Problem Set 3 Question 2

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Model

In a very reduced-form manner, consider individuals that have some demand for universities and $x^*(p_D, ...)$ for private universities over public universities where p_D is the inflation-adjusted difference between tuition rates, and ... are all other parameters that do not include the tuition ratio. Assume that x^* is decreasing in p_D. This means that as the difference grows, individuals demand private education less than public education. This is equivalent to assuming that private universities have higher tuition rates than public universities, and the substitution effect dominates the income effect for university education. We can see that equal (after adjusting for inflation) absolute changes in the tuition rates will not change p_D, and therefore not change the demand of the private or public universities relative to the other. It is possible however that total university demand may change.

1 How would a proportional increase in the tuition of both types of schools affect the market share of public schools?

A representative budget constraint for an individual would look like the following:

$$p_g g s + p_v (1-g) s + p_A A = M$$

Where p_g is the price of public universities, p_v is the price of private universities, and p_A is the price of all other goods lumped into an auxiliary good.

For a proportional increase in the tuition of α, the average price level will increase by $\beta < \alpha$ as education spending is only a portion of the overall budget. When we calculate the real tuition rates for each country, then they will have increased by roughly $\frac{1+\alpha}{1+\beta} > 1$ percent.

If tuition costs of private and public universities were initially p_1 and p_2 respectively, new prices would then be $(1+\alpha)p_1$ and $(1+\alpha)p_2$. The inflation adjusted difference between them would then be: $\left(\frac{1+\alpha}{1+\beta}\right)(p_1 - p_2) > p_1 - p_2$. This tells us that the inflation adjusted difference is larger the bigger the proportional increase in tuition. Since the relative demand of public universities increases on the difference of tuition costs (see model setup section), we conclude that the relatively cheaper option: public universities, will then get a higher market share relative to the initial state.

2 Assuming that proportional tuition changes did change market shares, would this substitution effect be inconsistent with the homogeneity restriction from demand theory?

No it is not inconsistent. Homogeneity is a property that involves the entire vector of prices that the consumer faces as well as his income. We must see an increase in all prices by the same amount as well as the income of all individuals to evaluate homogeneity properties. We have seen changes prices of only two goods which are only a portion of an individuals' budget. All that has happened is that two goods have become relatively more expensive. In general there is both a substitution and income effect when this occurs.

Alternatively, we could assume that students only have their education as the only good in their world. Then the proportional increase in the prices of the two goods would still not violate homogeneity, as the income is not being increased by the same proportion.

2.1 What if the household incomes also increased in the same proportion as tuition rates?

Consider the case where consumers only value spending on education. The only goods that the consumers consider buying are attending either public or private universities. The inflation rate could be assumed to be exactly the change in the tuition rates, as they take up the entire basket share of goods. In this case, we would see that $\frac{1+\alpha}{1+\beta} = 1$ so that the inflation-adjusted difference between the tuition rates has not changed. Therefore there would be no change to the market share of either type of university. This means there is no violation of homogeneity.

If we believe that the market share is changing, then there must be other goods in the consumers basket, and we would have to assume that they have all increased in the same proportion. Otherwise, the entire price vector $\boldsymbol{p}$ would not have changed proportionately and it would not be possible to evaluate homogeneity properties.

3 Is the quasilinear utility function below consistent with observations?

$$u(g,s) = \delta sg - sg\log(sg) - (1-g)s\log((1-g)s) + f(s) + s\log\left(\frac{s}{1+\exp(\delta)}\right) - t_g gs - t_v(1-g)s$$

The question that the representative consumer faces is to maximize this function subject to some budget constraint.

$$\begin{aligned} \max_{g,s} \quad & u(g,s) \\ \text{s.t} \quad & t_g gs + t_v s(1-g) = M \end{aligned}$$

One important thing to notice is that this budget constraint appears in the utility function, so we may apply it to simplify the utility function partially.

We are interested in whether or not this utility function displays the properties of individuals demand depending on the inflation-adjusted difference between the prices of the universities rather than the relative price. One way to model this is to change both of the prices by the same fixed amount, and determine if the optimal market share changes with this increase. If it does not, then they are making choices based on the difference rather than the relative price. To this end we add to the tuition prices a fixed amount of Δ. However, since the individuals only consume education, when the prices increase by Δ. Every individual has an increased expenditure of $s\Delta$. We wish to hold real income constant while we adjust the prices. To compensate for this inflation, we subtract this quantity to the expenditures to ensure that there is a constant real income. The consumer's problem has become:

$$\begin{aligned}\max_{g,s} \quad & \delta sg - sg\log(sg) - (1-g)s\log((1-g)s) + f(s) + \\ & s\log\left(\frac{s}{1+\exp(\delta)}\right) - M \\ \text{s.t} \quad & (t_g+\Delta)gs + (t_v+\Delta)s(1-g) - \Delta s = M\end{aligned}$$

After setting up the Lagrangian, We take the first order conditions. The solutions to these first order conditions are the functions g^*, s^* and λ^* that determine the optimal choices for various parameters.

$$\begin{aligned}0 &= s\left(\delta + \lambda\left(t_g - t_v\right) - \log\left(gs\right) + \log\left(s\left(1-g\right)\right)\right) \\ 0 &= g\delta - g\log\left(gs\right) + \lambda\left[g\left(t_g+\Delta\right) - \Delta + \left(1-g\right)\left(t_v+\Delta\right)\right] + \left(g-1\right)\log\left[s\left(1-g\right)\right] \\ &\quad + \log\left(\frac{s}{e^\delta+1}\right) + f'\left(s\right) \\ 0 &= gs\left(t_g - t_v\right) - m + st_v\end{aligned}$$

We are interested in whether or not $\frac{\partial g^*}{\partial \Delta} = 0$. We shall compute this value using the implicit function theorem, as we cannot explicitly solve these first-order conditions. The solution for this will be by using Cramer's rule.

The determinant of the Jacobian of this system is given by:

$$|J| = \begin{vmatrix} \frac{s}{g(1-g)} & \delta + \lambda\left(t_g - t_v\right) - \log\left(gs\right) + \log\left(s\left(1-g\right)\right) & s\left(t_g - t_v\right) \\ \delta + \lambda\left(t_g - t_v\right) - \log\left(gs\right) + \log\left(s\left(1-g\right)\right) & -f''(s) & gt_g - gt_v + t_v \\ s\left(t_g - t_v\right) & gt_g - gt_v + t_v & 0 \end{vmatrix}$$

Applying Cramer's rule to obtain the value of $\frac{\partial g^*}{\partial \Delta}$.

$$|J_\Delta| = \begin{vmatrix} 0 & \delta + \lambda\left(t_g - t_v\right) - \log\left(gs\right) + \log\left(s\left(1-g\right)\right) & s\left(t_g - t_v\right) \\ 0 & -f''(s) & gt_g - gt_v + t_v \\ 0 & gt_g - gt_v + t_v & 0 \end{vmatrix} = 0$$

Since there is a column of all zero values, we can see that $|J_\Delta| = 0$. Since we know that $|J| \neq 0$ as it is the Hessian of the utility maximization problem. Using the implicit function theorem we see that $\frac{\partial g^*}{\partial \Delta} = \frac{|J_\Delta|}{|J|} = 0$. This tells us that as the prices of the education change, but the inflation-adjusted difference between them remains constant, the market share does not change. This confirms that this utility function is consistent with the above observations.

4 How is the utility function above related to the logit demand model?

Let us believe that the valuations for public and private education are distributed Gumbel. These valuations are reduced by the inflation adjusted prices of attending each university. We shall assume that the scale parameters of both valuations are equal and normalized to 1.

Allow for the location parameter for public education: $\mu_v = \delta - t_v$ and the location parameter for private education: $\mu_g = -t_g$ If the tuition rates for each were equal, the valuations for private education would be δ higher on average.

The individual will choose to attend public university based on whether or not the difference in the valuations is positive or not. The difference of two Gumbel distributions is logistic. The location parameter will be $\delta + t_g - t_v$ and the scale parameter will remain normalized to 1. The cdf is given by:

$$\frac{1}{1 + \exp\left(-(x - \delta + t_g - t_v)\right)}$$

If this logistic random variable is positive, then the valuation for the private education is higher than the valuation for the public education. We can think of demand for private education by the probability that this realization is positive, for a given set of factor prices.

This probability is given by:

$$\Pr(V \leq 0) = \frac{\exp\left(t_g - t_v - \delta\right)}{\exp\left(t_g - t_v - \delta\right) + 1}$$

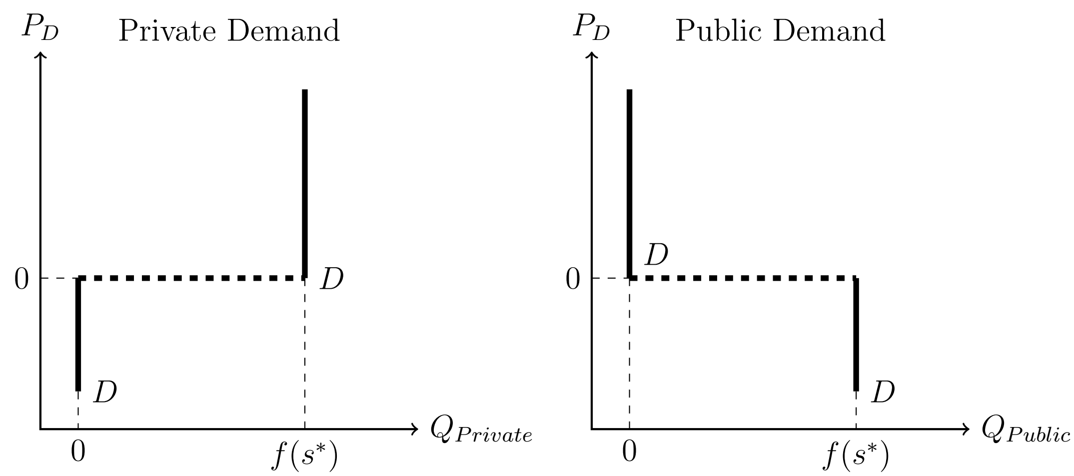
As the difference of the tuition rates $t_g - t_v = -(t_v - t_g)$ increases, the cost premium, $t_v - t_g$ of private education decreases. This means that private education and relatively cheaper. From the cdf we see that the probability of purchasing private education increases as the cost premium decreases.

For a quasilinear utility function, the demand for a good depends only on the prices and not on the income of the individual. This would imply that for a demand function x_i^*, $\frac{\partial x_i^*}{\partial M} = 0$. In the logit model, the probability that a student attends a type of university times the number of students gives the demand function for each type of university. Note that the probability of attending a particular type of university does not depend on the income of the student. It depends only on the preference of private education δ and the difference in the tuition rates. This displays the same properties of the quasilinear utility function in the percentage of people attending the university.

5 Other than the model(s) you used for (c) and (d), what is another way to model the dependence of market shares on price differences rather than ratios?

Consider a world where private and public education are perfect substitutes. If individuals are completely indifferent between consuming education at a private or public university, they will simply choose the university with the lowest price. In this case, the demand function will be based on whether or not the difference between the tuition rates is positive or negative. Absent other conditions, all students will consume the cheapest alternative, either public or private education. This is a strong form of substitution where one unit of the private university is worth the same as one unit of the public university (i.e. perfect substitutes with $\alpha = 1$).

Find below sample graphs of the demand functions against the inflation adjusted difference in the prices P_D.



6 If the federal government were considering whether its college tuition subsidies should be the same for all universities versus proportional to tuition, what are some of the reasons why private universities would favor the proportional subsidy? Are there any reasons why they would favor the constant subsidy?

Let the market share of the public universities be given by g. Assume that the university will spend its subsidy solely on reducing tuition for its students. Also, assume that when subsidies are said to be "the same" for all universities, it means that all universities will get a subsidy of "x dollars" per student, irrespective of the value of tuition. Similarly, when subsidies are said to be "proportional to tuition", it means that universities will receive a subsidy of "x percent" of tuition per student.

A proportional subsidy would effectively decrease the inflation adjusted tuition cost difference, and therefore, the market share of private universities would increase. In other words, private universities would favor a proportional subsidy because since their tuition costs are higher, they would perceive comparatively a higher benefit.

Alternatively, we could assume that the subsidy amount is a fixed quantity from the government perspective, and that the question is only how it should be allocated. In this scenario, the effective change in tuition costs would depend on the number of students in each university in addition to the allocation mechanism. If the mechanism was the "same subsidy" for either public or private school, there would be two possibilities, which are described below:

- If private colleges have a smaller share of the market compared to public colleges, their effective subsidy (i.e. change in tuition per student) will be greater than the effective subsidy for public colleges. Therefore, private colleges' market share would increase.
- If private schools have a larger market share than public colleges, the effective subsidy under the "same" mechanism would be less than the effective subsidy for public colleges. Therefore, the subsidy would actually decrease private colleges market share.

In short, private colleges would favor the "same subsidy" mechanism under the fixed subsidy amount scenario when they have a smaller market share than public colleges.

Conversely, if the mechanism for allocating subsidies were the "proportional subsidies" under the fixed subsidy amount assumption, there will be two additional scenarios, which are described below:

- If private colleges have a smaller share of the market, then they will ineluctably receive a higher effective subsidy. They would receive an bigger subsidy in absolute terms due to the higher tuition costs, but additionally the fact that their number of students is smaller would further increase the effective subsidy that private colleges receive compared to public schools. Consequently, private colleges' share of the market would increase.
- If private colleges have a larger market share than public colleges, the effective subsidy under the "proportional subsidies" would be uncertain. This occurs because even though the absolute subsidy that private colleges receive is larger due to the higher tuition costs, the actual effect is counter by the larger number of students. In particular, private colleges would favor this mechanism only when $\frac{1-g}{g} < \frac{t_v}{t_g}$. Where t_v, t_g are the tuition as before.

In conclusion, under the fixed subsidy assumption, private colleges will favor the proportional subsidy for certain when they have a smaller market share than public colleges. If they had a larger market share, private colleges' support in favor of the measure would depend on the given conditions for tuition costs and market shares.