

# Introduction to Machine Learning

INSA 2018

# General Information

# General Information

## Personal presentation

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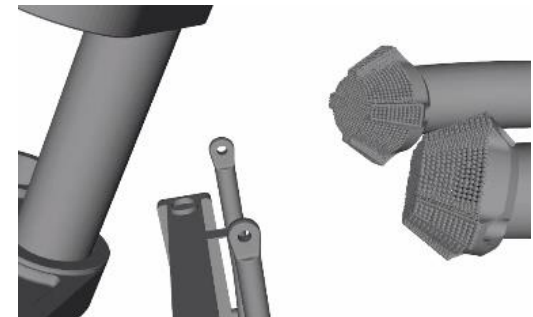
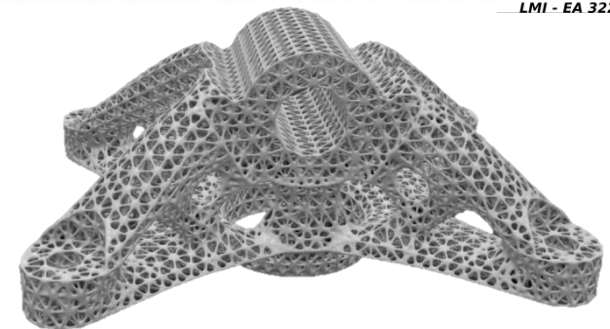
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- LMI INSA Rouen
  - PhD Thesis
- Shape Synthesis through Numerical Optimization



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# General Information

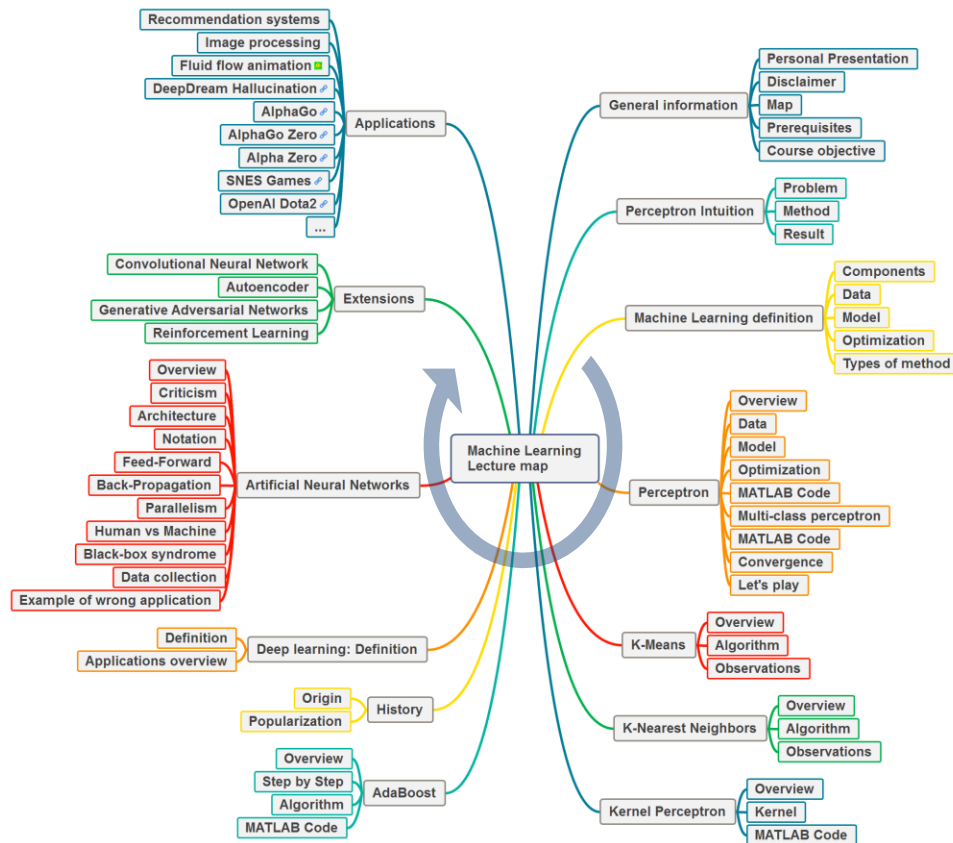
## Disclaimer

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- English
- Tu
- First iteration of this course
- Not from INSA

# General Information

## Map



# General Information

## Prerequisites

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### Calculus

Multivariable  
functions  
Partial derivatives

### Linear Algebra

Matrix-Vector  
arithmetic  
Multi-index notation

### Computer Science

Algorithmics  
Numerical  
optimization  
Parallelism

# General Information

## Course Objectives

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- Explore multiple different **Machine Learning** methods
- Work our way to **Artificial Neural Networks (ANN)**
- Understanding of underlying concepts and implementations
- Intuition on more complex systems
- **Pragmatism and critical thinking**

# Perceptron: Intuition



# Perceptron: Intuition

## Problem



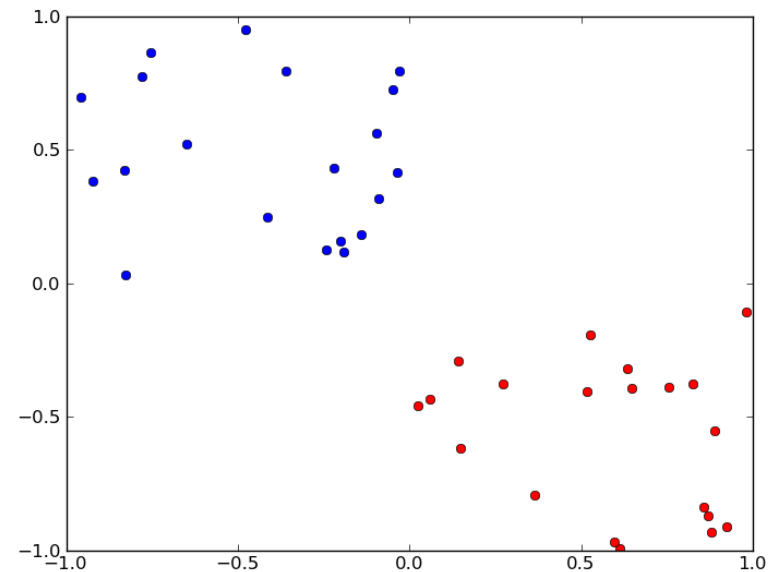
- Input
  - Feature vector
- Output
  - Discrete value
- Goal
  - Make a decision (Discrete value) based on the input data (Feature vector)



# Perceptron: Intuition

## Problem

- Input vector of 2 scalar value
  - Each possible input can be represented as a 2D point on a plane
- Output value
  - The output is a TRUE/FALSE answer based on the position of the 2D point

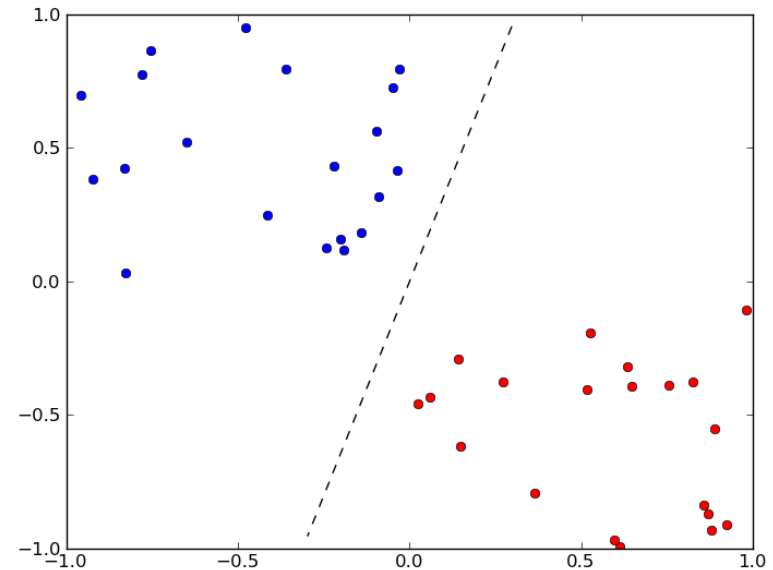


# Perceptron: Intuition

## Method

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- Model
  - The perceptron is a **linear classifier**
- Decision function / Activation function
  - Gives the TRUE/FALSE answer based on a data point and a given line
- Training algorithm
  - Method to choose a line that gives satisfying results when coupled with the decision function

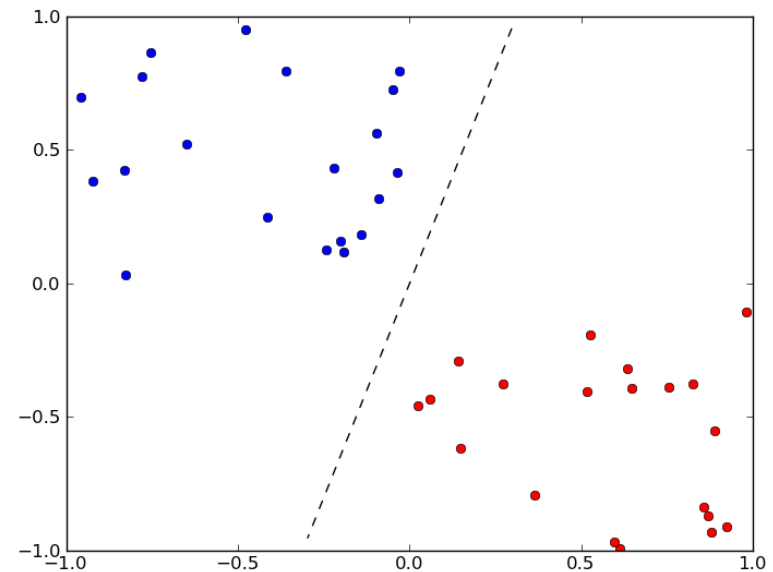


# Perceptron: Intuition

## Result

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- Potent generalizing model
- Slow to train
- Fast to evaluate



# Machine Learning : Definition

# Machine Learning : Definition

## Components

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### Data

What is provided to the machine ?

### Model

What does the machine do with it?

### Optimization

How does the machine learn to do it better ?

# Machine Learning : Definition

## Data

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- Training set

- List of data examples
  - “Individuals” in a population
  - Points in N-dimensional space



Feature vector

- Structured

- Examples tend to have the same size, with parameters in the same order

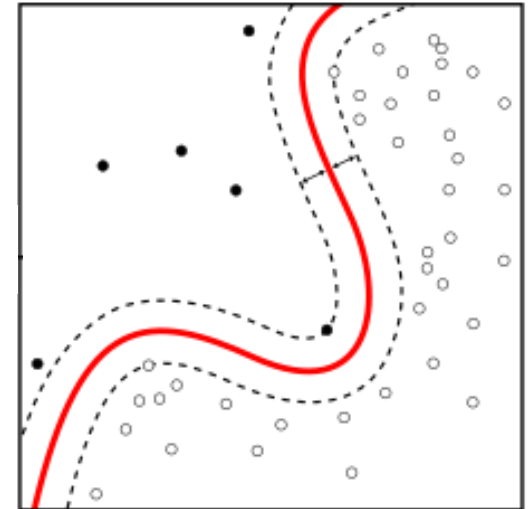
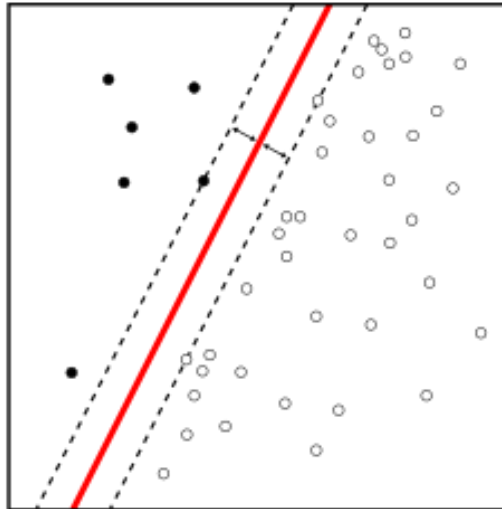
- Exhibits some behavior

- Assumption that the training set is a **representative sampling** of the N-Dimension Space **smooth underlying phenomenon**

# Machine Learning : Definition

## Model

- Complexity
  - Order
    - Linear
    - Non-Linear
  - Input space
  - Output space
- Genericity
  - Specific
  - General





# Machine Learning : Definition

## Optimization

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- Objective function / Cost function
  - Generally described as the **error of the model on the training data**
  - Also sometimes described by its inverse as the “fitness” of a model (GA)
- Error minimization
  - Differentiable
    - Gradient descent
  - Non-differentiable
    - Simulated Annealing
    - Particle Swarm Optimization
    - Genetic Algorithm

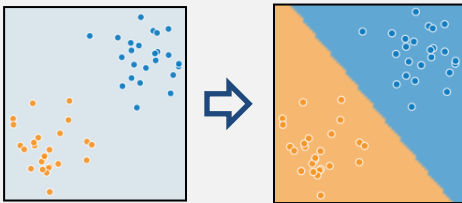
# Machine Learning : Definition

## Types of methods

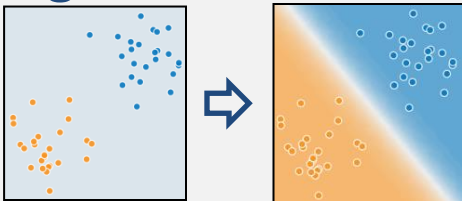


### Supervised

#### Classification

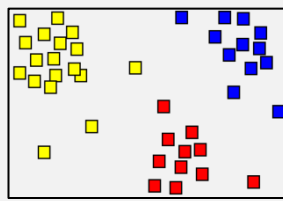
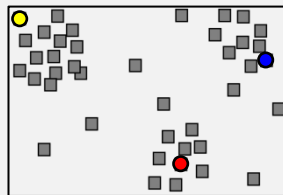


#### Regression



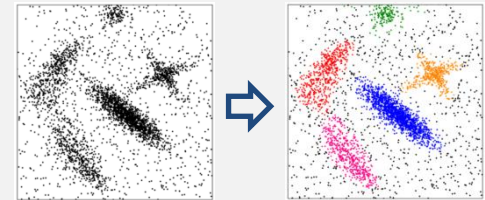
### Semi-supervised

#### Propagation

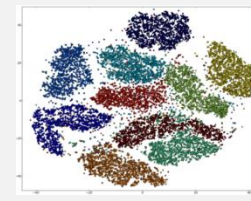


### Unsupervised

#### Clustering



#### Visualization



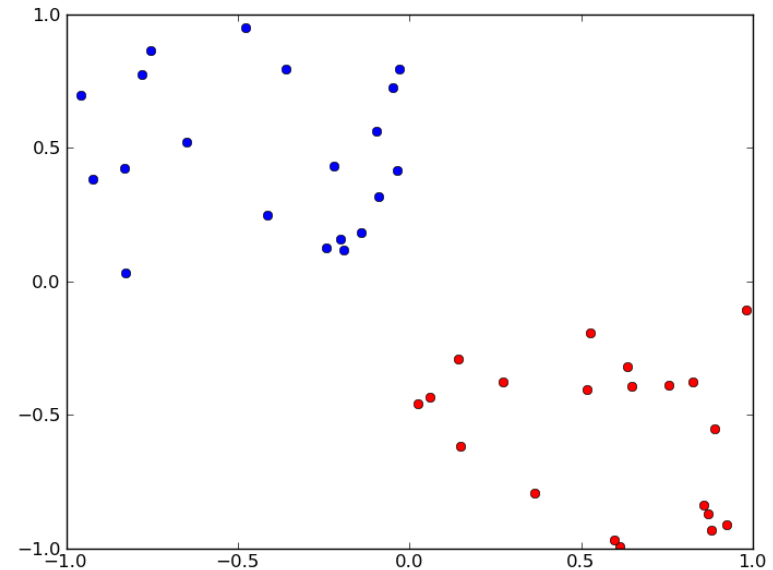
# Perceptron

# Perceptron

## Overview

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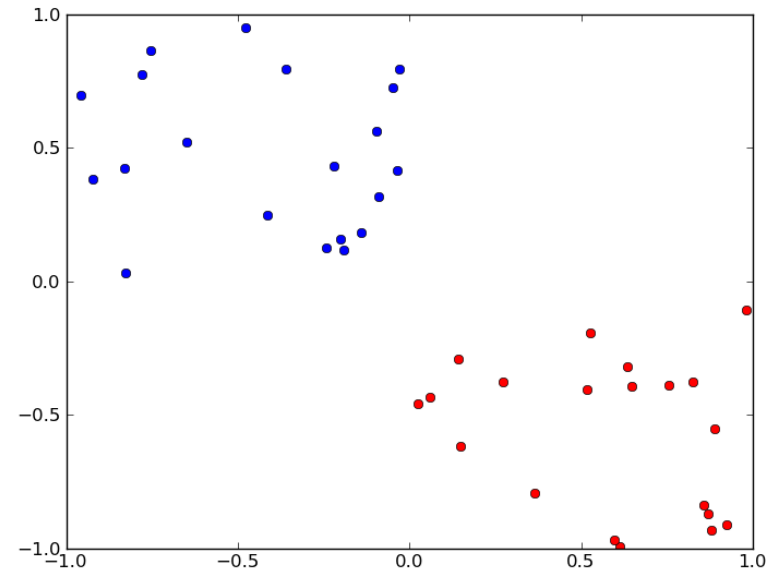
- Supervised
  - Classification
- Data
  - Feature vector
  - Label
- Model
  - Linear
- Optimization
  - Iterative method



# Perceptron

## Data

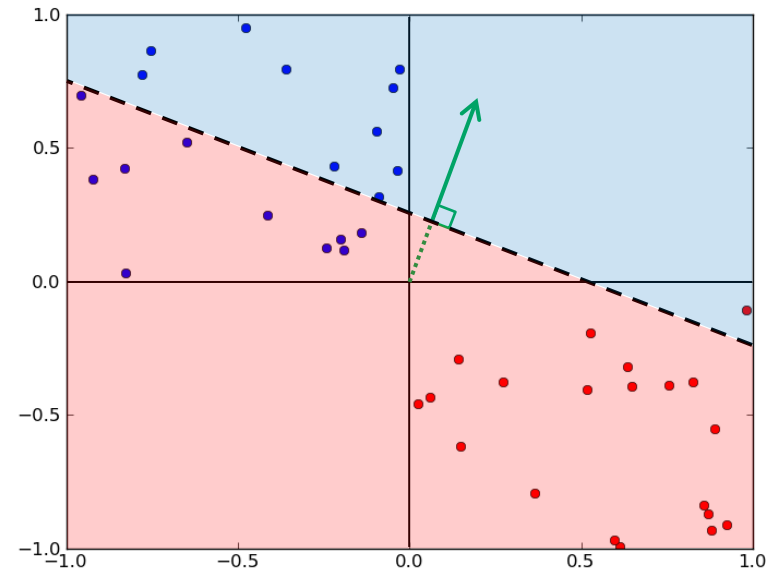
- Input vector of size N
  - Each possible input can be represented as a point in N-dimensional space
- Output value
  - The output is a BLUE/RED answer based on the position of the point
  - The value is represented by “+1” or “-1”



# Perceptron

## Model – Geometrical view

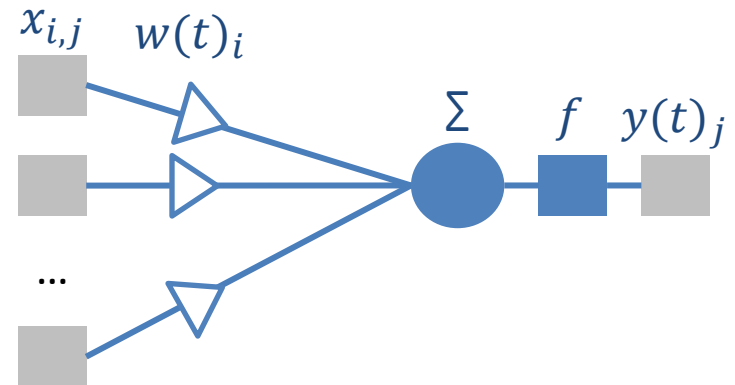
- The separating hyperplane needs  $N+1$  parameters to be described in  $N$ -dimensional Space
  - A vector, normal to the **hyperplane**
    - Called the **Weights**
  - A translation magnitude from the origin and along that **normal vector**
    - Called the **Bias**



# Perceptron

## Model – Mathematical view

- Let  $x_{i,j}$  be the  $i$  value of the  $j$  input sample point
- Let  $w(t)_i$  be the  $i$  weight value at time  $t$
- Let  $f$  be the activation function
- Let  $y(t)_j$  be the  $j$  output value at time
- Let  $d_j$  be the desired  $j$  output value



$$y(t)_j = f \left( \sum_{i=0}^n w(t)_i x_{i,j} \right)$$

Prediction function for the linear perceptron



Vector notation

# Perceptron

## Optimization

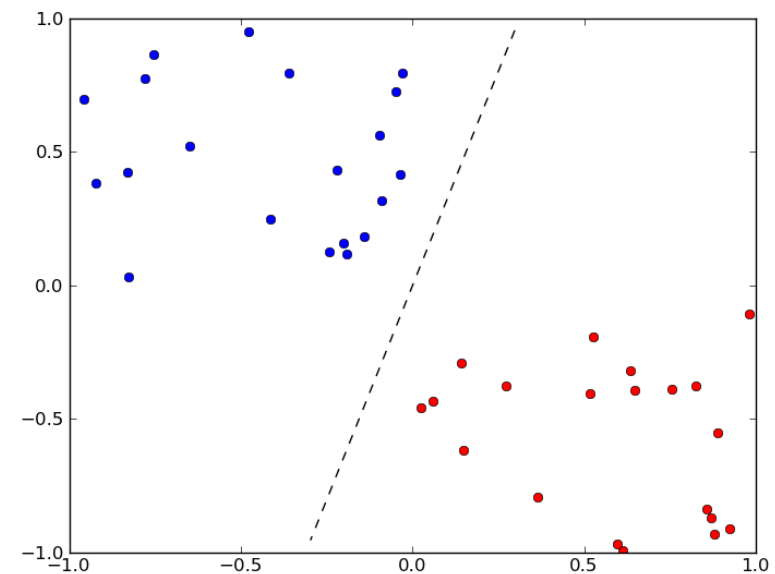
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- Let  $d_j$  be the desired  $j$  output value



Weight update intuition

Weight update rule for the linear perceptron

$$w(t+1)_i = w(t)_i + (d_j - y(t)_j)x_{i,j}$$

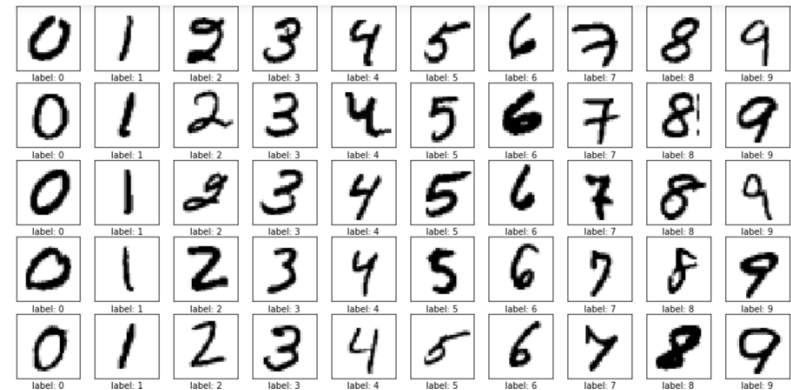




# Perceptron

## MATLAB Code

- MNIST Database
  - Handwritten digits
  - 10 classes
  - 70 000 labeled images
    - 60 000 training images
    - 10 000 test images
  - High dimension feature vectors
    - 16x16 or 28x28 grayscale images
- 2-classes
  - Detect “0” from “not 0”

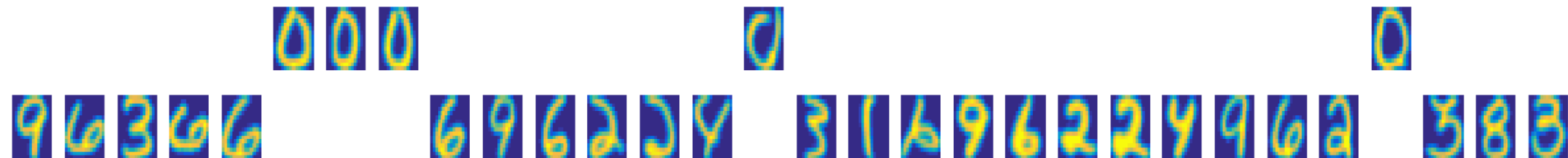


Examples of images in the MNIST database

# Perceptron

## MATLAB Code

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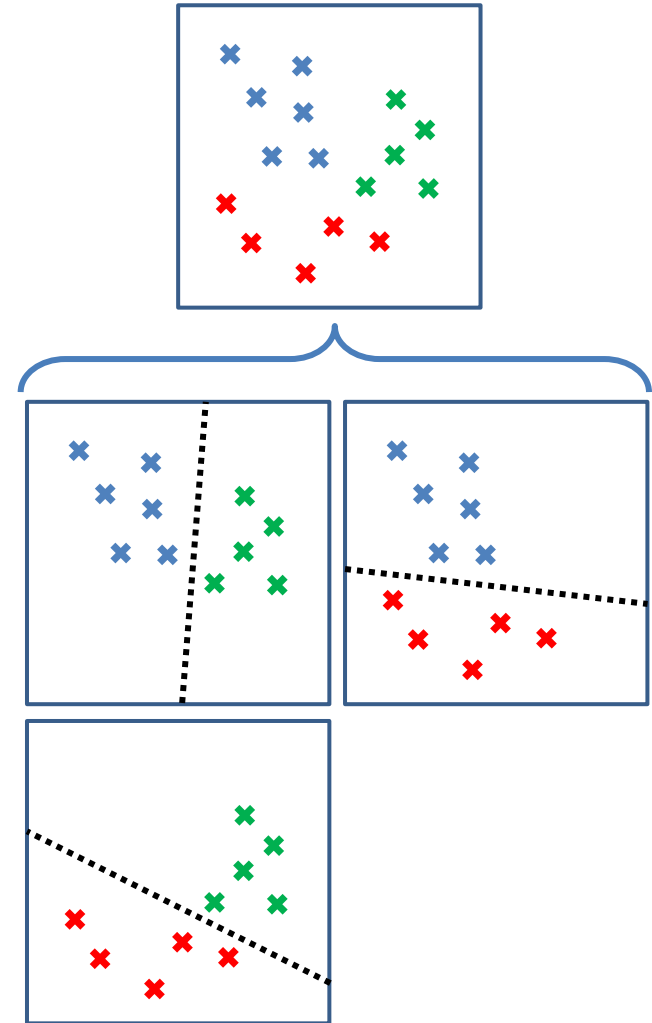


95.4% good predictions

# Perceptron

## Multi-class perceptron

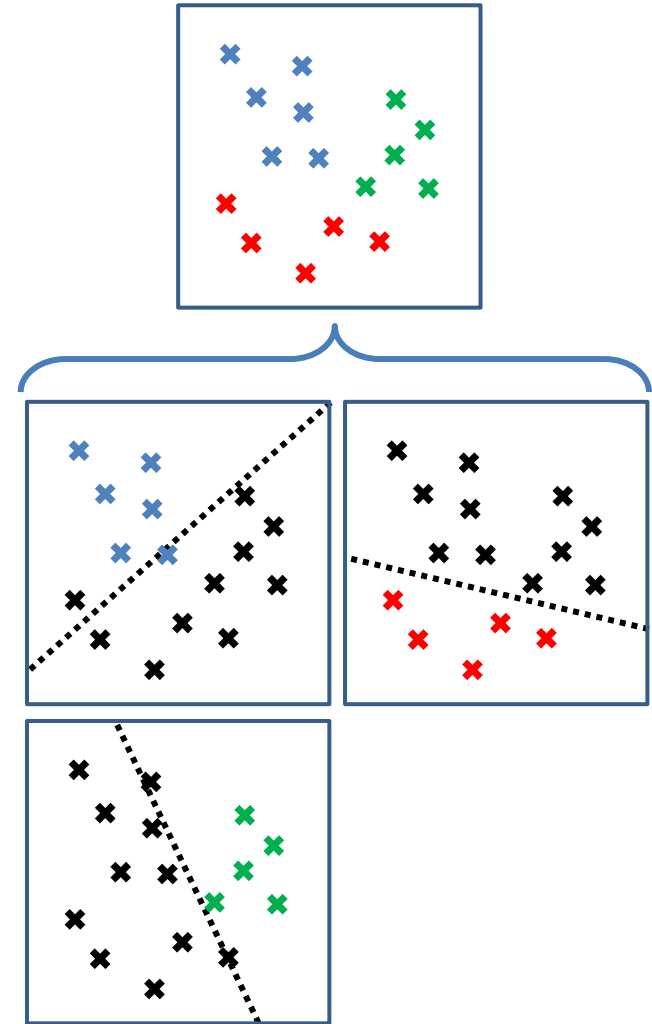
- The perceptron is built to solve 2 classes problems
- Many different strategies can extend its scope
  - 1 vs 1
  - 1 vs All
  - Tournament



# Perceptron

## Multi-class perceptron

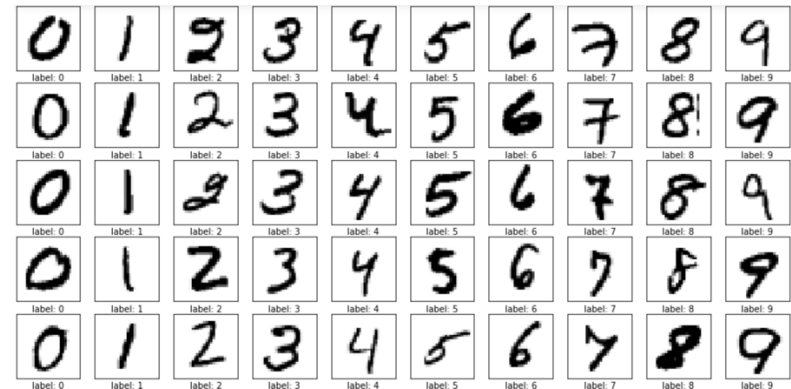
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# Perceptron

## MATLAB Code

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  - 10 classes
  - 70 000 labeled images
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    - 10 000 test images
  - High dimension feature vectors
    - 16x16 or 28x28 grayscale images
- 10-classes
  - 1 vs all strategy



Examples of images in the MNIST database

# Perceptron

## MATLAB Code

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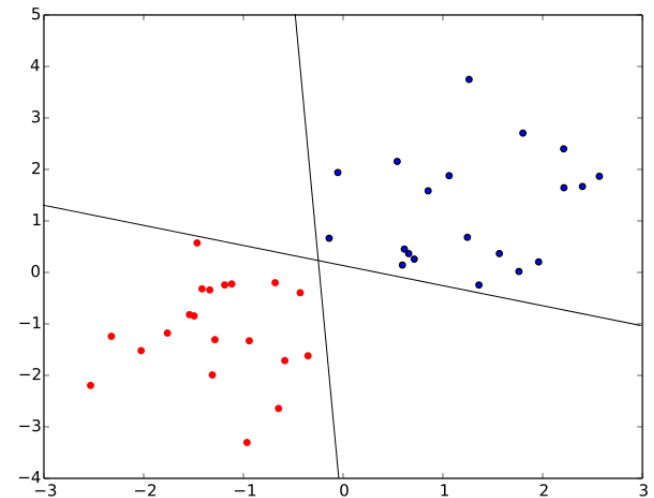


88.3% good predictions

# Perceptron

## Convergence

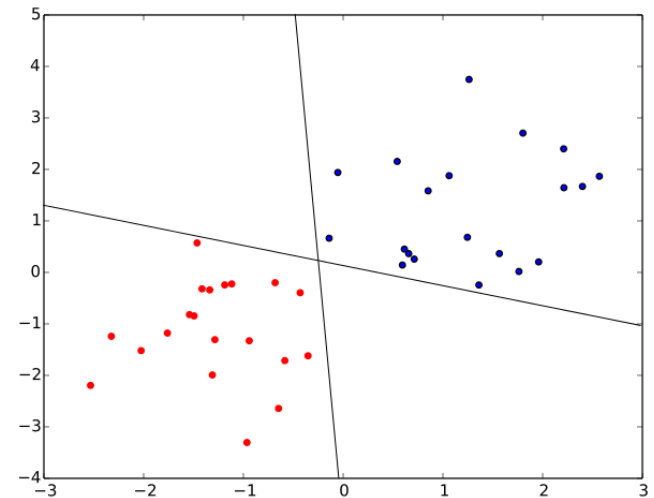
- Linearly Separable
  - Convergence in finite, upper bounded, number of steps
  - Dependency to training samples ordering
- Non-Linearly Separable
  - Generally suboptimal solution
  - Often oscillations
  - Sometimes, complete divergence



# Perceptron

## Activation function

- Heaviside step function as activation function
  - Multiple perfect solutions if linearly separable
  - Unstable if non linearly separable

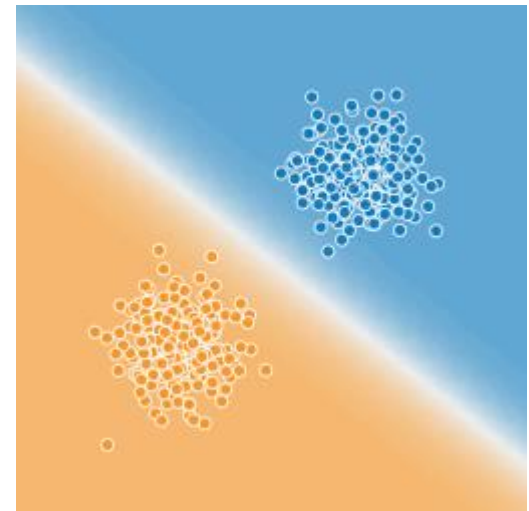
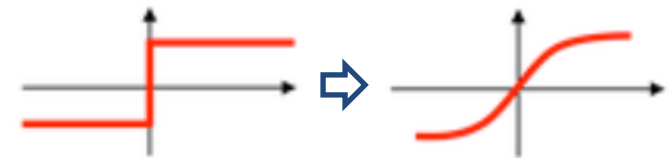




# Perceptron

## Activation function

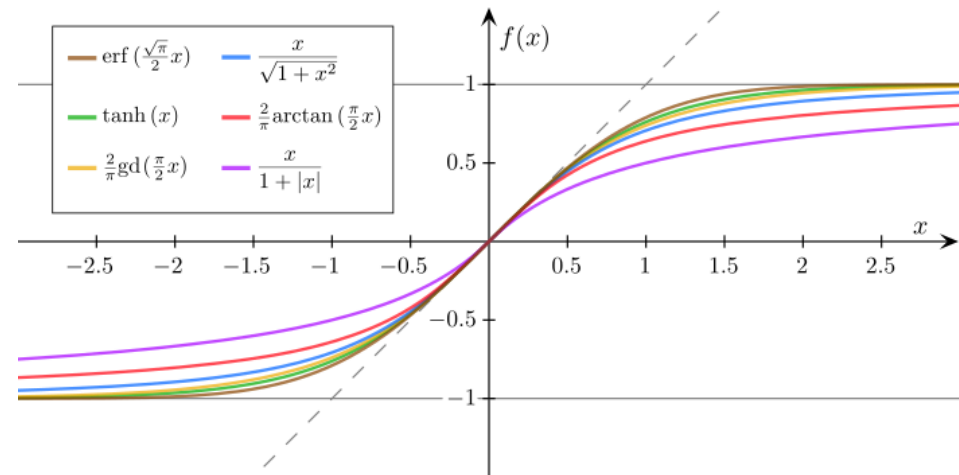
- Smooth step function as activation function
  - Convergence toward a solution maximizing the **distance to misclassification**
  - Generally stable regardless of linear separability
  - The perceptron becomes analogous to **Logistic Regression**



# Perceptron

## Activation function

- Multiple desirable properties:
  - Approximate identity near zero
  - Converges to +1 when  $x$  tends toward  $+\infty$
  - Converges to -1 when  $x$  tends toward  $-\infty$
- Many types of smooth activation functions

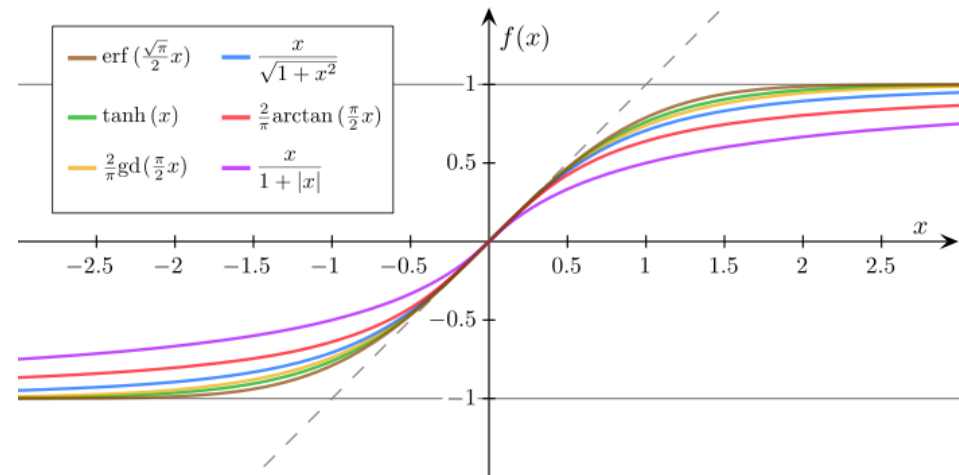


$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

# Perceptron

## Activation function

- Multiple desirable properties:
  - Differentiable
  - Monotonous
  - Monotonous derivative
- Optimization now require proper partial derivatives



$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

# Perceptron

## Let's play

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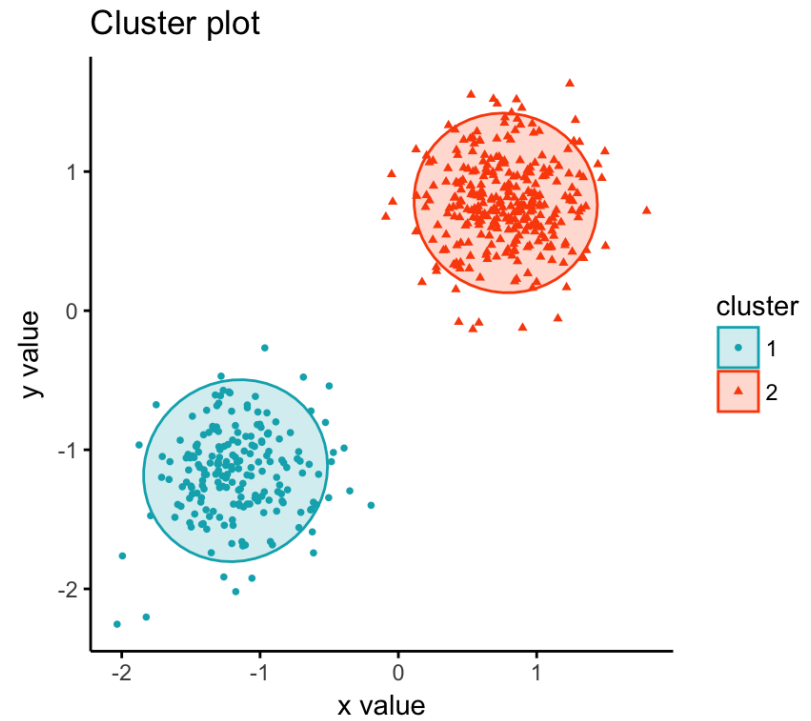
- [playground.tensorflow.org](https://playground.tensorflow.org)
  - Tool Description
  - Simple test
  - Noise
  - Overfitting
  - Non Linearity

# K-Means Clustering

# K-Means Clustering

## Overview

- Unsupervised
  - Clustering
- Data
  - Feature vectors
- Model
  - Centroid + Metric
- Optimization
  - Iterative method



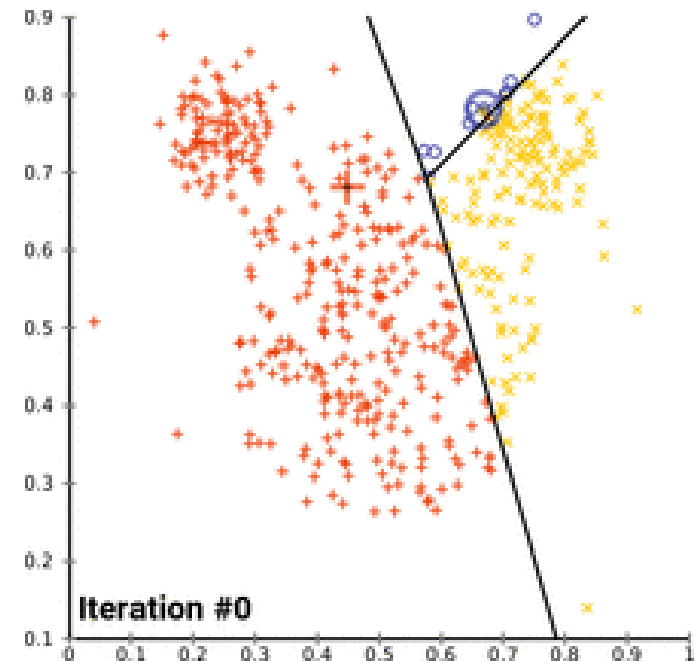
# K-Means Clustering

## Algorithm

- Initialization
  - Pick K random centroids
- Loop
  - Label each sample with the closest centroid
  - Move each centroid to the mean of their labeled samples



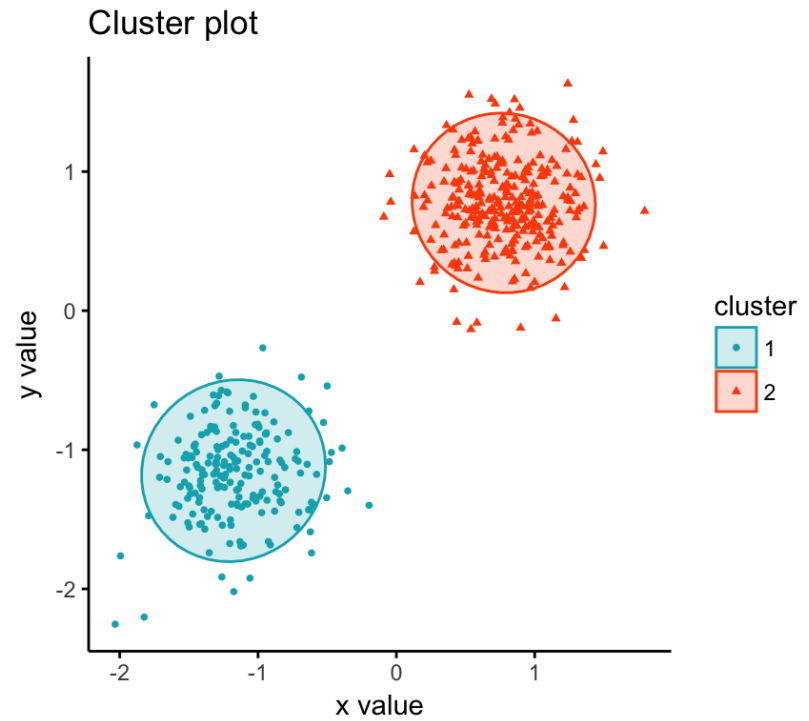
Step by step algorithm



# K-Means Clustering

## Observations

- Minimize intra class variance
- Maximize inter class variance
- Assumes compact clusters
- Link to Voronoi diagram
- Highly dependent on the chosen metric



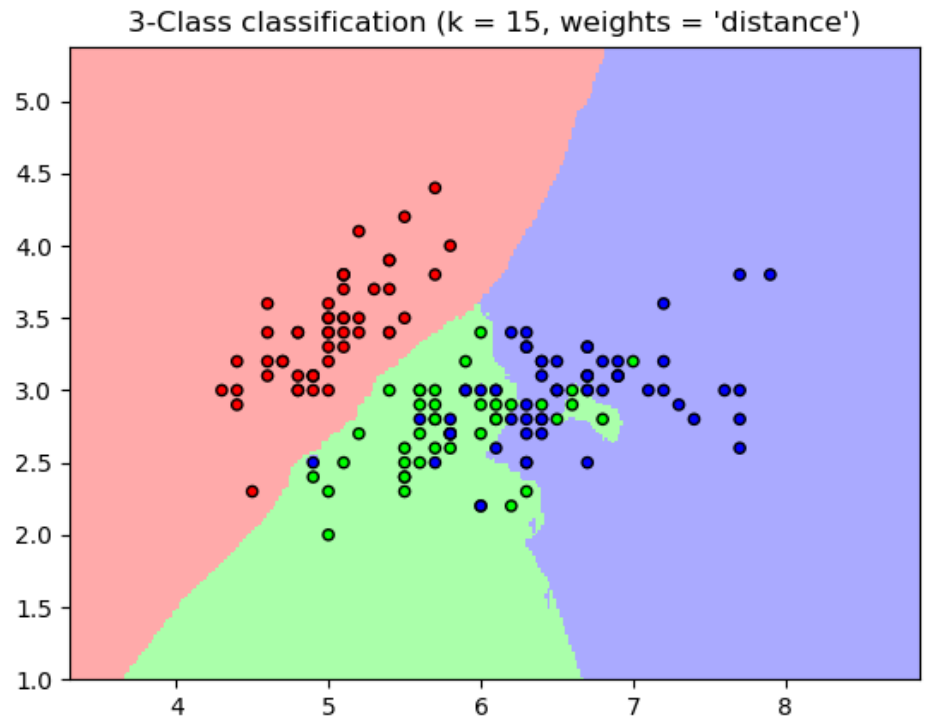


# K-Nearest Neighbors

# K-Nearest Neighbors

## Overview

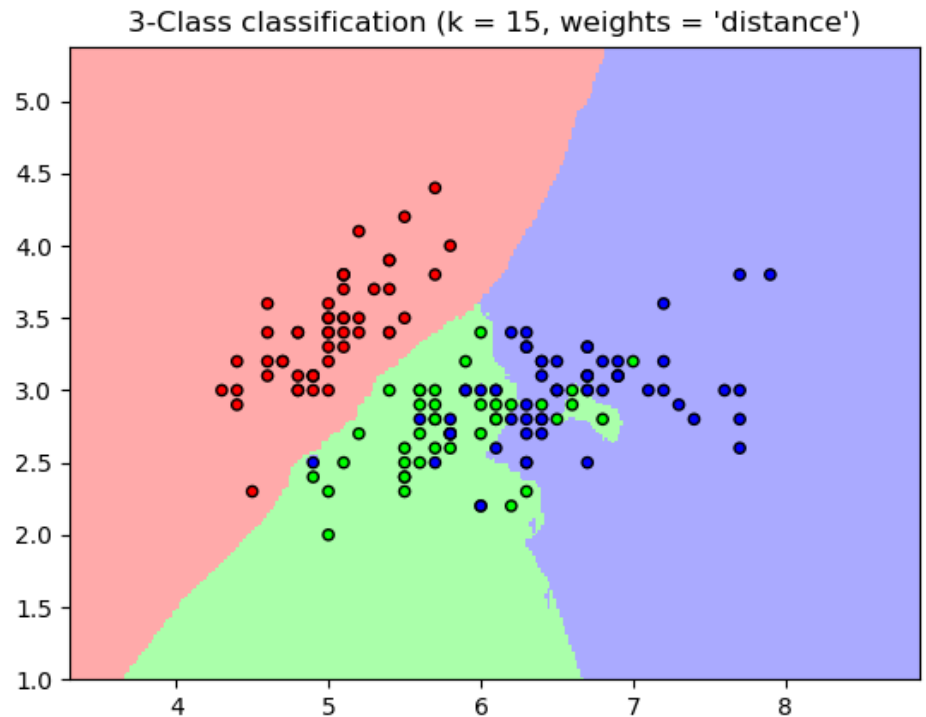
- Supervised
  - Classification
- Data
  - Feature vectors
  - Labels
- Model
  - Kernel based
- Optimization
  - None or Iterative method



# K-Nearest Neighbors

## Algorithm

- For a given test feature vector
  - Find the  $k$  closest training feature vectors
  - Take the class most present in  $k$  neighbors
  - Optional
    - Weight based on neighbors distances

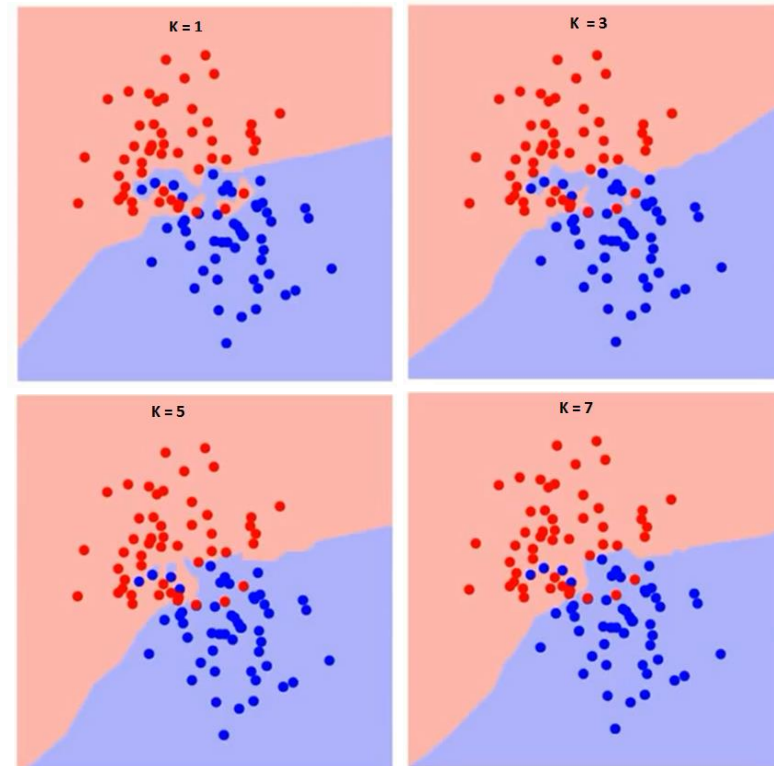


# K-Nearest Neighbors

## Observations



- Can easily handle any number of classes
- Smoothness controlled by “k”
- Highly dependent on the chosen metric
- Costly evaluation
  - Can be optimized by making the dataset sparse

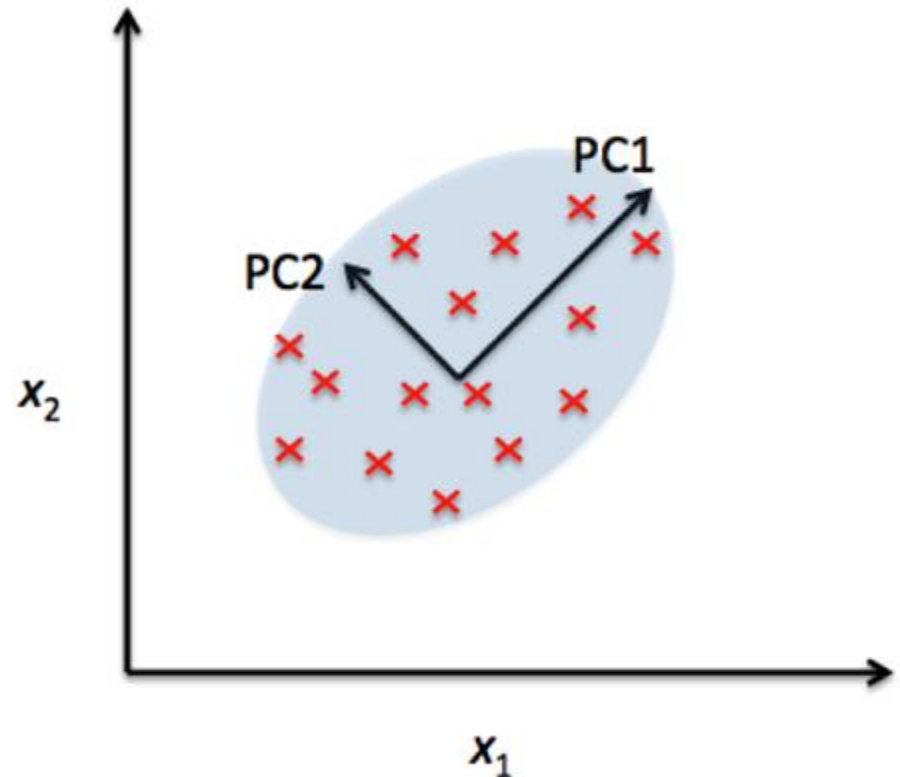


# Principal Component Analysis

# Principal Component Analysis

## Overview

- Unsupervised
  - Visualization
- Data
  - Feature vectors
- Model
  - Orthogonal linear transformation
- Optimization
  - Analytical or Iterative method



# Principal Component Analysis

## Algorithm

- Let  $p$  be the dimension of the data
- Let  $n$  be the number of data points
- Let  $\mathbf{X}$  be the  $p$  by  $n$  data matrix
  - $\mathbf{X}$  must have zero mean
- Let  $\mathbf{r}$  be the resulting first principal component of  $\mathbf{X}$

```

 $\mathbf{r}$  = a random vector of length  $p$ 
do  $c$  times:
     $\mathbf{s} = 0$  (a vector of length  $p$ )
    for each row  $\mathbf{x} \in \mathbf{X}$ 
         $\mathbf{s} = \mathbf{s} + (\mathbf{x} \cdot \mathbf{r})\mathbf{x}$ 
     $\mathbf{r} = \frac{\mathbf{s}}{|\mathbf{s}|}$ 
return  $\mathbf{r}$ 

```

# Principal Component Analysis

## Algorithm

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- Iterative algorithm is very efficient for high dimension data (when  $p$  is high)
  - Avoids computing the full covariance matrix
- Iterative algorithm is numerically instable after the first few principal components
  - Numerical error of each principal component computation is carried to the next
- This method is particularly well suited for **visualization** though **dimensionality reduction**



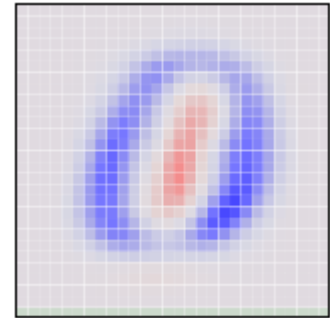
# Principal Component Analysis

## Example

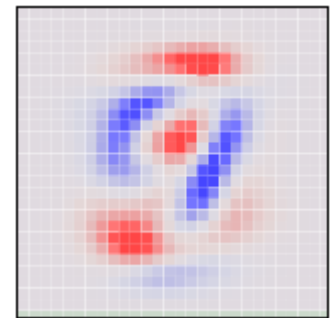
- Each principal component is a vector of size equal to the feature vector dimension
- If the input feature vectors are images, each principal component can also be seen as an image



2D example and color coding



First principal component in the MNIST Database



Second principal component in the MNIST Database

# Principal Component Analysis

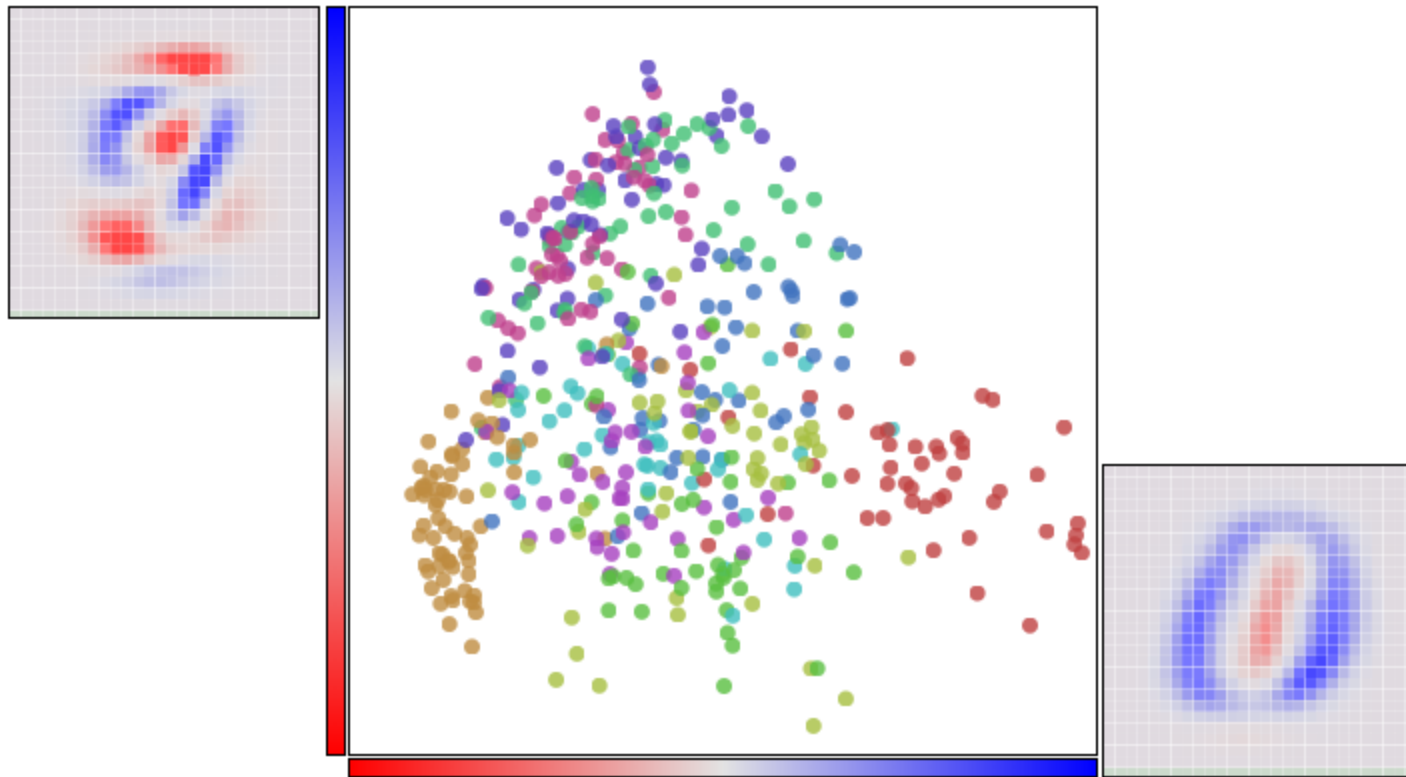
## Example

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# Principal Component Analysis

## Example



# Principal Component Analysis

## Example

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- <http://colah.github.io/posts/2014-10-Visualizing-MNIST/>

# Principal Component Analysis

## Implementation for images

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- Store all the images in a single matrix  $\mathbf{X}$ 
  - Each row corresponds to the pixel values of an image
  - The labels are NOT used
- Transform  $\mathbf{X}$  to give it zero mean
- Repeat 2 or 3 times :
  - Run the simple iterative algorithm to extract  $\mathbf{PC}$  the first principal component of  $\mathbf{X}$
  - Update  $\mathbf{X}$  by projecting each data point onto the hyperplane defined by  $\mathbf{PC}$
  - Store the value of  $\mathbf{PC}$  before computing the next principal component
- Visualize the resulting point cloud in 2D or 3D
  - Color code the points based on their label to see if some structure emerges



Projection ?

# Kernel Perceptron

# Kernel Perceptron

## Overview

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- Variant of traditional perceptron
  - Described in 1960
  - Capable of learning (almost arbitrary) nonlinear phenomena
- Type of instance-based learner
  - No explicit generalization, no fixed set of parameters to optimize
  - Hypothesis built directly by **combining training instances**
  - Can easily be extended with additional data
  - Example
    - K-nearest neighbors
- First of the **Kernel Machines**

# Kernel Perceptron

## Kernel

---

- The model is made of input instances
  - Good adaptability to different problems
  - Often sparse
- The prediction emerges from the **weighted sum of similarities** with instances from the training set
  - The similarity function is called **kernel** and implies a **metric to compare instances**
  - A vector often called **alpha** associates a “relevance score” to each instance of the training set
    - This defines the subset of training example later used to classify new data
- Other examples of kernel machines
  - Support Vector Machines (SVM)
  - Principal Component Analysis (PCA)
  - Gaussian Processes



# Kernel Perceptron

## Algorithm derivation

To derive a kernelized version of the perceptron algorithm, we must first formulate it in **dual form**, starting from the observation that the weight vector  $\mathbf{w}$  can be expressed as a **linear combination** of the  $n$  training samples. The equation for the weight vector is

$$\mathbf{w} = \sum_i^n \alpha_i y_i \mathbf{x}_i$$

where  $\alpha_i$  is the number of times  $\mathbf{x}_i$  was misclassified, forcing an update  $\mathbf{w} \leftarrow \mathbf{w} + y_i \mathbf{x}_i$ . Using this result, we can formulate the dual perceptron algorithm, which loops through the samples as before, making predictions, but instead of storing and updating a weight vector  $\mathbf{w}$ , it updates a "mistake counter" vector  $\alpha$ . We must also rewrite the prediction formula to get rid of  $\mathbf{w}$ :

$$\begin{aligned}\hat{y} &= \text{sgn}(\mathbf{w}^T \mathbf{x}) \\ &= \text{sgn} \left( \sum_i^n \alpha_i y_i \mathbf{x}_i \right)^T \mathbf{x} \\ &= \text{sgn} \sum_i^n \alpha_i y_i (\mathbf{x}_i \cdot \mathbf{x})\end{aligned}$$

# Kernel Perceptron

## Algorithm derivation

---

Plugging these two equations into the training loop turn it into the *dual perceptron* algorithm.

Finally, we can replace the **dot product** in the dual perceptron by an arbitrary kernel function, to get the effect of a feature map  $\Phi$  without computing  $\Phi(\mathbf{x})$  explicitly for any samples. Doing this yields the kernel perceptron algorithm:<sup>[4]</sup>

Initialize  $\alpha$  to an all-zeros vector of length  $n$ , the number of training samples.

For some fixed number of iterations, or until some stopping criterion is met:

For each training example  $\mathbf{x}_j, y_j$ :

$$\text{Let } \hat{y} = \text{sgn} \sum_i^n \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}_j)$$

If  $\hat{y} \neq y_j$ , perform an update by incrementing the mistake counter:

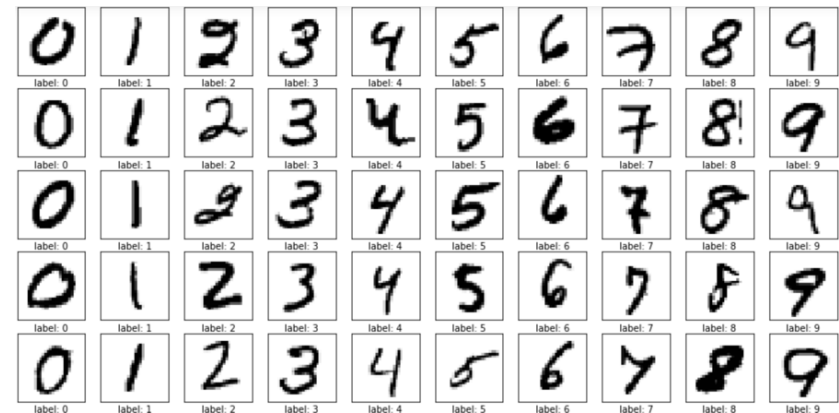
$$\alpha_j \leftarrow \alpha_j + 1$$

# Kernel Perceptron

## MATLAB Code



- MNIST Database
  - Handwritten digits
  - 10 classes
  - 70 000 labeled images
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  - High dimension feature vectors
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- 2-classes
  - Detect “0” from “not 0”
- 10-classes
  - 1 vs all strategy

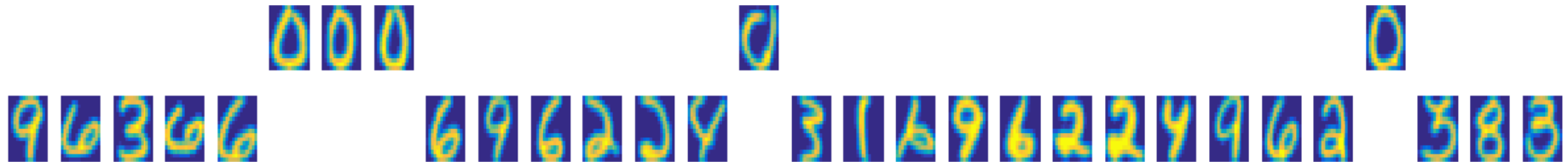


Examples of images in the MNIST database

# Kernel Perceptron

## MATLAB Code

---



98.7% good predictions

# Kernel Perceptron

## MATLAB Code

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93.4% good predictions

# AdaBoost

# AdaBoost

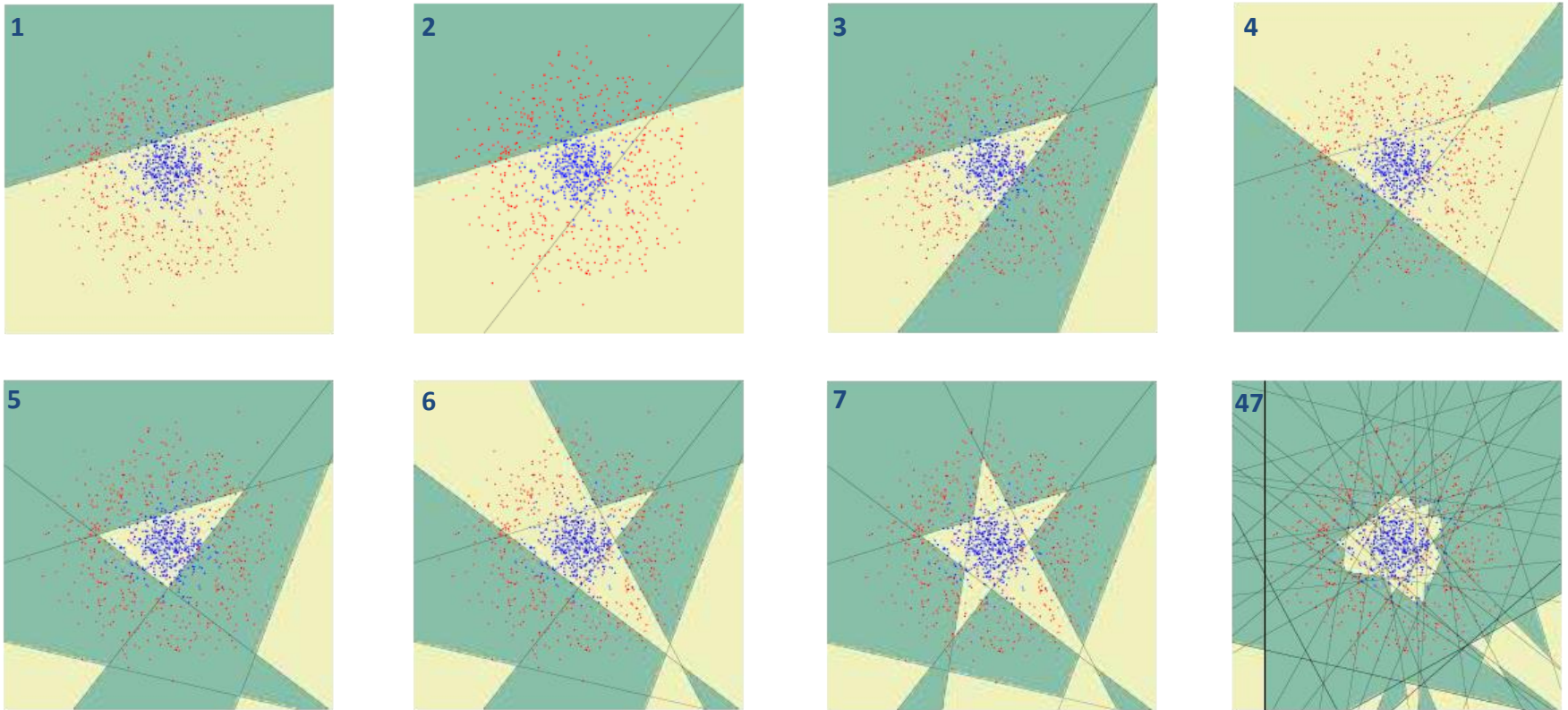
## Overview

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- Meta-Algorithm: 2003 Gödel Prize for Schapire and Freund (1996)
- **Adaptive Boosting**
  - Boosting
    - Combination of multiple **weak classifiers** to form one **strong classifier**
  - Adaptive
    - Favor misclassified instances in subsequent weak learners
    - Focus on “hard to classify” cases
- Many advantages
  - Fairly robust to overfitting
  - Low parameter count
  - High versatility
  - Fast and lightweight

# AdaBoost

## Step by Step





# AdaBoost

## Algorithm

For  $t$  in  $1 \dots T$ :

- Choose  $h_t(x)$ :
  - Find weak learner  $h_t(x)$  that minimizes  $\epsilon_t$ , the weighted sum error for misclassified points  $\epsilon_t = \sum_{\substack{i=1 \\ h_t(x_i) \neq y_i}}^n w_{i,t}$
  - Choose  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$
- Add to ensemble:
  - $F_t(x) = F_{t-1}(x) + \alpha_t h_t(x)$
- Update weights:
  - $w_{i,t+1} = w_{i,t} e^{-y_i \alpha_t h_t(x_i)}$  for all  $i$
  - Renormalize  $w_{i,t+1}$  such that  $\sum_i w_{i,t+1} = 1$

With:

- Samples  $x_1 \dots x_n$
- Desired outputs  $y_1 \dots y_n, y \in \{-1, 1\}$
- Initial weights  $w_{1,1} \dots w_{n,1}$  set to  $\frac{1}{n}$
- Error function  $E(f(x), y, i) = e^{-y_i f(x_i)}$
- Weak learners  $h: x \rightarrow [-1, 1]$

# AdaBoost

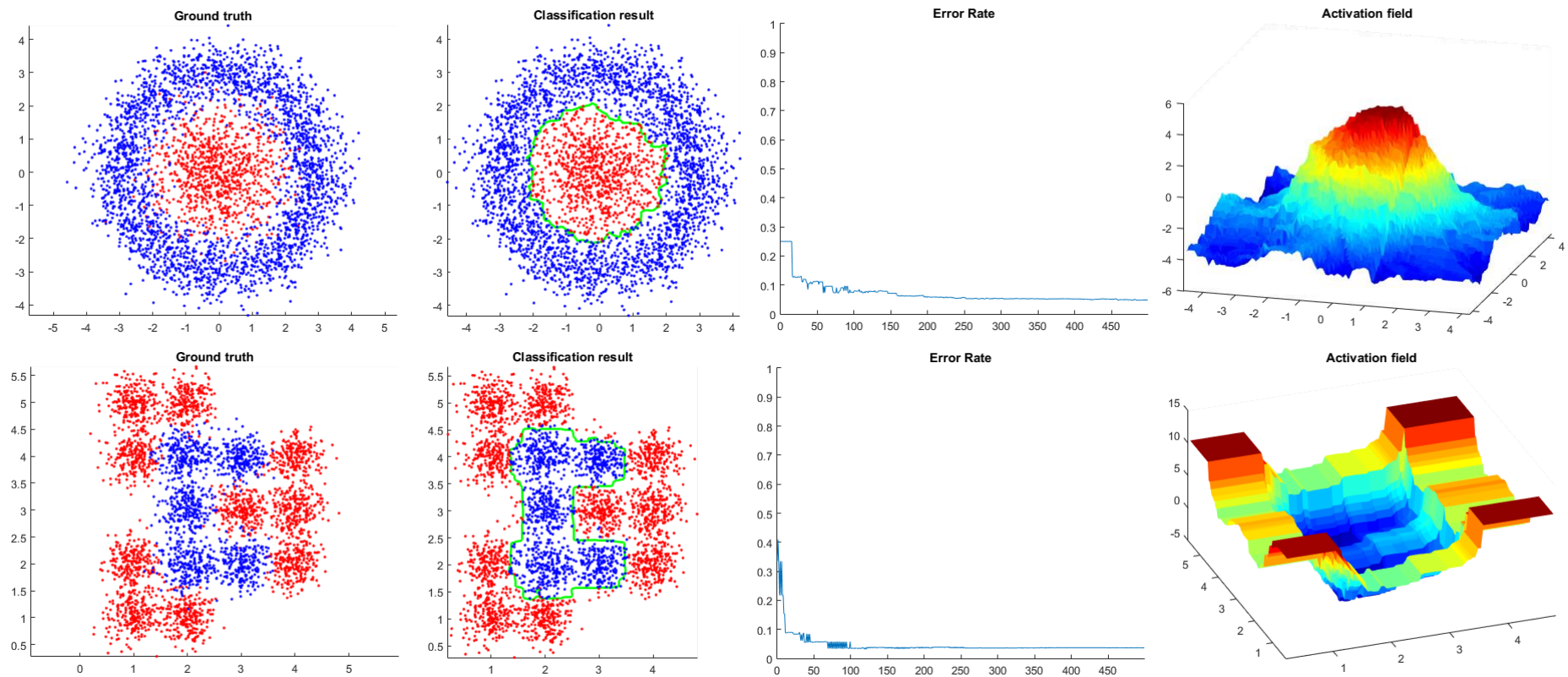
## MATLAB Code

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- Problem generation
  - 2D point distribution
  - Various patterns
- Weak classifier selection
  - Parallel separator
  - Linear perceptron
- Strong classifier construction
  - AdaBoost implementation
  - Weak learners stack + weight vector

# AdaBoost

## MATLAB Code

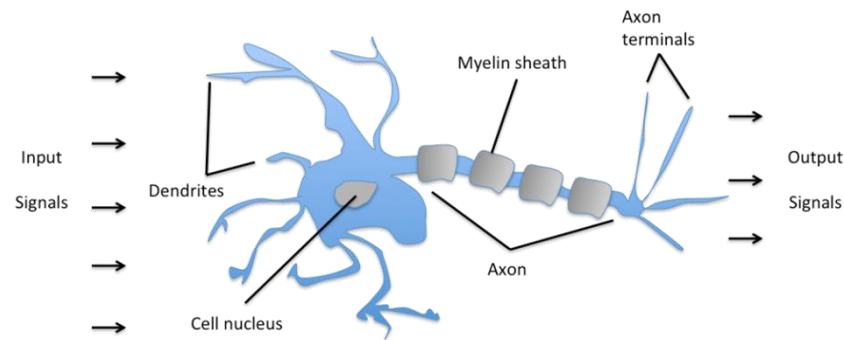


# History of ANNs

# History of ANNs

## Origin

- 1943 – First artificial neuron
  - Neurophysiologist Warren McCulloch
  - Mathematician Walter Pitts
  - Paper on the way neurons might work
  - Modeled by electrical circuit



# History of ANNs

## Origin

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- 1949 – First insight into neuron learning
  - Donald Hebb - The Organization of Behavior
  - Neurons have different signal “strengths”
  - Connections increase with repeated use
  - Neurons which fire together strengthen together



Neuron groups

# History of ANNs

## Origin

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- 1950 – First Artificial Neural Network ?
  - IBM Research Lab
  - Nathaniel Rochester
  - Computer simulation
  - Fail

# History of ANNs

## Origin

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- 1959 – First Artificial Neural Network !
  - Stanford
  - Bernard Widrow and Marcian Hoff
  - MADALINE (**M**ultiple **AD**aptive **LI**near **E**lements)
  - Adaptive filter that reduces echo on phone line



# History of ANNs

## Origin

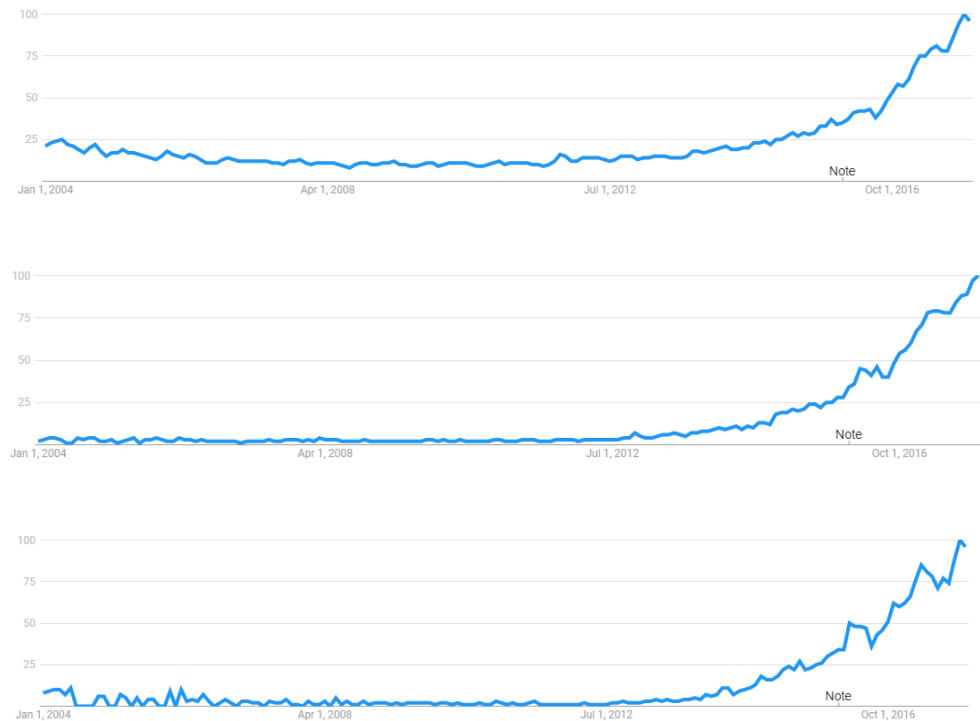
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- 1986 – **Back-Propagation** networks
  - 3 independent research groups
  - Distribution of error throughout the network
  - Multi layer networks
    - Slower than previous implementations
    - Better generalization capabilities

# History of ANNs

## Popularization

- “Machine Learning”
- “Convolutional neural network”
- “Deep Learning”



# History of ANNs

## Popularization

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### ○ Factors

- Availability of Data
  - Active and passive data gathering
- Computing Power
  - CPU, GPU, TPU
  - Parallel Computation libraries
- Wide applicability

# History of ANNs

## Popularization

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- Actors
  - Research groups
    - Universities
    - Research Institutions
  - Large Companies
    - Software
    - Hardware
  - Start-Ups

# Deep Learning : Definition

# Deep Learning : Definition

## Definition

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- Data
  - Supervised or Unsupervised
- Model
  - Cascade layers of nonlinear processing units for feature extraction
  - Multiple levels of representation and abstractions
  - Hierarchy of concepts
- Optimization
  - Backpropagation
  - Gradient descent

# Deep Learning : Definition

## Applications overview

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- Automatic speech recognition
- Image recognition
- Visual art processing
- Natural language processing
- Drug discovery and toxicology
- Customer relationship management
- Recommendation systems
- Bioinformatics
- Mobile advertising
- Image restoration

# Artificial Neural Network



# Artificial Neural Network

## Overview

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- Many variants of the same basic concept
  - Multilayer perceptron
  - Convolutional neural network
  - Memory networks
  - Cycle networks
  - Adversarial networks
  - Auto-encoders
  - ...
- Simple architecture
  - Allows for efficient parallelization
- Implemented in libraries for most programming languages
- High abstraction power
  - Universal function approximator

# Artificial Neural Network

## Criticism

- Lack of theoretical understanding
  - Capacity
  - Convergence
  - Architecture
  - Overfitting
  - Training
- Un-rigorous scientific community
  - Trend effect
  - Anthropomorphism
  - Easy publications
- Hardware
  - Specific needs
  - Power consumption

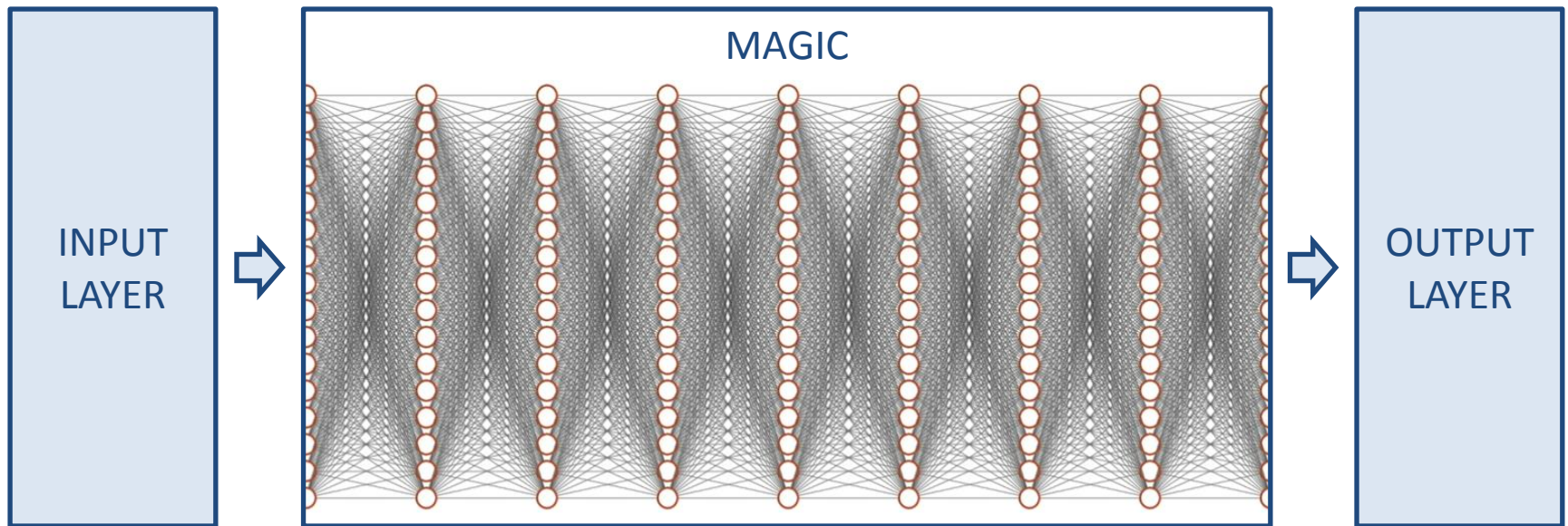


# Artificial Neural Network

## Architecture



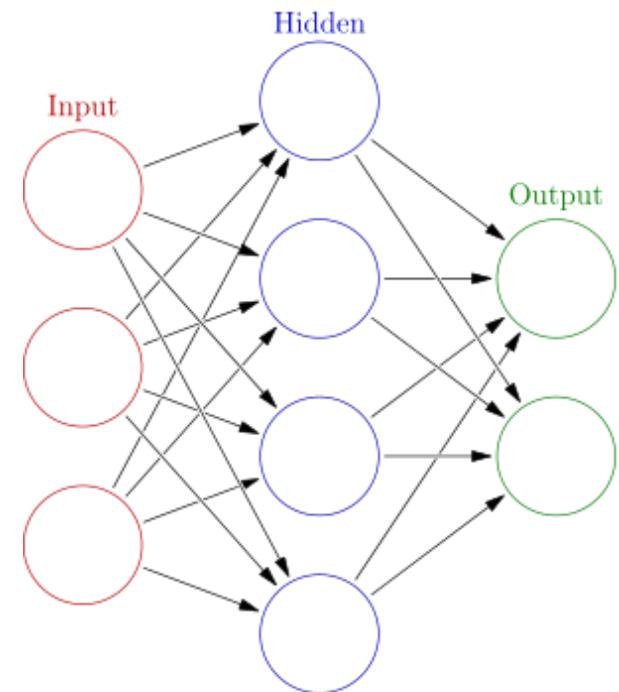
Recap architecture  
Perceptron & Adaboost



# Artificial Neural Network

## Notation

- Value of neuron  $k$  in layer  $L$  :  $\mathbf{a}_k^{(L)}$
- Value of bias for neuron  $k$  in layer  $L$  :  $\mathbf{b}_k^{(L)}$
- Value of weight going from neuron  $k$  in layer  $(L-1)$  to neuron  $j$  in layer  $L$  :  $\mathbf{w}_{j,k}^{(L)}$
- Weighted sum with bias for neuron  $k$  :  $\mathbf{z}_k^{(L)}$
- Activation function (sigmoid, logistic, tanh, ReLU, ...) :  $\sigma$
- Expected output value for neuron  $n$  :  $\mathbf{y}_k$
- Cost function :  $\mathbf{C}_0$



# Artificial Neural Network

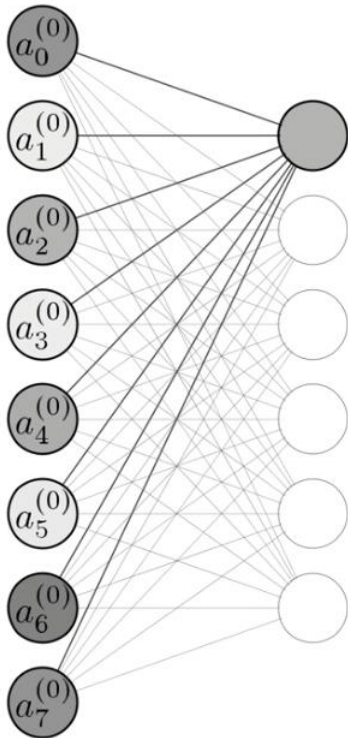
## Feed-Forward

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- The **Feed-Forward** algorithm is the process of computing the values of the **output layer** given :
  - The values of the **input layer**
    - Represented as a single vector
  - The values of the **weights**
    - Represented as a matrix between each layer
- Each neuron of the network simply contain a scalar value
  - The value of a neuron depends solely on the values of the previous layer and the weight in between
  - By feeding the values of each layer into the next one, we propagate the information forward though the network

# Artificial Neural Network

## Feed-Forward



Sigmoid

$$a_0^{(1)} = \sigma \left( w_{0,0} a_0^{(0)} + w_{0,1} a_1^{(0)} + \dots + w_{0,n} a_n^{(0)} + \underset{\substack{\uparrow \\ \text{Bias}}}{b_0} \right)$$

$$\sigma \left( \begin{bmatrix} w_{0,0} & w_{0,1} & \dots & w_{0,n} \\ w_{1,0} & w_{1,1} & \dots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{k,0} & w_{k,1} & \dots & w_{k,n} \end{bmatrix} \begin{bmatrix} a_0^{(0)} \\ a_1^{(0)} \\ \vdots \\ a_n^{(0)} \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix} \right) \quad \sigma(\mathbf{W}\mathbf{a}^{(0)} + \mathbf{b})$$

# Artificial Neural Network

## Feed-Forward

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- [playground.tensorflow.org](https://playground.tensorflow.org)
  - Solve the concentric distribution problem
  - Solve the checker pattern problem
  - Don't solve the spiral problem