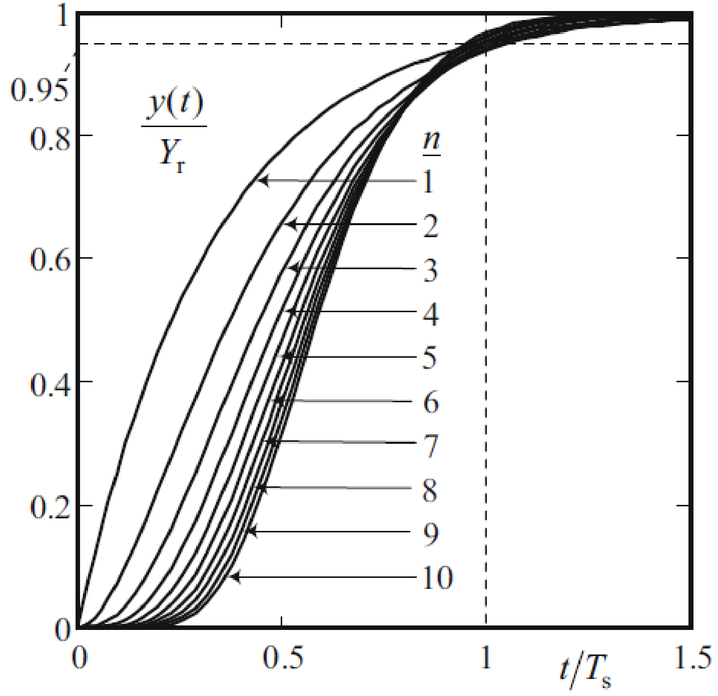
Classic control #4



Jacob Pedersen

Lecture

- Homework 10
- Lab this afternoon
- Integrator anti-windup
- Model based controller design
  - Pole placement
    - Ex IP controller
  - Settling time
  - Design the desired response using the settling time formula
  - Ex IP controller continued
    - Pole placement design with coincident (co-incident) poles
    - Pole placement design using the classic second order system definition (undamped natural frequency and damping ratio)
- Task

Reading

Reading for this lecture:

- Feedback Control (Dodds, S.J.)
  - Traditional controllers: Model based design
    - 4.1, 4.2, 4.3, 4.4 (I will only go through some of this section, if there is time), 4.5.3, 4.5.4, 4.5.5, 4.5.6, 4.6 (Browse through this at some point, some of it could maybe be useful for the report)

If we get time later, I will include some material from section 4.6

Homework 10

Full order system (the DC motors inductance is included)

1. Create a full order state space model of the DC motor system with viscous friction and a load input (only one DC motor without the extra inertia).
2. Generate a system transfer function of the full order state space model (Load torque TL= 0…basically removed)
   Input: Va
   Output: Omega (speed)
3. Compare all three: The Transfer function, Simscape and state space model against each other in a side-by-side Simulink simulation
   -Do they have the same response if the load input TL=0 and the input voltage is the same?

Read about the Masons rule in Feedback Control (Stephen dodds) A4.1.2. If viscous friction is included there will be two loop no-touching loops.
Remember all the loops are touching the forward path.
Remove the Coulomb friction in your Simscape model.

Reduced order state space model (the DC motors inductance is NOT included)

4. Create a reduced order state space model of the DC motor system with viscous friction and a load input (only one DC motor without the extra inertia).
5. Generate a system transfer function of the full order state space model (Load torque TL= 0…basically removed)
   Input: Va
   Output: Omega (speed)
6. Compare the transfer function and state space model against the full order Simscape model in a side-by-side Simulink simulation.

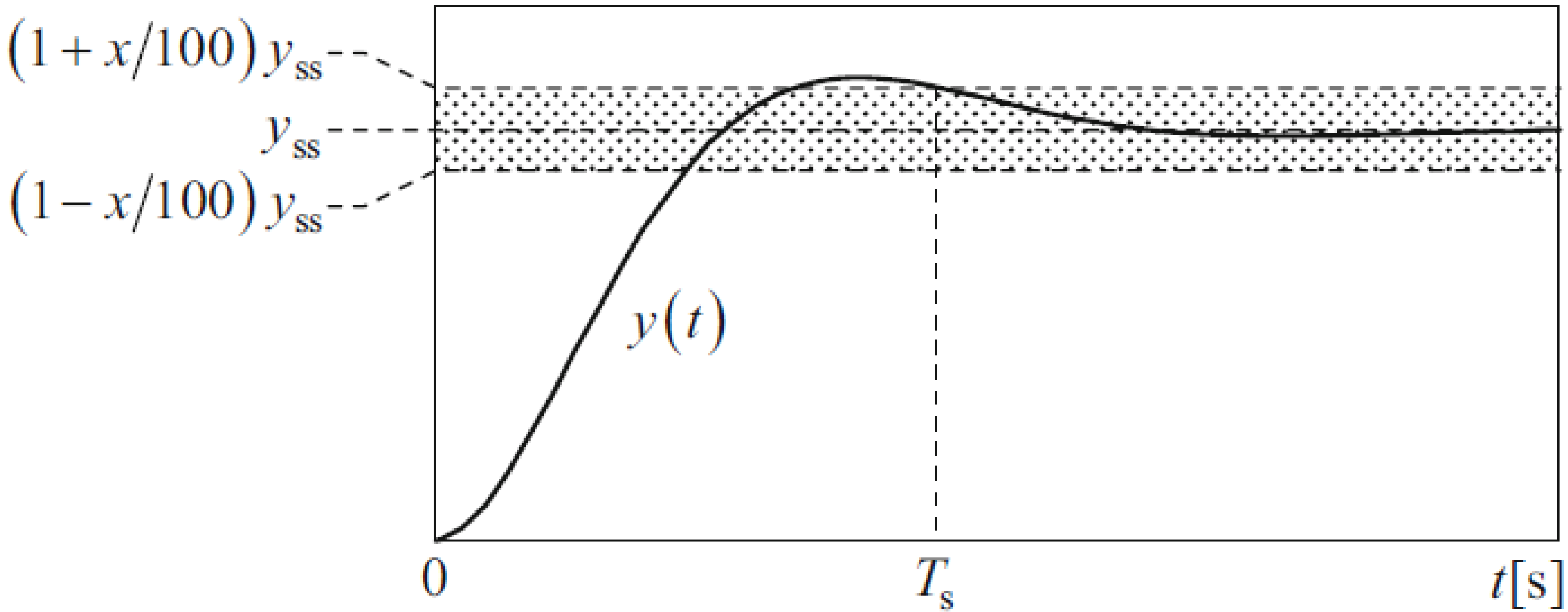
In the reduced order system there will only be one integrator. Input current is equal to ia=(va-eb)/Ra where eb is the back emf.

Remember the lab this afternoon

Model based design

Model based design (#1, #2)

Settling time

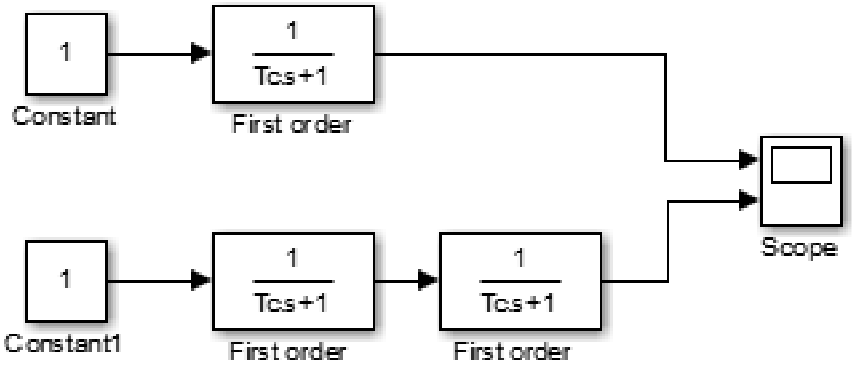


Definition 4.1 *The settling time, T_s, according to the x % criterion is the time for the step response, y(t), to reach and stay within a band having limits of* $y_{ss}\,(1 \pm x/100)$.

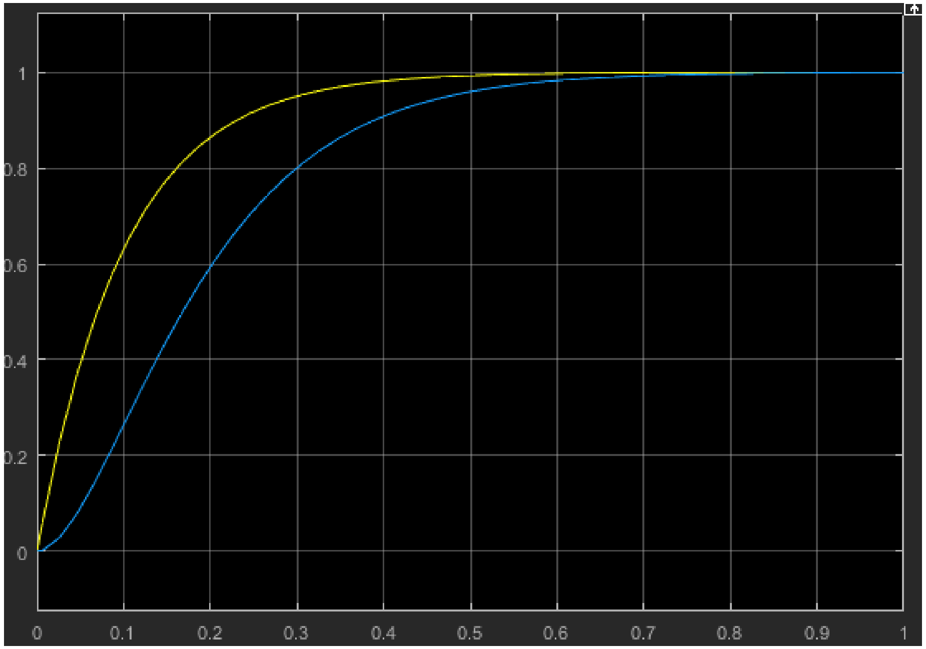
Settling time formula

Step responses with coincident closed loop poles

$$\frac{Y(s)}{Y_r(s)} = \left(\frac{1}{1+sT_c}\right)^n, \quad n = 1,\ 2,\ 3\ldots$$



Tc = 0.1 [s]



The 5 % formula #1

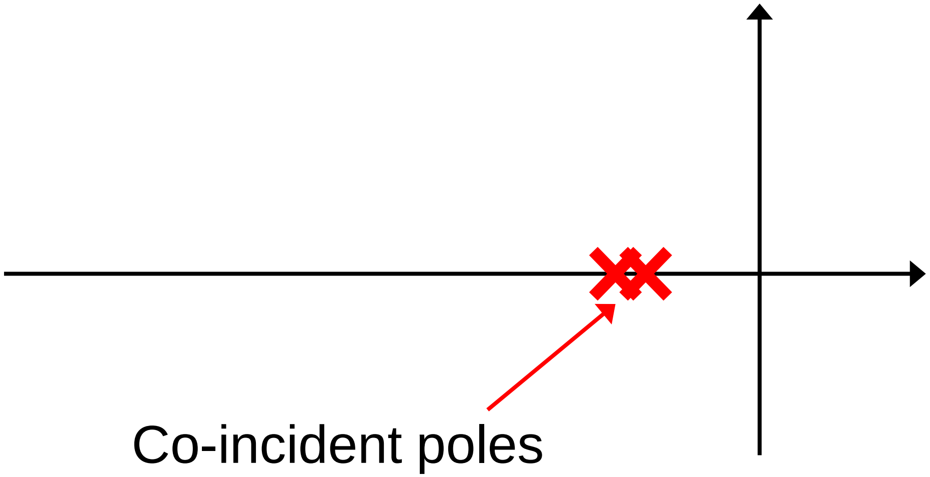
Designing a response for a n-order system

$$\left(s + \frac{1}{T_c}\right)^n = \left[s + \frac{1.5\,(1+n)}{T_s}\right]^n$$

where $T_s = 1.5\,(1+n)\,T_c$

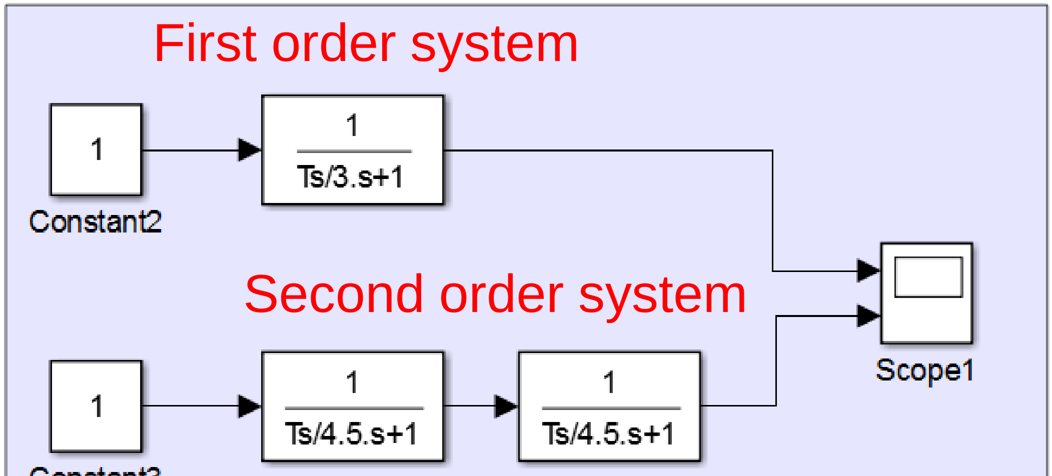
$$\frac{Y(s)}{Y_r(s)} = \left(\frac{1}{T_c s + 1}\right)^n, where\, T_c = T_s / 1.5(1+n)$$

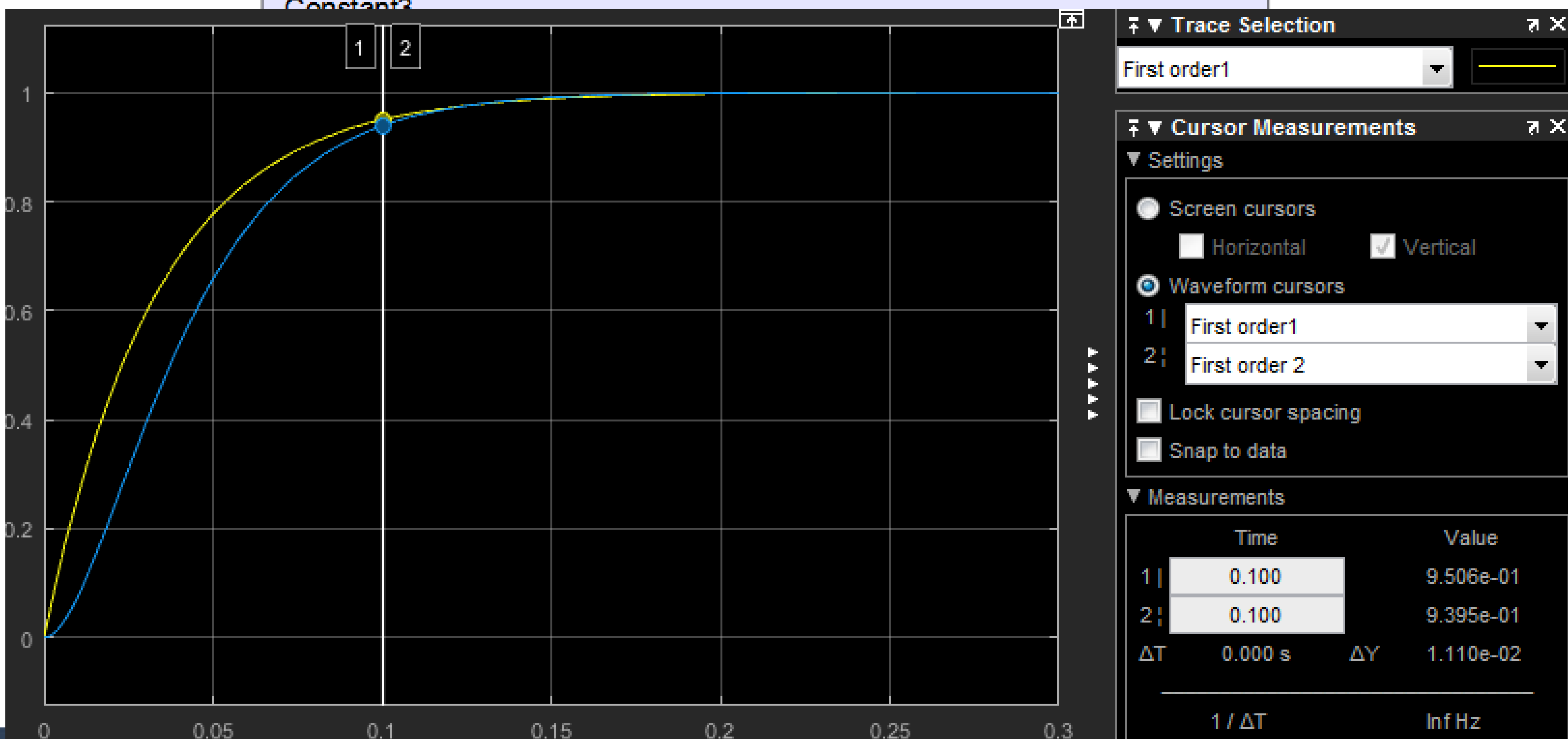
The 5 % formula #2



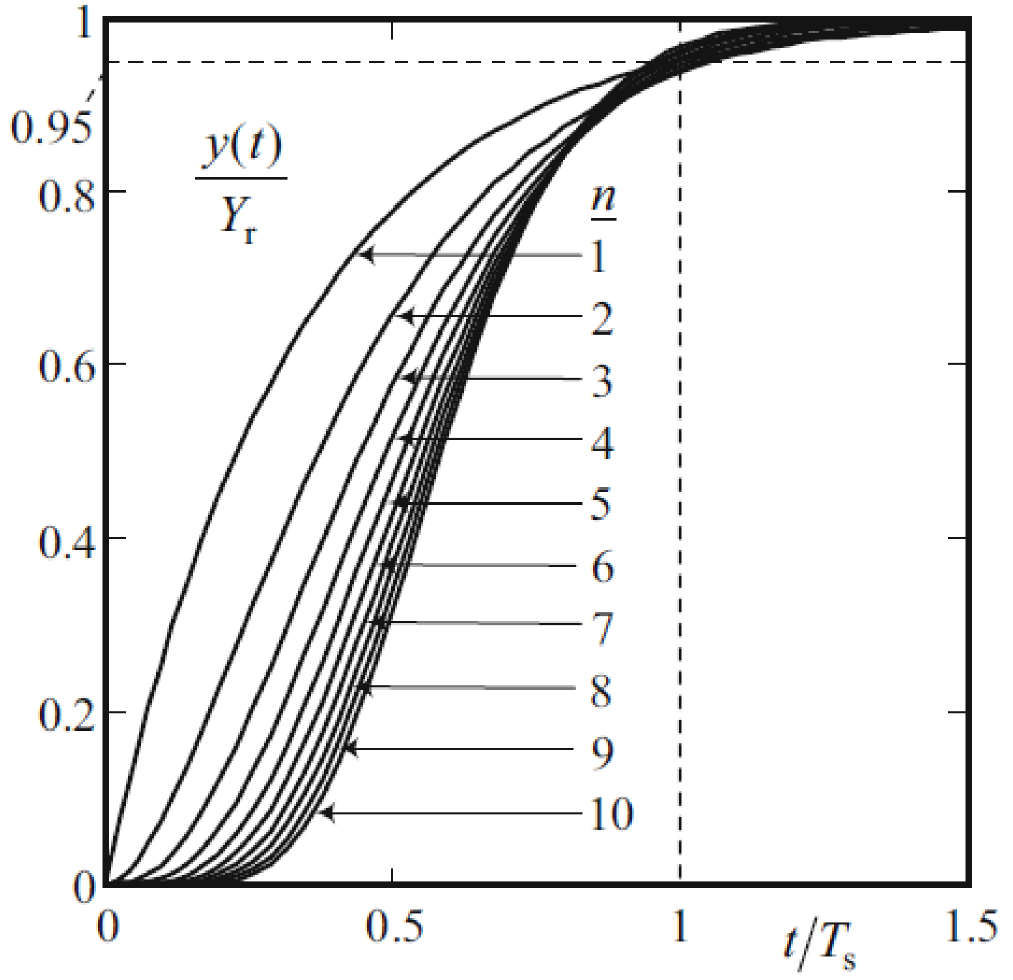
$$\left[s + \frac{1.5\,(1+n)}{T_s} \right]^n$$

The 5 % formula #3





The 5 % formula #4



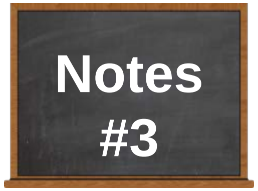
Design the desired response using the settling time formula #1

Designing a response for a n-order system

$$s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$$

r	1	2	3	4	5	6	7	8
d_0	α	α^2	α^3	α^4	α^5	α^6	α^7	α^8
d_1	–	2α	$3\alpha^2$	$4\alpha^3$	$5\alpha^4$	$6\alpha^5$	$7\alpha^6$	$8\alpha^7$
d_2	–	–	3α	$6\alpha^2$	$10\alpha^3$	$15\alpha^4$	$21\alpha^5$	$28\alpha^6$
d_3	–	–	–	4α	$10\alpha^2$	$20\alpha^3$	$35\alpha^4$	$56\alpha^5$
d_4	–	–	–	–	5α	$15\alpha^2$	$35\alpha^3$	$70\alpha^4$
d_5	–	–	–	–	–	6α	$21\alpha^2$	$56\alpha^3$
d_6	–	–	–	–	–	–	7α	$28\alpha^2$
d_7	–	–	–	–	–	–	–	8α

Table 4, p.850

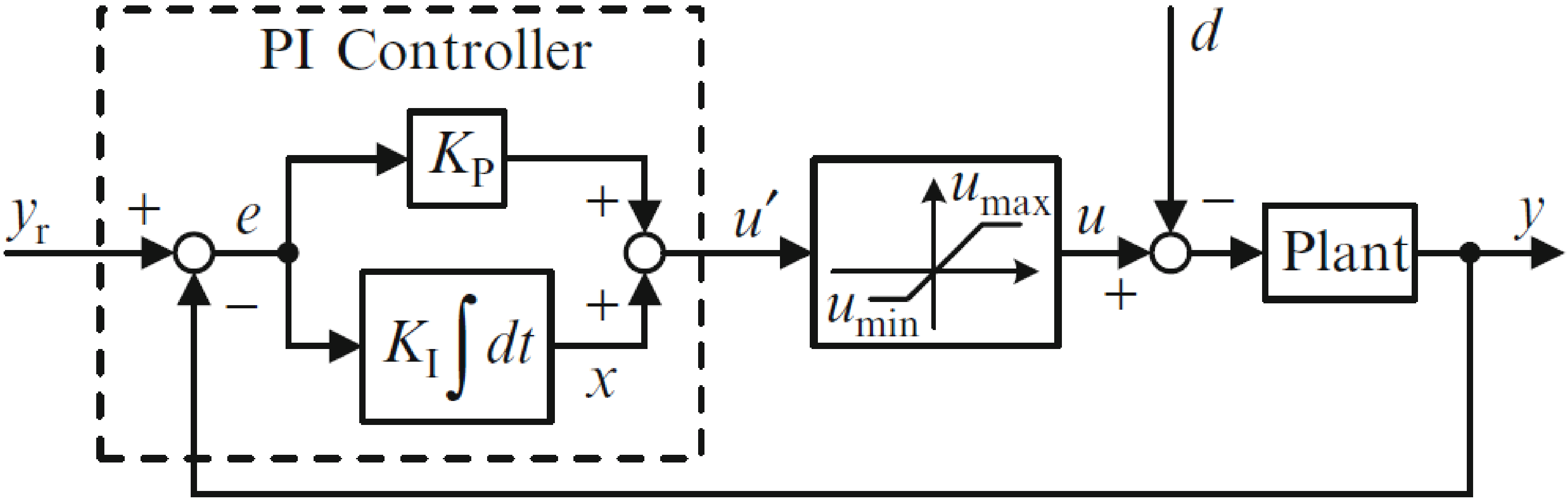


$$\alpha = 1.5\,(1+n)\,/\,T_{s5\%} \quad \text{or} \quad \alpha = 1.6\,(1.5+n)\,/\,T_{s2\%}$$

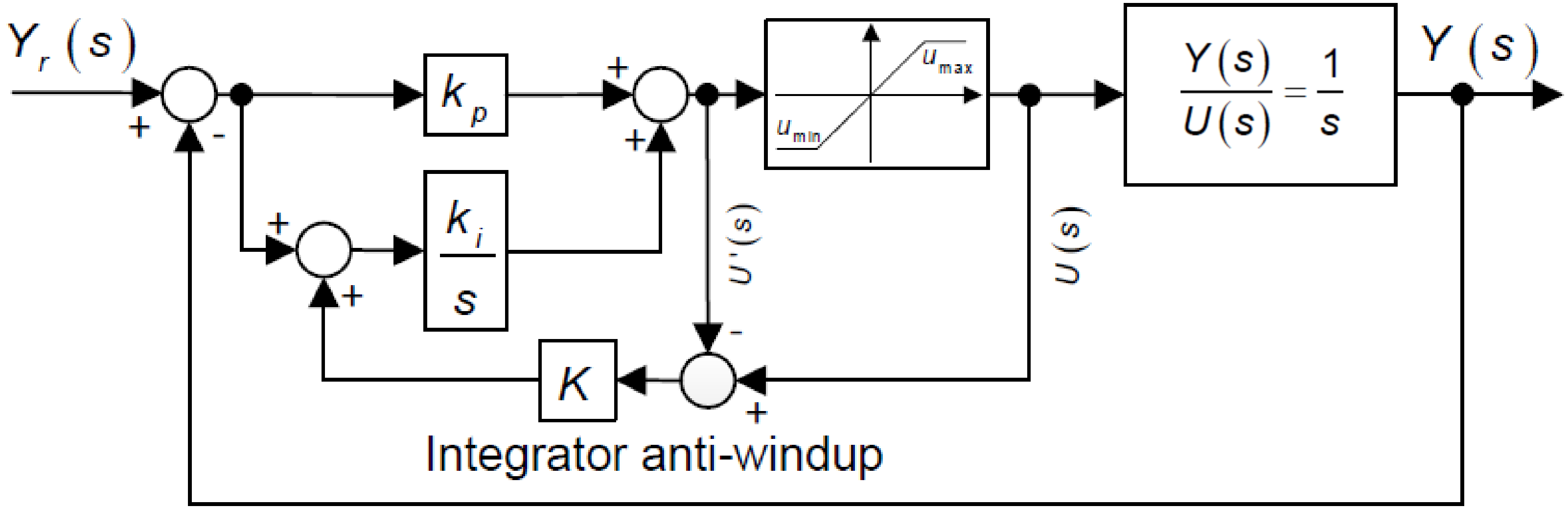
Model based design (#4, #5, #6)

Integrator anti-windup

Limitations in the control loop

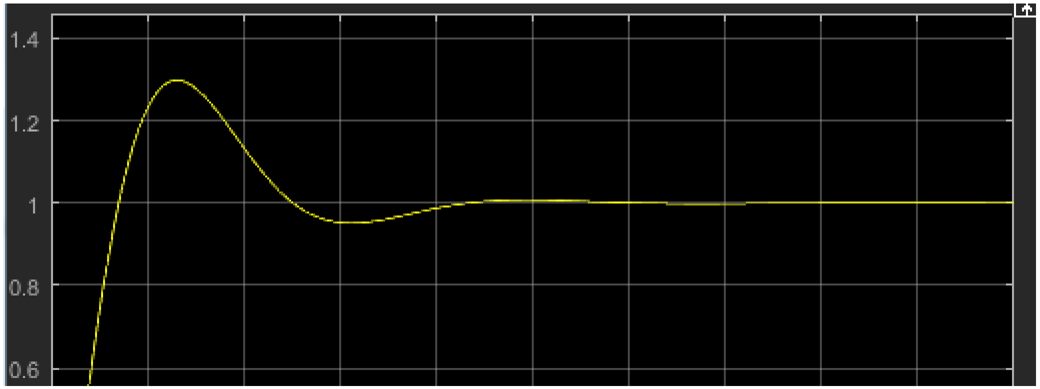


Method: Feedback loop #1

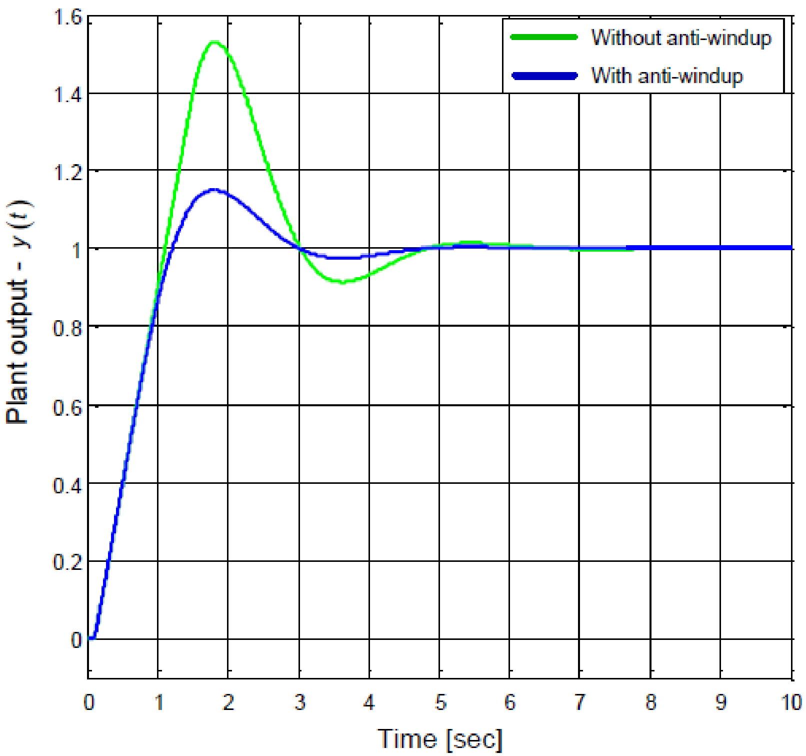


$$\lim_{K \to \infty} U'(s) = U(s)$$

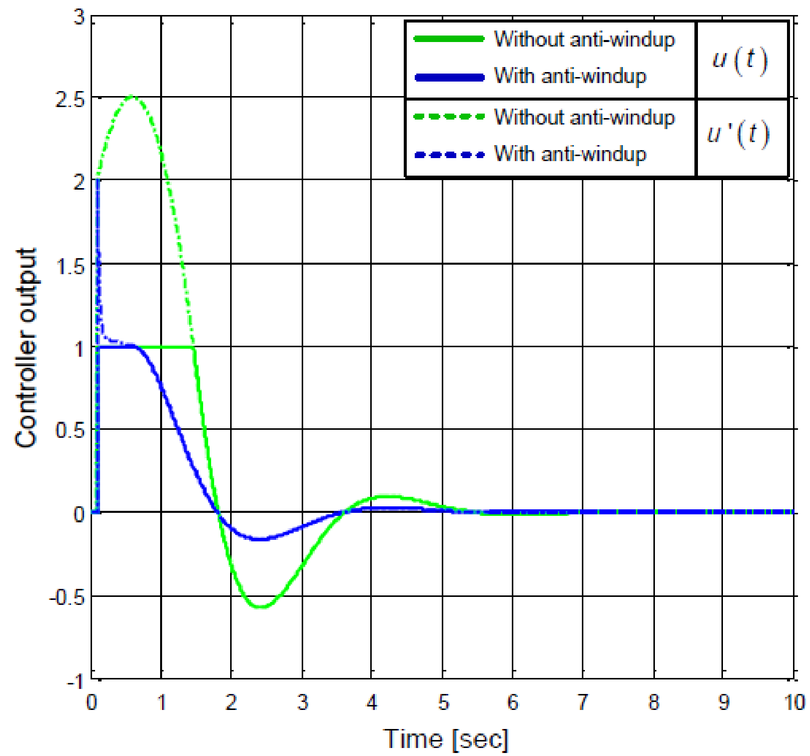
Method: Feedback loop #2



Without saturation



a) Plant output $y(t)$



b) Controller output $u(t)$

Task

- Build a PI controller with saturation (limits +/- 1) and integrator anti-windup + plant { $Y(s) = 1/s * U(s)$ }
  - Plot y(t) with and without saturation (anti-windup is disabled)
    - Tune the system to it hits the saturation limit
  - Plot y(t) with saturation and anti-windup is enabled
    - Tune k

Method: Integrator initialisation (a more practical approach)

Task

- Design a PI or IP controller for your reduced order DC motor system
  - Used the model of the closed loop system (the transfer function)
  - Design a desired closed loop system response using the Settling time formula (second order system)
  - Set the characteristic equations from the closed loop system transfer function equal to the desired closed loop system response (polynomial)
    - Isolate the controller gains
  - Validate the controller
    - Do you get the expected closed loop speed response?

Homework

- Design a IPD controller for your full order DC motor system
  - What order controller do you need for this?
    - To do a full pole placement (one control gain for each pole)
  - Used the model of the closed loop system (the transfer function)
  - Design a desired closed loop system response using the Settling time formula (third order system ??)
  - Set the characteristic equations from the closed loop system transfer function equal to the desired closed loop system response (polynomial)
    - Isolate the controller gains
  - Validate the controller
    - Do you get the expected closed loop speed response?

The End