<<Lecture14_Review_SS.pdf>>

Stat 324 – Introduction to Statistics for Engineers

LECTURE 14: EXAM 2 REVIEW

MIDTERM 2: TOOLS OF INFERENCE: HYPOTHESIS TESTING AND CONFIDENCE INTERVALS, ERRORS IN HYPOTHESIS TESTING, AND POWER.

Inference Procedures

- A confidence interval answers the question
  "what values are likely in the population"?
- A hypothesis test answers the question
  "is there sufficient evidence to reject an accepted model?"

(1) Identify the parameters of interest

HT:
(2) Write hypotheses
(3) Identify assumptions of the population based on sampling and observations from data
(4) Identify appropriate test
(5) Calculate test statistic and p value and df if appropriate
(6) Draw conclusion in context of the question

CI:
(2) Identify assumptions of the population based on sampling and observations from data
(3) Identify appropriate procedure
(4) Calculate Interval
(5) Interpret interval in context of the question .

HT/CI Decision Tree

1. Are there 2 independent samples taken or two?

A. 1 Sample

B. 2 Samples
Are they Independent or Matched?
If matched, find the differences and go to A.

2. What parameter is of interest?

Ai. Mean
Aii. Proportion of Success
Aiii. Median

Bi. Difference of Means
Bii. Difference in Proportions
Biii. Shifts in Distribution (Median)

3. What assumptions can be made?

Ai. Is population normal or sample size large?
t tests, t CI for mean, if unknown pop sigma;
z if known sigma

Ai If pops not normal and small sample size
Bootstrap CI/Test, Permutation Test
if large n, t, z, or boot give similar results

Aii. Is np and n(1-p) both >5, then z test for proportion

Aiii. If pop not normal? (Rank info)
Sign Test, Wilcoxon Signed rank Test

Bi. Are both populations Normal or sample sizes large?
Welch's t if no equal variance assumption
Pooled sd t if equal variance assumption
z if known sigma

Bi If pops not normal and small sample size
Bootstrap CI/Test
if large n, t, z, or boot give similar results

Bii. Is np and n(1-p) both >5, for both groups?
then z test/CI for diff in proportions

Biii: If both populations not normal (ordinal/interval data)
Wilcoxon Rank Sum Test

Mixed Practice 1: A survey dealing with customer attitudes towards packaging asked a random sample of 270 consumers, "Would you be willing to pay extra for tamper-resistant packages?" The number of yes responses was 189.

a. Identify the point estimate, then construct a 90% confidence interval for the proportion of all consumers who would pay extra for such packaging, after checking the necessary assumptions (and describing what they are).

$\hat{p} = \frac{189}{270} = 0.70$

assume SRS (simple random sample) of 270 consumers; respondents are independent
normality of $\hat{p}$: $\hat{p} * n = 189$ $(1-\hat{p}) = 81$ are both ≥ 5 so $\hat{p} \approx N$ by CLT

Obs prop $\pm Z_{\alpha/2} SE_{prop}$

$Z_{0.85} = 1.645$

$0.70 \pm 1.645(0.028) = (0.65, 0.745)$

$SE = \sqrt{\frac{0.70(0.30)}{270}} = 0.028$

b. How strong of evidence does this sample give us against a null of a .50 proportion of consumers being willing to pay extra for packaging, with the alternative that a higher proportion would pay more? π = proportion of consumers willing to pay more for packaging

$H_0: \pi = 0.50$

$H_A: \pi > 0.50$

$SE_{\hat{p}} = \sqrt{\frac{0.50(0.50)}{270}} = 0.030$

Assume independence
Assume normality of $\hat{p}$
$\pi * n = 0.50(270) = 135$ $(1-\pi)n = 135$
Since both > 5, $\hat{p} \approx N$ by CLT

$z = \frac{0.70 - 0.50}{0.030} = 6.67$

pvalue: $P(z \geq 6.67) \approx 0$ so very strong evidence to reject the null

Mixed Practice 2: Measurements on a number of physiological variables for a sample of 8 males and 8 female adolescent tennis players were reported in "Physiological and Anthropometric Profiles of Elite Prepubescent Tennis Players" (Sportsmed. (1984); 111-116). Results are summarized in the accompanying table:

Physiology	Boys		Girls	
	$\bar{x}$	s	$\bar{x}$	s
Shoulder Flexibility	214.4	12.9	216.3	18.0
Ankle Flexibility	71.4	4.1	72.5	9.3
Grip strength	-	-	-	-

a. Identify the point estimate, then construct a 95% confidence interval for the difference in shoulder flexibility between boy and girl shoulder flexibility (after stating what assumptions must be made).

2 independent samples $\quad Obs_{diff} \pm t * SE_{diff}$

Diff | G−B $\quad 216.3 - 214.4 = 1.9$

Assume populations are normal in order to make small sample t Confidence Interval

Assume 2 populations independent and obs within samples independence

Assume 2 population variances are equal $0.5 < \frac{12.9}{18} < 2$

$$s_p^2 = \frac{12.9^2(7) + 18^2(7)}{14} = 15.66^2$$

$$SE_{diff}: \sqrt{\frac{s_p^2}{n_B} + \frac{s_p^2}{n_G}} = 7.833$$

b. From your interval above; determine whether you would reject or fail to reject the hypothesis: $H_o: \mu_{SB} - \mu_{SF} = 0$ in favor of $H_A: \mu_{SB} - \mu_{SF} \neq 0$, Calculate an approximate p value.

$Obs_{diff} = 214.4 - 216.3 = -1.9 \qquad t_{obs} = \frac{-1.9 - 0}{7.833} = -0.2426$

$\frac{\alpha}{2} = 0.025 \quad t_{(14, 0.025)} = 2.145$

$2 * P(t_{14} \leq -0.2426) = 2 * P(t_{14} \geq 0.2426) = \text{pvalue} > 0.25$

so weak evidence against null

Mixed Practice 2: Measurements on a number of physiological variables for a sample of 8 males and 8 female adolescent tennis players were reported in "Physiological and Anthropometric Profiles of Elite Prepubescent Tennis Players" (Sportsmed. (1984); 111-116). Results are summarized in the accompanying table:

Physiology	Boys		Girls	
	$\bar{x}$	s	$\bar{x}$	s
Shoulder Flexibility	214.4	12.9	216.3	18.0
Ankle Flexibility	71.4	4.1	72.5	9.3
Grip strength	-	-	-	-

c. Determine whether you would reject for fail to reject the hypothesis: $H_o: \mu_{AB} - \mu_{AF} = 0$ in favor of $H_A: \mu_{AB} - \mu_{AF} \neq 0$. Where μ_{AB}, μ_{AF} are the mean ankle flexibility in adolescent tennis players. (after stating what Assumptions must be made).

$Obs_{diff} = 71.4 - 72.5 = -1.10 \qquad SE_{diff} = \sqrt{\frac{4.1^2}{8} + \frac{9.3^2}{8}} = 3.59 \qquad t_{obs} = \frac{-1.10 - 0}{3.59} = -0.306$

$$df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

$s_1^2/n_1 = \frac{9.3^2}{8} = 10.81125 \qquad s_2^2/n_2 = \frac{4.1^2}{8} = 2.10125$

$$\frac{(10.81125 + 2.10125)^2}{\frac{(10.81125)^2}{7} + \frac{(2.10125)^2}{7}} = 9.6219$$

$2 * P(t_9 \leq -0.306) = 2 * P(t_9 \geq 0.306) = \text{pvalue}$

d. A 95% confidence interval for the difference in mean grip strength(assuming equal variance) between boys and girls was given as: (-1.94, 5.34). What was the point estimate of the difference in mean grip strengths? What was the pooled standard deviation?

$Obs_{diff} = 23.9 - 22.2 = 1.7 \qquad SE = \sqrt{\frac{11.53}{8} + \frac{11.53}{8}} = 1.6977$

$\text{Pool var} = \frac{7 * 2.5^2 + 7 * 4.1^2}{14} = 11.53 \qquad Obs_{diff} \pm t_{(14, 0.025)} * SE = 1.7 \pm 2.145 * 1.6977 = (-1.94, 5.34)$

Mixed Practice 3: A company test-markets a new product in the Grand Rapids, Michigan and Witchita Kansas, metropolitan areas. The company's advertising in the Grand Rapids area is based almost entirely on television commercials. In Wichita, the company spends roughly equal dollar amount on a balanced mix of television, radio, newspaper, and magazine ads. Two months after the ad campaign begins, the company conducts surveys to determine consumer awareness of the product. Random samples from the Grand Rapids and Wichita markets were interviewed.

	Grand Rapids	Wichita
Number Interviewed	608	527
Number Aware	392	413

a. Determine an 80% CI for the regional difference in the proportions of all consumers who are aware of the product (after stating what assumptions must be made).

$608 - 392 = 216$ $\quad Obs_G = \frac{392}{608} = 0.6447$ $\quad Obs_W = \frac{413}{527} = 0.7836$

$\text{proportion}_{diff} = 0.7836 - 0.6447 = 0.1389$

$$\text{Proportion}_{SE} = \sqrt{\frac{0.6447 * (1-0.6447)}{608} + \frac{0.7836 * (1-0.7836)}{527}} = 0.026$$

b. Calculate the difference in the SE for the hypothesis test of H0: $\pi_G - \pi_W = 0$ vs HA: $\pi_G - \pi_W \neq 0$ compared to that used in the CI above.

$\text{pooled} = \frac{392 + 413}{608 + 527} = 0.709$ $\quad SE_{diff} = \sqrt{\frac{0.709 * (1-0.709)}{608} + \frac{0.709 * (1-0.709)}{527}} = 0.027$

Mixed Practice 4: Trace metals in drinking water affect the flavor of the water, and unusually high concentrations can poste a health hazard. The paper, "Trace Metals of South Indian River" reported trace-metal concentrations for both surface water and bottom water at six different river locations. Data on zinc concentrations (mg/L) is given here:

Location	**1**	**2**	**3**	**4**	**5**	**6**
Bottom Water	0.430	0.266	0.567	0.531	0.707	0.716
Top Water	0.415	0.238	0.390	0.410	0.605	0.609
Diff	0.015	0.028	0.177	0.121	0.102	0.107

a. Compute an appropriate 95% confidence interval to evaluate the difference in mean trace-metal concentrations (assuming normality of relevant population[s]).

mean diff = 0.0916   sd diff = 0.06068

$SE_{AVG} = \frac{0.06068}{\sqrt{6}} = 0.02477$ $t(5, 0.025) = 2.571$

$0.0916 \pm 2.57 * 0.02477 = (0.02799, 0.1553)$

b. Using this report as a preliminary study, the researchers would like to know the approximate power of a follow up $\alpha = 0.05$ hypothesis test of $H_o: \mu_D = 0$ vs $H_A: \mu_D > 0$ to detect a true mean difference of 0.20 if they use a sample size of 15 and assume that $\sigma = 0.061$.

$t\ 95\% = 1.96$ $\frac{1.96 * 0.061}{\sqrt{15}} = 0.03087$ $\frac{0.0259 - 0.20}{\frac{0.061}{\sqrt{15}}} = -11.0538$

R wilcoxn   P value = 0.01563

Mixed Practice 4 cont: reported trace-metal concentrations for both surface water and bottom water at six different river locations. Data on zinc concentrations (mg/L) is given here:

Location	1	2	3	4	5	6
Bottom Water	0.430	0.266	0.567	0.531	0.707	0.716
Top Water	0.415	0.238	0.390	0.410	0.605	0.609

c. If the scientist did not want to make a normality in the population(s) assumption, but was ok assuming the population[s] is[are] symmetric, what other test could they perform to determine the evidence of higher levels of trace-metal concentrations in bottom water compared to surface water?

boot strap

d. Perform each of the above mentioned, completing as much by hand for what is possible.

value	sample	Rank
10	a	1
10.2	a	2
10.7	a	3
11.1	b	4
11.3	b	5
11.5	b	6
12	b	7

$$U = 1+2+3 - R_{min}$$

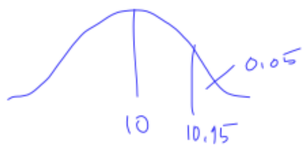
$$\frac{\underset{\text{sizeA}}{3} * \underset{\text{sizA}+1}{4}}{2}$$

Power: Reject Null when null is false for some specific truth ($\mu_A = 15$)

$H_0: \mu = 10$ $\quad \alpha = 5\%$ $\quad H_A: \mu > 10$

$P(\text{Reject } \mu = 10 \mid \mu_A = 15)$

RR



$\sigma = 2$

$n = 12$

$Z_{obs} > 1.645$ $\qquad \bar{X}_{obs} >$

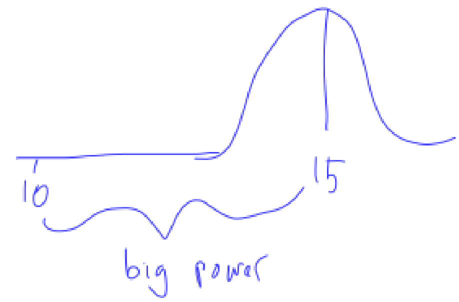
reject null

$$Z = \frac{\bar{X}_{obs} - \bar{X}_{exp}}{SE}$$

$$1.645 = \frac{\bar{X} - 10}{2/\sqrt{n}}$$

$$\bar{X}_{obs} = 1.645 \left(\frac{2}{\sqrt{12}}\right) + 10$$

$$\bar{X}_{obs} > 10.95$$



$$P(\bar{x} \geq 10.95 \mid \mu_A = 15)$$
$$= P\left(Z > \frac{10.95 - 15}{2/\sqrt{12}}\right)$$
$$= P(Z > -7)$$
$$= \text{pvalue} \approx 1$$

lost $= \frac{12}{17}$ improved $= \frac{4}{17}$ same $= \frac{1}{17}$

$n = 17$

H_A: students lose profficiency (med change < 0) sign test

H_0: students dont lose proficiency (median change $= 0$)

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$B^+ \sim Bin(16, 0.5)$ $B_{obs} = 4$

17-1 because 1 unchanged

$$\text{pvalue} = P(B \leq 4)$$