## Advanced Industrial Organization II Problem Set 1

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Due on January 22nd, at the beginning of the class.

## 1 Demand in a market with imperfect competition

Consider the following utility function of consumer i = 1, ..., N choosing j = 0, ..., J in market t = 1, ..., T:

$$u_{ijt} = \alpha p_{jt} + x_{jt}\beta + \xi_{jt} + \epsilon_{ijt},$$

where  $p_{jt}$  is price of product j in market t, and  $x_{jt}$ ,  $\xi_{jt}$  and  $\epsilon_{ijt}$  denote product characteristics. While consumers and producers observe  $x_{jt}$  and  $\xi_{jt}$ , the econometrician does not observe the latter.  $\epsilon_{ijt} \stackrel{iid}{\sim} T1EV$ , independent of all other variables, is only observed by consumer i in market t. Consumer i in market t chooses one product maximizing his utility:

$$y_{it} \equiv \operatorname{argmax}_{0 \le k \le J} u_{ijt}.$$

Price and product characteristics of the outside good, j=0, are normarized to 0, with the exception of  $\varepsilon_{i0t}$ . Let  $\theta=(\alpha,\beta')'$  be demand-side parameters.

There are f = 1, ..., F firms in a market, where each firm produces  $\mathcal{F}_f = \{j_1, ..., j_{J_f}\}$ .  $\{\mathcal{F}_1, ..., \mathcal{F}_F\}$  constitutes a partition of set $\{1, ..., J\}$ . The marginal cost of product j in market t is:

$$mc_{it} = w_{it}\gamma + \omega_{it}$$

where  $w_{jt}$  and  $\omega_{jt}$  are observed and unobserved cost components, respectively, in the econometrician's point of view. Assume there is at least one observed cost component not a part of  $x_{jt}$ . The profits of firm

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f in market t is:

$$\Pi_{ft} = \sum_{j \in \mathcal{F}_f} (p_{jt} - \text{mc}_{jt}) D_{jt}(p, x, \xi; \theta),$$

where  $D_{jt}(p, x, \xi; \theta)$  is demand of good j, in the firms' point of view. Firms simultaneously choose prices under static Nash-Bertrand competition.

- 1. Derive the aggregate demand system,  $D_{jt}(p, x, \xi; \theta)$ . (Hint: What do the firms observe?<sup>1</sup>)
- 2. Assume a Nash equilibrium exists. Derive the first order conditions of firm f and express it as an (implicit) function of  $p_{it}$ .
- 3. What does the above result imply in the relationship between  $p_{jt}$  and  $\xi_{jt}$ ?
- 4. Define

$$\hat{s}_{jt} = \frac{1}{N} \sum_{1 \le i \le N} 1\{y_{it} = j\},$$

for j=0,...,J in market t=1,...,T. Compute  $\text{plim}_{N\to\infty}\hat{s}_{jt}$  and discuss its relationship with  $D_{jt}(p,x,\xi;\theta)$ .

- 5. (Berry 1994) Suppose you only have market-level data:  $(\hat{s}_{jt}, p_{jt}, x_{jt}, w_{jt})_{jt}$ , instead of an individual-level dataset. Derive a linear regression equation of  $(\hat{s}_{jt}, p_{jt}, x_{jt})_{jt}$ . (Hint: Get rid of the denominator of  $D_{jt}(p, x, \xi; \theta)$ .)
- 6. Is the OLS estimator consistent for the demand-side parameters,  $\theta$ ? Why or why not?
- 7. Discuss potential advantages and disadvantages of the following variables as instruments: (1)  $w_{jt} \setminus x_{jt}$  (cost components not a part of  $x_{jt}$ ), (2)  $x_{-j,t}$  (characteristics of other products), (3)  $p_{j,-t}$  (prices of j in other markets).
- 8. In cereal\_ps3.xls, you are given a semi-fabricated dataset of Nevo (2000, 2001). In the dataset, product j is a unique identifier of 'firm' and 'brand,' and market t is that of 'city,' 'year' and 'quarter.'  $x_{jt}$  has two components, 'sugar' and 'mushy.' There are 20 instruments to be used for estimation ('z1'-'z20'). Assume that the data generating process follows the model we specified. Estimate  $\theta$  using an estimator you think it is consistent. Compare estimation results with and without brand and market dummies. Discuss the results.

<sup>&</sup>lt;sup>1</sup>Note that there is another interpretation of  $\varepsilon_{ijt}$  when deriving the aggregate demand: The suppliers observe  $\varepsilon_{ijt}$ , but a continuum of consumers of total mass N are distributed with Type 1 Extreme Value Distribution in the population. While its microfoundation is somehow different, two interpretations give the same result. See Anderson, de Palma and Thisse (1992) for a reference.

9. From  $D_{jt}(p, x, \xi; \theta)$ , derive own and cross-price elasticities. Do you find them restrictive? Why or why not? If you think they are, what could be potential remedies? Discuss briefly.