Problem Set 4.1

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Given: There is a market for housing with a range of quality levels. New high-quality housing alternatives are produced by a competitive industry with constant return to scale (i.e. constant marginal cost). There is a fixed stock of existing houses (H_{old} houses). Individuals differ in their level of income, but have the same preferences.

Assumed: Quality of houses is assumed to be a normal good, so that as income increases the quality choice increases. The number of inhabitants for the market considered (N_{pop}) is assumed to be greater than the number of existing houses (H_{old}), so that some inhabitants must occupy new houses. Additional, all consumers are assumed to have an income high enough so that they can afford renting a house. Since the producer of new houses exhibits a CRS function, it is assumed to make no profits ($\Pi_{new} = 0$) in equilibrium. Additionally, since it is given that new houses will be "high quality" houses, we assume that the derivative of the unit cost of production function for new houses is smaller than the hypothetical one for existing houses (i.e. $UC'_{new}(q) < UC'_{old}(q)$). We interpret the unit cost of increasing the quality in the context of maintenance frequency and amenities provided. Maintenance such as heating is more expensive to provide in older houses. We assume that they have lower unit cost to provide in new houses compared to older houses.

1 Part A

Q: Describe the equilibrium to this model. Who would tend to live in the new houses? What will determine the price of the old houses (which range in quality)?

We assumed that $H_{old} < N_{pop}$. However, we have not made any determinations on the consumers' income distribution. We will assume that there exist two types of consumers, rich and poor. Let $M_{rich} > M_{poor}$ and $N_{poor} + N_{rich} = N_{pop}$. Denote M_x as the income of group x.

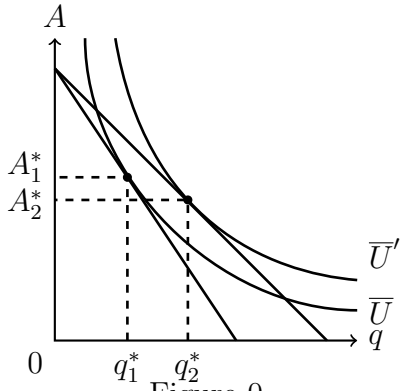

Figure 0

Consider the consumer's problem between the choice in the quality of the house and some auxiliary good that bundles all other choices. As the price of quality changes, a different quality is demanded. All of these choices map out the demand for quality of a particular individual. The individuals in the rich group have a higher income. Since quality in houses is normal, the rich group has a higher demand than the poor group.

Claim: Higher-income individuals tend to live in the new houses, and the relative price of old houses will depend on the relative scarcity of low-income individuals and existing houses.

Quality of houses are a normal good. For two different sets of income, the individual with the higher income will have choose a higher quality house. In equilibrium he will then occupy the higher quality house. The poorer individual will occupy the lower quality house, or the older house. We already established that $H_{old} < N_{pop}$. Nonetheless, it could be the case that $H_{old} > N_{poor}$. Some of the high income individuals would also occupy existing houses, or that $H_{old} < N_{poor}$, so that low income individuals occupy some new houses because old houses are scarce relative to the low income population.

Case 1: $H_{old} > N_{poor}$

In this case some high-income individuals will also occupy existing houses. For a given level of income (high), individuals must be indifferent between living in new and old houses. Consequently, old houses' prices will increase until the marginal quality cost function is tangent to the same level curve for the representative high income individual. In the equilibrium, additionally, poor individuals' level curve must be tangent to the same marginal quality cost curve for old houses. Otherwise, either there would be utility improvement opportunities and the allocation would not be a competitive equilibrium, or some of the poor individuals would not occupy a house. This case is shown below in Figure 1.

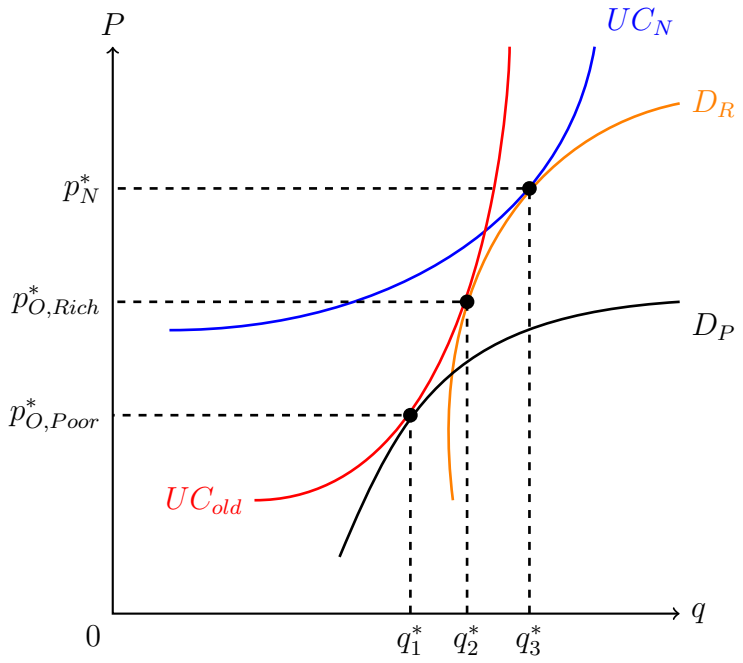

Figure 1: $N_{poor} < H_{old}$

Case 2: $H_{old} < N_{poor}$

In this case existing houses will be scarce relative to the low-income population so that some low-income individuals will have to live in new houses. In the equilibrium, high-income individuals will make a higher quality choice, while low-income individuals will make a lower quality choice and be indifferent between living in old and new houses. Therefore, old houses' prices will increase even further relative to case 1. This case is shown below in Figure 2.

In both cases higher income individuals tend to live in the new houses. Additionally, we observe that the relative price of old houses will depend on the relative scarcity of old houses and low-income individuals.

2 Part B

Q: How would an increase in the income of a group of consumers (say those in a given range of the income distribution) affect housing prices and the welfare of consumers at the different levels of income?

Maintaining the assumptions made in part A we can further analyze what could occur if any of the groups increased their income.

Again, we will have to study two cases within this frame, $N_{poor} < H_{old}$ (Case 1) and $N_{poor} > H_{old}$ (Case 2).

Case 1: $H_{old} > N_{poor}$

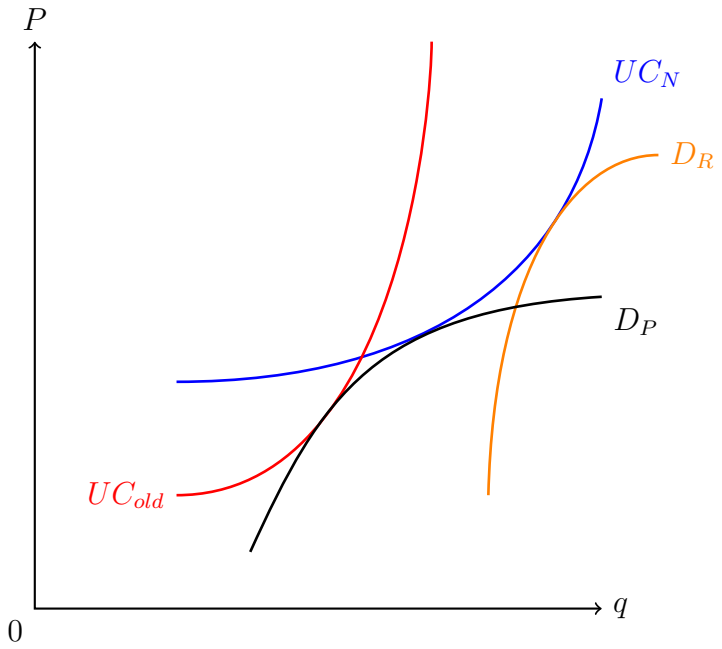

Figure 2: Both Buying New

An increase in the income of relative rich individuals (i.e. M_{rich}) would further shift their level curve towards a higher quality choices and increase the price of new houses. However, since in the equilibrium higher income individuals still need to be indifferent between new and some old houses (recall that some rich individuals would occupy existing houses in this case), the relative price of old houses for rich individuals would need to decrease. This would require a shift downward of the unit cost curve for existing houses (i.e. existing house owners' profits would decrease). Since the form of the curve of poor individuals has not changed, their quality choice will be the same. Nevertheless, since the unit cost curve for existing houses shifted down, the actual price for the same quality choice would decrease, making lower income individuals better off. This dynamic is shown below in Figure 3.

Additionally, within this case, it is possible that the income of poor individuals increased (i.e. M_{poor}), so that $M_{rich} > NewM_{poor} > PastM_{poor}$. In this case, although the quality choice of poor individuals would tend to increase, causing an increase in the price of old houses, there would be no shift in the old houses' unit cost curve (i.e. $C_{old}(q)$). Which would indicate that overall, existing house producers (or owners, since they are already build) would make a similar "profit". Nonetheless, one could imagine that additional income levels could be introduced in the model, as well as additional unit cost of production curves. In this case, shifts in unit cost curves could occur for certain income level increases (similar to what occurred in Figure 3 for an M_{rich} increase), but one would expect that since they tend to affect only cheaper alternatives, the overall effect on welfare would be reduced as the affected income level is poorer.

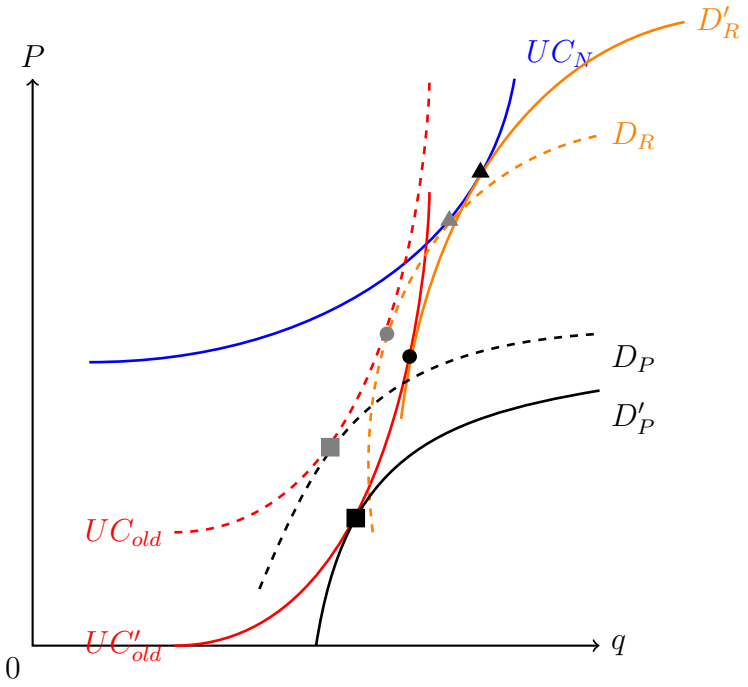

Figure 3.

Case 2: $H_{old} < N_{poor}$

In this case, if the M_{rich} increases, their quality choice (and the consequent housing price) would increase, but it would move along the fixed unit cost curve for new houses (recall $\Pi_{new} = 0$ and their marginal cost is constant). Nonetheless, it would have no effect on lower-income population's welfare. On the other hand, if there was an increase in M_{poor} so that $M_{rich} > NewM_{poor} > PastM_{poor}$, lower-income individuals would shift their choices towards higher qualities. Since poor individuals must be indifferent between new and old houses in the equilibrium, there will be a shift downward in the existing houses' unit cost curve. Overall, the welfare of poor individuals would increase, as the price that they would have to pay for the same quality choice would decrease. This is shown below in Figure 4.

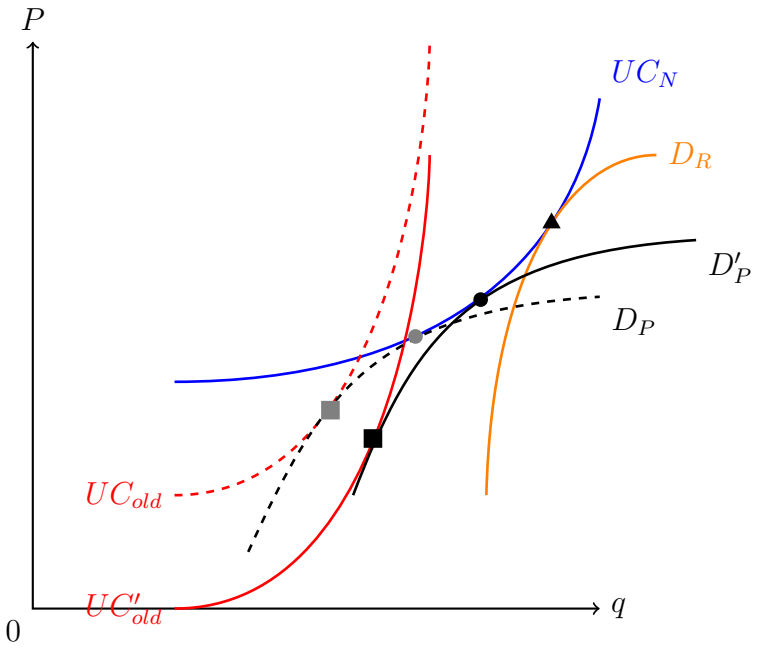

Figure 4.

Income increases for groups that are indifferent between the new and existing houses, have larger welfare effects than the groups that are only consuming one type of household.

3 Part C

Q: What would happen if the government replaced a number of the oldest houses with new ones through a process of urban renewal? Who would gain from that? Why? Could some people lose? How would the answers differ if individuals owned versus rented their housing?

In our simplified model, there are four representative agents involved: poor individual, rich individual, old house owner and new house provider. In equilibrium, given our framework, rich individuals are always either indifferent between occupying an old and a new house, or only living in new houses. Since the new houses provider exhibits CRS, the marginal cost of producing a new house would not increase, and therefore, rich individuals would never be better nor worse off. New house providers by definition, must have $\Pi = 0$, and therefore, would not be affected by the replacement process either. They would simply provide more houses at the same marginal cost without any profits. Old house owners, in general, will be worse off since they are earning a profit, and presumably they will not be earning one after their house is replaced. However, this would depend on the actual replacement mechanism (i.e. how much are they receiving?), so it is uncertain whether they would be better off.

If poor individuals had been indifferent between living in old and new houses (recall Part A - Case 2), they would not be affected by the replacement process either. Conversely, if they were using only old houses (recall Part A - Case 1), once a house is replaced, a poor individual will have to afford another house that he decided not to afford when she had the

alternative. Therefore, the poor individual will have to make a choice that was not maximizing their utility before, which could only make them worse off. If she had to chose a quality/price combination on the new houses unit cost curve, she would chose a lower quality product (recall $C'_N(q) < C'_O(q)$).

If individuals owned their houses, however, the initial "profit" that they were earning (recall that the unit cost curve for old houses was shifted upwards once new houses were introduced) could be either maintained, increased or destroyed during the replacement process. Depending on the actual replacement mechanism they could be either indifferent, better, or worse off. If they had to make any payments, for example, they could be worse off if those are not out-weighted my the benefits in quality/price they would receive from the new house, but that they decided not to afford before. Conversely, if they received a house that is strictly better quality/higher price at virtually no cost, they would be better off.

4 Part D

Q: Now assume that other aspects of life differ across houses in that the level of city services (e.g. police protection) differs across houses with the lower quality homes also receiving less police protection. How would a policy that mandated equal levels of services across areas affect outcomes? Who would gain from such a policy? Assume the amount of service is set equal to that of the highest quality homes, and that any additional spending is financed by lump sum taxes on consumers.

For simplicity-sake, we will assume that in this case consumers are only renting houses.

When a consumer rents a house, he now rents two goods. The quality of the house is bundled with the police protection alongside it. For each quality of house there is an associated level of police protection. At first, allow this quality of police protection to vary directly with the quality of house. Higher quality houses have higher quality police protection. Assume that this equilibrium fits the base graphs above.

Now the level of the police protection is no longer allowed to vary. Each quality house is now given the same protection as the highest quality houses were receiving before. Houses of lower quality now are relatively better, since the level of bundled police protection good is now higher. As lower quality houses are relatively better, people of all income levels demand them more. This means that lower quality houses are demanded more, and income of both groups drops by paying for this police force. This reduces the demand for all houses, and as housing quality is a normal good, biases both groups towards lower quality houses.

The rich group of people were consuming new houses with the highest level of police protection. Now they have less income available to them as they are being taxed as consumers. This leads them to consume lower quality houses. If there are rich people consuming the older houses, they change their consumption of older houses to be indifferent to the new houses. The group consuming the new houses is strictly worse off as it has less income, lower quality, and equal police protection. As they are indifferent to the group (possibly)

consuming old houses, all rich consumers are made worse off.

There is now relatively less cost to renting a lower quality house. The cost of lowering quality is no longer less police protection and lower quality. Poor people also have less income available to them since they are taxed. These two factors lead to poor people choosing a lower quality of house. Note that poor people do not choose the high level of police protection in the previous equilibrium. This tells us that they prefer higher quality of house to higher police protection, at their current level of consumption. Poor consumers are therefore worse off after the policy, as they were gaining an advantage from the lower quality homes having low police protection. This advantage was caused by them being less willing to pay for the police protection than the richer group, and thus benefiting more from police protection being reduced.

The land-lords of the old houses would gain from the policy. A lower quality house now has more demand for it, and therefore a higher rental price. This increases the returns to the land-lords and makes them strictly better off. Makers of the new homes continue to produce at zero profits and are unchanged by the policy.