Advanced Industrial Organization II Problem Set 2

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Due on February 19th, at the beginning of the class.

1 Berry, Levinsohn and Pakes (1995)

Consider the following utility function of consumer $i = 1, ..., N$ choosing $j = 0, ..., J$ in market $t = 1, ..., T$:

$$u_{ijt} = \alpha_i p_{jt} + x_{jt}\beta_i + \xi_{jt} + \epsilon_{ijt},$$

where α_i and β_i are consumer-specific and drawn from $F(\alpha_i, \beta_i; \theta)$, a known distribution up to θ. Assume further that $(\alpha_i, \beta_i^{'})^{'} \sim N(\theta_1, diag(\theta_2^2))$, where $\theta_1 = (\alpha, \beta')' = (E[\alpha_i], E[\beta_i^{'}])^{'}$ and $\theta_2 = (\sigma_\alpha, \sigma_{\beta_1}, ..., \sigma_{\beta_L})$. x_{jt} includes a constant. Let $\theta = (\theta_1^{'}, \theta_2^{'})^{'}$ be demand-side parameters. All specifications are the same as that in the problem set 1, unless otherwise noted.

In cereal_ps3.xls, you are given a semi-fabricated dataset of Nevo (2000, 2001). In the dataset, product j is a unique identifier of 'firm' and 'brand,' and market t is that of 'city,' 'year' and 'quarter.' Treat them as different products if either the firm or the brand identifier varies. x_{jt} includes two components, 'sugar' and 'mushy.' Brand and market dummies are not included. There are 20 instruments to be used for estimation ('z1'-'z20'). Assume that the data generating process follows the model we specified.

1. Derive the aggregate demand system, $D_{jt}(p, x, \xi; \theta)$.

2. From $D_{jt}(p, x, \xi; \theta)$, derive own and cross-price elasticities. Do you find them more flexible than those in the problem set 1? Why or why not?

3. (Monte Carlo integration) Compute the predicted market share, $s_{jt}(p, x, \xi; \theta)$, using the simulation method, at $\alpha = -20$, $\beta_{constant} = -4$, $\beta_{sugar} = 6$, $\beta_{mushy} = -1$, $\sigma_\alpha = 3$, $\sigma_{\beta_{constant}} = 1$, $\sigma_{\beta_{sugar}} = 2$,

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$\sigma_{\beta_{mushy}} = 1$, and $\xi_{jt} = 0$ for all j, t. Use $S = 100, 500, 1000$ simulation draws from iid standard normal and compare results. Fix a random seed for replicability.

4. (Inner loop) Define the mean valuations as follows: $\delta_{jt} \equiv \alpha p_{jt} + x_{jt}\beta + \xi_{jt}$. Compute δ_{jt} equating the observed ($\hat{s}_{jt}$) and predicted (s_{jt}) market shares, at the same parameter values of θ_2 as above. Using the above θ_1, compute and report the distribution of ξ_{jt} from δ_{jt}. Use the nested fixed point (NFP) algorithm to recover δ_{jt}. (Hint: At the beginning of the computation, draw a set of simulation draws just once and use them throughout the loop to avoid the 'moving target problem.')

5. (Outer loop) Write a code that returns a GMM objective value at any θ. Evaluate it at the above parameter values.

6. Estimate θ using the nested fixed point (NFP) algorithm. Try a weight matrix used by Nevo (2000, 2001): $Z'Z$, a 23-dimensional square matrix. Use any optimizer (e.g. KNITRO, IPOPT, fmincon) and a programming language you prefer. Try many starting values to make sure you attain the global minimum at your estimate. Do not use a code written by others. (You can choose to skip questions 3-5 and estimate the model using the MPEC algorithm, if you prefer to do so.) (Hint: Some derivative-based optimizers, which need first and second order derivatives to know where it moves to, attempt to approximate a derivative by evaluating a neighborhood of a desired point, if they are not provided by a programmer. While it is not recommended in empirics, in this part you can simply provide the optimizer with the GMM objective function and let the optimizer minimize it.)

2 Entry game with incomplete information

There are two firms, $i = 1, 2$ considering whether to enter market $t = 1, ..., T$. The firm would like to enter if and only if net utility of entry is nonnegative:

$$y_{it} = 1\{-\delta y_{-i,t} + \alpha x_t + \epsilon_{it} \geq 0\},$$

where $\varepsilon_{it} \overset{iid}{\sim} Logistic(0, 1)$ is independent across firms and markets. $x_t \in \{1, 2\}$ with $Pr(x_t = 1) = 0.5$ is also independent. Assume ε_{it} is the private information of firm i, and all the other variables are common knowledge to both firms. In this setting they play a static Bayesian Nash equilibrium. The econometrician observes $(y_{1t}, y_{2t}, x_t)_t$.

1. What is the firm i's strategy? What is the behavior of firm i in the other's point of view?

2. Compute all Bayesian Nash equilibria for $(\alpha, \delta) = (1, 1)$, $(3, 6)$, respectively. (Hint: Numerical solutions are fine.)

3. From now on, let $(\alpha, \delta) = (3, 6)$. Suppose both firms play the same, symmetric strategy. Estimate (α, δ) in the same spirit of Seim (2006) and run a Monte Carlo Simulation to see whether it is consistent. Consider $T = 1000$ and $S = 50$ for the simulation. Fix a random seed for replicability. (Hint: In constructing the likelihood, one's probability to enter the market depends on the other's decision. It can be obtained by solving a fixed point problem. Note that, as observed in part 2, their strategies also depend on (α, δ) and x. The number of unknowns is less with the symmetry assumption.)

4. Suppose the firms do not necessarily use the symmetric strategy. The probability that equilibrium $k = 1, ..., K$ is selected, if the model exhibits multiple equilibria, is $\lambda_{it}^k(x_t) \propto \exp(k/2)$, where the equilibria are ordered in a way the firm 1's choice probability in equilibrium k, $p_{1t}^k(x_t)$, satisfies $p_{1t}^k(x) \geq p_{1t}^{k+1}(x)$ for all $k = 1, ..., K-1$ and given x. Propose an estimator of (α, δ) and run a Monte Carlo Simulation to see whether it is consistent. (Hint: Consider the equilibrium selection probabilities as mixture weights. You need to introduce auxiliary parameters to consistently estimate the parameters of interest. Note that players are aware of which equilibrium is selected. Although not required, it would be a good exercise to estimate parameters using the estimator in part 3.)

5. Let u_t be an iid binary variable with $Pr(u_t = 1|x_t) = \frac{1}{1+x_t}$, which is unobserved to the econometrician. Let the equilibrium selection probabilities be $\lambda_{it}^k(x_t, u_t) \propto \exp((k + u_t)/2)$. Does the estimator you propose above give consistent estimates? Discuss.