Speed control of a PMSM motor based on the new disturbance observer

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Abstract—**This paper is concerned with speed control of a permanent magnet synchronous motor (PMSM) based on the new disturbance observer. The unknown disturbance is compensated so that faster speed response and smaller speed ripple can be achieved in the presence of a time-varying load. The proposed disturbance observer has a finite memory structure and thus is robust to model uncertainties. Also, due to the simple structure of the observer, it can be easily implemented.**

I. INTRODUCTION

The disturbance observer is widely used in motor control systems [1] [2] [3] [4] [5]. The disturbance observer estimates unknown disturbance (i.e., load torque in motor control systems) and compensate the unknown load torque based on its estimation. Using the disturbance observer, we can obtain faster speed responses and smaller ripple in the presence of a time-varying load.

Most disturbance observers are model-based. Since the system model contains some uncertainties [6], the disturbance estimation should be robust to the model uncertainties. Otherwise the disturbance estimate may diverge and the system can be unstable. However, this robustness issue has not been considered much in disturbance observers. In this paper, a finite memory disturbance observer is proposed. The finite memory estimation is often used in constructing robust estimator [7] [8]. The idea used to derive the proposed finite memory disturbance observer is based on the results in [9], where a deadbeat control of continuous-time systems is considered.

From the viewpoint of implementation, the disturbance observer should be simple. This is particularly true for motor control systems, where computational power is often limited. The proposed disturbance observer has very simple structure.

The organization of the paper is as follows. In Section II, a new disturbance observer is proposed and its properties are investigated. In Section III, how to choose design parameters in the proposed disturbance observer is discussed. In Section IV, the disturbance observer is applied to a permanent magnet synchronous motor (PMSM) control system. Conclusion is given in Section V.

II. DISTURBANCE OBSERVER

Consider the following system

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + b(u(t)+d(t)) + w(t) \\ y(t) & = & cx(t) + v(t) \end{array} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, u is the control input, d is unknown disturbance to be estimated, and y is the output. Process noise w and measurement noise v are zero-mean white gaussian noises, which satisfy

$$\begin{array}{rcl} \mathrm{E}\{w(t)w(s)'\} & = & Q\delta(t-s), \\ \mathrm{E}\{v(t)v(s)'\} & = & R\delta(t-s), \\ \mathrm{E}\{v(t)w(s)'\} & = & 0. \end{array} \tag{2}$$

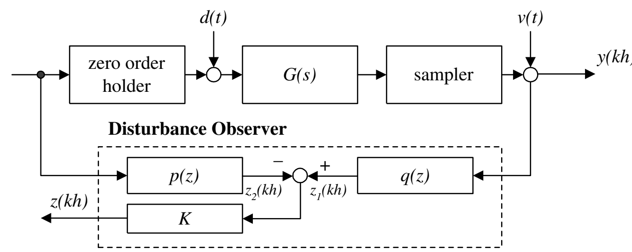

Fig. 1. Structure of the disturbance observer

The objective of this paper is to construct a disturbance observer, which estimates unknown disturbance $d(t)$ and compensates its effect. The system (1) is assumed to be controlled by a digital controller with a zero order holder (with the hold period h) and a sampler (with the sampling period h). The structure of the disturbance observer is given in Fig. 1, where $p(z)$ and $q(z)$ are defined by

$$\begin{array}{rcl} p(z) & \triangleq & p_1 z^{-1} + p_2 z^{-2} + \cdots + p_N z^{-N} \\ q(z) & \triangleq & q_0 + q_1 z^{-1} + q_2 z^{-2} + + \cdots + q_N z^{-N}, \end{array} \tag{3}$$

respectively. From Fig. 1, $z(kh)$, $z_1(kh)$, and $z_2(kh)$ are given by

$$\begin{array}{rcl} z(kh) & = & K(z_1(kh) - z_2(kh)) \\ z_1(kh) & = & \displaystyle\sum_{i=0}^{N} q_i y((k-i)h) \\ z_2(kh) & = & \displaystyle\sum_{i=1}^{N} p_i u((k-i)h). \end{array} \tag{4}$$

Design parameters p_i, q_i, and K are chosen so that $z(kh)$ becomes the estimate of $d((k-1)h)$. Due to the sampler, there is one step delay in the disturbance observer: thus $z(kh) = \hat{d}((k-1)h)$ instead of $z(kh) = \hat{d}(kh)$. Note the simple structure of the disturbance observer: $z(kh)$ is just a linear combination of delayed output $y((k-i)h)$ and delayed input $u((k-i)h)$.

Basic principles of the proposed disturbance observer will be given in Theorem 1 and 2. Let W be the set of eigenvalues of A:

$$W = \{s_i \in \mathbb{C}, i = 1, \cdots, n \mid \det(sI - A) = 0\}$$

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where it is assumed that all eigenvalues of A are distinct. Coefficients q_i are chosen to satisfy the following equation:

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & e^{-s_1 h} & \cdots & e^{-s_1 N h} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & e^{-s_n h} & \cdots & e^{-s_n N h} \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ \vdots \\ q_N \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5}$$

where $N \geq n$ is a design parameter. The matrix in (5) is a Vandermonde matrix [10] and when $N = n$, the matrix is nonsingular. Hence there always exist q_i satisfying (5).

Lemma 1: If coefficients q_i satisfy (5), the following is satisfied.

$$\sum_{i=0}^{N} q_i e^{A(N-i)h} x((k-N)h) = 0. \tag{6}$$

Proof: Let v_j be an eigenvector of A corresponding to an eigenvalue s_i. Since s_i are distinct, $\dim \mathrm{span}\{v_1, \cdots, v_n\} = n$; hence for any $x((k-N)h)$, there exist α_j, $j = 1, \cdots, n$ such that

$$x((k-N)h) = \sum_{j=1}^{n} \alpha_j v_j. \tag{7}$$

Using this relationship, we obtain

$$\begin{aligned}
&\sum_{i=0}^{N} q_i e^{A(N-i)h} x((k-N)h) \\
&= \sum_{i=0}^{N} q_i e^{A(N-i)h} \sum_{j=1}^{n} \alpha_j v_j \\
&= \sum_{j=1}^{n} \alpha_j \left(\sum_{i=0}^{N} q_i e^{A(N-i)h} v_j \right) \\
&= \sum_{j=1}^{n} \alpha_j \left(\sum_{i=0}^{N} q_i e^{ANh} e^{-Aih} v_j \right) \\
&= \sum_{j=1}^{n} \alpha_j \left(\sum_{i=0}^{N} q_i e^{ANh} e^{-s_i ih} v_j \right) \\
&= \sum_{j=1}^{n} \alpha_j e^{ANh} \left(\sum_{i=0}^{N} q_i e^{-s_i ih} \right) v_j = 0.
\end{aligned}$$

The last equality is from (5), which implies

$$\sum_{i=0}^{N} q_i e^{-s_i ih} = 0.$$

■

Coefficients p_i are chosen to satisfy the following equation:

$$p_i = c \sum_{j=0}^{i-1} q_i e^{A(i-j-1)h} \int_0^h e^{Ar} b \, dr. \tag{8}$$

Theorem 1: If q_i and p_i satisfy (5) and (8) respectively, then for $k > N$,

$$\begin{aligned}
z(kh) = K \Bigg\{ &\sum_{i=0}^{N} q_i v((k-i)h) \\
&+ \sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} \{w(r) + bd(r)\} \, dr \Bigg\}.
\end{aligned} \tag{9}$$

Proof: From (1), we obtain

$$\begin{aligned}
x(kh) &= e^{ANh} x((k-N)h) + \int_{(k-N)h}^{kh} e^{A(kh-r)} bu(r) \, dr \\
&+ \int_{(k-N)h}^{kh} e^{A(kh-r)} \left(bd(r) + w(r) \right) \, dr.
\end{aligned}$$

Noting $u(kh + r) = u(kh)$, $0 \leq r < h$, we have

$$\begin{aligned}
&\int_{(k-N)h}^{kh} e^{A(kh-r)} bu(r) \, dr \\
&= \int_0^{Nh} e^{A(Nh-r)} bu((k-N)h + r) \, dr \\
&= \sum_{j=1}^{N} \int_{(j-1)h}^{jh} e^{A(Nh-r)} b \, dr \, u((k-N+j-1)h) \\
&= \sum_{j=1}^{N} \int_0^h e^{A(Nh-r-(j-1)h)} b \, dr \, u((k-N+j-1)h) \\
&= \sum_{j=1}^{N} e^{A(N-j)h} \int_0^h e^{A(h-r)} b \, dr \, u((k-N+j-1)h) \\
&= \sum_{j=1}^{N} e^{A(N-j)h} \int_0^h e^{Ar} b \, dr \, u((k-N+j-1)h).
\end{aligned}$$

Thus we have

$$\begin{aligned}
&y((k-i)h) \\
&= cx((k-i)h) + v((k-i)h) \\
&= ce^{A(N-i)h} x((k-N)h) \\
&+ c \sum_{j=1}^{N} e^{A(N-j)h} \int_0^h e^{Ar} b \, dr \, u((k-N+j-1)h) \\
&+ \int_{(k-N)h}^{kh} ce^{A(kh-r)} \left(bd(r) + w(r) \right) \, dr + v((k-i)h) \\
&= y_1 + y_2 + y_3 + y_4
\end{aligned}$$

where $y_1 \sim y_4$ denote four terms of the last equation. From the definition of $z_1(kh)$, we have

$$\begin{aligned}
z_1(kh) &= \sum_{i=0}^{N} q_i y((k-i)h) \\
&= \sum_{i=0}^{N} q_i y_1((k-i)h) + \sum_{i=0}^{N} q_i y_2((k-i)h) \\
&+ \sum_{i=0}^{N} q_i y_3((k-i)h) + \sum_{i=0}^{N} q_i y_4((k-i)h).
\end{aligned} \tag{10}$$

From Lemma 1, we have

$$\sum_{i=0}^{N} q_i y_1((k-i)h) = 0. \tag{11}$$

The second term in (10) is given by

$$\begin{aligned}
&\sum_{i=0}^{N} q_i y_2((k-i)h) \\
&= \sum_{i=0}^{N} q_i c \sum_{j=1}^{N} e^{A(N-j)h} \int_0^h e^{Ar} b\, dr\, u((k-N+j-1)h) \\
&= c \sum_{j=1}^{N} \left(\sum_{i=0}^{N-j} q_i e^{A(N-i-j)h} \int_0^h e^{Ar} b\, dr \right) \\
&\quad u((k-N+j-1)h).
\end{aligned}$$

Renaming the summation variable j as i and i as j, we obtain

$$\begin{aligned}
&\sum_{i=0}^{N} q_i y_2((k-i)h) \\
&= c \sum_{i=1}^{N} \left(\sum_{j=0}^{N-i} q_i e^{A(N-i-j)h} \int_0^h e^{Ar} b\, dr \right) \\
&\quad u((k-N+i-1)h) \\
&= \sum_{i=1}^{N} \left(c \sum_{j=0}^{i-1} q_i e^{A(i-j-1)h} \int_0^h e^{Ar} b\, dr \right) u((k-i)h) \\
&= \sum_{i=1}^{N} p_i u((k-i)h) = z_2(kh).
\end{aligned}$$

The third term in (10) is given by

$$\sum_{i=0}^{N} q_i c \int_{(k-N)h}^{kh} c e^{A(kh-r)} \left(bd(r) + w(r) \right)\, dr.$$

Summarizing the above results, we obtain (9). ■

Note that there is no $x((k-i)h)$ term in (9); that is, $z(kh)$ removes effect of state initial conditions and $z(kh)$ consists of only disturbance d and noises v and w.

It is assumed that the disturbance d varies slowly with respect to Nh; then $d((k-N)h+r) \approx d((k-1)h)$, $0 \le r \le Nh$. With this assumption, we have

$$\begin{aligned}
&\sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} d(r)\, dr \\
&\approx \textstyle\sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} dr\, d((k-i)h).
\end{aligned} \tag{12}$$

The slowly-varying disturbance assumption is satisfied if the disturbance is load change in motor control systems, where typical sampling frequency is $10 \sim 100$ KHz and load change frequency is usually less than 1 KHz.

Let K be defined by

$$K \triangleq 1/\left\{ \sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} b\, dr \right\}, \tag{13}$$

then from (9) and (12), we have

$$z(kh) \approx d((k-1)h) + z_3(kh) \tag{14}$$

where

$$\begin{aligned}
z_3(kh) &\triangleq \sum_{i=0}^{N} q_i v((k-i)h) \\
&\quad + \sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} w(r) dr.
\end{aligned} \tag{15}$$

If noise term z_3 does not exist, $z(kh)$ becomes an estimate of $d((k-1)h)$. In practice, however, the noise term z_3 is inevitable. In that case, it is desirable to reduce effect of z_3. Since v and w are zero mean, from (15) we have $\mathrm{E}\{z_3(kh)\} = 0$ regardless of choice of q_i. However, the variance of $z_3(kh)$ depends on the choice of q_i, which is explicitly shown in the next theorem.

Theorem 2: Variance of $z_3(kh)$ is given by

$$\begin{aligned}
&\mathrm{E}\{z_3(kh)' z_3(kh)\} \\
&= q' \left(H + \begin{bmatrix} \mathrm{Tr}\, R & & \\ & \ddots & \\ & & \mathrm{Tr}\, R \end{bmatrix} \right) q
\end{aligned} \tag{16}$$

where $q \triangleq [\ q_0, \cdots, q_N\ ]'$ and the (i,j)-th element of H is given by

$$\begin{aligned}
H_{ij} \triangleq\ & \int_0^{Nh-\max(ih,jh)} \mathrm{Tr}\left(C e^{A((N-i)h-r)} Q \right. \\
& \left. e^{A'((N-j)h-r)} C' \right) dr.
\end{aligned}$$

Proof: Let $z_4(kh)$ and $z_5(kh)$ be defined by

$$\begin{aligned}
z_4(kh) &= \sum_{i=0}^{N} q_i v((k-i)h) \\
z_5(kh) &= \sum_{i=0}^{N} q_i \int_{(k-N)h}^{(k-i)h} c e^{A((k-i)h-r)} w(r) dr.
\end{aligned}$$

Since v and w are uncorrelated, we have

$$\begin{aligned}
&\mathrm{E}\{z_3(kh)' z_3(kh)\} \\
&= \mathrm{E}\{(z_4(kh) + z_5(kh))'(z_4(kh) + z_5(kh))\} \\
&= \mathrm{E}\{z_4(kh)' z_4(kh)\} + \mathrm{E}\{z_5(kh)' z_5(kh)\}.
\end{aligned}$$

From (2), we have

$$\begin{aligned}
&\mathrm{E}\{z_4(kh)' z_4(kh)\} \\
&= \mathrm{E}\{\sum_{i=0}^{N} \sum_{j=0}^{N} q_i q_j v((k-i)h)' v((k-j)h)\} \\
&= \mathrm{E}\{\sum_{i=0}^{N} \sum_{j=0}^{N} q_i q_j\, \mathrm{Tr}\, (v((k-i)h) v((k-j)h)')\} \\
&= \sum_{i=0}^{N} q_i^2\, \mathrm{Tr}\, R.
\end{aligned}$$

Note that $z_5(kh)$ can be written as follows:

$$\begin{aligned}
&z_5(kh) \\
&= \sum_{i=0}^{N} \int_0^{(N-i)h} c e^{A((N-i)h-r)} b w(r + (k-N)h)\, dr.
\end{aligned}$$

Thus we have

$$
\begin{aligned}
&\mathrm{E}\left\{z_5(kh)'z_5(kh)\right\} = \mathrm{E}\left\{\mathrm{Tr}\left(z_5(kh)z_5(kh)'\right)\right\} \\
&= \sum_{i=0}^{N}\sum_{j=0}^{N} q_i q_j \int_0^{(N-i)h}\int_0^{(N-j)h} \mathrm{Tr}\Big(ce^{A((N-i)h-r)} \\
&\quad \mathrm{E}\left\{w((k-N)h+r)w((k-N)h+r)'\right\} \\
&\quad e^{A'((N-i)h-r)}c'\Big)\, dr\, ds \\
&= \sum_{i=0}^{N}\sum_{j=0}^{N} q_i q_j \int_0^{(N-i)h}\int_0^{(N-j)h} \mathrm{Tr}\Big(ce^{A((N-i)h-r)} \\
&\quad Q\delta(r-s)e^{A'((N-i)h-r)}c'\Big)\, dr\, ds \\
&= \sum_{i=0}^{N}\sum_{j=0}^{N} q_i q_j \int_0^{N-\max(i,j)h} \mathrm{Tr}\Big(ce^{A((N-i)h-r)} \\
&\quad Qe^{A'((N-i)h-r)}c'\Big)\, dr \\
&= \sum_{i=0}^{N}\sum_{j=0}^{N} q_i q_j H_{ij}.
\end{aligned}
$$

■

III. Disturbance observer design

In this section, how to choose design parameters in the disturbance observer (4) is discussed. Design parameters in (4) are N, q_i, p_i and K. The value h in (4) is the same as that of the sampling period. Thus the value h can also be a design parameter if the sampling period can be chosen freely.

Design parameter q_i must satisfy (5), where $N \geq n$ is a necessary condition to the existence of q_i. Thus N should be larger or equal to n. If $N > n$, the q_i sets satisfying (5) are not unique and there are degrees of freedom. These degrees of freedom can be used to minimize the variance of noise term $z_3(kh)$. Minimization of (16) over the vector q subject to (5) is a linear quadratic optimization, which can be computed efficiently. Once N and q_i are determined, p_i and K can be immediately determined from (8) and (13).

In case that h is a design parameter, h affects both disturbance estimation speed and noise sensitivity. From (5), q_i tends to become larger as $h \to \infty$. Hence the noise effects in (14) becomes larger (see (16)). On the other hand, the estimation speed of disturbance d becomes fast (see (4)) as $h \to \infty$. Hence h should be chosen so that the estimation is fast enough and not too sensitive to noise.

Now the design procedure for the proposed disturbance observer is summarized.

step	procedure
1	determine the sampling period h
2	determine the number of measurement point N
3	compute q_i by minimizing (16) subject to (5)
4	compute p_i from (8)

IV. Motor control using the disturbance observer

The proposed disturbance observer is applied to the motor control system to compensate the load torque. The objective of the system (see Fig. 2) is to maintain the motor speed at a constant reference speed regardless of the load torque. The mechanical dynamics of a motor are described by

$$\frac{dw}{dt} = -\frac{B}{J}w + \frac{1}{J}(T_e - T_L) \tag{17}$$

where J and B are the inertia of the rotor and the viscous friction coefficient, respectively. The electromagnetic torque T_e is given by $T_e = k_t i_q$, where i_q represents the q-phase current in the synchronous frame. The load torque T_L is considered as an unknown disturbance to the system dynamics.

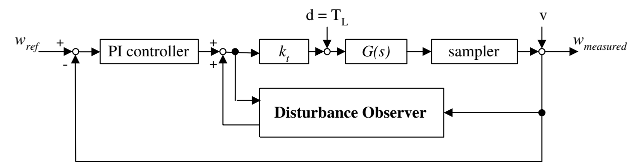

Fig. 2. Load torque compensation using the disturbance observer

A PI controller is used to control the motor and the disturbance observer is used to compensate the unknown load torque quickly.

In the first simulation, step load torque (see Fig. 3.(a)) is applied to the motor. Fig. 3.(b) shows the speed of the motor when the disturbance observer is not used. It can be seen that the motor speed deviates much from the reference speed: the maximum deviation is as high as 19. Fig. 3.(c) shows the speed of the motor when the disturbance observer is used to compensate the load torque. It can be seen that the maximum deviation is less than 3. One undesirable side-effect of the disturbance observer is fluctuation of the motor speed even without the load disturbance. This can be attributed to the fact that the disturbance estimation contains the noise term ($z_3(kh)$ in (14)). The effect of noise term can be partially reduced by using large N: simulation experiences reveal that the larger N becomes, the smaller the variance of z_3.

Fig. 4 shows the simulation results when sinusoidal torque load is applied. Similar to the step load torque case, the disturbance observer can significantly reduce the load torque effect in the speed of the motor (compare Fig. 4.(b) and Fig. 4.(b)).

Simulations are also performed with inaccurate motor dynamics ($0.5J \sim 1.5J$). The simulation results are similar to those in Fig. 3 and Fig. 4. Hence the proposed disturbance is robust to the model uncertainties. This robustness is due to the finite memory structure of the disturbance observer (see (4)). The disturbance is estimated at each step instead of being estimated recursively. Thus model uncertainties are not accumulated in the estimate, which can make the system unstable.

Simulation parameters used in this paper are as follows: motor ($J = 0.00135$, $B = 0$, $k_t = 1$), noise variance ($Q = 0.1$, $R = 1$), sampling period ($h = 0.001$), PI controller gain ($K_p = 0.02$, $K_i = 0.05$), $N = 1$.

V. Conclusion

The new disturbance observer is proposed, whose advantages are (i) finite memory estimation (robust to model uncertainties), (ii) simple structure, (iii) explicit consideration of sample-and-hold digital implementation. It is shown that load

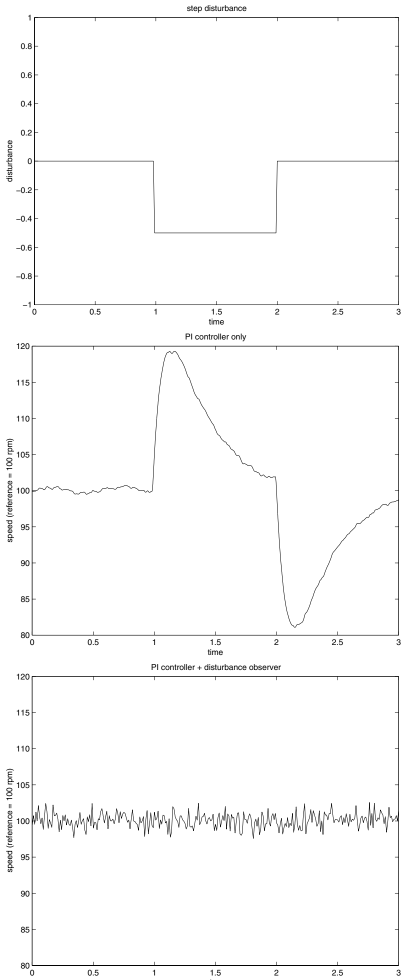

Fig. 3. (a) disturbance $d(r)$ (b) speed response (without the observer) (c) speed response (with the observer)

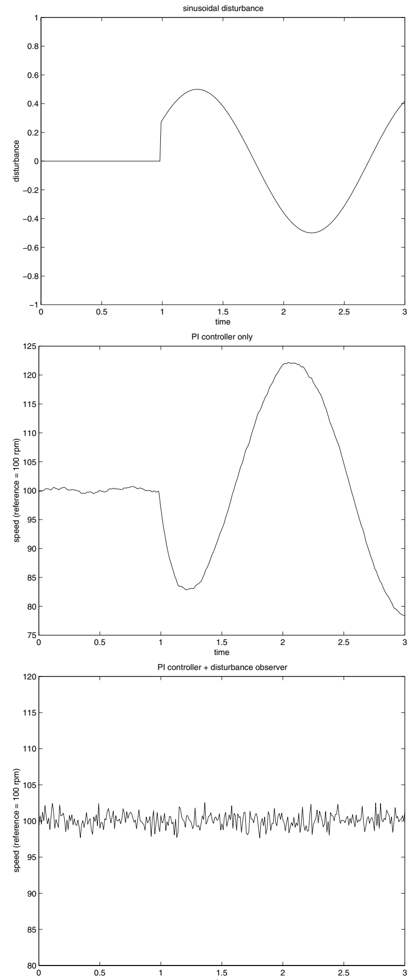

Fig. 4. (a) disturbance $d(r)$ (b) speed response (without the observer) (c) speed response (with the observer)

torque can be effectively compensated: the regulated output is not affected much by time-varying load torque. One disadvantage is that the disturbance estimation tends to be affected by the measurement noise, which can be partially overcome by using large N.

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