Problem Set 6 - Question 2

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What can you say about the relationship between prevalence and new infections? Separate your answer into a mechanical component, related to random pairings of partners and a behavioral component.

We shall consider a model of the dynamics in which there are three types of actors. Actors who do not have syphilis, actors who have contracted syphilis but are not yet contagious, and actors who have syphilis and are still contagious. Any individual actor knows which state he is in, but not the state of whomever he is pairing with. All individuals are aware of the amount of contagious individuals. All actors are aware of y_t. Let us normalize the number of people in the world to 1. In this way, if all people sought the same amount of encounters, the probability of obtaining syphilis would be given by $p = \frac{y_t}{N} = y_t$.

Every individual therefore has a state, 1 for healthy, $i \in \{2, 3, ...x, x + 1\}$ for individuals with the disease that are not yet contagious, and $x+2$ for contagious. All information about the individual is summarized by these states. Individuals in state 1 move to state 2 if they contract the disease, and otherwise remain in state 1. All individuals in state 2 through $x+1$ move to the next state with probability 1. Contagious individuals in state $x+2$ remain there with probability $1-r$ and move to the healthy state with probability r.

All actors in the model receive the same benefit from a sexual encounter, and individuals only differ between the costs they face for this encounter. Each individual does face a constant cost per encounter taken as a monetary amount. This can be rationalized either in time spent in seduction, or in actual costs spent on dates. This is just a fixed cost among dates, and ensures that individuals that bear no risk of obtaining the disease would not desire infinite hook-ups. Healthy individuals face some cost β if they contract the disease, which is the time discounted present cost of obtaining the disease. This β accounts for all the utility and costs associated with having the disease, in expectation.

This cost function is state dependent, as individuals that have syphilis cannot obtain the disease a second time. These individuals still face the costs of an encounter, but not the disease risk. Let us believe that there is some guilt factor associated with giving syphilis to another person. Call this level γ. This guilt is only felt if syphilis is translated to another person. In expectation, this is $\alpha(1-p)\gamma$. Non-contagious individuals that have the disease do not bear the guilt, while contagious individuals do. This means that non-contagious diseased actors have the least costs to copulation.

The benefit of α sexual encounters is given by $u(\alpha)$ which is an increasing and concave function. Individuals in equilibrium will copulate to the point where marginal benefit equals marginal cost. Note that the benefit is the same for every individual regardless of state. All that differs between individuals is the marginal cost.

The healthy individuals' choice does not depend on what the sick people are doing. They have no knowledge of the choices made by the sick parties, and only observe the quantity of infected individuals. They do not even discover the costs of their actions until x periods into the future.

When they decide to engage in the sexual marketplace, they face a different probability of contracting the disease. The contagious, and not yet contagious sick individuals will choose different levels of copulation in general than the healthy individuals. The probability of them actually obtaining the disease is then a mixture of each groups' choice of copulation. Let us call this endogenous probability P^*

There are y_t contagious people, who choice to fornicate a_c times. The healthy individuals fornicate α times, and the non-contagious but infect individuals fornicate a_n times. These choices are given by:

$$\begin{aligned} \alpha &= \arg\max_{\alpha} \quad u(\alpha) - \delta\alpha - \beta * (1 - (1-p)^{\alpha}) \\ a_c &= \arg\max_{a_c} \quad u(a_c) - \delta a_c - \alpha(1-p)\gamma \\ a_n &= \arg\max_{a_n} \quad u(a_n) - \delta a_n \end{aligned}$$

The probability that a healthy individual contracts the disease from a single encounter is given by

$$P^* = \frac{a_c y_t}{a_c y_t + \alpha h_t + a_n(1 - y_t - h_t)}$$

where h_t is the number of healthy individuals at the time t. This is not simply the number of contagious people, as we see each group of people make different choices about the amount of people they choose to sleep with.

We can see that the choices of the sick individuals is only present in the level of P^*, as the dynamics of sick people are completely irrelevant of their choice of fornication. People who are not contagious will become contagious with probability one, and the contagious individuals become healthy with probability r. Neither are endogenous to their choices, and they can only affect the dynamics of healthy individuals.

For $x = 3$, the transition probability matrix for this system is given below:

$$\begin{pmatrix} (1-P^*)^{\alpha^*} & 1-(1-P^*)^{\alpha^*} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ r & 0 & 0 & 0 & 1-r \end{pmatrix}$$

Where P^* is given as above, and α^* is the choice for only the healthy individuals.

How many prevalence steady states are there?

There always exists the trivial steady state, where $y_{ss} = 0$. If there are no infected individuals, nobody can obtain the disease, and individuals will copulate without fear of disease.

In steady state, individuals always make the same choices. $P^* = P_{ss}$, $\alpha = \alpha_{ss}$ and $y_t = y_{t+1} = y_{ss}$.

Under these assumptions, the transition probability matrix is constant over time in the steady state. We can apply the balance equations to obtain the steady state distribution. This can be done by calculating the left eigenvalues of the transition probability matrix, and taking the eigenvector corresponding to $\lambda = 1$. Normalizing this eigenvector gives us the stationary distribution. This stationary distribution is given by

$$\pi_1 = \frac{r}{r + xr(1-P_{ss})^{\alpha_{ss}} + (1-P_{ss})^{\alpha_{ss}}}$$
$$\pi_i = \frac{r(1-P_{ss})^{\alpha_{ss}}}{r + xr(1-P_{ss})^{\alpha_{ss}} + (1-P_{ss})^{\alpha_{ss}}}$$
$$\pi_{x+2} = \frac{(1-P_{ss})^{\alpha_{ss}}}{r + xr(1-P_{ss})^{\alpha_{ss}} + (1-P_{ss})^{\alpha_{ss}}}$$

Regardless of the value of P_{ss}, the probability of obtaining the disease in steady state is, there is a single non-trivial steady state. This Markov process is irreducible and therefore it makes sense to compute the steady state in the way we have done.

There are always two steady states, the trivial steady state and the disease-ridden steady state.

Can incentives explain why prevalence follows a cycle?

The incentives in this problem are strange in the sense that once an individual contracts the disease, he does not become contagious for several periods. This means that the probability of being infected is based in part by the probability of being infected in the past.

Note that changes in the probability of being infected disincentivise the healthy individuals from fornicating, and incentivise the contagious to procreate more. Those that do not have syphilis are at more risk with a random fornication, and the contagious feel less guilt, so they will procreate more. This acts to increase the probability of being infected from a random encounter as a secondary effect. Both of these combine to ensure that the costs of sexual encounters increase for healthy individuals.

As the probability of being infected rose in the past, healthy individuals changed their behavior to fornicate less. However, this could not affect the amount of infected individuals for x years. Imagine a shock occurs where there are suddenly 10 percent more infected people at a time t. Suddenly the probability of being infected has risen, but for the next $x-1$ years, the people who were infected previously are the only people who become contagious. The growth of the system is unaffected until x years later, where less people become infected. This causes the chance of being infected to decrease, but the behavior based on this decrease will not be shown in the dynamics for x years.

For the years until this behavioral change is made apparent, the contagious rate will continue along its previous trajectory.

This x year delay distortion in the incentives will cause the prevalence to cycle. Once the rate has decreased to a certain amount, more people will copulate, and it will take x years for the prevalence to begin to increase. This will lead to another decrease and so on.

The larger the value of x, the greater the cyclical behavior, as it takes longer for changes in behavior to reflect back into the incentives. Incentives always change x years after the behavior does. Therefore for there to be a cycle and any sense of delay, we need x to at least be 1 or greater. If x is zero, people are able to react instantly, and the question is trivial.

How is your answer different if you recognize that people differ in terms of the number of partners that they have?

Let us believe that individuals are heterogeneous across the benefit that they receive from a fornication session. The individuals that receive more benefit will choose to plunder more booty, and the individuals that receive less benefit will choose to fornicate less.

If people differ in the number of partners that they have, more sexually active people are more likely to contract the disease. This manifests itself as different choices of α for different people. This means that the people more likely to contract the disease, denoted w, are more likely to be the ones infected. As the infection rate rises, these people are more disproportionately infected. The remaining people are less likely to do deed already, and seeing the higher infection rates, become even less likely to fornicate.

This does not change the behavioral trends being cyclical. The effect caused by heterogeneity is still delayed by x time periods. The expected number of healthy individuals that become diseased has decreased, but any change in this number takes x years to impact the actual number of infected individuals. Regardless of whether or not this change is effected by a change in the distribution of the healthy individuals or the choices that the healthy individuals are making. This simply makes the effects sharper within the cycles. As the number of infected people rises, there is a larger decrease in the expected quantity of hook-ups. This is caused by the behavioral response just as in the homogeneous case, but also because the remaining healthy people are less likely to hook up. This leads to a larger decrease in sexual actions than under homogeneity. When the infected number begins to drop, it will drop by more than if the population was homogeneous, and likewise when it increases again.

There is no change in the qualitative behavior of the dynamic system. Only the magnitude of deviations during each period have changed, the behavior is still periodic with the same pattern of increasing and decreasing.

What would have been different about the prevalence series if prostitution had been legal?

Let us believe that people are capable of screening prostitutes for syphilis. This is the only real distinction that occurs when legalizing prostitutes, as illegal prostitutes still exist regardless. But now they have access to a legal system and rights to ensure their ability to screen. This allows prostitutes to become a substitute for hook-ups which presumably cost more money, but do not run the risk of carrying the disease.

This means that when there are high amounts of syphilis, demand for the prostitutes will be high, and when the disease is low, demand for the prostitutes will be relatively lower.

Assuming that the benefit of fornication is equal between hookups and prostitutes, the decision is made where the cost premium of a prostitute is equal to the monetary cost of the disease times the probability of being infected. Healthy individuals are then indifferent between dating and using prostitutes, so there will be some supply of prostitutes at the given price.

Individuals in general will face a lower risk of obtaining the disease, as they are indifferent between a prostitute and a hook-up, and there is some supply of prostitutes at the given price level. The individuals that utilize the prostitutes will face no risk of the disease where they once had a risk, so the overall level of the disease will fall.

This will reduce the price of the prostitutes, decreasing the quantity supplied. This will lead to less individuals using the prostitutes than before, but still less diseases overall than when prostitution was not an institution. We would still expect a cyclical behavior, but the existence of prostitutes mutes its magnitude. There is an effect of less prostitutes being available at low levels of the disease causing the muting to be less than initially expected. However this secondary effect cannot dominate the primary effect of there being less disease-sharing hookups.

Does the emergence of AIDS help explain why syphilis prevalence dropped to new lows?

Yes. The cost of a random hookup increases when there is another disease in play. AIDS is also considered to have a much more negative effect on the health of an individual than syphilis. This means that individuals would be much more sensitive to hooking up with people, as the cost of contracting AIDS was very high.

Each individual now faces some probability ϕ of obtaining AIDS from a possible hookup. This cost is faced by all parties, as regardless of having syphilis, one can still contract AIDS. As there is no cure for AIDS, the dynamics do not allow for people to become healthy again, and the present cost of obtaining that disease is much higher. Denote this cost by Θ

$$\begin{aligned}
\alpha &= \arg\max_{\alpha} \quad u(\alpha) - \delta\alpha - \beta(1-(1-p)^{\alpha}) - \Theta(1-(1-\phi)^{\alpha}) \\
a_c &= \arg\max_{a_c} \quad u(a_c) - \delta a_c - \alpha(1-p)\gamma - \Theta(1-(1-\phi)^{a_c}) \\
a_n &= \arg\max_{a_n} \quad u(a_n) - \delta a_n - \Theta(1-(1-\phi)^{a_n})
\end{aligned}$$

We can see that the cost for all groups has increased, so there will be less quantity of hook ups provided by all three groups. We cannot justify that the probability of obtaining the disease from a single hook-up has increased. Since the utility function is concave, non-contagious infected individuals decrease their fornication the most. It may be that healthy individuals reduce their hook ups less than contagious individuals, but either way the probability of obtaining the disease is indeterminate.

Though it is likely that the decrease in the amount of copulation by the healthy individuals would offset this lowered probability, it cannot be shown to occur in all cases. Consider very concave utility functions, high costs of syphilis, AIDS, and guilt. This would mean that the non-contagious individuals reduce their fornication by a large amount, and the other two groups reduce it by little. Now the probability of obtaining the disease from a random hook-up is much higher. This would lead to a higher amount of syphilis, despite every group desiring less hook ups.

We do know that the amount of copulation has decreased for a certainty. However, because every group responds to this added cost of possibly obtaining AIDS in a different way, it can be that healthy individuals are more at risk of obtaining the disease. The existence of non-contagious infected individuals allowed for healthy individuals to be shielded from the contagious individuals, but since that group responds most strongly to the presence of AIDS, healthy individuals can be at more risk.

However, if all groups respond near the same proportion, we would expect the presence of AIDS to reduce the prevalence of syphilis, as all groups change their amount of copulation, and the probability of a healthy individual obtaining the disease would not change by much. In this case, syphilis cases reduce, and we get the behavior noted in the time series data give.