<<Lecture11_SS.pdf>>

Stat 324 – Introduction to Statistics for Engineers

LECTURE 11: SIGN TEST FOR MEDIAN AND HYPOTHESIS TESTS FOR 1 SAMPLE PROPORTIONS

Sign Test for Medians (means if symmetric data)

If the data do not seem to be from a normal population and the sample size is small, an alternative to the bootstrap is the sign Test.

Technically it is a test for the median, but in symmetric data, it is equivalent to test for a mean.

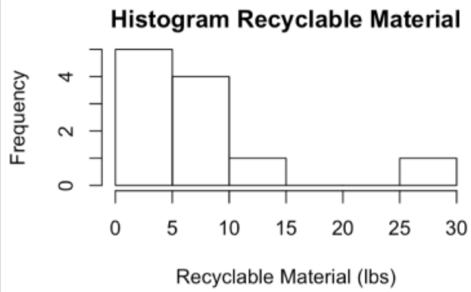
E.g: A city trash department is considering separating recyclables from trash to save landfill space and sell the recyclables. Based on data from other cities, **if more than half the city's households produce 6 lbs or more of recyclable material per collection period**, the separation will be profitable. A random sample of 11 households yields these data on recyclable material per household in pounds:

14.2, 5.3, 2.9, 4.2, 1.8, 6.3, 1.1, 2.6, 6.7, 7.8, 25.9

Sign Test for Medians (means if symmetric data)

E.g: …**if more than half the city's households produce 6 lbs or more of recyclable material per collection period**, the separation will be profitable. A random sample of 11 households yields…

14.2, 5.3, 2.9, 4.2, 1.8, 6.3, 1.1, 2.6, 6.7, 7.8, 25.9



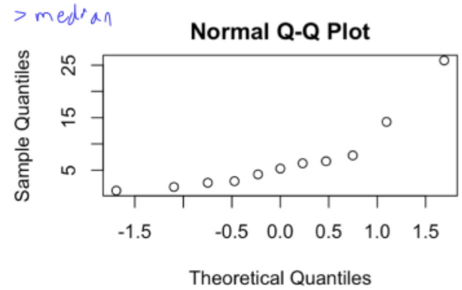
mean > median

mean(recycle)=7.164

median(recycle)=5.4

Sd(recycle)=7.20226

IQR(recycle)=4.5



Both plots show __right skew__ which gives us some evidence that the amount of recyclable materials in the population may not be normal. Since we have n=11, the CLT is questionable, and our question is really about the value of the median of the population.

Sign Test for Medians (means if symmetric data)

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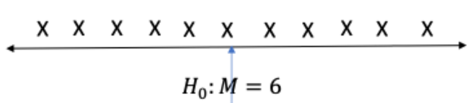
14.2, 5.3, 2.9, 4.2, 1.8, 6.3, 1.1, 2.6, 6.7, 7.8, 25.9

Let M be the population median, we then test:

$H_0: M = 6$ and H_A: __$M > 6$__ (since we want evidence of profitability)

We have a parameter, now we need a test statistic.

If $H_0: M = 6$ is true , the sample should have __about half__ the observations greater than 6 and half less than 6. The probability of observing a value greater than 6 for each of the households should be __0.5__.



Sign Test for Medians (means if symmetric data)

E.g: …**if more than half the city's households produce 6 lbs or more of recyclable material per collection period**, the separation will be profitable. A random sample of 11 households yields…

$H_0: m = 6$

Considering whether a value is above or below the median null, is equivalent to the sign when $x_i - M_o$ is calculated.

Value(X)	**1.1**	**1.8**	**2.6**	**2.9**	**4.2**	**5.3**	**6.3**	**6.7**	**7.8**	**14.2**	**25.9**
Sign($x_i - M_o$)	-	-	-	-	-	-	+	+	+	+	+

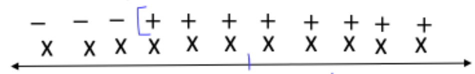
Let N -= Number of negative signs, N+= Number of positive signs

*Notice, if there are ~ the same number of + and – signs, we should have weak evidence against null/ a big p value.

If $H_0: M = 6$ *is true*, $\boxed{N+ \sim Bin(n, 0.5)}$ and $N- \sim Bin(n, 0.5)$ $N+ \sim Bin(11, 0.5)$

*n is number of values that aren't exactly the median (*remove exact matches from sample)*

If $H_A: M > 6$, then we would expect more than half of the observations to be greater than 6 (and fewer than half to be <6).



true med > 6

$H_0: M = 6$

Sign Test for Medians (means if symmetric data)

E.g: …**if more than half the city's households produce 6 lbs or more of recyclable material per collection period**, the separation will be profitable. A random sample of 11 households yields…

Value(X)	**1.1**	**1.8**	**2.6**	**2.9**	**4.2**	**5.3**	**6.3**	**6.7**	**7.8**	**14.2**	**25.9**
Sign($x_i - M_o$)	-	-	-	-	-	-	+	+	+	+	+

Let B=# Number of households that produce ~~6 lbs or more~~ more than 6 lbs per collection period (values above median).

$B \sim Bin(11, 0.5)$. Our observed test statistic is b=5 (+) $b_{obs} = 5$

Strong evidence against H_0 in favor of $H_A: Med > 6$ would be too many + (or too few -)

pvalue $= P(B \geq 5 \mid H_0) = P(B=5) + P(B=6) \ldots + P(B=11) = 0.726$ so weak evidence against null

$= \binom{11}{5}(0.5)^5(0.5)^6 + \binom{11}{6}(0.5)^6(0.5)^5 \ldots + \binom{11}{11}(0.5)^{11}(0.5)^0$

no evidence to suggest Median > 6

In R:

not specified so 0.5

sum(dbinom(x=5:11, size=11, prob=.5)) or

binom.test(x=5, n=11, p=.5, alternative="greater")

Sign Test for Medians (means if symmetric data)

Summary:
Suppose $X_1, \ldots X_n$ is a simple random sample from a population with median M. To test that M has a specified value, M_o,

1. State null and alternative hypotheses: $H_0: M = M_o$ and H_A

2. Check assumptions (independent observations? Symmetric?)

3. Find differences from the median, $X_1 - M_0, \ldots X_n - M_o$ and the observed test statistic b=# of positive signs.

4. Find the p-value, which is the probability for $B \sim Bin(n, .5)$ depending on H_a:
   $H_a: M > M_o \implies p - value = P(B \geq b)$. (Too many positives)
   $H_a: M < M_o \implies p - value = P(B \leq b)$ (Too few positives)
   $H_a: M \neq M_o \implies p - value = 2 * \min(P(B \leq b), P(B \geq b))$ (Too many or few positives)

5. Draw conclusion by comparing p value to α

Info about a Population Proportion π

Recall: A firm has an information file on each of a large number of clients. Call the population proportion of files with errors π. The CEO decides that if π > .15, it will be worthwhile to review and fix every file. An SRS of size n = 100 is taken with the same result as before:

Files with an error: 20; files without errors: 80, $\hat{p} = \frac{20}{100} = 0.20$ (have an error)

Define parameter of interest and Pick hypotheses:

We want to test:
H_o: $\pi = 0.15$. vs H_A: $\pi > 0.15$, where π: true proportion of files with an error

Choose Significance Level (what are the trade offs between Type 1 and Type II error?):

$\alpha = 0.05$

What do we know about our sample estimator $\hat{p}$?

Info about a Population Proportion π

What do we know about our sample estimator $\hat{p}$? $\hat{p} = \frac{X}{n}$ *Where* $X \sim Bin(n, \pi_0)$ under H_0

success

size sample

$$E\left(\hat{p} = \frac{X}{n}\right) = \frac{E(X)}{n} = \frac{n\pi_0}{n} = \pi_0 \quad \text{and} \quad SD\left(\hat{p} = \frac{X}{n}\right) = \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$$

unbiased

and if $n\pi_0 > 5$ and $n(1-\pi_0) > 5$,

then $\hat{p} = \frac{X}{n} \approx N(\pi_0, \frac{\pi_0(1-\pi_0)}{n})$ by the CLT So $Z = \frac{\hat{p}-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \approx N(0,1)$

We will be using a **z test statistic**

*Notice we are using a continuous distribution N to approximate something discrete- there are some continuity corrections we can use, but with large sample sizes changes negligible.

Info about a Population Proportion π

For $H_0: \pi_0 = 0.15$, n =100

$$E\left(\hat{p} = \frac{X}{n}\right) = 0.15 \text{ and } SD\left(\hat{p} = \frac{X}{n}\right) = \sigma_{\hat{p}} = \sqrt{\frac{0.15 * 0.85}{100}} = 0.0357$$

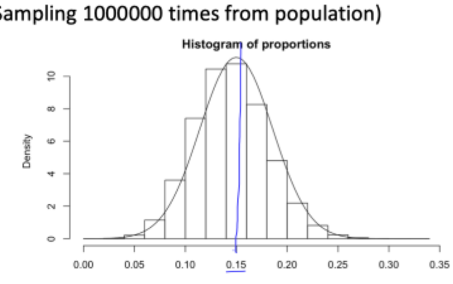
$100 * .15 = 15$ and $100 * 0.85 = 85 > 5$, so

> 5

$\hat{p} = \frac{X}{n} \approx N(0.15, \frac{0.15(0.85)}{100}) = N(0.15, 0.357^2)$ and $Z = \frac{\hat{p} - 0.15}{0.0357} \approx N(0,1)$

```
proportions<-count_data/100
hist(proportions, freq=FALSE)
mean(proportions)
1] 0.1499963
sd(proportions)
1] 0.03571459
```

(Sampling 1000000 times from population)



Testing about a Population Proportion π

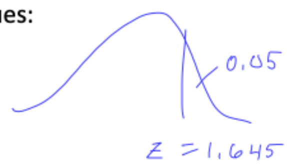
We want to test:

$H_o: \pi = 0.15.$ vs $H_A: \pi > 0.15$, where π: true proportion of files with an error at $\alpha = 0.05$

For $H_0: \pi_0 = 0.15$ n=100 $\quad \hat{p} = \frac{X}{n} \approx N\left(0.15, \sqrt{\frac{.15(0.85)}{100}}\right)$ so $Z = \frac{\hat{p}-0.15}{0.0357} \approx N(0,1)$

$\hat{p}_{obs} = \frac{20}{100} = 0.2$

Critical Values:



RR: $z_{0.05} > 1.645$

RR: $\hat{p}_{0.05} > 0.287$

H_0

$1.645 = \frac{\hat{p} - 0.15}{0.0357}$

$\hat{p} = 1.645(0.0357) + 0.15$

$= 0.287$

P Value:

$Z_{obs} = \frac{0.20 - 0.15}{0.0357} = \frac{0.20 - 0.15}{\sqrt{\frac{0.15(0.85)}{100}}} = 1.4$ $\quad P(Z \geq 1.4) = 1 - P(Z < 1.4) = 1 - 0.9192$
$= 0.0807$

Conclusions:

$\hat{p}_{obs} = 0.20$ is not in RR; insufficient evidence to reject null

pvalue $= 0.0807 > 0.05$; insufficient evidence at 5% level to reject the null

Comparing Confidence Intervals and Hypothesis Tests for π

[failed to reject

- Does the same relationship exist that a conclusion based on a 2-sided $100(1 - \alpha)\%$ CI will match that made by a 2-sided hypothesis test at the α level?

Not always:

Test Statistic : $z = \frac{\hat{p} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$ $\qquad$ Confidence Interval: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

They give similar information about the population parameter, but the link isn't perfect because of the standard error formulas used

Standard error for tests of proportion contain info about the null, while standard error for tests of means do not.

Comparing Sign and Proportion Test

Here is a SRS of 20 component lifetimes (in hours):

1.7, 3.3, 5.1, 6.9, 12.6, 14.4, 16.4, 24.6, 26.0, 26.5, 32.1, 37.4, 40.1, 40.5, 41.5, 72.4, 80.1, 86.4, 87.5, 100.2

Do a **1-sample proportion test** and a **sign test** to see if there is strong evidence that the population median lifetime is larger than 15 hours after checking that assumptions are met.

(1) 1-Sample Proportion Test

(2) Sign Test

Comparing Sign and Proportion Test

SRS of 20 component lifetimes (in hours):

1.7, 3.3, 5.1, 6.9, 12.6, 14.4, 16.4, 24.6, 26.0, 26.5, 32.1, 37.4, 40.1, 40.5, 41.5, 72.4, 80.1, 86.4, 87.5, 100.2

(is there is strong evidence that the population median lifetime is larger than 15 hours?)

(1) 1-Sample/Population Proportion Test for π = proportion over 15 hours

Define Hypothesis and parameter of interest: $H_0: \pi = 0.50 \quad H_A: \pi > 0.50$

Check Assumptions 1.00s independent? assume yes SRS?

$n\pi > 5 \quad n(1-\pi) > 5$

does CLT give us normality of $\hat{p}$

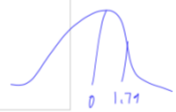
$20 * 0.5 = 10 > 5 \quad 20 * (1-0.5) = 10 > 5$

Calculate P Value under H_0: $\hat{p} \approx N\left(0.50, \left(\sqrt{\frac{0.5*0.5}{20}}\right)^2\right) = N(0.5, 0.1118^2) \quad \hat{p}_{obs} = \frac{14}{20} = 0.70$

$P(\hat{p} \geq 0.70 \mid H_0) \approx P\left(Z \geq \frac{0.70 - 0.50}{\underbrace{0.1118^2}_{SE\hat{p}}}\right) = P(\underbrace{Z \geq 1.79}_{2 \text{ dec}}) = 1 - P(Z < 1.79)$

$= 1 - 0.9633$

$= 0.0368$



Draw conclusions in context

moderate evidence against null, we have sufficient evidence at $\alpha = 0.05$ to reject the null (since $0.0368 < 0.05$). Evidence suggests the median lifetime larger than 15 hours.

Comparing Sign and Proportion Test

SRS of 20 component lifetimes (in hours):

1.7, 3.3, 5.1, 6.9, 12.6, 14.4, 16.4, 24.6, 26.0, 26.5, 32.1, 37.4, 40.1, 40.5, 41.5, 72.4, 80.1, 86.4, 87.5, 100.2 = 14

way above 15 but only counts one value above 15 so lose spread

(is there is strong evidence that the population median lifetime is larger than 15 hours?)

(2) Sign Test for Median

Define Hypothesis and parameter of interest: $H_0: Med = 15 \quad H_A: Med > 15$

Check Assumptions Independence? ✓ under H_0: $B \sim \text{binom}(20, 0.50)$

let B = # of lifetimes above 15

Calculate P Value $B_{obs} = 14 \quad P(B \geq 14 \mid H_0) = P(B = 14) + P(B = 15) + \ldots P(B = 20) = 0.05766$ binom.test(14,

$\binom{20}{14}(0.5)^{20} + \ldots = 0.05766$

Draw conclusions in context we have insufficient evidence at the 5% level to reject the null

notice we got 0.05766 with sign test and 0.0368 with proportion test

on different sides of 5% but only 2% difference between tests