# Discussion 8 Review

1. When the data is drawn from a population that has a normal distribution and $\sigma$ is unknown, use a t-test. To test:

$$H_0 : \mu = \mu_0$$
$$H_A : \mu \neq \mu_0$$

at the $100\alpha\%$ level based on a sample of size $n$, use one of the following methods:

- Using the rejection region method, determine the value $t_{(n-1,\alpha/2)}$ so that:

$$P(-t_{(n-1,\alpha/2)} \leq t \leq t_{(n-1,\alpha/2)}) = 1 - \alpha.$$

Then compute $t_{obs} = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$. Reject the null if $t_{obs} < -t_{(n-1,\alpha/2)}$ or $t_{obs} > t_{(n-1,\alpha/2)}$.

- Using the p-value method, compute

$$p - value = P(t_{(n-1)} < -|t_{obs}|) + P(t_{(n-1)} > |t_{obs}|).$$

Reject if $p - value < \alpha$.

- Using the CI method, find a $t$-based $100(1-\alpha)\%$ CI for $\mu$. If $\mu_0$ is in the interval, do not reject the null. If it is not in the interval, reject the null.

2. When the data is drawn from a population that does not have a normal distribution but $n$ is large enough that the CLT applies, test just as for the $t$-test, but use the normal distribution.

3. When the data is not normal and $n$ is too small to use the CLT, use one of the following methods:

- Bootstrap. Test:

$$H_0 : \mu = \mu_0$$
$$H_A : \mu > \mu_0$$

by doing the following:

(1) Draw a random sample $x_1$, $x_2$, ..., $x_n$ of size $n$ from the population and compute $t_{obs} = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$.

(2) Draw a simple random sample, with replacement, of size $n$, from the sample data. Call these observations $x_1^*$, $x_2^*$, ..., $x_n^*$. Often this means that the same data point will be repeated twice in the resampling.

(3) Compute the mean and sd of the resampled data. Call these things $\bar{x}^*$ and $s^*$.

(4) Compute the statistic $\hat{t} = \frac{\bar{x}^* - \bar{x}}{\frac{s^*}{\sqrt{n}}}$.

(5) Repeat steps 2-4 $B$ times, where $B$ is a large number, and compute $\hat{t}$ from each one. This is an approximation to the sampling distribution of $t$.

(6) Let $m$ be the number of values of $\hat{t}$ that are greater than or equal to $t_{obs}$. The bootstrap p-value is $m/B$.

- Sign Test. If $M$ is the population median, test:

$$H_0 : M = M_0$$
$$H_A : M > M_0$$

by computing $b$ = the number of observations strictly larger than $M_0$. If any observations are equal to $M_0$, remove them. The p-value is then $P(B \geq b)$, where $B \sim Bin(n, 0.5)$.

4. When making a test about population proportion $\pi$ based on a sample of size $n$, if $n(\pi_0) > 5$ and $n(1 - \pi_0) > 5$, then test:

$$H_0 : \pi = \pi_0$$
$$H_A : \pi \neq \pi_0.$$

by computing the sample proportion $p$, and then finding:

$$z_{obs} = \frac{(p - \pi_0)}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

Then the p-value is $P(Z < -|z_{obs}|) + P(Z > |z_{obs}|)$. Reject if $p - value < \alpha$.