HW4

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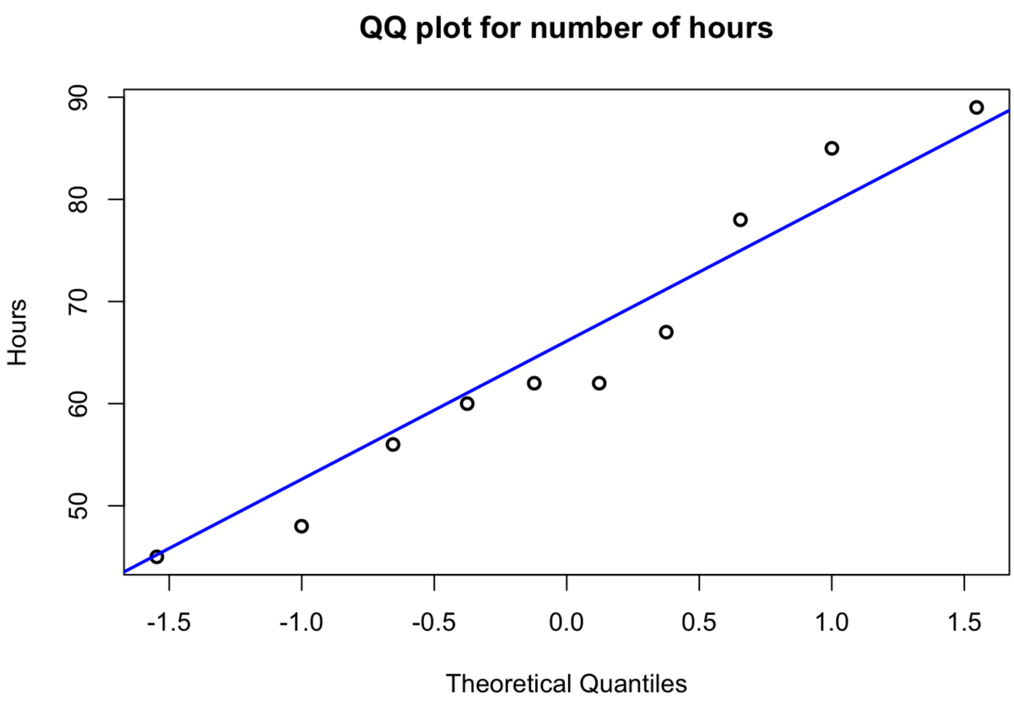
Problem 1

The data below record the number of hours a team of workers takes to assemble a custom-built motorcycle. The data are recorded for 10 different teams each assembling a motorcycle. 89 78 48 85 67 45 60 62 62 56

a. Create a QQplot in R and comment on the assumption that the population of times to assemble a motorcycle is well-approximated by a normal distribution.

```
hours = c(89, 78, 48, 85, 67, 45, 60, 62, 62, 56)
qqnorm(hours,lwd=2,main="QQ plot for number of hours", ylab = "Hours")
qqline(hours,col="blue",lwd=2)
```



From the QQ-plot it is observed most of the observations are very close to a linear line and hence the data is well approximated by a normal distribution.

b. Construct a 90% confidence interval by hand for the mean time it takes a team of workers to assemble a custom-built motorcycle.

```
sd(hours)
```

```
## [1] 14.76332
```

```
mean(hours)
```

```
## [1] 65.2
```

```
# Use qnorm() on &alpha; / 2
# So 0.10 / 2 = 0.05
qnorm(0.05)
```

```
## [1] -1.644854
```

$\bar{X} \pm z(\alpha / 2) * \sigma / \sqrt{n}$
n = 10
μ = 65.2

SD = 14.76332
Margin of Error = 1.644 * (14.76332 / √10)
μ ± Margin of Error
= (57.52487, 72.87513)

c. Construct the same interval above using R's t.test() command.

```
t.test(hours, conf.level = 0.90)
```

```
##
##  One Sample t-test
##
## data:  hours
## t = 13.966, df = 9, p-value = 2.095e-07
## alternative hypothesis: true mean is not equal to 0
## 90 percent confidence interval:
##  56.64198 73.75802
## sample estimates:
## mean of x
##      65.2
```

d. By how much does the confidence interval width of a 95% interval differ from that of a 90% interval for this data (can complete in R or by hand)?

```
t.test(hours, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  hours
## t = 13.966, df = 9, p-value = 2.095e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  54.63896 75.76104
## sample estimates:
## mean of x
##      65.2
```

The new interval is wider, but we are more confident that it contains the true mean.

e. Suppose instead, the manager had performed a hypothesis test at the 10% level of the null that μ = 60 hours vs the alternative that μ < 60. Compute the pvalue for his hypothesis test and summarize the conclusion that he would have drawn from the sample that he observed.

H_0: μ = 60
H_A: μ < 60
Z test statistic value < $Z_{0.90}$ = -1.644854, or obs $\bar{X}$ < 65.2 with sample size of 10
P($\bar{X}$ > 65.2) = P(Z > 65.2 - 59 / (14.76332 / √10))
P(Z > 1.328029) = 1 - P(Z <= 1.328029)
= 1 - 0.908
= 0.092

9% chance to fail to reject H_0 when we should have rejected it.

Problem 2

An automobile club which pays for emergency road services (ERS) requested by its members wishes to estimate the proportions of the different types of ERS requests. Upon examining a sample of 2927 ERS calls, it finds that 1499 calls related to starting problems, 849 calls involved serious mechanical failures requiring towing, 498 calls involved flat tires or lockouts, and 81 calls were for other reasons.

a. Estimate the true proportion of ERS calls that involved serious mechanical problems requiring towing and construct a 95% confidence interval after checking that conditions have been met. Interpret your interval in context.

(849 / 2927) = 0.29
True proportion = 0.29
95% CI = $Z_{\alpha / 2}$ * √ (0.29 * (1 - 0.29)) / n
= 1.96 * √ (0.29 * 0.71) / 2927
= 1.96 * √0.00007
= 1.96 * 0.0084

= 0.0164
0.29 ± 0.0164
= (0.2736, 0.3064)
We are 95% confident that the proportion of calls related to serious mechanical problems requiring towing will fall between (0.2736, 0.3064).

b. Calculate a 98% confidence interval for the true proportion of ERS calls that related to starting problems after checking that conditions have been met. Interpret your interval in context.

(1499 / 2927) = 0.51
0.51 ± Margin of Error
Margin of Error = $Z_{\alpha / 2} * \sqrt{}\ 0.51 * (1 - 0.51) / n$
Z table at 0.98 is 2.33
= 2.33 * $\sqrt{}$ (0.51 * 0.49) / 2927
= 2.33 * 0.00924
= 0.02153
0.51 ± 0.02153
= (0.4885, 0.5315)
We are 98% confident that the proportion of calls related to starting problems will fall between (0.4885, 0.5315).

Problem 3

At the Hawaii Pineapple Company, managers are interested in the size of the pineapples grown in the company's fields. Last year, the mean weight of the pineapples harvested from one large field was 31 ounces with a standard deviation of 4 ounces. A different irrigation system was installed in this field after the growing season. Managers wonder if the the mean weight of pineapples grown in the field this year will be different from last.

a. Write out the null Ho and alternative hypotheses Hain terms of the population mean μ.

H_0: μ = 31
H_A: μ =/= 31

b. If the managers choose to use a significance level of 0.05 and assume σ = 4, identify their power to detect a increase of 2 ounces in the mean (μa = 33) if they look at a sample that is 30 pineapples this year and use the two-sided alternative. Also identify the probability of making a type 2 error with true μa = 33.

α = 0.05
σ = 4
n = 30
H_0: μ = 31
H_A: μ = 33
Power is given by P(Reject H_0 | H_0 is false)
Z = ±1.96
$\bar{X}$ - μ / (σ / $\sqrt{n}$) = 1.96
$\bar{X}$ - 31 / (4 / $\sqrt{30}$) = 1.96
31 ± 1.431382
$\bar{X}$ = 29.57, 32.43
Reject H_0 if $\bar{X}$ < 29.57 AND $\bar{X}$ > 32.43
Power = P(Reject H_0 | μ_a = 33)
= P($\bar{X}$ < 29.57 OR $\bar{X}$ > 32.43 | μ_a = 33)
= P(Z < 29.57 - 33 / (4 / $\sqrt{30}$) OR Z > 32.43 - 33 / (4 / $\sqrt{30}$))
= P(Z < -6.70 OR Z > -0.7805)
β = Probability of type 2 error
= 1 - Power = 0.2177

c. Draw pictures of the null and alternative distributions of the means and shade the areas that correspond to (i) Type 1 error, (ii) Type 2 error, and (iii) Power from part (b). (The online applet from the notes may help.)

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d. Explain Type 1 and Type 2 errors of the test in context.

Type 1 error: We conclude that the mean weight of pineapples grown in the field this year different from last year, when actually it is not different.
Type 2 error: We conclude that the mean weight is not different from last year when actually it is different.

e. What sample size should the managers use to ensure their test has power of at least 0.9 to detect μ_a = 33 (assuming σ = 4)?

β = 0.10
α = 0.05
Check 1 - 0.10 = 0.90 in Z table
$Z_{0.10}$ = 1.285

$Z_{0.05} = 1.645$
$n = (4 * (1.645 + 1.285) / 31 - 33)^2$
$= 34.3396$
~ 34

f. Explain why the managers may prefer to compute a 95% confidence interval instead of a two-sided 5% hypothesis test.

The two-sided test at $\alpha = 0.05$ rejects H_0 in favor of H_a. The corresponding 95% confidence interval does not include 31 as a plausible value of the parameter μ. In other words, the test and interval lead to the same conclusion about H_0. But the confidence interval provides much more information: a set of plausible values for the population mean.

Prblem 4

A scientist is doing a preliminary study to try to determine the sample size necessary for her larger study. She would like to show that the mean in her population of interest is larger than 12 and is starting with the assumption that Ho : μ = 12 which is will be testing at the α = 0.1 level. She takes a random sample of n=10 from her population and checks the QQplot and sample histogram. From the graphs, the normality of the population assumption is pretty well met. Her sample mean is $\bar{X} = 14.2$ and sample standard deviation is s = 4.88.

a. Compute the critical values and rejection region for the appropriate test statistic and $\bar{X}$ and p value of the appropriate test. Draw a conclusion in the context of the study.

Test statistic $t = \sqrt{10}(\bar{X} - 12) / s$
$\bar{X} = 14.2$
$s = 4.88$
$\sqrt{10}(14.2 - 12) / 4.88$
$= 1.4256$
Critical value of $t_{(9\ ,\ 0.10)} = 1.383$
p-value $= P(t > 1.4256 \mid t \sim t_9) = 0.0939 < 0.10$
Hence we reject H_0 at 0.10 level of significance and conclude that the true mean is larger than 12.

b. Suppose she is interested in the specific alternative that μA = 15 and will still be conducting a one-sided test. Compute the power of a follow up hypothesis test to correctly reject the null if in fact μA = 15 is true with a sample size of 40. Use the sample standard deviation above to estimate σ, use z critical values, and α = 0.1.

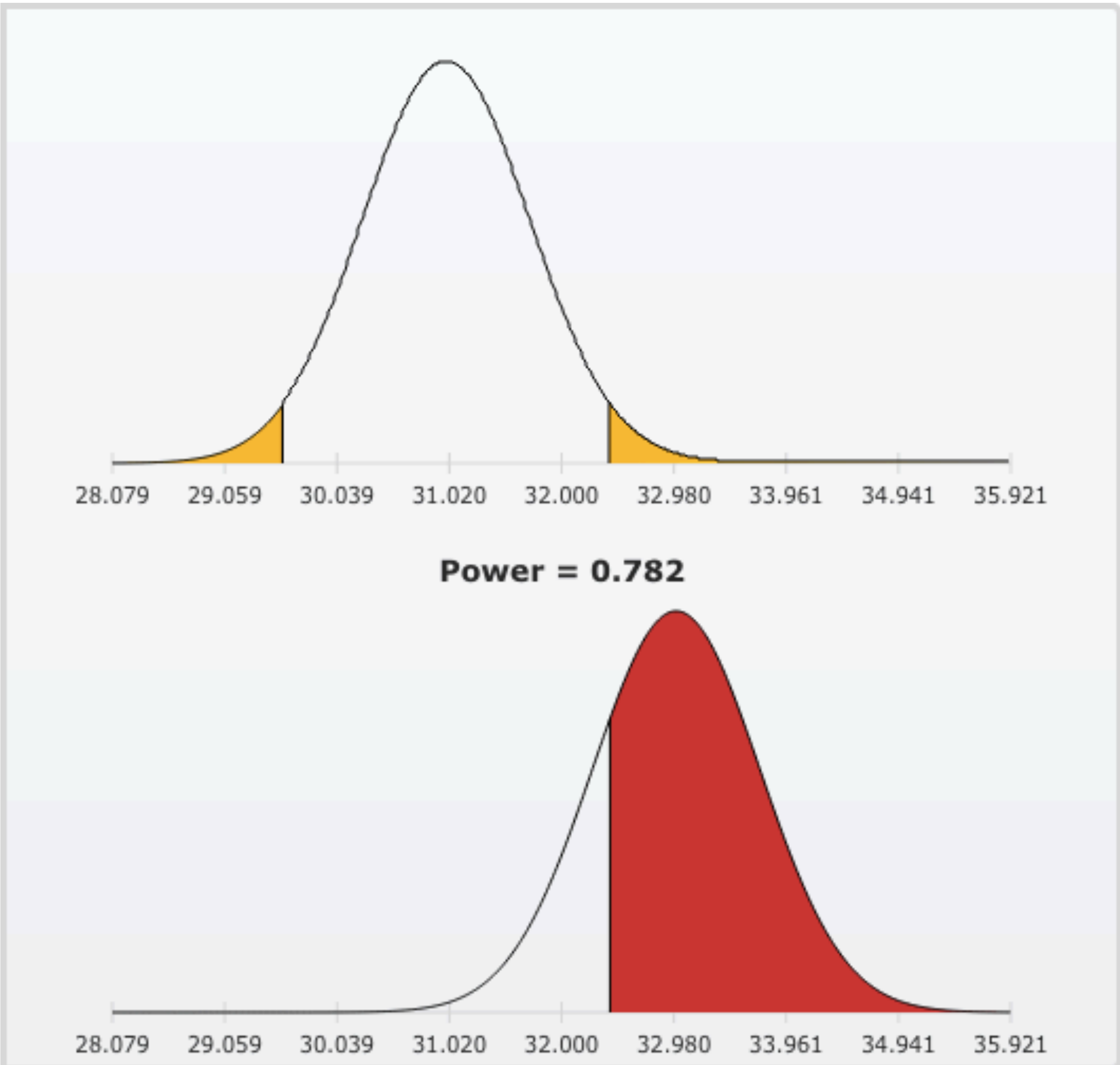
We reject H_0 at 0.10 level of significance if:
$\sqrt{10}(\bar{X} - 12) / s > 1.383 \Rightarrow \bar{X} > 12 + 1.383(s) / \sqrt{10}$
Power of test $\mu_A = 15 = P(\bar{X} > 12 + 1.383(s) / \sqrt{10} \mid \mu_A = 15)$
$= P(\sqrt{10}(\bar{X} - 12) / s > (\sqrt{10}(12 + 1.383(s) / \sqrt{10}) - 15) / s \mid \mu_A = 15)$
$= P(\sqrt{10}(\bar{X} - 12) / s > (-0.5610 \mid (\sqrt{}(\bar{X} - 15)) / s) \sim t_9)$
$= 0.7058$

c. She will use s = 4.88 as her best guess of σ so she can use a Z test simplification. Approximately what sample size would be required to achieve a power of 0.85 if the true population mean is μA = 15? Give your answer as the smallest whole number that meets the criterion.

Assume $\sqrt{n}(\bar{X} - \mu_A) / s \sim N(0,1)$
Power of test at $\mu_A = 15$
$= P(\sqrt{n}(\bar{X} - 15) / s > \sqrt{n}(12 + ((1.383 * 4.88) / \sqrt{n}) - 15) / 4.88 \mid \mu_A = 15)$
$= 1 - \Phi(\sqrt{n}(12 + ((1.383 * 4.88) / \sqrt{n}) - 15) / 4.88)$
$= 0.85 = \Phi(1.0364)$
$\sqrt{n}(12 + ((1.383 * 4.88) / \sqrt{n}) - 15) / 4.88 = -1.0364$
Or, $11.80667 / \sqrt{n}$
$= 11.80667 / 3$
$= 15.48861$
~ 16

d. If she decided to perform the test at a significance level of α = 0.05 instead of α = 0.10, how would that effect her power of detecting μA = 15? (no calculations needed)

If she plans to perform the test using t-statistic instead of z statistic we need a larger sample size to reach the power 0.85. Since t distribution is leptokurtic and symmetric about zero whereas standard normal distribution is symmetric about zero but mesokurtic i.e. t distribution is more picked than standard normal distribution. Therefore the tailed probability is small for t distribution than standard normal distribution.


28.079
29.059
30.039
31.020
32.000
32.980
33.961
34.941
35.921
Power = 0.782
28.079
29.059
30.039
31.020
32.000
32.980
33.961
34.941
35.921