Solutions Discussion 4: Normal RVs and Combining RVs

1. Weights of female cats of a certain breed are well approximated by a normal distribution with mean 4.1 kg and standard deviation of 0.6 kg $X \sim (4.1, 0.6^2)$.

   (a) What proportion of female cats have weights between 3.7 and 4.4 kg? *P(3.7<X<4.4) implies P(-0.6666667<Z<0.5), .6915-.2514= 0.4401, pnorm(4.4, 4.1, .6)-pnorm(3.7, 4.1, .6)=0.4389699*

   (b) A certain female cat has a weight that is 0.5 standard deviations above the mean. What proportion of female cats are heavier than this one? *Z score of 0.5 means .6915 area below, so about 1-.6915=0.3085 heavier*

   (c) How heavy is a female cat whose weight is on the 80th percentile? *.80 percentile means z score of z=0.84 so X=.6*.84+4.1=4.604, qnorm(.8, 4.1, .6)=4.604973*

   (d) A female cat is chosen at random. What is the probability that she weighs more than 4.5 kg? $P(X > 4.5) = P(Z > (4.5 - 4.1)/.6) = P(Z > 0.6666667) = 1 - 0.7475075 = 0.2525$ *pnorm(4.5, 4.1, .6)=0.7475075*

   (e) Ten female cats are chosen at random from a large population. What is the probablity that exactly 2 of them weigh more than 4.5 kg? Assume that the weights of the 10 cats are independent (large population size). *X=Binomial(10,0.2525).* $P(X = 2) = 10C2 * 0.7475075^8 * (1 - 0.7475075)^2 = 0.2796565$

   (f) Ten female cats are chosen at random from a large population. What is the probablity that their average weight is more than 4.2 kg? Assume that the weights of the 10 cats are independent (large population size). $\mu_{\bar{x}} = 4.1$; $\sigma_{\bar{X}} = \frac{0.6}{\sqrt{10}} = 0.1897$. $P(\bar{X} > 4.2) = P(Z > \frac{4.2-4.1}{0.1897}) = P(Z > 0.5271) = 0.29906$

   (g) Suppose 1 female cat is selected at random from the given distribution and 1 male cat of that same breed is also selected at random. Male weights are well approximated by a normal distribution with mean of 4.3 and standard deviation of 0.2; $Y \sim (4.3, 0.2^2)$. Define a new random variable: $D = Y - X$. Describe the distribution of D and calculate the probability that $D > 0$ assuming X and Y are independent. What does this value mean in the context of the problem? $\mu_D = 4.3 - 4.1 = 0.2$ *and* $Var(D) = Var(Y) + Var(X) = 0.2^2 + 0.6^2 = 0.4$ *so* $SD(D) = \sqrt{.4} = 0.632$ *so* $D \sim N(0.2, .4)$ *Since linear combination of normal RV is normal.*$P(D > 0) = P(Z > \frac{0-.2}{.632}) = P(Z > -0.316) = 1 - 0.376 = 0.624$ *This is the probability of getting a male cat that weighs more than a female cat if we have a random pair of one male and one female cat.*

2. A large auto dealership keeps track of sales and lease agreements made during each hour of the day. Let X=the number of cars sold at the East Side Madison location and Y=the number of cars leased at the Milwaukee location during the first hour of business on randomly selected Fridays. Based on previous records, the probability distribution of X and Y are as follows. Note: $\mu_X = 1.1$, $\sigma_X = 0.943$ and $\mu_Y = 0.7$ and $\sigma_Y = 0.64$.

Cars Sold X_i	0	1	2	3	Cars Cars Leased Y_i	0	1	2
$P(X = x_i)$	0.3	0.4	0.2	0.1	$P(Y = y_i)$	0.4	0.5	0.1

(a) Define $T = X + Y$. Find and interpret μ_T in context. $\mu_T = 1.8$. *On average, this dealership sells or leases 1.8 cars in the first hour of business on Fridays.*

(b) Compute σ_T assuming that X and Y are independent. Is this is a good assumption? $\sigma_T = \sqrt{.943^2 + .64^2} = 1.14$. *Not sure. If an individual leases a car they are not likely to also purchase a car from the other location. Also knowing how many cars were purchased at the one location, may give us information about how many may be leased at the second location - but we don't know. Are they competing goods or not? Would people who are buying a car consider leasing one instead? This independence assumption would be something I would want to discuss with an economist.*

(c) The dealer's manager receives a \$500 bonus for each car sold and a \$300 bonus for each car leased. Find the mean and standard deviation for the manager's total bonus for the first hour of business on Fridays assuming X and Y are independent. $\mu_B = 500 * 1.1 + 300 * .7 = \$760$ *and* $\sigma_B = \sqrt{500^2(.943)^2 + 300^2 * (.64)^2} = \$509.09$

(d) Define $D = X - Y$. Find and interpret μ_D. Can we easily compute $P(D > 0)$? $\mu_D = E(X) - E(Y) = 0.4$ *On Average, this dealership sells 0.4 cars more than it leases during the first hour of business on Fridays. We cannot easily compute that probability without writing out the full pmf of* $D = X - Y$. *We cannot use our normal calculations because the distribution of D is not approximately normal (neither X nor Y are approximately normal.)*