Discussion 4: Normal RVs and Combining RVs

1. Weights of female cats of a certain breed are well approximated by a normal distribution with mean 4.1 kg and standard deviation of 0.6 kg $X \sim (4.1, 0.6^2)$.

   (a) What proportion of female cats have weights between 3.7 and 4.4 kg?

   (b) A certain female cat has a weight that is 0.5 standard deviations above the mean. What proportion of female cats are heavier than this one?

   (c) How heavy is a female cat whose weight is on the 80th percentile?

   (d) A female cat is chosen at random. What is the probability that she weighs more than 4.5 kg?

   (e) Ten female cats are chosen at random from a large population. What is the probablity that exactly 2 of them weigh more than 4.5 kg? Assume that the weights of the 10 cats are independent (large population size).

   (f) Ten female cats are chosen at random from a large population. What is the probablity that their average weight is more than 4.2 kg? Assume that the weights of the 10 cats are independent (large population size).

   (g) Suppose 1 female cat is selected at random from the given distribution and 1 male cat of that same breed is also selected at random. Male weights are well approximated by a normal distribution with mean of 4.3 and standard deviation of 0.2; $Y \sim (4.3, 0.2^2)$. Define a new random variable: $D = Y - X$. Describe the distribution of D and calculate the probability that $D > 0$ assuming X and Y are independent. What does this value mean in the context of the problem?

2. A large auto dealership keeps track of sales and lease agreements made during each hour of the day. Let X=the number of cars sold at the East Side Madison location and Y=the number of cars leased at the Milwaukee location during the first hour of business on randomly selected Fridays. Based on previous records, the probability distribution of X and Y are as follows. Note: $\mu_X = 1.1$, $\sigma_X = 0.943$ and $\mu_Y = 0.7$ and $\sigma_Y = 0.64$.

Cars Sold X_i	0	1	2	3	Cars Cars Leased Y_i	0	1	2
$P(X = x_i)$	0.3	0.4	0.2	0.1	$P(Y = y_i)$	0.4	0.5	0.1

(a) Define $T = X + Y$. Find and interpret μ_T in context.

(b) Compute σ_T assuming that X and Y are independent. Is this is a good assumption?

(c) The dealer's manager receives a \$500 bonus for each car sold and a \$300 bonus for each car leased. Find the mean and standard deviation for the manager's total bonus for the first hour of business on Fridays assuming X and Y are independent.

(d) Define $D = X - Y$. Find and interpret μ_D. Can we easily compute $P(D > 0)$?