HW2

Teryl Schmidt | Discussion 312 | 9072604920

9/23/2018

Problem 1

A system contains two components, A and B, connected in series, as shown in the diagram. Assume that A and B function independently. For the system to function, both components must function.

a. If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions.

```
# P(System functions) = P(A ∩ B)
# = P(A) * P(B)

(1 - 0.05) * (1 - 0.03)
```

```
## [1] 0.9215
```

the probability of the system functioning is 0.9215 or 92%

b. If both A and B have probability of p of failing, what must the maximum value of p be so that the probability that the system functions is at least 0.90?

P(System funcions) = P(A ∩ B)

= P(A) * P(B)

= (1 - p) * (1 - p)

= (1 - p)^2

(1 - p)^2 = 0.90

p = 1 - sqrt(0.90)

= 0.05

Problem 2

A local taxi company owns 10 taxis. Three are randomly selected for compliance with emission standards (without replacement) and all are found to not be in compliance. The company claims all their other cars are in compliance.

a. Calculate the probability of choosing the three taxis not in compliance, if in fact 7 out of the 10 are in compliance.

```
# P(3 non-compliant) = (3 / 10) * (2 / 9) * (1 / 8)
(3 / 10) * (2 / 9) * (1 / 8)
```

```
## [1] 0.008333333
```

= The probability of picking 3 non-compliant taxis is 0.00833

b. Comment on the believability of the company's claim. Could be true, but the probability of picking the only 3 that are not compliant is so low that it is very unlikely that the rest of the taxis are compliant.

Problem 3

From the service records over the past year, the manager of an auto detailing shop has estimated the following partial probability distribution for X= the number of customers asking for service per day at his shop. X | 0 | 1 | 2 | 3 | 4 | 5 | P(X=x) | 0.04 | ? | 0.34 | 0.20 | 0.11 | 0.06 |

a. What does the probability of 1 customer asking for service P(X = 1) have to be to make this a probability distribution?

```
# All probabilities must add up to 1

1 - (0.04 + 0.34 + 0.20 + 0.11 + 0.06)
```

```
## [1] 0.25
```

The probability of P(X = 1) is 0.25

b. Suppose the shop currently has the capacity to serve up to 3 customers a day. What is the probability that one or more customer needs to be turned away?

```
# P(X > 3)
# P(X = 4) + P(X = 5)

(0.11 + 0.06)
```

```
## [1] 0.17
```

The probability of P(X > 3 4) is 0.17

c. What is the probability that the shop's capacity is not fully utilized on a day?

```
# P(X < 3)
# P(X = 0) + P(X = 1) + P(X = 2)

(0.04 + 0.25 + 0.34)
```

```
## [1] 0.63
```

The probability of P(X < 3) is 0.63

d. By how much must the capacity be increased so the probability of turning a customer away is no more than 0.10? The capacity must be set to 5 because the probability is 0.06 which is less than 0.10

Problem 4

A box contains five slips of paper. These slips are marked $1, $1, $1, $10, and $25. The winner of a contest will select two slips of paper at random with replacement and will then get the larger of the dollar amounts on the two slips. Define a random variable W=the amount awarded.

a. Determine the probability distribution of W (write out the pmf).

Slips = 1, 2, 3, 4, 5

Amount = 1, 1, 1, 10, 25

w = 1: (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) (3,1) (3,2) (3,3)

w = 10: (1,4) (2,4) (3,4) (4,1) (4,2) (4,3) (4,4)

w = 25: (1,5) (2,5) (3,5) (4,5) (5,1) (5,2) (5,3) (5,4) (5,5)

W = w = 1: (1, 1) (1, 1) (1, 1) (1, 1) (1, 1) (1, 1) (1, 1) (1, 1) (1, 1)

w = 10: (1, 10) (1, 10) (1, 10) (10, 1) (10, 1) (10, 1) (10, 10)

w = 25: (1, 25) (1, 25) (1, 25) (25, 1) (25, 1) (25, 1) (10, 25) (25, 10) (25, 25)

b. What is the least probable outcome for the amount won?

The lowest probability is $10

c. What is the expected value for the amount won?

P(X = 1): 9 / 25 = 0.36

P(X = 10): 7 / 25 = 0.28

P(X = 25): 9 / 25 = 0.36

E(X) = (1 * 0.36) + (10 * 0.28) + (25 * 0.36) = 12.16

The expected outcome is 12.16

d. What is the variance and standard deviation for the amount won?

VAR = (0.36 * (1 - 12.16)^2) + (0.28 * (10 - 12.16)^2) + (0.36 * (25 - 12.16)^2) = 105.4944

SD = 10.27105

e. What is the probability that the amount won is more than one standard deviation away from the expected value?

```
# Mean
1 * (0.36) + 10 * (0.28) + 25 * (0.36)
```

```
## [1] 12.16
```

```
# SD
(0.36 * (1 - 12.16)^2) + (0.28 * (10 - 12.16)^2) + (0.36 * (25 - 12.16)^2)
```

```
## [1] 105.4944
```

```
sqrt(105.4944)
```

```
## [1] 10.27105
```

```
# E(X) +/- SD
12.16 - 10.27105
```

```
## [1] 1.88895
```

```
12.16 + 10.27105
```

```
## [1] 22.43105
```

```
# Range = 1.88895 to 22.43105
# P(X = 1) + P(X = 25)
0.36 + 0.36
```

```
## [1] 0.72
```

Probability is 0.72 or 72%

Problem 5

For each of the following questions, say whether the random process is reasonably a binomial process or not, and explain your answer. As part of your explanation, you will want to comment on the potential validity of each of things that must be true for a process to be a binomial process. If it is a binomial process, identify n : the number of Bernoulli trials and π the probability of success.

a. A fair die is rolled until a 1 appears, and X denotes the number of rolls.

A fixed number of trials - No, not a fixed number of trials

Each trial is independent of the others - Yes, current roll does not depend on previous

There are only two outcomes - No, 6 outcomes

The probability of each outcome remains constant from trial to trial - Yes, 1/6 every roll

This is not a Bernoulli trial.

b. Ten different basketball players each attempt 1 free throw and X is the total number of successful attempts.

A fixed number of trials - Yes, 10 players

Each trial is independent of the others - Yes, current trail does not rely on previous

There are only two outcomes - Yes, make the basket or not

The probability of each outcome remains constant from trial to trial - No, players ahve different skill level

This is not a Bernoulli trial.

c. It has been reported that nation-wide, one-third of all credit card users pay their bills in full each month. Let X be the number of people in a sample of 25 randomly chosen credit card users in Madison who pay their bill in full on a given month.

A fixed number of trials - Yes, 25 people

Each trial is independent of the others - Yes, people are random so they have no effect on whether eachothers credit card bill is paid

There are only two outcomes - Yes, either paid or not

The probability of each outcome remains constant from trial to trial - Yes, if people are truly random

This is a Bernoulli trial.

d. Let X be the number of months out of a randomly chosen year that one randomly chosen credit card user in Madison pays their bill in full.

A fixed number of trials - Yes, 12 months

Each trial is independent of the others - Yes, each month does not depend on previous

There are only two outcomes - Yes, either paid for the month or not

The probability of each outcome remains constant from trial to trial - Yes

This is a Bernoulli trial.

Problem 6

Exit polling has been a controversial practice in recent elections, since early release of the resulting information appears to affect whether or not those who have not yet voted do so. Suppose that 90% of all registered Wisconsin voters favor banning the release of information from exit polls in presidential elections until after the polls in Wisconsin close. A random sample of 25 Wisconsin voters is selected (You can assume that the responses of those surveyed are independent).

a. What is the probability that more than 23 favor the ban?

```
# P(X > 23) = P(X = 24) + P(X = 25)
#                              25!
# SUM from 24 to 25  ------------------ * (0.90)^x * (1 - 0.90)^25 - x
#                        x! * (25 - x)!

# x = 24, 25

(factorial(25) / (factorial(24) * factorial(25 - 24))) * (0.90^24) * (1 - 0.90)^(25 - 24) + (factorial(25)
/ (factorial(25) * factorial(25 - 25))) * (0.90^25) * (1 - 0.90)^(25 - 25)
```

```
## [1] 0.2712059
```

```
dbinom(24, 25, 0.90) + dbinom(25, 25, 0.90)
```

```
## [1] 0.2712059
```

```
# Probability of success π = 0.90
# 1 - π = 1 - 0.90
# = 0.10
# P(X = x) = (0.10)^(x - 1) * 0.90

# P(more than 23 favor ban)
# P(X > 23)
# P(24) + P(25)
# (25 choose 24) * (0.90^(24)) * (0.10^(25 - 24)) + (25 choose 25) * (0.90^(25)) * (0.10^(25 - 25))
# (25 * (0.90^24) * (0.10^1)) + (1 * (0.90^25) * (0.10^0))
```

The probability is 0.2712059

b. What is the probability that at least 23 favor the ban?

```
# P(X >= 23) = P(X = 23) + P(X = 24) + P(X = 25)
#                              25!
# SUM from 23 to 25  ------------------ * (0.90)^x * (1 - 0.90)^25 - x
#                        x! * (25 - x)!

# x = 23, 24, 25

(factorial(25) / (factorial(23) * factorial(25 - 23))) * (0.90^23) * (1 - 0.90)^(25 - 23) + (factorial(25)
/ (factorial(24) * factorial(25 - 24))) * (0.90^24) * (1 - 0.90)^(25 - 24) + (factorial(25) / (factorial(25
) * factorial(25 - 25))) * (0.90^25) * (1 - 0.90)^(25 - 25)
```

```
## [1] 0.5370941
```

```
dbinom(23, 25, 0.90) + dbinom(24, 25, 0.90) + dbinom(25, 25, 0.90)
```

```
## [1] 0.5370941
```

```
# P(X >= 23)
# P(23) + P(24) + P(25)
# (25 choose 23) * (0.90^(23)) * (0.10^(25 - 23)) + (25 choose 24) * (0.90^(24)) * (0.10^(25 - 24)) + (25 ch
oose 25) * (0.90^(25)) * (0.10^(25 - 25))

# (300 * (0.90^23) * (0.10^2)) + (25 * (0.90^24) * (0.10^1)) + (1 * (0.90^25) * (0.10^0))
```

The probability is 0.5370941

c. What are the mean value and standard deviation of the number who favor the ban in a sample of size 25?

```
# Mean
25 * (0.90)
```

```
## [1] 22.5
```

```
# SD
sqrt(25 * 0.90 * (1 - 0.90))
```

```
## [1] 1.5
```

Mean = 22.5 | SD = 1.5

d. Is it probable that fewer than 23 in the sample favor the ban with the assertion that 90% of the populace favors the ban? (Hint: Consider $P(X < 23)$ when $\pi = 0.90$)

```
# P(X < 23)
# P(0) + P(1) + P(2) . . .  + P(22)

pbinom(22, 25, 0.90, TRUE)
```

```
## [1] 0.4629059
```

The probability is 0.4629059

Problem 7

On the question of whether Interstate 90 should become a tollway, suppose the probability that a randomly selected person in Wisconsin favors tolling is 0.4, opposes it is 0.5, and has no opinion is 0.1. If six people are interviewed and they respond independently, find the probability that:

a. Five favor tolling:

```
# P(0) = N N N N N N = (0.50)^6
# P(1) = Y N N N N N = (0.40 * (0.50^5)) * 6
#        N Y N N N N
#        N N Y N N N
#        . . .

# (0.4)^5 * (0.5+0.1) * 6

# (n choose b) *    π^b   * (1 - π)^(n - b)   |  n = # trials b = # successes
# (6 choose 5) * (0.40^5) * (1 - 0.40)^(6 - 5)
6 * (0.40^5) * (1 - 0.40)^(6 - 5)
```

```
## [1] 0.036864
```

```
dbinom(5, 6, 0.40)
```

```
## [1] 0.036864
```

The probability is 0.036864

b. Two oppose tolling:

```r
# (n choose b) *     π^b    * (1 - π)^(n - b)   |   n = # trials b = # successes

# (6 choose 2) * (0.50^2) * (0.40 + 0.10)^(6 - 2)
15 * (0.50^2) * (0.40 + 0.10)^(6 - 2)
```

```
## [1] 0.234375
```

```r
dbinom(2, 6, 0.50)
```

```
## [1] 0.234375
```

The probability is 0.234375

c. More than 4 favor tolling:

```r
# (6 choose 5) * (0.40^5) * (0.50 + 0.10)^(6 - 5) + (6 choose 6) * (0.40^6) * (0.50 + 0.10)^(6 - 6)
# OR
# (n choose b) *     π^b    * (1 - π)^(n - b)   |   n = # trials b = # successes
# 6 * (0.40^5) * (1 - 0.40)^(6 - 5) + 1 * (0.40^ 6) * (1 - 0.40)^(6 - 6)

(6 * (0.40^5) * (0.50 + 0.10)^(6 - 5)) + (1 * (0.40^6) * (0.50 + 0.10)^(6 - 6))
```

```
## [1] 0.04096
```

```r
dbinom(5, 6, 0.40) + dbinom(6, 6, 0.40)
```

```
## [1] 0.04096
```

The probability is 0.04096

d. At least one has no opinion on tolling:

```r
# 1 - (6 choose 0) * (0.10^0) * (0.50 + 0.40)^(6 - 0)
1 - (1 * (0.10^0) * (0.40 + 0.50)^(6 - 0))
```

```
## [1] 0.468559
```

```r
dbinom(1, 6, 0.10) + dbinom(2, 6, 0.10) + dbinom(3, 6, 0.10) + dbinom(4, 6, 0.10) + dbinom(5, 6, 0.10) + db
inom(6, 6, 0.10)
```

```
## [1] 0.468559
```

```r
pbinom(5, 6, 0.90)
```

```
## [1] 0.468559
```

The probability is 0.468559