Discussion 4 Review

1. The Binomial RV, which is usually denoted by B, is then defined as the total number of successes out of n many trials with probability π of success on any given trial. The shorthand to define a random variable as a Binomial RV with parameters π and n is

$$B \sim Bin(n, \pi)$$

and the probability of observing b sucesses is:

$$p(B = b) = \frac{n!}{b!(n-b)!}\pi^b(1-\pi)^{n-b}$$

and the expectation and variance are:

$$E(B) = n\pi \quad \text{and} \quad \text{VAR}(B) = n\pi(1-\pi)$$

.

2. The **expectation** of an RV X, denoted $E(X)$ or μ_X, is like the mean of the population. The expectation of a discrete RV X is:

$$\mu_X = E(X) = \sum_x x \cdot p(x)$$

The **variance** of an RV X, denoted $\text{VAR}(X)$, or σ_X^2 is like the variance of the population. The variance of a discrete RV X is:

$$\sigma_X^2 = \text{VAR}(X) = \sum_x p(x) \cdot (x - E(X))^2$$

3. The normal distribution has the following properies:

   (a) The normal is symmetric around the mean, μ.

   (b) The total area under the curve is 1.

   (c) The area under the curve between $\mu - \sigma$ and $\mu + \sigma$ is about 0.68; the area under the curve between $\mu - 2\sigma$ and $\mu + 2\sigma$ is about 0.95.

   (d) If X is a normal RV, the pdf of X is:

$$f(x) = \tfrac{1}{\sqrt{2\pi\sigma^2}}e^{(\frac{-(x-\mu)^2}{2\sigma^2})}$$

   (e) A normal RV X is denoted $X \sim N(\mu, \sigma^2)$, and the expectation and variance are:

$$E(X) = \mu \text{ and } \text{VAR}(X) = \sigma^2 \text{ (so } SD(X) = \sigma).$$

(f) If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$.

(g) If $X \sim N(\mu, \sigma^2)$, $\mathbb{P}(X \leq x) = \mathbb{P}(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) = \mathbb{P}(Z \leq \frac{x-\mu}{\sigma})$.

(h) If $Z \sim N(0, 1)$, then $X = Z\sigma + \mu \sim N(\mu, \sigma^2)$.

4. Some rules of expectation and variance follow:

   (a) $E(c) = c$.

   (b) $E(c * X) = c * E(X)$.

   (c) $E(X + c) = E(X) + c$.

   (d) $E(X + Y) = E(X) + E(Y)$.

   (e) $VAR(c) = 0$.

   (f) $VAR(c * X) = c^2 VAR(X)$.

   (g) $VAR(X + c) = VAR(X)$.

   (h) If X and Y are independent, $VAR(X + Y) = VAR(X) + VAR(Y)$.