Internship Repport

Non regular Nodes method for Euler Gas Equation

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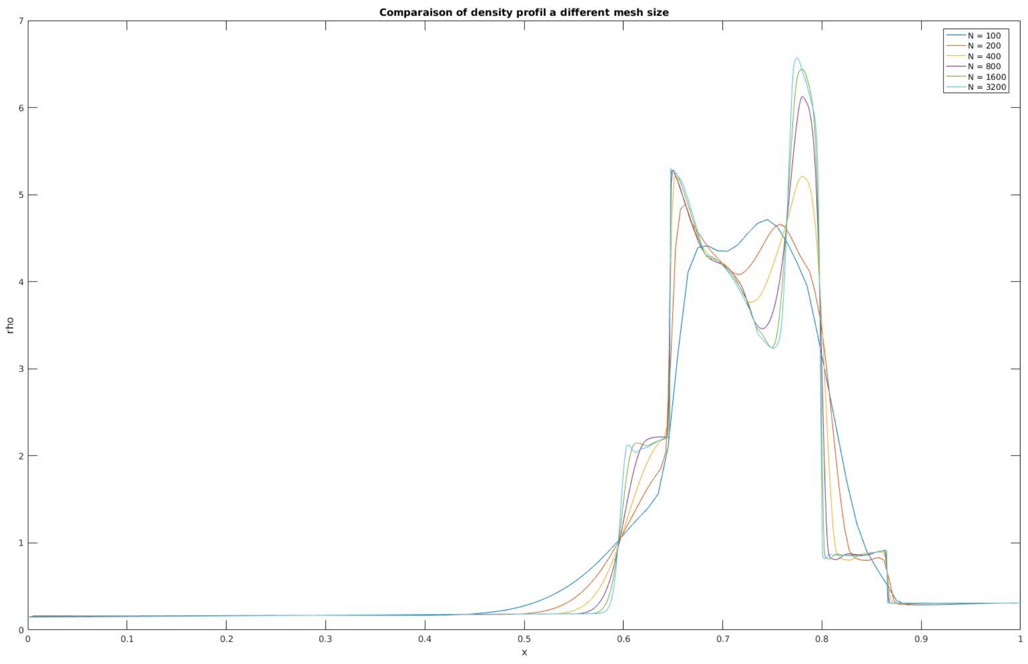

Author :
Timothée SCHMODERER

Referent :
Gregor GASSNER
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Contents

1 Introduction

2 Euler Gas Equation

Let's begin with a brief introduction about the euler gas equations (see [4]).
These equations were first described by Euler in 1757. They describe fluid behaviour when the flow is adiabatic, without momentum exchange by viscosity or enerny exchange by thermal conduction. That is to say, it especially work well with gas.
For a one dimensionnal system, a flow in a tube, the equations are written as follow :

$$\boxed{\frac{\partial}{\partial t}\begin{pmatrix}\rho \\ \rho u \\ E\end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix}\rho u \\ P+\rho u^2 \\ u(E+P)\end{pmatrix} = 0 \qquad \forall x \in \Omega,\ \forall t \geq 0} \tag{1}$$

Actually, these equations are just a formulation for the conservation of mass, momentum and energy across time and space.

Convention : This system illustrates a convention we will systematically make. The partial derivative are intends to act on each components of the vectors.

Where :

1. ρ stands for the fluid density is strictly positive.
2. P stands for the fluid presure and is also strictly positive.
3. E is the total energy of the system
4. u is the fluid velocity which could be either positive (if gas is moving toward increasing value in space) or negative.

All this variable depends of time and space. We ommit this dependance for lighter notation.

The system (1) is close by a fourth equation, an equation of state which bound the four variable together :

$$\boxed{E = \frac{P}{\gamma - 1} - \frac{\rho u^2}{2}} \tag{2}$$

Where γ is the heat capacity ratio (or adiabatic index) of the fluid. It will be taken equal to : 1.4.
We will also need to compute the speed of sound in the gas, which is givern by :

$$c = \sqrt{\frac{\gamma P}{\rho}} \tag{3}$$

In general there is no analytic solution of the system (1 - 2). That's why we need numerical scheme that can good approximate the solution. Moreover usually we can evaluate a numerical scheme by comparing the result of a computation to the analytics solution but as we don't know it, we will have to consider different kind of convergence and error analysis

of our numerical scheme.
The system (1) describe non linear hyperbolics partial differential equations. A new difficulty is that the solution could present discontinuities, or in a more physical vocabulary, chock waves. This is a major problem to construct numerical method to capture them.

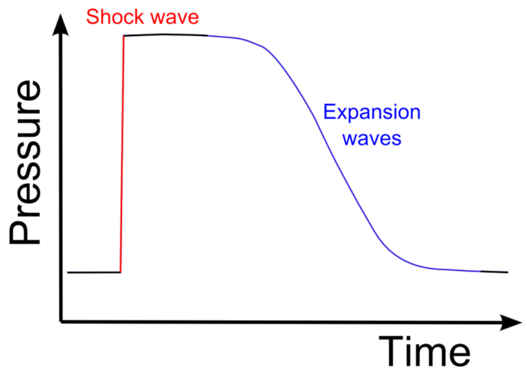

FIGURE 1 – Instance of chock waves

To put the system 1 is a more compact way, let $U = \begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}$ be the state vector of the system and $f(U) = \begin{pmatrix} \rho u \\ P + \rho u^2 \\ u(E+P) \end{pmatrix}$ be the flux vector such as (1) rewrite as :

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} = 0$$

This a general notation for hyberbolic systems. For instance, if we choose f to be the identity then we get the advection equation.

2.1 The problem

In this etud, the domain is taken to be : $\Omega = [0,1]$ and the time in $[0, 0.1]$ The problem is closed with proper boundary and initial conditions :

$$\begin{aligned}
u(0,t) = u(1,t) &= 0 \qquad \forall t \in [0,0.1] \\
\left.\frac{\partial \rho}{\partial x}(x,t)\right|_{x\in\{0,1\}} &= 0 \qquad \forall t \in [0,0.1] \\
\left.\frac{\partial P}{\partial x}(x,t)\right|_{x\in\{0,1\}} &= 0 \qquad \forall t \in [0,0.1] \\
u(x,0) &= 0 \quad \forall x \in \Omega \\
\rho(x,0) &= 1 \quad \forall x \in \Omega \\
P(x,0) &= \begin{cases} 1000 & \text{if } 0 < x < 0.1 \\ 0.01 & \text{if } 0.1 < x < 0.9 \\ 100 & \text{if } 0.9 < x < 1 \end{cases}
\end{aligned} \tag{4}$$

Which we can interpret as solid wall conditions on both extremities of the tubes.

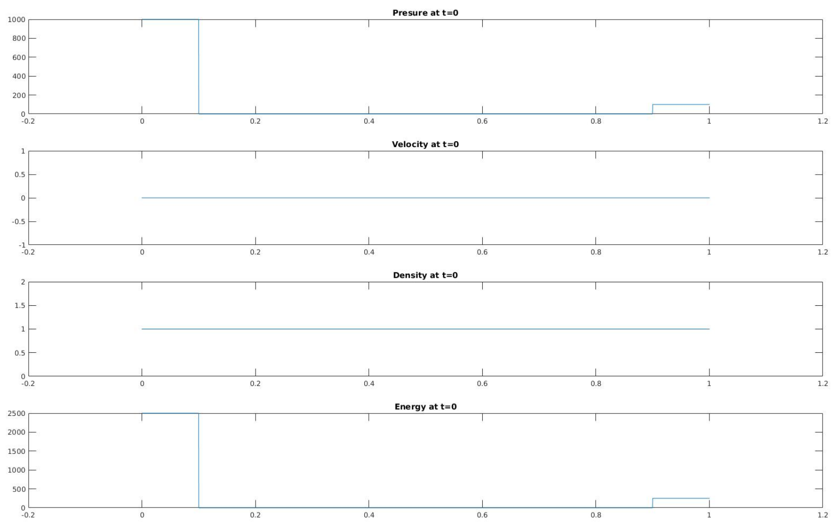

FIGURE 2 – Initial conditions of the problem

2.2 Expecting results

The initial condition suggests that at $t = 0$ there is some kind of valves at $x = 0.1$ and $x = 0.9$ that are oppened.
Basics physical considerations, based on the fact that the distribution of energy tends to be homogeneous in the domain, make us thought that the left side will be moving to the right, the right side to the left. So two wave of gas are moving towards each other. But what happend when this two wave coliddes ?
In fact, this example come from [1] so by their computation we can expect a profil like :

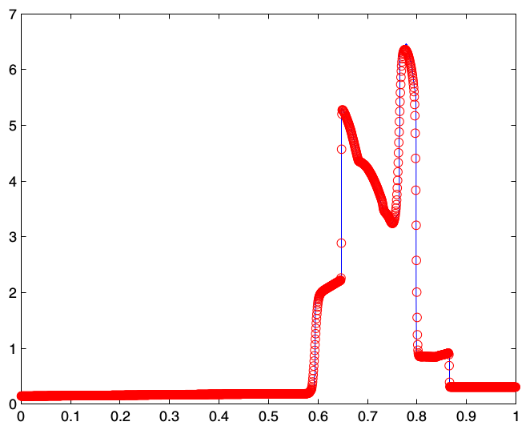

FIGURE 3 – Expecting result at $t = 0.038$

3 Numerical scheme

The goal is to implement the scheme describe in [2] on the example 5.1.4 from [1]. Notice that the method describe in [2] concentrate on moving boundary in two dimensions. But we forgott that and focus on the case the boundaries are fixed and the dimension is one.

3.1 General method for hyberbolic system

The method describe here is the one called "finite volume" (see [?]). This method is used to numerically solve partial differential equations (like finite differences or finite elements), particulary hyperbolic ones but with recent developement elliptic and parabollic too. The partial differential equation is approximately solved by using in mesh made of "finite volume", little segments in our 1D case.
The heart of this method is the divergence theorem, as our hyperbolic equation have a divergence term by using this theorem we can transform volum integral of the divergence term in flux integral at the cells interfaces. We then approximate this flux by constructing the numerical flux.

3.1.1 Cells and Nodes

Choose a integer N (obviously not zero, we won't go far), and let divide Ω into N equals grid "cells" (Figure 4), let x_i be the cell-centers and Δx be the cell's width.

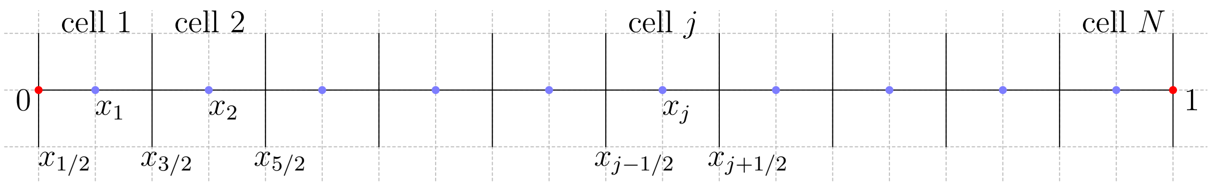

FIGURE 4 – Cells construction on Ω

We denote by $x_{j\pm1/2}$ the cells interfaces : $x_j \pm \dfrac{\Delta x}{2}$.

A simple reasonnement will lead us to the main idea for hyberbolic system. Let us integrate over one cell the partial derivative equation (2) :

$$\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{\partial U}{\partial t} dx = -\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{\partial f(U)}{\partial x} dx$$
$$\frac{d}{dt}\int_{x_{j-1/2}}^{x_{j+1/2}} U dx = -\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{\partial f(U)}{\partial x} dx$$

Here the magic appears, on left side we almost reconize the mean value of U in the cell, and on the right side the computation is obvious since we integrate a space derivative with respect to space. Hence :

$$\Delta x \frac{d\bar{U}_j}{dt} = -(f(U(x_{j+1/2}, t)) - f(U(x_{j-1/2}, t)))$$

Where $\bar{U}_j$ is the mean value of U in the jth cell, and $\Delta x = \dfrac{1}{N}$ is the cell length.

Remark : Look how logical this is, the time variation of the mean value is equal of the flux differences in the cell.

The difficulty that arrises is that we know the values of U in the cell center by setting it to be the mean value of U in the cell. However we need to have the values of U at the cells interfaces in order to compute the right member.

3.1.2 Reconstruction

To solve the precedent problem we have to "guess" the value of U at the cell interfaces with the knowledge of value in the center. The first possibility is to consider U to be constant over each cell :

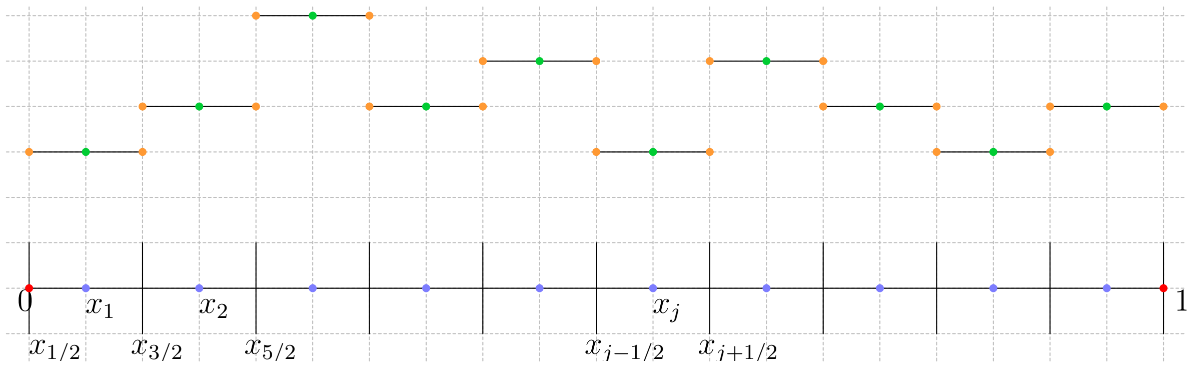

FIGURE 5 – Constant interpolation of U

Thus another problem arise, we construct two value for U at the cells interfaces, which one we should choose ?
Second possibility is to consider piecewise linear function between centers :

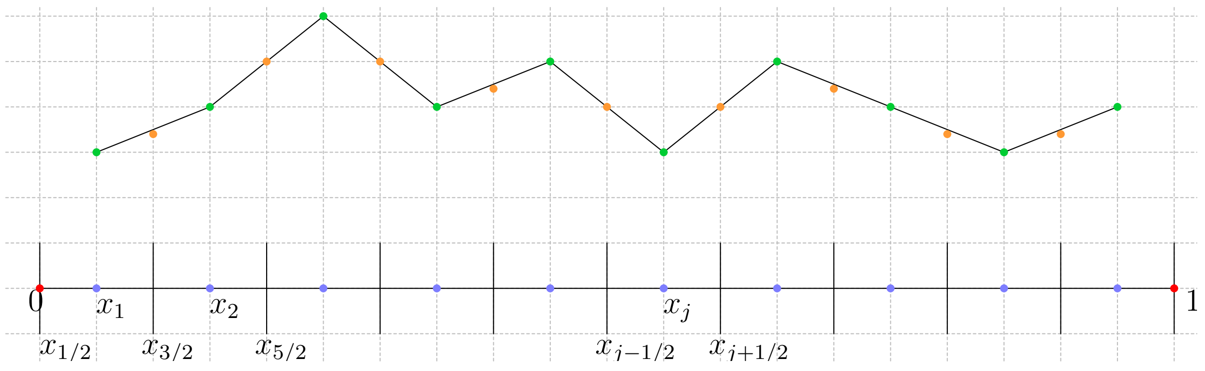

FIGURE 6 – Linear Interpolation of U

We solve the problem of multiple value at interfaces but we see that we would need a special treatement for the boundary.
In fact, there infinetly many ways to do this reconstruction procedure (with paraboloids, cubics, splines ...) so I will now focus on the scheme propose in the paper I am interested in [2].

The goal is to get the following equations :

$$\boxed{\frac{dU_j}{dt} = -\frac{\hat{f}_{j+1/2} - \hat{f}_{j-1/2}}{\Delta x} \qquad j = 1, \dots, N \qquad (5)}$$

Where, of course, $f_{j\pm1/2} = f(U(x_{j\pm1/2}, t))$. And the hat ˆ denote that this is our numerical approcimation of the flux.
We compute the $\{\hat{f}_{j\pm1/2}\}$ as follows :

1. Compute the spectral radius of the jacobian at $x = x_j$. Analytics computation lead us to know that for compressible Euler equation it is :

$$a_j = |u_j| + c_j$$

2. We split the flux function f as :

$$f(U_j) = f_j^+ + f_j^- \qquad f_j^\pm = \frac{1}{2}\left(f(U_j) \pm a_j U_j\right)$$

3. We compute the slopes in each cells using the minmod function :

$$(f_x)_j^\pm = minmod\left(\theta\frac{f_j^\pm - f_{j-1}^\pm}{\Delta x}, \frac{f_{j+1}^\pm - f_{j-1}^\pm}{2\Delta x}, \theta\frac{f_{j+1}^\pm - f_j^\pm}{\Delta x}\right)$$

   Where $\theta \in [1, 2]$ is a parameter that control te amount of numerical dissipation, larger values of θ typically lead to sharper resolution of discontinuities, but may cause some oscillations, it will be taken at 1.5.
   The *minmod* function is defined by :

$$minmod(a, b, c) = \begin{cases} \min(a, b, c) & \text{if } a > 0,\ b > 0 \text{ and } c > 0 \\ \max(a, b, c) & \text{if } a < 0,\ b < 0 \text{ and } c < 0 \\ 0 & \text{else} \end{cases}$$

4. Construct f^E and f^W as :

$$f_j^E = f_j^+ + \frac{\Delta x}{2}(f_x)_j^+ \qquad f_j^W = f_j^- - \frac{\Delta x}{2}(f_x)_j^-$$

5. Finally :

$$\hat{f}_{j+1/2} = f_j^E + f_{j+1}^W \quad \hat{f}_{j-1/2} = f_{j-1}^E + f_j^W$$

3.2 Boundary condition and Ghost Nodes

While presenting the scheme I haven't discuss on which nodes we have to apply the method. Obviously there is no probleme when we are in the middle of the domain problems arrises when we come close to the boundaries.
In order to achieve the method two ghosts point are needed on each side, with value dicted by the boundary conditions.
Why two ? Remember that we want values at each cell's interfaces. Then for instance to get $\hat{f}_{1-1/2}$ we will need f_0^E so at this point we need one ghost cell left. Moreover to get f_0^E we will need to get the slopes for $j = 0$ then we might need $f_{-1}^{\pm}$ in the *minmod* function. So one more ghost cell.
I describe the comportement on the left side, the same can be applied to the right side.

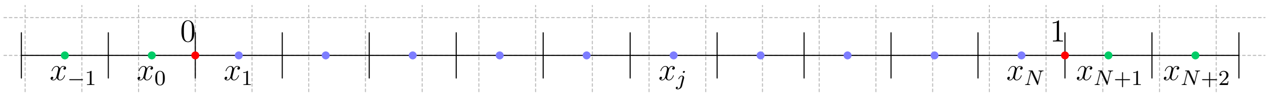

FIGURE 7 – Ω with ghost points

By the boundary condition we get the value of ρ, u and P at the two ghost cells centers :

$$\begin{aligned} \rho_0 &= \rho_1 & u_0 &= -u_1 & P_0 &= P_1 \\ \rho_{-1} &= \rho_2 & u_{-1} &= -u_2 & P_{-1} &= P_2 \end{aligned}$$

Which in term of U is the following :

$$U_0 = \begin{pmatrix} U_1^1 \\ -U_1^2 \\ U_1^3 \end{pmatrix} \qquad U_{-1} = \begin{pmatrix} U_2^1 \\ -U_2^2 \\ U_2^3 \end{pmatrix}$$

where the upperscript is the denotes the i-th component.
The whole method can be summarize in this array (red are ghost values, and dot show for what j we can compute the value) :

cell	-1	0	1	2	$\dots$	j	$\dots$	$N-1$	N	$N+1$	$N+2$
a_j	·	·	·	·	·	·	·	·	·	·	·
$f^{\pm}$	·	·	·	·	·	·	·	·	·	·	·
$f_j^{\pm} - f_{j-1}^{\pm}$		·	·	·	·	·	·	·	·	·	·
$f_{j+1}^{\pm} - f_{j-1}^{\pm}$		·	·	·	·	·	·	·	·	·	
$f_{j+1}^{\pm} - f_j^{\pm}$	·	·	·	·	·	·	·	·	·	·	
$(f_x)^{\pm}$		·	·	·	·	·	·	·	·	·	
f^E		·	·	·	·	·	·	·	·	·	
f^W		·	·	·	·	·	·	·	·	·	
$\hat{f}_{j+1/2}$		·	·	·	·	·	·	·	·		
$\hat{f}_{j-1/2}$			·	·	·	·	·	·	·	·	
$\dfrac{dU_j}{dt}$			·	·	·	·	·	·	·		

3.3 Integration of the semi-discrete system

Once we get the scheme 5 we have to numerically integrate it. Here I present two method. The easiest way to do it is with the Euler forward scheme. We took a time discretization $\{t_n = n\Delta t\}$ and we approximate the time derivative by :

$$\frac{dU_j}{dt} \approx \frac{U_j^{n+1} - U_j^n}{\Delta t}$$

Thus the final discrete equation we get is :

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\left(\hat{f}_{j+1/2} - \hat{f}_{j-1/2}\right)$$

However this method present one major inconvenient, depending on the choice of Δt the numerical method could be unstable (that is to say, unbounded oscillations appears in the numerical solution). Thus we will have to determine the so called Courant's number which bound the time step to the space step.
Moreover, even with a "good" choose of Δt some oscillation could occurs which are only numerical. this oscillations are due to the choice of the integration scheme. To handle this problem I used a Strong Stability Preserving Runge-Kutta method of order 3 (SSP - RK). It presents two main advantages. First the courant number is the same as the one with the Euler scheme, and second, this method is TVD (total variation diminution) :

$$TV(U^{n+1}) \leq TV(U^n) \qquad TV(U^n) = \sum_j |U_{j+1}^n - U_j^n|$$

I implemented the following scheme from ([3]) :

$$\begin{aligned} u^{(1)} &= u^n + \Delta t L(u^n) \\ u^{(2)} &= \frac{3}{4}u^n + \frac{1}{4}u^{(1)} + \frac{1}{4}\Delta t L(u^{(1)}) \\ u^{n+1} &= \frac{1}{3}u^n + \frac{2}{3}u^{(2)} + \frac{2}{3}\Delta t L(u^{(2)}) \end{aligned} \tag{6}$$

Where the L operator is the one that give us the second member. This scheme is SSP with the same Courant's number as the Euler Forward scheme and third order in time.

Courant's number : This number is a constant C for which the method is stable and such as :

$$\frac{|u|\Delta t}{\Delta x} \leq C$$

Numerical experiment suggests that $C = 0.437$ was good. Then as we know that approximately the velocity is bound by 23 we can choose the time step to be :

$$\Delta t = \frac{0.437}{23}\Delta x$$

4 Numerical Experiment for regular nodes distribution

4.1 Data Representation

I hope no one here is a purist mathematician beacause I will abuse with the "dot" notation of Matlab meaning component by component operation. Choosing Matlab for programming

language was a logical first choice to do, because many operation are transparent in Matlab. However we should keep in mind that greater efficiency could be achieve with mor basic language.
With a very natural way, the partial differential equation is describeb by a vector U whose size is : 3 row times (N+4) colums. Why N+4 ? Because there is N internal nodes and 2 ghost points on each sides.
Thus in only on structure I could represent my system at time t. For instance for $N = 10$ cells and $t = 0$ I get the structure :

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.025 & 2500 & 2500 & 0.025 & 0.025 & 0.025 & 0.025 & 0.025 & 0.025 & 0.025 & 0.025 & 250 & 250 & 0.025 \end{pmatrix}$$

Let recall that with that structure we have information in the center of the cell and we want achieve information at cells interfaces.

4.2 Implementation

I will describe the code implementing the method in Matlab language [1]. I recall that all the operation should be thinking made in one cell. The choice of the matlab language offers us the "element by element" operations which I allow us to compute all the cell in the same operation.

First is teh function implementing the flux : $\begin{pmatrix} \rho u \\ \rho u^2 + P \\ u(E+P) \end{pmatrix}$. Given the vector of state U we can get easily u, ρ and E but we will need the adiabatic constant γ to compute the pressure with the equation of state (2). Thus the flux is given by :

$$f(U) = U.*u + \begin{bmatrix} 0_{\mathbb{R}^{N+4}} \\ P \\ P.*u \end{bmatrix}$$

```
function y = f(U,gamma)
  % Velocity
  v = U(2,:)./U(1,:);
  % Pressure
  P = (gamma-1)*(U(3,:)-0.5*U(1,:).*v.*v);
  % Flux
  y = U.*v+[zeros(size(P));P;P.*v];
end
```

Code 1 – Implementation of the flux function - f.m

Second easy step is to compute the speed of the sound in the domain. Same argumentation leads us to compute density and pressure from the vector of state and the adiabatic constant :

$$c = \sqrt{\gamma P./\rho}$$

1. Actuaally, it will not works with matlab beacause I write it in Octave and this software is more tolerant. But minor changes will make it work with Matlab.

```
function c = speedofsound(U,gamma)
  % Density
  rho = U(1,:);
  % Pressure
  P = (gamma-1)*(U(3,:)-0.5*(U(2,:).^2)./rho);
  % Speed of sound
  c = sqrt(gamma*P./rho);
end
```

Code 2 – Implementation of the computation of the speed of sound - speedofsound.m

Another minor function is the one that construct the initial condition of pressure :

$$P_0(x) = \begin{cases} 1000 & \text{if } x \in]0, 0.1[\\ 0.01 & \text{if } x \in]0.1, 0.9[\\ 100 & \text{if } x \in]0.9, 1[\end{cases}$$

However the function here is not well implemented because it will search thrre times in the x array where are the value. This would be avoid by making only one loop over x. But, I let the function in this form for lighter code.

```
function y = P0(x)
  y = zeros(size(x));
  y(find(0.0 < x & x < 0.1)) = 1000;
  y(find(0.1 < x & x < 0.9)) = 0.01;
  y(find(0.9 < x & x < 1.0)) = 100;
end
```

Code 3 – Implementation of the initial condition of pressure - P0.m

One important function in the reconstruction process in the *minmod* function. This is maybe the function I am the less proud about because it is certainly using the full power offers by the data structure, but it is not the speedest way to do it and it is strongly dependant of this particular case.

$$minmod(a,b,c) = \begin{cases} min(a,b,c) & \text{if } a > 0,\ b > 0 \text{ and } c > 0 \\ max(a,b,c) & \text{if } a < 0,\ b < 0 \text{ and } c < 0 \\ 0 & \text{else} \end{cases}$$

```
function y = minmod(a,b,c)
  y = zeros(size(a));
  % Index where the three numbers are positive
  iM = find(a > 0 & b > 0 & c > 0);
  y(iM) = min(min(a(iM),b(iM)),c(iM));
  % Index where the three numbers are negative
  im = find(a < 0 & b < 0 & c < 0);
  y(im) = max(max(a(im),b(im)),c(im));
end
```

Code 4 – Implementation of the minmod function - minmod.m

Finally the hearth of the method is contain in this last function file. Given U, the adiabatic constant, the minmod reconstruction constant and the mesh size, it will produce the second member in 5. I just change the input values of the *minmod* function. In deed, in the paper they divided this inputs by Δx but then the results is multiply by Δx to get f^E and f^W. So to gain six division and two multiplication, I directly cancel them.

```
function q = qf_uniform(U,gamma,theta,dx)
  fU = f(U,gamma);
  c = speedofsound(U,gamma);
  a = abs(U(2,:)./U(1,:)) + c;

  % Compute f+ & f-
  fp = 0.5 * (fU + a.*U);
  fm = 0.5 * (fU - a.*U);

  % Compute df+
  dfp0 = theta*(fp(:,2:end-1) - fp(:,1:end-2)); % Option 1
  dfp1 = (fp(:,3:end) - fp(:,1:end-2))/2; % Option 2
  dfp2 = theta*(fp(:,3:end) - fp(:,2:end-1)); % Option 3

  dfp = minmod(dfp0,dfp1,dfp2);

  % Compute df-
  dfm0 = theta*(fm(:,2:end-1) - fm(:,1:end-2)); % Option 1
  dfm1 = (fm(:,3:end) - fm(:,1:end-2))/2; % Option 2
  dfm2 = theta*(fm(:,3:end) - fm(:,2:end-1)); % Option 3

  dfm = minmod(dfm0,dfm1,dfm2);

  fE = fp(:,2:end-1) + 0.5*dfp;
  fW = fm(:,2:end-1) - 0.5*dfm;

  % Compute f+0.5 and f-0.5
  fphalf = fE(:,2:end-1) + fW(:,3:end);
  fmhalf = fE(:,1:end-2) + fW(:,2:end-1);

  % Compute second member
  q = (fphalf - fmhalf)/dx;
end
```

Code 5 – Implementation of the computation of the second member - qf_uniform.m

And least the main function is setting all the parameters for the study of this case and process the SSP Runge Kutta method.

```
function main_euler_uniform(N)
  % Constantes
  a = 0;
  b = 1;
  theta = 1.5;
  gamma = 1.4;
  T = 0.1;
  dx = (b-a)/N;
  x = [a-dx/2:dx:b+dx/2];
  x = [x(1)-dx,x,x(end)+dx];
  cfl = 0.437/23;
  dt = cfl*dx;
  s = size(x);
  niter = ceil(T/dt);
  % Initialisation in Omega at t=0
  tmprho = ones(1,s(2)-4);
  tmpv = zeros(1,s(2)-4);
  tmpP = P0(x(3:end-2));

  % Apply boundary condition to fill the ghost cells
  rho = [tmprho(2),tmprho(1),tmprho,tmprho(end), tmprho(end-1)];
  v = [-tmpv(2),-tmpv(1),tmpv,-tmpv(end),-tmpv(end-1)];
  P = [tmpP(2),tmpP(1),tmpP,tmpP(end),tmpP(end-1)];
  E = P/(gamma-1) + 0.5*rho.*v.*v;
  % Construction of the vector of state
  U = [rho;rho.*v;E];

  % U at the next time step
  U1 = zeros(size(U));
  % Reserve memory space for the RK method
  U1_1 = zeros(size(U));
  U1_2 = zeros(size(U));

  for t = 1:niter

    % SSP RK order 3
    q = qf_uniform(U,gamma,theta,dx);
    U1_1(:,3:end-2) = U(:,3:end-2) - dt*q;
    U1_1(:,[1 2 end-1 end]) = [1;-1;1].*U1_1(:,[4 3 end-2 end-3]);

    q1 = qf_uniform(U1_1,gamma,theta,dx);
    U1_2(:,3:end-2) = 0.75*U(:,3:end-2) + 0.25*U1_1(:,3:end-2) -
       0.25*dt*q1;
    U1_2(:,[1 2 end-1 end]) = [1;-1;1].*U1_2(:,[4 3 end-2 end-3]);

    q2 = qf_uniform(U1_2,gamma,theta,dx);
    U1(:,3:end-2) = (1/3)*U(:,3:end-2) + (2/3)*U1_2(:,3:end-2) -
       (2/3)*dt*q2;
    U1(:,[1 2 end-1 end]) = [1;-1;1].*U1(:,[4 3 end-2 end-3]);
    c = speedofsound(U,gamma);
    % Loop
    U = U1;

    if sum(imag(c) > 0) >0 % in case of instability of the method,
       this criterion will stop the loop
      break;
    end
  end
end % end function
```

Code 6 – Implementation of the main function - main_euler_uniform.m

Note that each at each Runge-Kutta time step I apply the boundary condition to U.

4.3 Results

I recall that more results and some animations are available at https://github.com/tschmoderer/euler-prj.

4.3.1 Chock capture

I ran many cases with different mesh size. The bigger the mesh is, the more accurate is the solution computed.

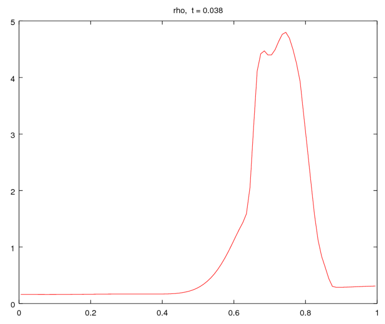

FIGURE 8 – Chock at $t = 0.038$ for $N = 100$ nodes

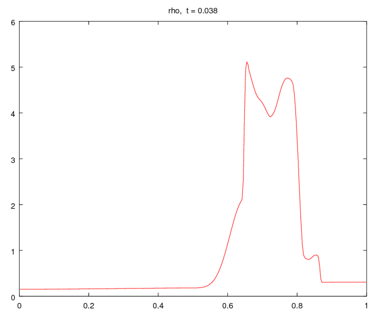

FIGURE 9 – Chock at $t = 0.038$ for $N = 300$ nodes

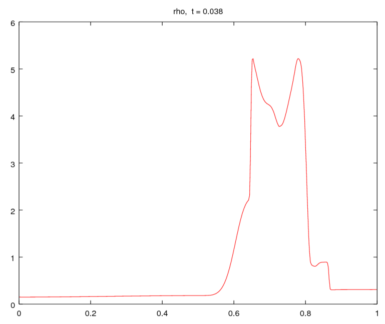

FIGURE 10 – Chock at $t = 0.038$ for $N = 400$ nodes

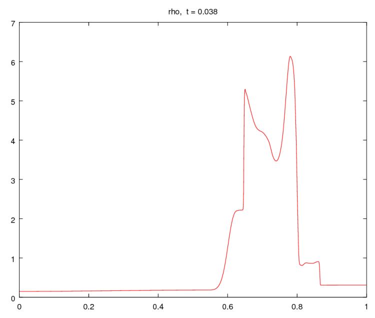

FIGURE 11 – Chock at $t = 0.038$ for $N = 800$ nodes

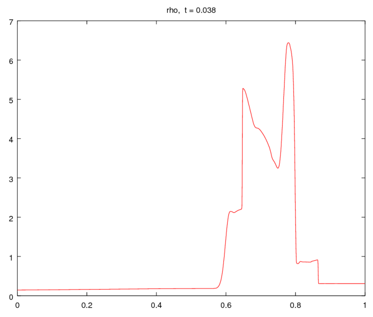

FIGURE 12 – Chock at $t = 0.038$ for $N = 1600$ nodes

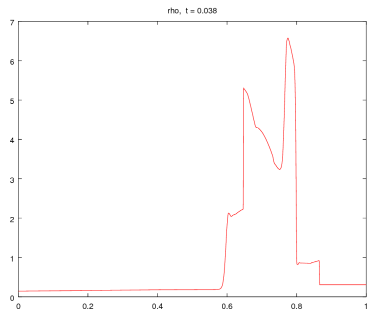

FIGURE 13 – Chock at $t = 0.038$ for $N = 3200$ nodes

4.3.2 Euler vs Runge Kutta

Let's illustrate the influence of the choice of the integration method. We can see on figure 14 that some socillations occurs in the computed solution with Euler's forward scheme, but that there is no on figure 15 where the solution was computed with a third order SSP Runge-Kutta method.

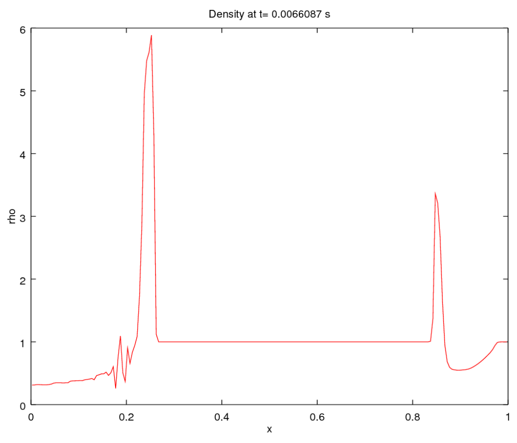

FIGURE 14 – Density with Euler forward scheme

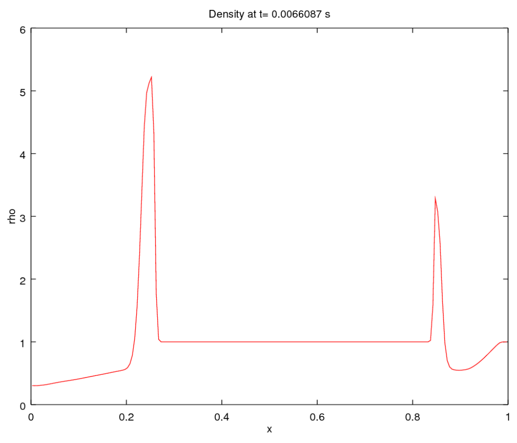

FIGURE 15 – Density with the SSP RK scheme

4.4 Error and convergence

As we saw earlier, finding convergence rate is quite difficult. Because we don't know the analytics solution, we have to adopt a particular strategy.

4.4.1 Rate of convergence

5 Adaptation on non regular nodes distribution

5.1 My adaptation

5.2 Legendre Gauss ...

6 Conclusion

Références

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Table des figures

List of Codes