1. **(4 points)** A diagnostic test is used to determine whether a patient has arthritis. A treatment will be prescribed only if the doctor thinks the test gives enough evidence to suggest the patient has arthritis. The hypotheses might be stated as follows:

$$H_0\text{: The patient does not have arthritis}$$
$$H_A\text{: The patient has arthritis}$$

Label each of the following statements as "Type 1 Error", "Type 2 Error" or "No Error" as appropriate.

1 point each

(a) Diagnosing arthritis in a patient who has arthritis. *No Error*

(b) Failing to diagnose arthritis in a patient who has arthritis. *Type 2 Error*

(c) Diagnosing arthritis in a patient who does not have arthritis. *Type 1 Error*

(d) Failing to diagnose arthritis in a patient who does not have arthritis. *No Error*

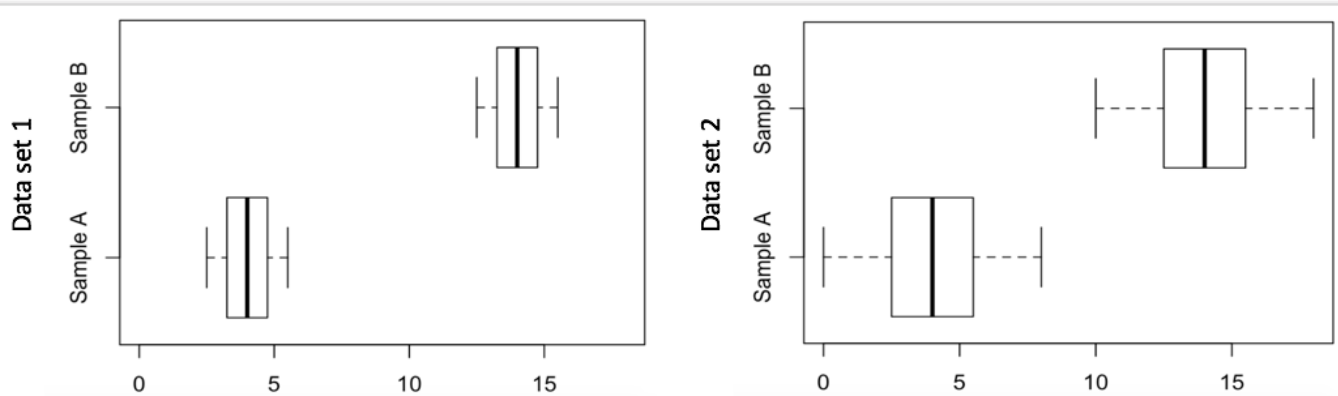
1 point each

2. **(3 points)** Which of the following statements is/are true about the t distribution with ν degrees of freedom (assume $\nu > 1$)? Select all that apply.

(a) The t-distribution is symmetric for all ν TRUE

(b) The t-distribution with ν degrees of freedom has heavier tails (higher variance) than the t-distribution with $\nu + 1$ degrees of freedom TRUE

(c) As ν increases, the t-distribution more closely resembles a Standard Normal distribution. TRUE



3. **(4 points)** Consider two data sets, data set 1 and data set 2. Each data set has two independent samples: a Sample A with mean $\bar{X}_A = 4$ and $n_a = 7$ values and a Sample B with $\bar{X}_B = 14$ and $n_b = 8$ values. Suppose the hypotheses $H_0 : \mu_A - \mu_B = 0$ and $H_A : \mu_A - \mu_B \neq 0$ are tested on **data set 1** with a Wilcoxon (p-value= 0.0014) and a t test with equal variance assumption (p-value=0.0002). If the same hypotheses are tested on data set 2 with both tests, circle the correct word to fill in the blanks:

2 point each

(a) The p value of the t test will be ______ than/to 0.0002. smaller, larger, equal *larger*

(b) The p value of the Wilcoxon will be ______ than/to 0.0014. smaller, larger, equal *equal*

4. A lumber company typically only agrees to log a plot of land if they have strong evidence that a large proportion of the trees on the plot have usable wood. A consultant to the lumber company takes a random sample of 80 trees from Plot A and determines a confidence interval of (0.659, 0.821) for the proportion of trees on Plot A that have useable wood, however she never reports the level of confidence.

   (a) **(3 points)** Determine the percent of usable wood that was observed on Plot A in that sample.
   (0.821+0.659)/2=0.74

   (b) **(4 points)** Determine the level of confidence for the interval given above.
   *(0.821+0.659)/2=0.74 so ME=0.081=z*SE, proportion of success: .74; proportion of failure: .26;* $SE = \sqrt{\frac{.74 * .26}{80}} = 0.049$ *Since* $SE = 0.049$ *z=ME/SE=0.081/0.049=1.64 so 90% CI:* $.74 \pm 1.645 * 0.049 = (0.659, 0.821)$.

   (c) **(3 points)** Suppose two other plots of land, Plot B and Plot C are sampled and they observe that 81 out of the 120 trees on Plot B and 72 of the 95 trees on Plot C have useable wood. Identify the appropriate computation to determine the test statistic for the hypothesis test of $H_O : \pi_B - \pi_C = 0$ and $H_A : \pi_B - \pi_C \neq 0$

      i. $\frac{.675 - .758 - 0}{\sqrt{\frac{.675 * .325}{120} - \frac{.758 * .242}{95}}}$

      ii. $2 * \frac{.675 - .758 - 0}{\sqrt{\frac{.675 * .325}{120} + \frac{.758 * .242}{95}}}$

      iii. $2 * \frac{.675 - .758 - 0}{\sqrt{\frac{.712 * 0.288}{120} + \frac{.712 * 0.288}{95}}}$

      iv. $\frac{.675 - .758 - 0}{\sqrt{\frac{.712 * 0.288}{120} + \frac{.712 * 0.288}{95}}}$

      v. $\frac{.675 - .758 - 0}{\sqrt{\frac{.712 * 0.288}{120} - \frac{.712 * 0.288}{95}}}$

   D. Plot B: 81/120=0.675 and Plot C:72/95=/758; pooled: (81+72)/(120+95)=0.7116
   +3 if chose correct answer.

5. **(4 points)** Identify whether each of the tests are appropriate for hypothesis tests with 2 independent, 2 matched (dependent), or either (depending on the set up) sampling. *+1 point each*

   (a) Welch's T test independent, dependent, either *independent*

   (b) Bootstrap independent, dependent, either *either*

   (c) Wilcoxon Signed-Rank independent, dependent, either *dependent*

   (d) Sign Test independent, dependent, either *dependent*

6. A city manager audits the parking tickets issued by paarking officers. In the past, the number of improperly issued tickets per officer had a normal distribution with mean $\mu = 185$ and standard deviation $\sigma = 23.6$. The city manager suspects the mean number of improperly issued tickets has increased because of a change in night parking regulations. She would like to test the following hypotheses for μ the mean number of improperly issued tickets per officer:

$$H_0 : \mu = 185 \text{ and } H_A : \mu > 185$$

(a) **(4 points)** What is the range of observed sample mean values that would lead to a conclusion that there has been an increase in the mean number of improperly issued tickets at an $\alpha = 0.1$ level if the manager plans to take a sample size of 20 officers and assumes that the standard deviation is still $\sigma = 23.6$?
$Z = 1.285 \textit{ so } \bar{X} = 185 + 1.285 * \frac{23.6}{\sqrt{20}} \textit{ so } \bar{x} > 191.78$

(b) **(4 points)** What power does the hypothesis test above have at the $\alpha = 0.1$ level to reject the null if in fact the true mean number of improperly issued tickets this year is 195 (assume sample size and standard deviation of part a)?
$Power{=}P(Reject H_o|H_a : \mu = 195) = P(\bar{x} > 191.78|\mu_a = 195) = P(z > \frac{191.78-195}{23.6/\sqrt{20}}) = P(Z > -0.610) = 0.729$. *So power* $\approx .729$

(c) **(3 points)** Which of the following actions would increase the power of the hypothesis test described above? (Select all that apply)
+1.5 for each correct; -1 for each incorreclty chosen; minimum of 0

i. Having a larger population standard deviation
ii. If the true mean number of improperly issued tickets this year is 205 TRUE
iii. Taking a sample of size 30 officers instead of 20. TRUE
iv. Lowering the originally set α level to 5%.

(d) **(3 points)** Consider a 1-sample z test and confidence interval for means. Explain why when the lower bound of a [two-sided] 95% confidence interval is greater than the mean μ_0 specified in the null hypothesis ($H_0 : \mu = \mu_0$), that same sample would give us sufficient evidence at the 5% level to reject the null in favor of the **one-sided greater** alternative ($H_A : \mu > \mu_0$). (some algebra or pictures regarding the rejection region may help)
Since we know lower bound is above μ_0, *we can write:* $\bar{x} - z_{0.025}(1.96) * \frac{SD}{\sqrt{n}} > \mu_0$. *Re writing this in terms of* $\bar{x} > \mu_0 + z_{0.025}(1.96) * \frac{SD}{\sqrt{n}}$. *We know the rejection region for the one-sided test in terms of z is* $z_{obs} > z_{.05}(1.645)$ *which in terms of* $\bar{x} > \mu_0 + z_{0.05}(1.645) * \frac{SD}{\sqrt{n}}$. *Since* $\bar{x} > \mu_0 + z_{0.025}(1.96) * \frac{SD}{\sqrt{n}} > \mu_0 + z_{0.05}(1.645) * \frac{SD}{\sqrt{n}}$ *implies* $\bar{x}$ *is in the RR for the 1 sided hypothesis test*

7. A scientist wants to test whether the mean glutamic acid in the core tissue is greater than that in the outter (pericarp) tissue. Ten ripe mangoes were sampled and the amount of glutamic acid (mg/100g fresh tissue) was determined spectrophotometrically in both the outter layer (pericarp) and the core tissue of each mango. The summary statistics of the data are presented below:

Area	Sample Mean	Sample Median	Sample SD	Size
Outter	10.40	10.23	0.90	10
Core	11.20	11.14	1.07	10
Diff:Core-Outter	0.80	0.71	0.99	10

(a) **(3 points)** Explain what assumption(s) we must assume are met to perform the relevant t test and what graphs or summary measures we could use to evaluate them.

I will do a 1 sample t test since we have a matched pair set up. We have to assume the population of differences is approximately normal and each difference is also independent. We also assume we do not know the population standard deviation. We could look at the qqnorm plot of the differences to see if we have evidence that the population of differences is approximately normal.

(b) Perform the appropriate t hypothesis test at an alpha = 5% level regarding the research question.

i. **(3 points)** Hypotheses and parameter[s] of interest:
Let D=Core-Outside: so μ_d is the true mean difference glutamic acid in Core-Outside $H_o : \mu_D = 0$ *vs* $H_A : \mu_D > 0$

ii. **(3 points)** Test statistic and p value (Show computations):
$t_{obs} = \frac{0.80-0}{0.99/\sqrt{10}} = 2.55$. *p value*$=.01 < P(T_9 > 2.55) < .02$.*Exact values in R: t = 2.5568, df = 9, p-value = 0.01543*

iii. **(3 points)** Draw a conclusion in the context of the question:
Since our observed p value is smaller than $\alpha = 0.05$*, we have sufficient evidence to reject the null. Evidence suggests higher levels of mean glutamic acid in the core tissue than the outter (pericarp) tissue.*

(c) **(4 points)** Suppose that instead of performing the t hypothesis test, the researcher wanted a [two-sided] confidence interval from a "bootstrap t" distribution. Some relevant quantiles are given for the bootstrap distribution below. Construct a 95% confidence interval using the bootstrap t distribution.

Percent Below:	2.5%	5%	10%	90%	95%	97.5%
Bootstrap t value:	-2.64	-2.12	-1.54	1.26	1.67	2.06

ANSWER: obs difference: .80 and SE: $\frac{0.99}{\sqrt{10}} = 0.313$ 95% *CI: .80-2.06*0.313=0.155; .80+2.64*0.313=1.626*

Summary Statistics included for reference:

Area	Sample Mean	Sample Median	Sample SD	Size
Outter	10.40	10.23	0.90	10
Core	11.20	11.14	1.07	10
Core-Outter	0.80	0.71	0.99	10

(d) **(6 points)** Suppose that instead of either of the analyses above, the researcher ignored the study design and chose to do a two independent sample t test, with hypotheses: $H_0 : \mu_{core} - \mu_{out} = 0$ vs $H_0 : \mu_{core} - \mu_{out} > 0$ **assuming equal population variances**. Describe whether each of the following values would increase, decrease, or not change (You need to include some computations or clear explanation for your answer) compared to part b.

- numerator of observed test statisic: would stay the same at 0.80-0
- denominator of observed test statisic: increase: $SE = .99 * \sqrt{1/10 + 1/10} = 0.4427$ where $s_p = \sqrt{\frac{9*(0.90^2)+9*(1.07)^2}{18}} = \sqrt{\frac{17.59}{18}} = \sqrt{0.977} = 0.99$
- observed t test statistic: decrease $\frac{.80-0}{0.4427} = 1.81$
- degrees of freedom: increase: would now be 18
- p value: between 2.5% and 5%; increase

8. **(3 points)** Suppose we are interested in constructing a confidence interval for a difference in population means with two independent samples. Choose one option for **each** of the following characteristics that will create a confidence interval with the **narrowest/shortest** width?

+1 point each; larger, smaller, lower

(a) ______ sample sizes smaller, larger *Large*

(b) populations with ______ standard deviation lower, higher *low*

(c) ______ confidence level lower, higher *low*