Discussion 7 Review

1. Hypothesis Testing Concepts

    - Hypothesis testing involves making a guess about a parameter before gathering data, and then seeing if the data is consistent with our guess.

    - In hypothesis testing, there are two competing hypotheses. One hypothesis, called the **null**, usually denoted H_0, is the uninteresting result. The null is assumed true unless there is evidence to the contrary. Usually the null specifies a single value for a parameter. The other hypothesis, called the **alternative**, usually denoted H_A, is usually what we would like to show. The alternative usually specifies a range of possible values for the parameter.

    - Data are gathered, and a quantity is computed called the **test statistic**. The specific formula for the test statistic will depend on the parameter being tested and the nature of the sampling. The test statistic is an RV. The numerical value of the test statistic (its realization) will be used as evidence to decide between the null and alternative.

    - If the test statistic indicates that the null is likely false, we say that the statistic falls in the **rejection region** and we **reject** the null. If the test statistic offers insufficient evidence against the null, we say that we **do not reject** the null.

    - The following table indicates the possible outcomes of a test:

	Reject H_0	Not Reject H_0
H_0 True	Type I Error / α	Correct
H_0 False	Correct	Type II Error / β

    - $\alpha = P(Reject\ H_0 | H_0\ true)$. Smaller values of α are better.

    - $\beta = P(Not\ reject\ H_0 | H_0\ false)$. Smaller values of β are better.

    - Power $= 1 - \beta = P(Reject\ H_0 | H_0\ false)$. Larger values of power are better.

    - If the sample size is held fixed, if we adjust our rejection region to decrease α, β will go up, and vice versa. The only way to decrease both α and β simultaneously is to increase the sample size.

    - The **p-value** is the probability of a test statistic realizing to a value that is as or more extreme than the one actually observed, when the null hypothesis is true. Smaller p-values indicate relatively more evidence against the null hypothesis.

    - The p-value required to cause a rejection of the null is called the **significance level** of the test.

2. When the data is drawn from a population that has a normal distribution and σ is unknown, use a t-test. To test:

$$H_0 : \mu = \mu_0$$
$$H_A : \mu \neq \mu_0$$

at the $100\alpha\%$ level based on a sample of size n, use one of the following methods:

- Using the rejection region method, determine the value $t_{(n-1,\alpha/2)}$ so that:

$$P(-t_{(n-1,\alpha/2)} \leq t \leq t_{(n-1,\alpha/2)}) = 1 - \alpha.$$

Then compute $t_{obs} = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$. Reject the null if $t_{obs} < -t_{(n-1,\alpha/2)}$ or $t_{obs} > t_{(n-1,\alpha/2)}$.

- Using the p-value method, compute

$$p - value = P(t_{(n-1)} < -|t_{obs}|) + P(t_{(n-1)} > |t_{obs}|).$$

Reject if $p - value < \alpha$.

- Using the CI method, find a t-based $100(1-\alpha)\%$ CI for μ. If μ_0 is in the interval, do not reject the null. If it is not in the interval, reject the null.

3. When the data is drawn from a population that has a normal distribution and σ is known, the sample size n required to achieve power $1 - \beta$ for a test of $H_0 : \mu = \mu_0$ vs. $H_A : \mu \neq \mu_0$ when the real μ is μ_A at level α is approximately:

$$n = (\frac{\sigma(z_{\alpha/2}+z_{\beta})}{\mu_0-\mu_A})^2.$$