HWK4_Soln

Accuracy Points: 35; Completion Points: 10; Total Points: 45

1. The data below record the number of hours a team of workers takes to assemble a custom-built motorcycle. The data are recorded for 10 different teams each assembling a motorcycle.

   89 78 48 85 67 45 60 62 62 56

   (a) Create a QQplot in R and comment on the assumption that the population of times to assemble a motorcycle is well-approximated by a normal distribution.

   Answer: *QQplot of data shown at bottom. We see some departures from straight line, but nothing too crazy. This amount of "non-linearily" is very possible in a sample of 10 from a normal population (as see in question 8)*

   (b) Construct a 90% confidence interval by hand for the mean time it takes a team of workers to assemble a custom-built motorcycle.

   Answer: ***4 points: 2: t, 2: interval*** *Since we do not have population sd, we will approximate it with sample sd* $s_x = 14.76$ *and use a t interval. CI: sample.mean* $\pm t_{.05,9}\frac{SD}{\sqrt{n}}$ =$65.2 \pm 1.83 * \frac{14.76}{\sqrt{10}} = 65.2 \pm 8.54 = (56.66, 73.74)$

   (c) Construct the same interval above using R's t.test() command.

   Answer: ***2 points*** *Values will be very similar. t.test code given below.*

   (d) By how much does the confidence interval width of a 95% interval differ from that of a 90% interval for this data?

   Answer: ***2 points*** *Width of 90: 73.76-56.64=17.116; Width of 95: 75.76104-54.63896=21.12208. Total width difference: 4.006*

   (e) Suppose instead, the manager had performed a hypothesis test at the 10% level of the null that $\mu = 60$ hours vs the alternative that $\mu < 60$. Compute the pvalue for his hypothesis test and summarize the conclusion that he would have drawn from the sample that he observed.

   Answer: ***6 points: 2 points for*** t_{obs}***, 2 points for correct p; 2 points for conclusion*** *Since we do not have population sd, we will approximate it with sample sd* $s_x = 14.76$ *and perform a t test (since our sample size is small).* $P(\bar{X} < 65.2) = P(T_9 < \frac{65.2-60}{14.76/\sqrt{10}}) = P(T_9 < 1.11) = 0.8529$ *or* $0.85 <$ *pvalaue* $< .90$ *off of table. This sample offered no evidence against the null (in the direction of the alternative hypothesis). The manager would fail to reject the null; insufficient evidence to suggest the mean time to assemble a custom-built motorcycle is less than 60 hours.*

2. An automobile club which pays for emergency road services (ERS) requested by its members wishes to estimate the proportions of the different types of ERS requests. Upon examining a sample of 2927 ERS calls, it finds that 1499 calls related to starting problems, 849 calls involved serious mechanical failures requiring towing, 498 calls involved flat tires or lockouts, and 81 calls were for other reasons.

(a) Estimate the true proportion of ERS calls that involved serious mechanical problems requiring towing and construct a 95% confidence interval after checking that conditions have been met. Interpret your interval in context.

Answer: We need to check that $n\hat{\pi} = 849 > 5$ *and* $n(1-\hat{\pi}) = 2078 > 5$. *Both of which is true, so we can use the Normal approximation according to the CLT:* 95% *CI:* $\frac{849}{2927} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.29 \pm 1.96\sqrt{\frac{.29(.71)}{2927}} = 0.29 \pm 1.96 * 0.0084 = 0.29 \pm 0.016464 = (0.274, 0.306)$. *We are* 95% *confident the interval from 0.274 to 0.306 captures the true percent of ERS calls that involve serious mechanical problems.*

(b) Calculate a 98% confidence interval for the true proportion of ERS calls that related to starting problems after checking that conditions have been met. Interpret your interval in context.

Answer: ***6 points: 1 point for assumptions check, 3 points for correct CI; 2 for correct interpretation.*** *We need to assume that assume the observaations are independent and an SRS. We also check that* $n\hat{\pi} = 1499 > 5$ *and* $n(1-\hat{\pi}) = 428 > 5$ *and since both are true, so we can use the Normal approximation according to the CLT:* 98% *CI:* $\frac{1499}{2927} \pm z_{.01}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.512 \pm 2.33\sqrt{\frac{0.512(0.488)}{2927}} = 0.512 \pm 2.33 * 0.00924 = 0.512 \pm 0.0215 = (0.4905, 0.5335)$. *We are* 98% *confident the interval from 0.4905 to 0.5335 captures the true percent of ERS calls that involve starting problems.*

3. At the Hawaii Pineapple Company, managers are interested in the size of the pineapples grown in the company's fields. Last year, the mean weight of the pineapples harvested from one large field was 31 ounces with a standard deviation of 4 ounces. A different irrigation system was installed in this field after the growing season. Managers wonder if the the mean weight of pineapples grown in the field this year will be different from last.

(a) Write out the null H_o and alternative hypotheses $H_a$in terms of the population mean μ.

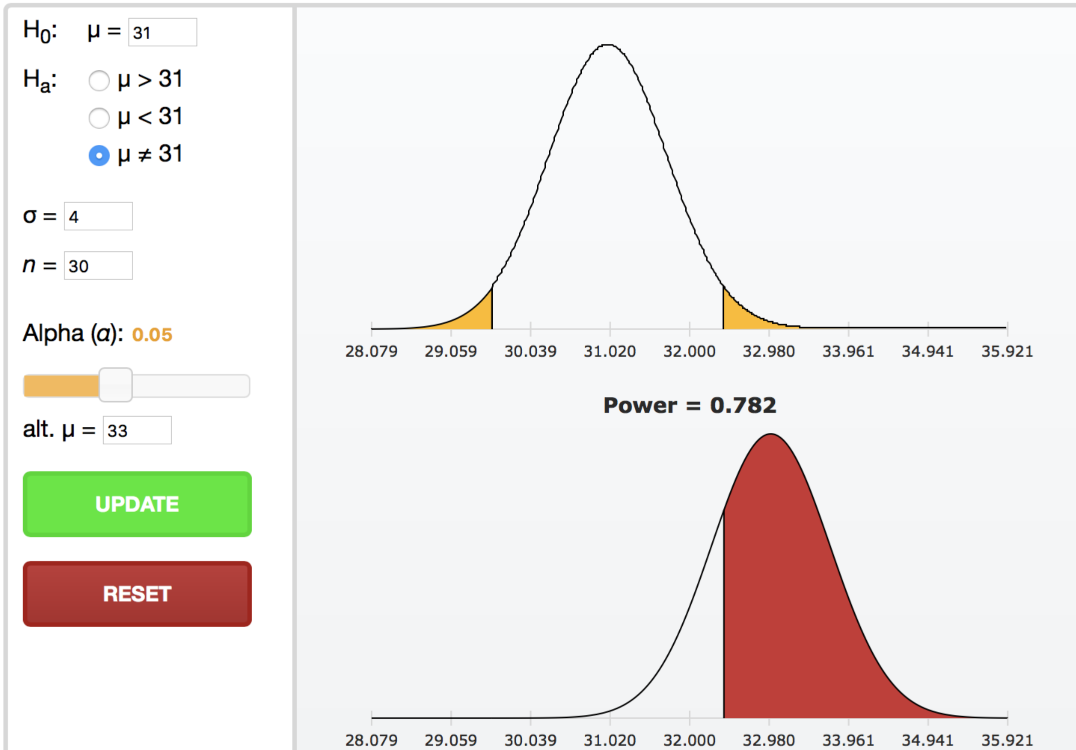
3 points: 1 for correct parameter; 2 for correct hypoth designation *Answer:*$H_o : \mu = 31$ *and* $H_a : \mu \neq 31$

(b) If the managers choose to use a significance level of 0.05 and assume $\sigma = 4$, identify their power to detect a mean increase of 2 ounces ($\mu_a = 33$) if they look at a sample that is 30 pineapples this year and use the two-sided alternative. Also identify the probability of making a type 2 error with true $\mu_a = 33$.

6 points: 3 points for correct power; 3 points for type 2 error as compliment of power*Critical values of:* $z_{.025} = 1.96, z_{.975} = -1.96$ *so the correponding critical values:* $\bar{X} = -1.96 * \frac{4}{\sqrt{30}} + 31 = 29.56862$ *and* $\bar{X} = 1.96 * \frac{4}{\sqrt{30}} + 31 = 32.43138$. *If true mean is 31+2=33,* $power(\mu_a = 33) = P(\bar{X} > 32.43138|\mu_a = 33) + P(\bar{X} < 29.56862|\mu_a = 33) = P(Z > \frac{32.43138-33}{4/\sqrt{30}}) + P(Z < \frac{29.56862-33}{4/\sqrt{30}}) = 0.7818967 + 1.309687e-06 = 0.7818967$. $P(Type2error) = 1 - 0.7818967 = 0.2181033$

(c) Draw pictures of the null and alternative distributions of the means and shade the areas that correspond to (i) Type 1 error, (ii) Type 2 error, and (iii) Power from part (b). (The online applet from the notes may help.)

3 points: 1 for each correct labeling (Yellow should be labeled type 1 error; Red should be labeled power, and white in lower graph - compliment of power should be labled type 2 error)



(d) Explain Type 1 and Type 2 errors of the test in context.

ANSWER: Type 1 error: get a sample value more extreme than expected with a mean of 31 so conclude evidence suggests there is a change in this year's mean weight, when in actuality there is not. Type 2 error: get a sample value very likely with a true mean of 31 so conclude evidence suggests there is not a change in this year's mean weight, when in actuality there was.

(e) What sample size should the managers use to ensure their test has power of at least 0.9 to detect a mean increase of 2 ounces (assuming $\sigma = 4$)?

3 points: 1 for correct $z_{\alpha/2}$, 1 for correct z_β, 1 for putting values together in correct formula or getting correct value through simulation *ANSWER:* $n \approx (\frac{\sigma(z_{\alpha/2}+z_\beta)}{\mu_o-\mu_a})^2$. $\sigma = 4$, $z_{\alpha/2} = z_{.025} = 1.96$, $z_{beta} = z_{.10} = 1.281552$ *so* $n \approx (\frac{4(1.959964+1.281552)}{33-31})^2 = 42.0297$. *You can check n=43 on the simulation pushes power above .90. Note, if students didn't retain so many decimal points, you'd get a n=42, and that is fine.*

(f) Explain why the managers may prefer to compute a 95% confidence interval instead of a two-sided 5% hypothesis test.

ANSWER: A CI gives us a similar reject/fail to reject conclusion and also gives us a range of plausible values.

4. A scientist is doing a preliminary study to try to determine the sample size necessary for her larger study. She would like to show that the mean in her population of interest is larger than 12 and is starting with the assumption that $H_o : \mu = 12$ which is will be testing at the $\alpha = 0.1$ level. She takes a random sample of n=10 from her population and checks the QQplot and sample histogram. From the graphs, the normality of the population assumption is pretty well met. Her sample mean is $\bar{X} = 14.2$ and sample standard deviation is $s = 4.88$.

   (a) Compute the critical values and rejection region for the appropriate test statistic and $\bar{X}$ and p value of the appropriate test. Draw a conclusion in the context of the study.

   Answer: $H_o : \mu = 12$ *and* $H_A : \mu > 12$. *Critical values:* $t_{.1,9} = 1.383$*; RR:* $t_9 > 1.383$, $\bar{X} = 1.383 * \frac{4.88}{\sqrt{10}} + 12 = 14.13423$ *RR:*$\bar{X} > 14.13$. *Pvalue:* $P(T_9 > \frac{14.2-12}{\frac{4.88}{\sqrt{10}}} = P(T_9 > 1.425617) = 0.09386302$. *Sufficient evidence at the* $\alpha = 10\%$ *level to suggest the true mean in her population is larger than 12.*

   (b) Suppose she is interested in the specific alternative that $\mu_A = 15$ and will still be conducting a one-sided test. Compute the power of a follow up hypothesis test to correctly reject the null if in fact $\mu_A = 15$ is true with a sample size of 40. Use the sample standard deviation above to estimate σ, use z critical values, and $\alpha = 0.1$.

   Z critical values: $z > z_{.1} = 1.285$*; so* $\bar{X} > 1.285 * \frac{4.88}{\sqrt{40}} + 12 = 12.9915$ *so* $Power_{\mu_a=15} = P(\bar{X} > 12.9915|\mu_a = 15) = P(Z > \frac{12.9915-15}{4.88/\sqrt{40}}) = P(Z > -2.603047) = 0.99538$

   (c) She will use $s = 4.88$ as her best guess of σ so she can use a Z test simplification. Approximately what sample size would be required to achieve a power of 0.85 if the true population mean is $\mu_A = 15$? Give your answer as the smallest whole number that meets the criterion.

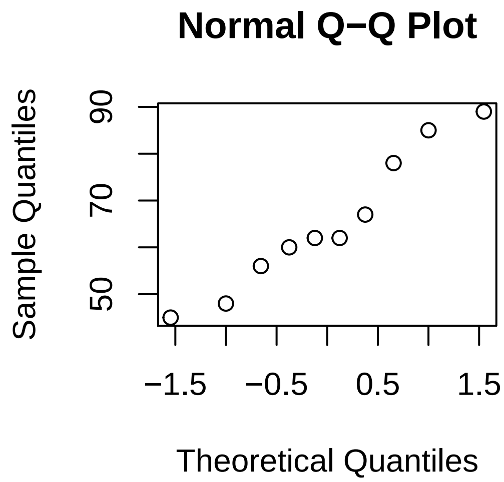
   Answer: $\sigma = 4.88$, $z_\alpha = z_{.10} = 1.285$ *and* $z_\beta = z_{.15} = 1.035$ *so* $n \approx (\frac{4.88(1.285+1.035)}{3})^2 \approx 14.24$. *n=15.*

   (d) If she decided to perform the test at a significance level of $\alpha = 0.05$ instead of $\alpha = 0.10$, how would that effect her power of detecting $\mu_A = 15$? (no calculations needed)

   Answer: setting a lower alpha is requiring stronger evidence for rejecting the null, thus the power of our test to detect the same $\mu_A = 15$ *has decreased.*

QQplot for Question #1:

```
motor.data<-c(89,78,48,85,67,45,60, 62, 62,56)
qqnorm(motor.data)
```




```r
mean(motor.data) #65.2
```

```
## [1] 65.2
```

```r
sd(motor.data) #14.76
```

```
## [1] 14.76332
```

```r
qt(.95, df=9)  #1.83
```

```
## [1] 1.833113
```

```r
t.test(motor.data, conf.level=.90)
```

```
##
##  One Sample t-test
##
## data:  motor.data
## t = 13.966, df = 9, p-value = 2.095e-07
## alternative hypothesis: true mean is not equal to 0
## 90 percent confidence interval:
##  56.64198 73.75802
## sample estimates:
## mean of x
##      65.2
```

```r
t.test(motor.data, conf.level=.95)
```

```
##
##  One Sample t-test
##
## data:  motor.data
## t = 13.966, df = 9, p-value = 2.095e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  54.63896 75.76104
## sample estimates:
## mean of x
##      65.2
```

```r
t.test(motor.data, conf.level=.90, alternative = "less", mu=60)
```

```
##
```

```
##  One Sample t-test
##
## data:  motor.data
## t = 1.1138, df = 9, p-value = 0.8529
## alternative hypothesis: true mean is less than 60
## 90 percent confidence interval:
##      -Inf 71.65677
## sample estimates:
## mean of x
##      65.2
```