Price Theory 1 - Problem Set 2 - Question 1

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Setup

Consider a world with three agents: Consumers, Firms producing food, and firms producing cars. We assume the standard quasi-concave preferences etc.

Technology in this world is linear, and the only factor of production is labor. Assume that food firms produce m units of food per unit labor and that car firms produce c units of cars per unit labor. Individuals are also able to produce food at the same rate of m units of a food per unit time.

Consumers

Consumers are able to produce food at home, producing m units of food per unit time. They are able to work at either car or food firms freely and receive wages w_v, w_f respectively. However they view food produced in the market and from their house as perfect substitutes.

The question they face is:

$$\begin{aligned} \max_{\ell, f_H, f_m, v} \quad & U(f_H + f_m, v) \\ \text{s.t.} \quad & \ell \in [0,1] \\ & f_H = (1-\ell)m \\ & \ell \overline{w} = p_v v + p_f f_m \end{aligned}$$

Where ℓ is the amount of time they spend working, f_H is food produced at home, f_m is food bought at market, and v are the vehicles bought at market. $\overline{w}$ is the maximum of the two wages earned by working for the food and vehicle firms.

Rearranging the constraints and using $\ell = 1 - \frac{f_H}{m}$.

$$\begin{aligned} \max_{\ell, f_H, f_m, v} \quad & U(f_H + f_m, v) \\ & \overline{w} = p_v v + p_f f_m + \frac{\overline{w}}{m} f_H \end{aligned}$$

This shows that the time spent working to produce food at home is the cost that could be earned by working. This is the shadow price of working at home to produce food. The consumer values food produced from either place equally, thus he will purchase the market with lower price.

Firms

There are two different groups of firms in competitive markets, one group which produces cars, and one which produces food. Both firms only take labor as an input, and produce output linearly. The firms are competitive and receive zero profits. Their profit functions are given below.

$$p_f m L_f - w_f L_f = 0$$
$$p_v c L_v - w_v L_v = 0$$

Consumers can move between working at either type of firm without cost. If the wages paid by different sectors differ, then consumers will all work for the firms with the higher wages. Therefore the only equilibrium in wages is where the wages are equal. Thus: $w_v = w_f = w$

$$p_f m = w_f = w = w_v = p_v c$$
$$\frac{p_f}{p_v} = \frac{c}{m} = \frac{\frac{1}{m}}{\frac{1}{c}}$$

It takes a worker $\frac{1}{m}$ time to make a unit of food, so the relative benefit, when wages are equal must equal the relative cost in time that it takes to produce one unit in either sector. The relative price that each firm sets in equilibrium will be exactly the ratio of time in which its good can be produced.

Part A

In equilibrium the consumers problem simplifies as the wages are equal, and $w = p_f m$. This allows us to simplify his objective:

$$\max_{f_H, f_m, v} \quad U(f_H + f_m, v)$$
$$\text{s.t.} \quad w = p_v v + p_f(f_m + f_H)$$

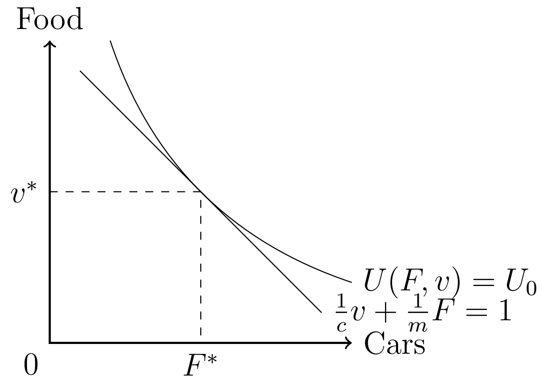
This shows that every consumer is indifferent between consuming food produced at the market or at home, and in equilibrium is indifferent between producing food produced at market or at home. Every possible allocation where $f_H + f_m = F$ are equally optimal for the consumer. This arises from the fact that consumers are no more efficient at producing food in the firm as they are at home, so consumers face the same cost in terms of time. We may replace $f_H + f_m$ with F and reach an equivalent problem for the consumer.

$$\max_{F, v} \quad U(F, v)$$
$$\text{s.t.} \quad w = p_v v + p_f F$$

This is the classical consumers problem, and for well-behaved utility functions its solution is understood to be:

$$\frac{\frac{\partial U}{\partial v}}{\frac{\partial U}{\partial F}} = \frac{p_v}{p_f} = \frac{m}{c} = \frac{\frac{1}{c}}{\frac{1}{m}}$$

Therefore the relative benefits of consuming the goods must be equal to the relative costs of consuming those goods, which is exactly the ratio of the time spent to produce either of those goods (the implicit costs).



However, this cannot pin down time usage. From the initial constraints, it is known that the time spent working will be given by:

$$\ell = 1 - \frac{f_H}{m}$$

In equilibrium, the individual is indifferent between producing and consuming food at home or the marketplace. It is equivalent to think about the consumers spending all of his time working, and then spending some of the money earned on food in the marketplace as it is to believe that the consumer buys no food at the marketplace, working only to buy cars.

There is a minimum time required to spend working. The consumer cannot substitute car production in the household, so he must work to earn the wages required to purchase the cars on the market. Let v^* be the optimal amount of cars used.

$$\ell_{min} = v^* \frac{p_v}{w} = \frac{v^*}{c}$$

This is the amount of time required to earn the money to buy the optimal amount of cars. The consumer is indifferent between working this amount and any amount above it, capped at one.

Note that the consumer's budget constraint could be rewritten as:

$$1 = \frac{1}{c}v + \frac{1}{m}F$$

That is, the prices faced by the consumer are equal to the time required to produce a single unit of that item. Therefore as m goes up, individuals find producing food less costly, and believe that they have relatively more time. If we believe that the "income effect" of having what appears to be more time is dominated by the substitution effect, then the individual will produce more food. The same logic applies for an increase in c.

Part B

It does not matter if meals are produced in the market or at home. In equilibrium, the cost of substituting between making food at home and buying the food is the same to the consumer, and the benefit is clearly the same. As long as food consumption is positive, there are an infinite number of equilibria where different amounts of time are spent producing food in the firm and in the household.

Part C

These taxes affect the revenues earned by each of the firms. Their zero-profit conditions become:

$$.9p_f m = w_f = w \qquad p_f = \frac{10w}{9m} \qquad \frac{p_f}{p_v} = \frac{c}{m}$$
$$.9p_v c = w_v = w \qquad p_v = \frac{10w}{9c}$$

We still have the same relationship for the relative prices as before. Let us examine the new consumer's problem:

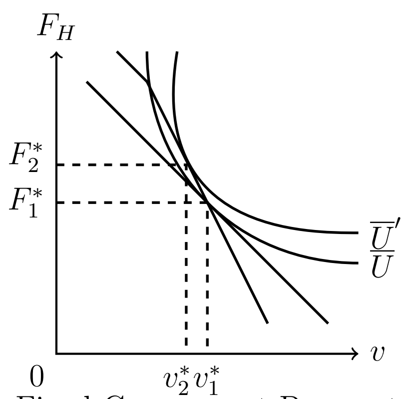
$$\begin{aligned} \max_{v, f_m, f_H} \quad & U(f_H + f_m, v) \\ \text{s.t} \quad & R + w\ell = p_v v + p_f f_m \\ & \ell \in [0,1] \end{aligned}$$

Applying the equilibrium prices to our constraints:

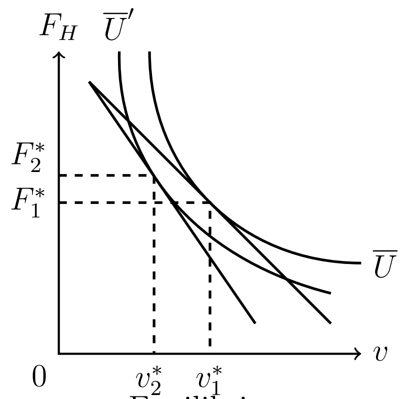
$$\begin{aligned} \max_{v, f_m, f_H} \quad & U(f_H + f_m, v) \\ \text{s.t} \quad & 1 + \frac{R}{w} = \frac{10}{9c}v + \frac{10}{9m}f_m + \frac{1}{m}f_H \\ & f_H \in [0, m] \end{aligned}$$

In terms of time, the price of the market goods is relatively more expensive to the individual, and their income has been raised. As Household production of food is relatively cheaper than market production of food, all food consumption will take place in the home. Individuals will only work to consume cars, and will work $c = \frac{10w}{9}v^*$ hours.

The wage relative to the price of cars that the consumer faces is lower, but their income is subsidized by the government paying them some quantity of money. This amount cannot be normalized by the wage amount because each individual is setting his workload individually. As a result, producing food in the home is relatively cheaper, as he still receives this tax load when he works less. Labor is competitive, he has no power in controlling the total amount of work, and thus his taxable income.



Fixed Government Payment



Equilibrium

One important concept seen on this graph is that the original consumption is always a choice for the worker. If he did not adjust his behavior, his wage plus subsidy would allow him to purchase the same bundle at market. It will never be the case that he is worse off under this tax. To face a higher utility, he must substitute food for cars. This tells us that food consumption will rise and that car consumption will fall. This will cause the government revenue to drop, and the individuals receive less revenue from the government. Then they must adjust their behavior accordingly. This process will repeat.

They have less income than before, so their budget line lowers. If they elected to work 0 hours, they could produce the same amount of food at home as pre-tax as the equilibrium government payout would be 0. Cars are relatively more expensive, so the slope of the budget line is steeper. This gives us the equilibrium budget line on the graph on the right. and the lower equilibrium utility level $\overline{U}'$. Consumers must work less, and therefore receive a lower income because of the government than before the tax. This limits their possible consumption of vehicles to less than it was before the tax, causing an inward shift of their budget set. In equilibrium they reach a lower level of utility than before the tax. If we believe that cars are a normal good, consumers will strictly consume less cars in the post-tax equilibrium. Food is relatively cheaper, but income has decreased, so the change in food consumption will be indeterminate.

Part D

Now the consumers face a cost of switching between working for the firm that produces food and the firm that produces cars. The cost is k units of time that they must spend learning how the use the machinery in order to begin working for the car-producing firm. Note that if the consumer elects to face this fixed cost, he will spend all of his working time working to produce cars. If he chooses not to, all of his working time will be spent producing food. He earns a higher wage working for car production, so there is no reason for him to split his time and earn the lower wage after facing the fixed cost.

Since there is an extra cost to working at only one firm, we should not expect the wages to be equal in equilibrium. The car firms will provide a higher wage in order to attract workers to pay this cost of time spent learning. Both firms however will continue to earn zero profits in equilibrium.

$$\begin{aligned} p_v c L_v - w_v L_v &= 0 \qquad p_f m L_f - w_f L_f = 0 \\ p_v c &= w_v \qquad p_f m = w_f \end{aligned}$$

In equilibrium, the consumer will be indifferent between working at producing cars, and working at producing food. Otherwise all consumers would work for one firm, and there would be no production of the other good. This means that either choice of job should live on the same indifference curve.

If the consumer were to work for the food producing firm, the question is as before: But with the only choice of working is producing food for the market. He does face a higher price in time for cars. This occurs because $p_v c = w_v > w_f$ in our equilibrium. The mechanism of his choice remains the same, and he will still maximize utility as if he were facing two goods: food and cars each with a price determined by the market. The shadow price of making food at home remains equal to the market price of food.

If the consumer elects to work in car-production, his utility maximization problem is as follows:

$$\begin{aligned} \max_{f_H, f_m, v, \ell} \quad & U(f_H + f_m) \\ \text{s.t.} \quad & w_v \ell = p_f f_m + p_v v \\ & (1 - \ell - k)m = f_H \\ & \ell \in [0, 1-k] \end{aligned}$$

The second constraint can be rewritten as: $\ell = 1 - \frac{f_H}{m} - k$. Applying this to the first constraint gives us:

$$\begin{aligned} (1-k)w_v &= p_f f_m + p_v v + \frac{w_v}{m} f_H \\ (1-k)w_v &= \frac{w_f}{m} f_m + \frac{w_v}{c} v + \frac{w_v}{m} f_H \\ (1-k) &= \frac{w_f}{w_v} \frac{1}{m} f_m + \frac{1}{c} v + \frac{1}{m} f_H \end{aligned}$$

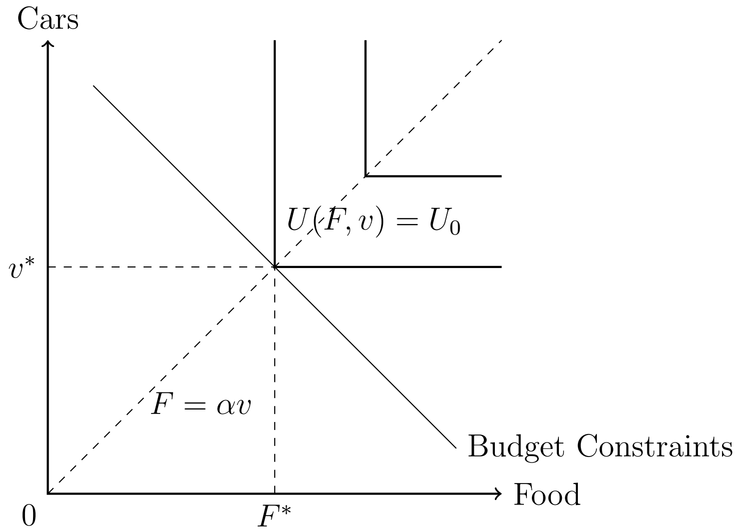
Contrast this to the budget curve faced by the consumer who works for the food firms.

$$\begin{aligned} w_f &= p_f f_m + p_v v + \frac{w_f}{m} f_H \\ w_f &= \frac{w_f}{m} f_m + \frac{w_v}{c} v + \frac{w_f}{m} f_H \\ 1 &= \frac{1}{m} f_m + \frac{w_v}{w_f} \frac{1}{c} v + \frac{1}{m} f_H \end{aligned}$$

As the wages offered to the workers in the car workers are higher, $\frac{w_f}{w_v} < 1$ and the price of food in the market is less than the shadow price of producing food in the home. Car workers will no longer be indifferent between producing food at home and in the market, and will buy all of their food at market. This also implies that they will spend all of their time working i.e. $\ell = 1 - k$.

When we assume that cars and meals are consumed in fixed proportions, we assume the goods are perfect complements. Let α be the proportion of food that consumers buy relative to the number of cars. One important property of the Leontief is that the only place where a cost minimizing consumers will choose to be is the vertex of the indifference curve.

Note that consumers must be indifferent between working for either the car or food firms. Since the only optimal consumption bundle on an indifference curve is the vertex, both types of consumers must have the same consumption bundles. Food working consumers are indifferent between working and buying their food, and producing the food at home. For the sake of the plot, assume that they choose to buy all their food at the market. They face the same prices in the market, so for the same consumption they must have equal income. This means that either consumer faces the same budget constraint. This implies $w_v(1-k) = w_f$.



This will be governed by the following system:

$$(1-k) = \frac{w_v}{w_f m} f_m + \frac{1}{c} v$$
$$\frac{w_v}{w_f} = 1 - k$$
$$f_m = \alpha v$$

Note that k, m, c, α are fixed in this scenario, and $f_m, v, \frac{w_f}{w_v}$ are three unknowns in a system of three equations. As long as this system is consistent, it will determine the consumption and relative wages.

Clearly everyone will work, as their utility with no consumption of cars is zero due to the Leontief utility function. Cars cannot be produced at home so these must be bought with wages. Therefore all types of workers will choose to work, and the ones that work producing cars will spend all of their available time working, and then purchase food in the market.

The consumers that produce food will surely work some minimum amount, which earns them enough wages to pay for their consumption of cars. They are indifferent between working more and using this wage to buy food at the market, and producing the food at home, so nothing can be said about the exact amount that they choose to work.

Part E

Solving this system of equations, we arrive at:

$$\frac{w_v}{w_f} = (1-k)$$
$$v = \frac{mc(1-k)}{(1-k)\alpha c + m}$$
$$f_m = \frac{cm\alpha(1-k)}{(1-k)\alpha c + m}$$

Immediately we can see that k has no effect on the amount of consumption of the consumers, and can only affect the relative wages of the two types of consumers. As $k \downarrow 0$, we can see that the relative wage converges to 1. This takes us to the scenario solved in part A.

$$\frac{\partial \frac{w_v}{w_f}}{\partial k} = -1$$

This tells us that the magnitude of a change in k has on the relative wage is equal to the change in k. This occurs from the Leontief utility function enforcing that both consumers face the same budget constraint.

The relative wage is proportional to the relative prices for the consumer:

$$\frac{p_v}{p_f} = \frac{w_v m}{w_f c} = (1-k)\frac{m}{c}$$

As k rises, the relative prices of cars fall, scaled by the technology. The larger the fixed cost, the less expensive cars are relative to food.

Part F

The above analysis is independent of the magnitude of k so its result apply for both a small and a large value of k.

For any $k \in (0,1)$ we will find that all consumers will work. The consumer will have some portion of free time, call it ϵ. This time can be spent working to earn $w_v\epsilon$, and this can be used to buy some amount of food and cars. Due to the Leontief utility function, consuming the small amount is strictly better than consuming nothing.

The consumers that work for the car firm will continue to work their entire time, and buy food from the market as it is cheaper than producing the food at home. The consumers working for the food firms are still indifferent between buying their food at the market and making the food at home. Some food at least will always be produced and sold at market since there is a known demand for food, by the workers who build cars, and supplied by the food workers using this money to buy cars. There could be more demand and supply where the workers produce food at the firm and consume that amount spending their entire wage, but this will still be in equilibrium. As such, we do not know how much work the food workers will choose to do. We do know that the car workers will spend all of their available time $(1-k)$ to produce cars. There is still a minimum amount of work that the food producers will work, which is enough work to earn the wages to buy the amount of cars they choose to produce.