ps4

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November 2018

1 ps4

[...incomplete...]

2 q2

2.1 a

$\Rightarrow$
Take $\mathbb{A}\mathbb{X} = \mathbb{I}$

$$E(\tilde{\beta}|X_1, ..., X_n) = E(\mathbb{A}\mathbb{Y}|X_1, ..., X_n) \quad (1)$$
$$= \mathbb{A}E(\tilde{\beta}|X_1, ..., X_n) \quad (2)$$

Note, this follows as $E(f(x)y|x) = E(f(x))E(y|x)$ Now, we have that this becomes

$$\begin{aligned} &= \mathbb{A}'(E(Y_1|X_1, ..., X_n) \dots E(Y_n|X_1, ..., X_n))' \\ &= \mathbb{A}'(E(Y_1|X_1) \dots E(Y_n|X_n))' \\ &= \mathbb{A}'(X_1\beta \dots X_n\beta)' \\ &= \mathbb{A}'\mathbb{X}\beta \\ &= \mathbb{I}\beta = \beta \end{aligned}$$

$\Leftarrow$
Suppose $E(\tilde{\beta}|X_1, ..., X_n) = \beta$. Then, we have, from the equalities above, that

$$E(\tilde{\beta}|X_1, ..., X_n) = \mathbb{A}'\mathbb{X}\beta$$

Thus,

$$\begin{aligned} &\mathbb{A}'\mathbb{X}\beta - \beta = 0 \Rightarrow \\ &(\mathbb{A}'\mathbb{X} - \mathbb{I})\beta = 0 \end{aligned}$$

This implies, again that $\mathbb{A}\mathbb{X} = \mathbb{I}$

2.2 b

$$\begin{aligned}
Var(\tilde{\beta}|X_1,...,X_n) &= E((\mathbb{A}\mathbb{Y})^2|X_1,...,X_n) - E(\mathbb{A}\mathbb{Y}|X_1,...,X_n)^2 \\
&= \mathbb{A}(E(\mathbb{Y})^2|X_1,...,X_n) - E(\mathbb{Y}|X_1,...,X_n))\mathbb{A} \\
&= \mathbb{A}((E(Y_1^2|X_1)...E(Y_n^2|X_n))' - (E(Y_1|X_)^2...E(Y_n|X_n)^2)')\mathbb{A} \\
&= \mathbb{A}(Var(Y_1|X_1)...Var(Y_n|X_n))\mathbb{A} \\
&= \mathbb{A}(\sigma^2(X_1)...\sigma^2(X_n))\mathbb{A} & = \mathbb{A}\mathbb{D}\mathbb{A}
\end{aligned}$$

3 q7

3.1 a

Note that this follows simply from the random assignment. Because individuals are not aware of their assignment before the experiment and have equal likelihood of being assigned to treatment or control groups, their probability of being assigned to the treatment is independent of their α_i and β_i. Thus, D_i is independent of (a_i, β_i)

3.2 b

Note, that we can write down the β (i.e from class) as $E(D_i, D_i')E(D_i, Y_i)$, which, in the case of binary regression

$$\begin{aligned}
beta &= Var(D_iD_i')^{-1}Cov(D_iY_i) \\
&= Var(D_iD_i')^{-1}Cov(D_i\alpha_i + \beta_iD_i) \\
&= Var(D_iD_i')^{-1}(Cov(D_i\alpha_i) + Cov(\beta_iD_iD_i))
\end{aligned}$$

Since (α_i, β_i) are independent of D_i, the first term is just 0, and the second term is $E(\beta_i)E(D_iD_i')$. Thus, we have:

$$\begin{aligned}
&= Var(D_iD_i')^{-1}Cov(\beta_i)Cov(D_iD_i') \\
&= Var(D_iD_i')^{-1}Cov(D_iD_i')E(\beta_i) \\
&= E(\beta_i)
\end{aligned}$$

Using, the above, note we can also solve for α:

$$\alpha = E(y - betaD_i)$$

$$\begin{aligned}
\alpha &= E(Y_i) - \beta E(D_i) \\
&= E(\alpha_i + \beta_iD_i) - \beta E(D_i) \\
&= E(\alpha_i) + E(\beta_iD_i) - \beta E(D_i)
\end{aligned}$$

Again, by the independence of β_i and D_i, we get that this equals

$$\begin{aligned}
&= E(\alpha_i) + E(\beta_i)E(D_i) - \beta E(D_i) \\
&= E(\alpha_i) + \beta E(D_i) - \beta E(D_i) \\
&= E(\alpha_i)
\end{aligned}$$

3.3 q3

Note that since we aren't given homoskedasticity[1], we can use calculate robust standard errors:

$$\begin{aligned}
\hat{\Omega} &= (\frac{1}{n}\sum_{i=1}^{n} X_i X_i')^{-1}(\frac{1}{n}\sum_{i=1}^{n} X_i X_i'\hat{\epsilon}_i)(\frac{1}{n}\sum_{i=1}^{n} X_i X_i')^{-1} \\
&\to \Omega
\end{aligned}$$

Now, note that the errors in β are just the (2,2) component of Ω.

$$C_n = [\hat{\beta}_n - \Phi^{-1}(1-\alpha/2) \times \sqrt{\frac{\hat{\Omega}_{2,2}}{n}}, \hat{\beta}_n + \Phi^{-1}(1-\alpha/2) \times \sqrt{\frac{\hat{\Omega}_{2,2}}{n}}]$$

[1] In fact, in this setup that as α_i and β_i depend on i, it is probable that the errors are not homoskedastic