Price Theory I: Problem Set 3 Question 1

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1 Setup

We are imagining a business where an owner is looking to hire a manager, and can select between a relative and a more talented contractor (sub-scripted by "R" and "C" respectively). He will offer a portion of the reported profits to the manager when he makes a job offer. He will keep $\beta \in (0,1)$ of the reported profits and the manager will get the rest, $(1-\beta)$. The true profits are $u+T$ where T is the talent of the manager and profits are increasing in T. A twist on this is that the manager is able to steal from the profits before it is split, but he faces a cost of stealing per dollar: $\gamma \in (0,1)$. Additionally, there is an altruistic relationship between the owner and the relative–this means that one of the two counts the other's utility as his own (preferring his own utility more, of course). There are a couple of cases that we can imagine.

Before beginning the model, however, we shall adjust some notation to make things simpler. From here on out (T_C+u) is normalized to one. And therefore, $(T_R+u) = 1-L$, where $L = T_R - T_C$, the difference between the contractor's talent and the relative's talent.

An important fact is that we assume that both managers have outside options. We suppose that they are both capable of achieving a wage of W_R–their reservation wage–even if they are not hired.

2 Model

First consider an **altruistic relative**–we will call this case one. In this scenario, we can model the utility functions of the actors as follows:
The owner's

$$
\begin{aligned}
u_O &= \beta(1-e_C) \\
&\text{or} \\
u_O &= \beta(1-e_R-L)
\end{aligned}
$$

where $e_i \in (0,1)$ is the amount of money the manager steals and i is either the relative or contractor.
The contractor:

$$
\begin{aligned}
u_C &= (1-\beta)(1-e_C) + e_C - \gamma e_C \\
u_C &= (1-\beta) + e_C(\beta-\gamma).
\end{aligned}
$$

And finally, the altruistic son:

$$u_R = \alpha[(1-\beta)(1-L-e_R)+e_R-\gamma e_R]+(1-\alpha)[\beta(1-e_R-L)]$$
$$u_R = \alpha[(1-\beta)(1-L)+e_R(\beta-\gamma)]+(1-\alpha)[\beta(1-e_R-L)].$$

Importantly, the relative leans toward being selfish, which means that $\alpha \in (.5, 1)$.

If it is the **owner who is altruistic**, we can quickly see how these change the model–we will call this case two. The utility of the owner becomes:

$$u_O = \alpha[\beta(1-e_R-L)]+(1-\alpha)[(1-\beta)(1-L)+e_R(\beta-\gamma)]$$
$$or$$
$$u_O = \alpha[\beta(1-e_C)]+(1-\alpha)W_R$$

where the first equation shows the owner's utility if his relative is working for him, and the latter shows his relative making his reservation wage.

In this scenario, the relative's utility would simply be:

$$u_R = (1-\beta)(1-L)+e_R(\beta-\gamma).$$

And lastly, it is obvious that the utility of the contractor is unchanged–this guy is never altruistic.

3 Part A

If we take a look again at the utilities of the managers, we can immediately see when they will, and when they won't steal. First, check when the owner is not altruistic–which is case 1.

3.1 Case 1

Due to this, for the non-altruistic people, their utility is equivalent to their profit: Π_i where $i \in \{C, R, O\}$. Consider the owner's profit: in this case, he is only interested in his own profit. And since the contractor can always provide a higher amount of profit to the owner (he is more talented, whereas the relative is maybe a little slow), using a Bertrand argument, we can see that the relative will never steal since the contractor can always offer a better deal. If the relative doesn't steal, and contractor steals the amount of the difference in talent, then the owner should be indifferent between them. Thus, to make things simple (rather than assuming the contractor steals some increasingly small ϵ less than the difference in talent, which is boring), we will assume that the owner likes the spunk of the thief and rewards him with a job.

$$\Pi_C = (1-\beta)+e_C(\beta-\gamma)$$

and the partial with respect to e_C

$$\frac{\partial \Pi_C}{\partial e_C} = \beta-\gamma$$

Thus, when $\beta > \gamma$ he will steal. He will steal until the owner's profit is the same as it would have been with the relative of dubious talent (i.e. $e_C = L = T_C - T_R$). He will continue to steal that exact same amount, until reaching a $\tilde{\beta}$, which is a β so large that he makes the same as he would with his outside options–mathematically, we mean:

$$\Pi_C = (1 - \tilde{\beta}) + L(\tilde{\beta} - \gamma) = W_R$$

After that point, he must steal more than L to maintain at least $\Pi_C = W_R$. We can see this easily in graphical form. Consider **Figure 1**.

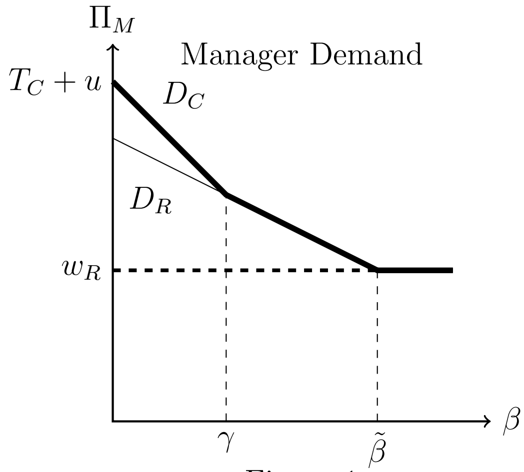

Figure 1

There we can easily track the story from left to right: the contractor doesn't steal until the shares of the owner that he would get from stealing outweigh the costs–when $\gamma = \beta$, and then steals at $e_C = L$ until $\tilde{\beta}$ at which he has to increase the amounts he steals to compensate himself up to his outside option, namely W_R.

Because we are interested in when the owner can keep people from stealing profits, we need to see this story from the owner's perspective. **Since the owner chooses β, under what conditions would he choose $\beta \leq \gamma$ thus causing the manager to not hide profits?** To answer this, consider **Figure 2** and **Figure 3**.

There is a simple condition in which you get the result in Figure 2 instead of the result in Figure 3. Namely, that condition is when:

$$(1 - L)(\gamma - \tilde{\beta}) > \gamma L$$
$$\implies 1 + \frac{\tilde{\beta}(L-1)}{\gamma} > 2L$$

Intuitively, this is a question about if, when there is theft, a larger share for the owner will be enough to compensate him for the theft while keeping the manager's profit at or above his reservation wage. Thus, in Figure 2 the owner would always maximize his profit by choosing $\beta^* = \tilde{\beta}$. Naturally, if the inequality is flipped, then we would get Figure 3. When they are equal, there would be two potential equally attractive choices.

Clearly, however, when we get the result in Figure 3, the owner could–and would–keep the manager from stealing because he would maximize his own profits by selecting $\beta = \gamma$.

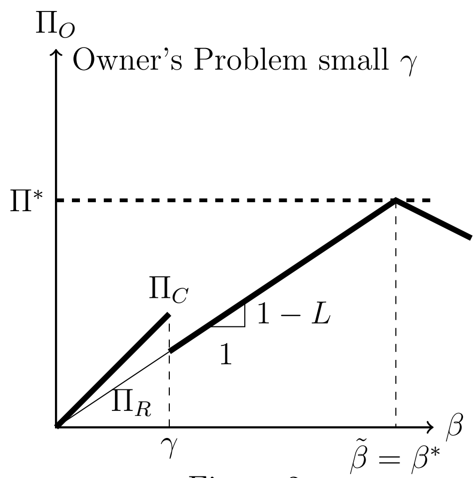

Figure 2

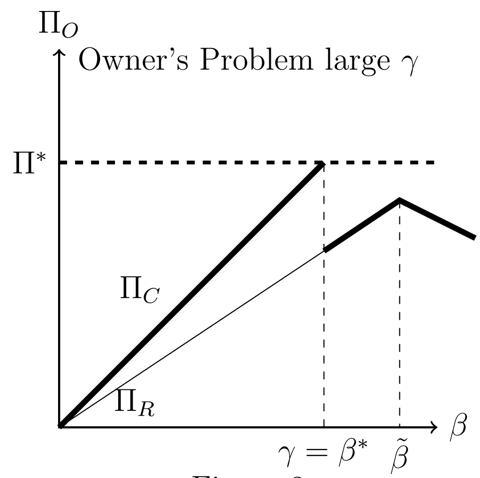

Figure 3

Therefore, the condition in which the owner can keep the manager from stealing is when L, γ, and $\tilde{\beta}$ are such that

$$1+\frac{\tilde{\beta}(L-1)}{\gamma}>2L.$$

3.2 Case 2

There is a second case however. Namely, when the owner is altruistic and the relative is not. There is again a similar condition for when the owner is able to keep his manager from stealing. There is an obvious one: $\gamma>\tilde{\beta}$, which would just mean that the cost of stealing was so high, it would never occur until after the reservation wage was past. Because this is a trivial solution, we will assume that $\gamma<\tilde{\beta}$.

Even with this assumption, there are two cases that are possible. To begin to see this, realize that because the owner is altruistic, for any β small enough he will always hire his relative–because he actually gets some utility from it. We will have the relative stealing once $\beta>\gamma$, just like the contractor. And he will steal down to the amount that the owner will be indifferent between them again (recall that our owner likes some moxie, and thus will always choose the thief all else equal). Unfortunately for our burgling relative, the talented contractor will become able to do more for the owner once his line intersects the relative's. This is demonstrated below in Figures 4 and 5–the lines intersect at η. At this point, the relative couldn't outperform the contractor even if he didn't steal. However, if $\eta<\tilde{\beta}$, then our question about how the owner could keep the manager from stealing would be moot–he couldn't ever do it because the contractor would always steal the difference. Thus, only in a very particular scenario in Figure 5 will we see the owner able to keep people from stealing.

The intuition behind the argument is the same as for the previous case. Can the owner make his share large enough to compensate for the fall in profit before raising his β to $\tilde{\beta}$? Our condition for no theft mathematically is a bit messy:

$$\begin{aligned}
\tilde{\beta} - \gamma &> \Pi_{OR} - \Pi_{OC} \\
\Pi_{OR} &= \alpha\gamma(1-L) + (1-\alpha)(1-L) \\
\Pi_{OC} &= \alpha - (1-\alpha)W_R = \alpha - (1-\alpha)(1-\tilde{\beta}) \\
\implies \tilde{\beta} - \gamma &> \alpha\gamma(1-L) + (1-\alpha)(1-L) - \alpha - (1-\alpha)(1-\tilde{\beta})
\end{aligned}$$

Unfortunately, there isn't a convenient way of simplifying this. Π_{OR} is the profit of the owner if he hires the relative at $\beta = \gamma$ and the relative chooses not to steal. The Π_{OC} is the profit the owner gets from hiring the contractor at $\beta = \gamma$. Now, we argue that he is in fact hiring the relative here, but since he is actually indifferent between them at this point, it is equivalent, and easier to view it this way. Note that the owner still gets utility from the fact that the relative would make a reservation wage. If the inequality were flipped, then we know we would get theft.

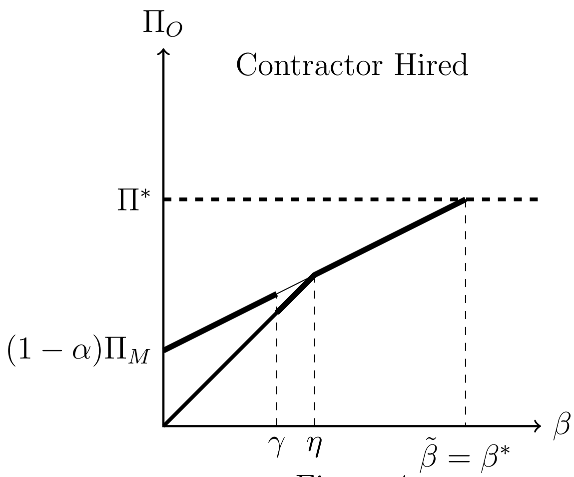

Figure 4

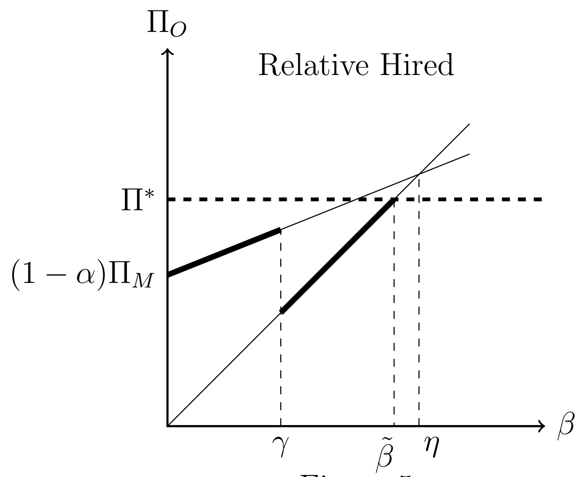

Figure 5

4 Part B

From here one out, we will assume that the owner cannot keep his managers from stealing. This implies that for a non-altruistic owner:

$$1 + \frac{\tilde{\beta}(L-1)}{\gamma} > 2L. \tag{1}$$

And for an altruistic owner:

$$\tilde{\beta} - \gamma > \Pi_{OR} - \Pi_{OC}$$

or, if we plug in for the profits,

$$\tilde{\beta} - \gamma > \alpha\gamma(1-L) + (1-\alpha)(1-L) - \alpha - (1-\alpha)(1-\tilde{\beta}). \tag{2}$$

Given these conditions, do we expect the owner's son to hide profits? This depends on multiple factors. First, if the owner is not the altruistic one (Case 1), then the son would never be hired (the guy just isn't talented enough). So he could not steal.

In Case 2, where the owner is altruistic, the son would steal if he was hired. This would depend, as it did above, on where η is. Again, consider Figures 4 and 5. In Figure 4, for βs greater than η, the contractor is able to offer enough additional profit to make up for the fact that the owner is altruistic and is thus hired. Figure 5 shows the opposite, and in this case the relative would steal from his altruistic father.

5 Part C

The above conclusions hold for the relative being a son-in-law who has an altruistic relationship with his wife. We could imagine the owner having an altruistic relationship with his daughter, and therefore there is indirect altruism for his son-in-law (or the other way around). However, this is effectively just choosing a different parameter for altruism. Meaning, α would just be multiplied through by another parameter, but the results wouldn't depend on those–the actual values of the realizations may be different, but the analysis is identical.

6 Part D

If we say that the relative is not altruistic, but instead feels guilt about stealing (modeled as higher γ), does the answer to Part B change? In short, no because Part B asks us to restricted our analysis to cases where the owner's optimal choice includes theft (i.e. the relations in equations (1) and (2) are satisfied).

Consider Case 1 where the owner is not altruistic. Since we found that the relative would never steal in this case, changing the cost of stealing for him would have no effect at all.

In Case 2, we have the same result. Because our analysis is restricted, the only item of consequence is what η is, and that will not depend on γ.

Of course, if we drop our restriction, then we may get different results. In fact, the restrictions are less likely to hold for large γs. This is obvious from the equations:

$$1 + \frac{\tilde{\beta}(L-1)}{\gamma} > 2L,$$

and, for an altruistic owner:

$$\begin{gathered}\tilde{\beta} - \gamma > \alpha\gamma(1-L) + (1-\alpha)(1-L) - \alpha - (1-\alpha)(1-\tilde{\beta}) \\ \Longrightarrow \\ \tilde{\beta} + \gamma(L-1-\alpha) > (1-\alpha)(1-L) - \alpha - (1-\alpha)(1-\tilde{\beta}).\end{gathered}$$

It is immediately clear that the left hand side of these equations decrease in γ, and are therefore less likely to hold for large γs. Thus, if we allow these conditions to slip, then we may begin to get results where there is no theft (which is explored more in Part F).

7 Part E

The owner prefers to hire his less talented son only when he is altruistic. When the owner is altruistic, and sufficiently so such that the intersection between the utility of the owner and the profit of the contractor occurs after $\tilde{\beta}$, he chooses to hire his son.

8 Part F

If we think about social institutions that encourage honesty, it is natural to view this as a higher γ level for everyone. This is similar to Part D with one exception. In Case 1 (where the owner is not altruistic), the manager would actually have a different γ (Part D was specific to the relative who is never manager in Case 1). However, the only role γ can play in Case 1 is in deciding whether the equilibrium β^* is $\tilde{\beta}$ or γ. And, because Part B requires that we ignore the cases where theft is not committed (i.e. the relations in equations (1) and (2) are satisfied), any change in γ has no effect at all on the analysis–unless we allow γ to increase until the optimal β is γ as illustrated in Figure 3.

Consequently, we need to drop our restrictions for us to get substantially different results. When we do this, we find some interesting facts. First, the managers would always be better off. For a large γ, we could get scenarios where the owner chooses $\beta^* = \gamma$, as we can see in Figure 3 (shown below). In such a case, the manager is always strictly better off–consider Figure 1 (shown below). At γ his profits are higher than they are at the only other option for a β^*, namely $\tilde{\beta}$, because he would only be making his reservation wage at that point.

We can also see that the owner is strictly better off as well. Notice that Π_O evaluated at $\tilde{\beta}$ will have the same value regardless of what γ is. But, if γ is large enough, the owner will be able to do better (as shown by Π^* in the figure).

In conclusion, if we restrict our analysis to when theft is occurring, changes in γ are uninteresting because the changes are bounded. However, if we allow it to rise to high enough levels, then we can get some results. Namely, that for high enough γ the owner and the manager are strictly better off, and therefore social institutions that encourage honesty have an effect of making everyone better off and lowing costs of effort because no one steals.

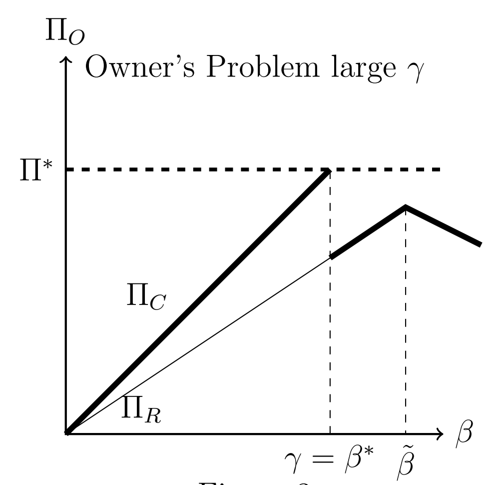

Figure 3

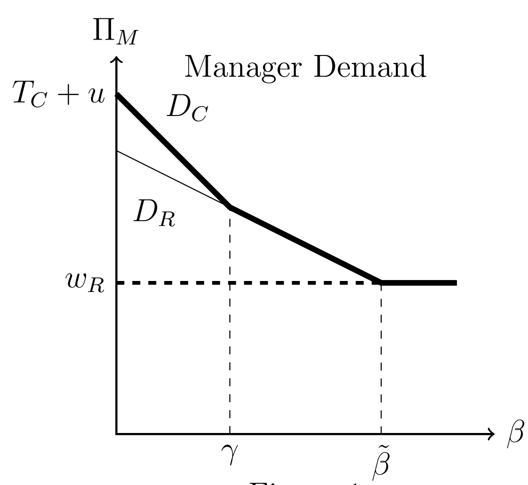

Figure 1