forward with the test anyway, just to see:* $t_{obs} = \frac{2.058824-0}{3.732804/\sqrt{17}} = 2.2741$. *Compared to a T distribution with 16 degrees of freedom we get the p value:* $P(T_{16} > 2.27409) = 0.018542$.

BOOTSTRAP ANSWER: same hypotheses. Bootstrap is not making any normality in population assumption; p value= 188/10000=0.0188. This test is giving us slightly less evidence against the null. Both tests give us enough evidence at the 5% level to reject the null of no difference in means and suggest that the average difference in sales is higher than 0.

(c) Perform a sign test and the relevant Wilcoxon (with R) to test the null hypothesis that the distribution of differences is symmetric about a median of 0 ($\mu_D = 0$) and the alternative that the distribution of differences is shifted to the right of 0 ($\mu_D > 0$). Compare the conclusions of the tests.

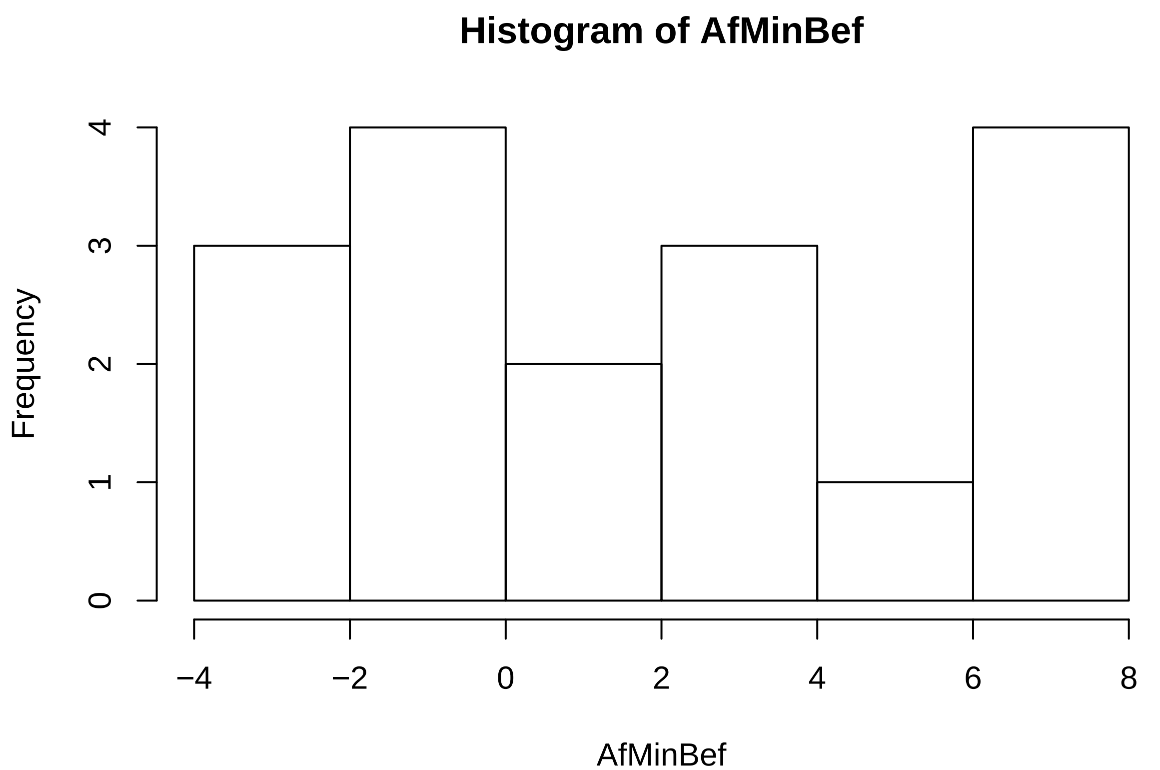
SIGN TEST ANSWER: There are 17 observations, but only 15 of them are non zero. So under our null that the median is 0; the test statistic B=Number of Increases follows $B \sim Bin(15, .5)$. $b_{obs} = 10$. *To test a shift to the right, we compute:* $P(B \geq 10) = P(B = 10) + P(B = 11) + P(B = 12) + ... + P(B = 15) = 0.1508789$.*This test gives us insufficient evidence against the null. Insufficient evidence that median is larger than 0.*

Wilcoxon Signed Rank Test: There are 17 observations, but only 15 of them are non zero (we will see an error that says it caannot compute exact p-value with ties). We get an observed V of 94.5 and p value of 0.02614. This test gives us sufficient evidence against the null to suggest the true location shift is greater than 0. The difference in the results comes from the sign test ignoring a lot of information - it converted numberic values to just + and - which loses a lot of information. Typically we will get the "best informatin" by leaving values as numeric when possible.

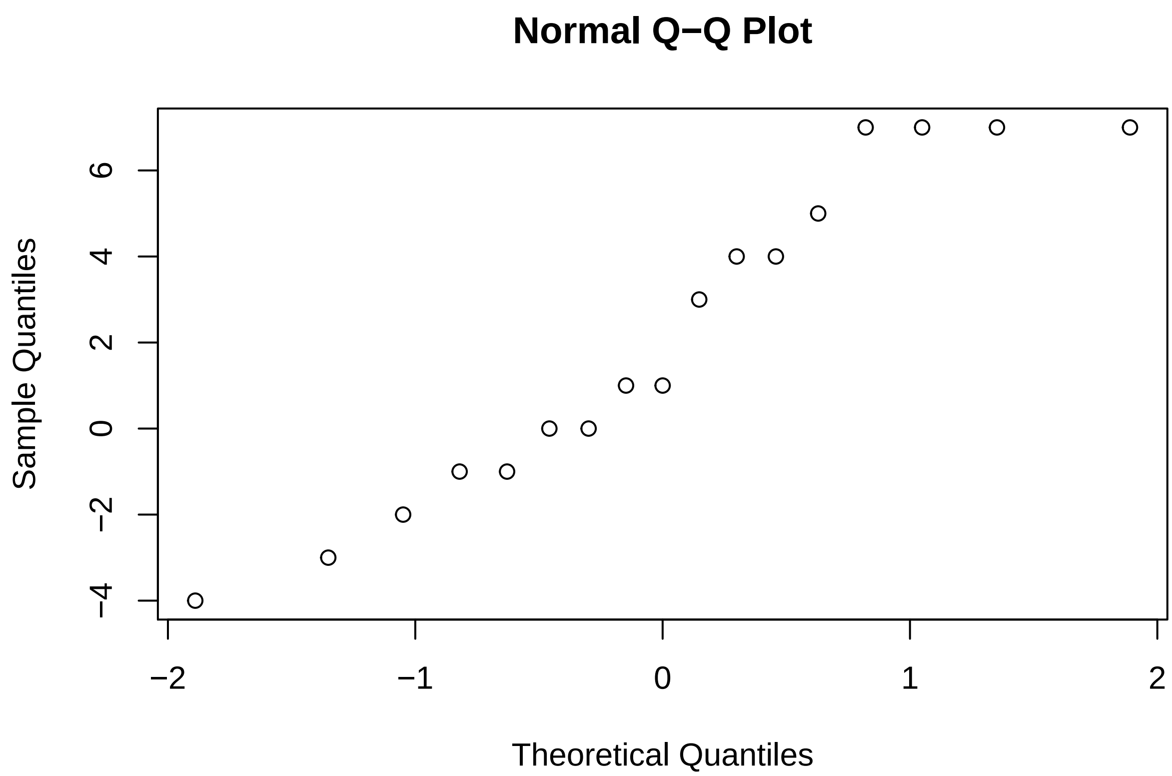
```r
Before<-c(3, 8, 7, 10, 6, 7, 12, 7, 6, 8, 9, 7, 10, 7, 4, 5, 7)
After<-c(7, 12, 14, 8, 5, 7, 8, 6, 13, 9, 6, 8, 10, 10, 11, 12, 12)
(AfMinBef<-After-Before)
```

```
##  [1]  4  4  7 -2 -1  0 -4 -1  7  1 -3  1  0  3  7  7  5
```

```r
hist(AfMinBef)
```




```r
qqnorm(AfMinBef)
```




```
(mD=mean(AfMinBef))   # 2.058824;
```

```
## [1] 2.058824
```

```
(sD=sd(AfMinBef))   #3.732804
```

```
## [1] 3.732804
```

```
(nD=length(AfMinBef))   #17
```

```
## [1] 17
```

```
(tdiff.obs<-(mD-0)/(sD/sqrt(nD)))   #2.274094
```

```
## [1] 2.274094
```

```
pt(2.274094, df=nD-1, lower.tail=FALSE)   #p-value = 0.01854195
```

```
## [1] 0.01854195
```

```
t.test(After, Before, paired=TRUE, alternative="greater")   #t = 2.1619, df = 16, p-value = 0.01854
```

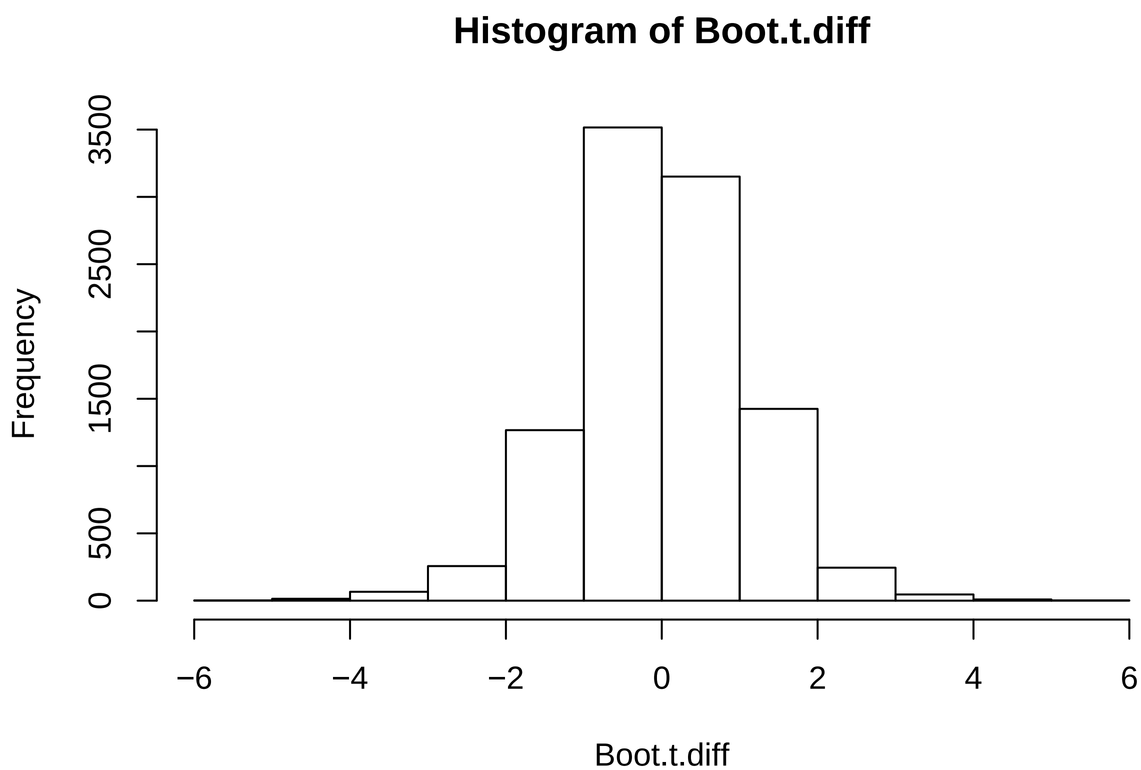
```
##
##  Paired t-test
##
## data:  After and Before
## t = 2.2741, df = 16, p-value = 0.01854
## alternative hypothesis: true difference in means is greater than 0
## 95 percent confidence interval:
##  0.4782089       Inf
## sample estimates:
## mean of the differences
##                2.058824
```

```r
t.test(Before, After, paired=TRUE, alternative="less")  #t = -2.2741, df = 16, p-value = 0.01854
```

```
##
##  Paired t-test
##
## data:  Before and After
## t = -2.2741, df = 16, p-value = 0.01854
## alternative hypothesis: true difference in means is less than 0
## 95 percent confidence interval:
##        -Inf -0.4782089
## sample estimates:
## mean of the differences
##               -2.058824
```

```r
#Bootrap Function
bootstrap = function(x, n.boot) {
  n = length(x)
  x.bar <- mean(x)
  t.hat <- numeric(n.boot) # create vector of length n.boot zeros
  for(i in 1:n.boot) {
    x.star <- sample(x, size=n, replace=TRUE)
    x.bar.star <- mean(x.star)
    s.star <- sd(x.star)
    t.hat[i] <- (x.bar.star - x.bar) / (s.star / sqrt(n))
}
  return(t.hat)
}

set.seed(1)
B=10000
Boot.t.diff<-bootstrap(AfMinBef, B)
hist(Boot.t.diff)
```




```
(m.above<-sum(Boot.t.diff >= tdiff.obs))  #188
```

```
## [1] 188
```

```
(m.below<-sum(Boot.t.diff <= tdiff.obs))  #9812
```

```
## [1] 9812
```

```
(p.value<-m.above/B)  #0.0188
```

```
## [1] 0.0188
```

```
sum(dbinom(10:15, 15, 0.5))
```

```
## [1] 0.1508789
```

```
binom.test(10,15, alternative="greater")
```

```
##
##  Exact binomial test
##
## data:  10 and 15
## number of successes = 10, number of trials = 15, p-value = 0.1509
## alternative hypothesis: true probability of success is greater than 0.5
## 95 percent confidence interval:
##  0.4225563 1.0000000
## sample estimates:
## probability of success
##              0.6666667
```

```
wilcox.test(After, Before, alternative="greater", paired=TRUE)
```

```
## Warning in wilcox.test.default(After, Before, alternative = "greater",
## paired = TRUE): cannot compute exact p-value with ties
```

```
## Warning in wilcox.test.default(After, Before, alternative = "greater",
## paired = TRUE): cannot compute exact p-value with zeroes

##
##  Wilcoxon signed rank test with continuity correction
##
## data:  After and Before
## V = 94.5, p-value = 0.02614
## alternative hypothesis: true location shift is greater than 0
```