Lessons from Behavioral Economics: Cumulative Prospect Theory Applied to Online Lotteries

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1 Research Question

Many structural models for individuals behavior have become unpopular among economists because of strict assumptions of rationality on behavior. In this paper I examine a relaxation of this behavior and investigate whether or not it can be used to explain behavior better than other rational models. I hope to shed light on a question of whether a class of behavioral models, cumulative prospect theory, is helpful in considering mechanisms that utilize randomness. I use field data from a collection of markets for lotteries as well as secondary markets for each of their contents to determine valuations, and build a model of risk preferences from this. I seek to answer the question of whether or not this model provides additional predictive and explanatory power in examining behavior of individuals buying lotteries. I aim to analyze which components of this model of behavior are most important in these lotteries being valued so much above their expected value.

The question of interest is whether or not Cumulative Prospect theory is able to better explain decision making in low-risk lotteries than Expected Utility theory, and if so, what mechanism allows for this better explanation. In particular, the robustness of the parameters in terms of their out-of-sample fit is also of interest, as different researcher's calibrations of the model have found quite different results.

In essence, I have a field experiment and wish to determine from this data whether or not cumulative prospect theory can explain individual's behavior better than traditional "rational" models of consumer behavior. This is an ideal setting for such a project as these lotteries combine many elements that behavioral economists would term "irrational". These lotteries feature many high value but extremely low (order of 10^{-5}) probability outcomes, the potential market is mainly teenagers, and the cost of participation is low ($2.50-4.00). In such a setting, one would expect a large gain from predictive power employing a behavioral model of risk.

If there are indeed large gains to be had from considering this behavioral model over the expected utility framework, it may be that many behavioral notions of irrationality, such as bidding behavior in auctions can be explained. To generalize these results from small-scale lotteries to distinct fields such as auctions, there must be some notion of robustness of the parameter estimates, and I intend to investigate whether or not these parameters are dependent on the structure of the lottery, or are primitives in the purest sense.

2 Literature Review

? presents a broad review of where Prospect theory has been applied, as well as its problems with its application, particularly in the choice of a reference point, which appears to be very significant, but there is little guidance on what to choose beyond possibly the expected value of the lottery. Applications of the model, originally proposed by **?** exist mostly in finance and insurance. I intend to extend this body to look at the behavior of non-expert individuals in a market scenario. I believe that this area has not had many applications, likely because of the rarity of quality data outside of these fields.

The literature on Cumulative Prospect theory, the main structure of this model is primarily focused on experimental data. The literature began with the paper by **?** that suffered from problems relating to stochastic dominance, and was updated in 1992 with their paper on cumulative prospect theory.

? give a discussion on the interpretation and development of the probability weighting function used in cumulative prospect theory as well as several forms and their ensuing interpretations. The different parameters are identified with respect to psychological phenomena found relevant to the decision making process. Thus "identification" in the reduced-form sense of interpretation of the parameters become much more possible, as there are many dimensions of the model.

While most of the literature has been focused on the experimental setting, or high level financial decision making such as **?**. **?** presents an application of prospect theory to real-world data. He uses data on homeowner's choices on deductibles for home-insurance policies as a measure of moderate financial risks. The main body of the paper focuses on the standard expected utility framework, but it is extended to cumulative prospect theory in a discussion. There is however no empirical work with prospect theory on the data, as there is substantial heterogeneity within the data.

? form a structural model based on a discrete choice model, and non-parmetrically estimate the utility function as evidence for the existence of probability weights in insurance choices. This can be explained by cumulative prospect theory's decision weighting scheme. Much of the literature on experiments in this field also shows that the results are sensitive to the experimental conditions, **?**

? discuss the use of preference for lottery-payoffs for encouraging behaviors, which is relevant to my market. However the literature is underdeveloped, likely due to its legal nature in the United States.

This paper builds on some literature in the Industrial Organization Field that is based around applying behavioral models to structural estimation. Bajari and Hortacsu examine whether or not traditional models of auction behavior can explain auction bidding behavior compared to adaptive models of learning and quantal response equilibrium. However, rather than examining experimental data to draw conclusions, I intend to use field data.

The demand estimation framework that I intend to employ is the discrete-choice demand framework introduced by McFadden (1971). This paper was extended to demand estimation with a shock in **?** that is the model I intend to emulate for my discrete choice estimation. Heterogeneity was introduced into this framework in the seminal paper by **?** that develops the multinomial logit demand system that is common in demand estimation today. This alleviates some of the theoretical problems that are created by the structure of the discrete choice estimation, namely independence of irrelevant alternatives.

Estimation of these models began with the strategy first suggested by **?** commonly referred to as the Nested Fixed Point Algorithm, but has recently been superseded by the Mathematical Programming under Equality Constraints suggested by **?**. This algorithm performs extremely well under sparse Hessian and gradients, of which my method contains many. This will allow for significantly easier estimation of the demand system.

There is a focus on using Discrete choice models with prospect theory that has appeared in the Travel Behavior Literature. Such as Li,Hensher 2006 and De Palma, Picard, Waddell 2007 and Avineri and Prashker 2005. This literature, inspired by **?**, uses different behavioral models of individual behavior under lotteries directly in the utility specification of discrete choice. The paper discusses the different function forms and choices made by the researched in choosing the behavioral model to estimate choices. The emphasis on the behavioral models, in particular De Palma, is on improving prediction rather than better explanatory power for the model. In particular, all of these exercises use simulated data or laboratory experiments to evaluate consumer's decision making, rather than estimating parameters from market choices that are observed. These problems manifest themselves in a lack of a measure of willingness to pay as well and reference point, making estimation of the parameters difficult when it is even attempted. Li also notes that the existing literature on transport fails to address individual specific heterogeneity as well.

3 Data

The Data come from the *Steam Community Market*, a system of continuous double auctions that operates as a competitive market. Individuals place buy and sell orders which are then matched according to the seller's price. The lotteries of interest are sold on the market, and each of their contents are sold individually as well. Since this structure of a market converges quickly to a competitive equilibrium, we have high quality data on the equilibrium price and quantity of not only the lottery, but for each of its contents. The specific data examined are market transaction history for all items sold on the *Steam Community Market* for Counter-Strike: Global Offensive.

Counter-Strike Global Offensive is a first-person shooter game where one team (terrorists) attempt to plant a bomb and defend it while the counter-terrorists attempt to defuse the bomb. Each team has specific guns that they are able to purchase at the start of every round. The in-game cost, game balance, and meta-game all contribute to the popularity of each weapon. Players may choose to purchase purely cosmetic "skins" for their weapons which change the appearance of their weapon when they buy it. These skins are sold in lotteries called weapon crates which are dropped randomly to players in-game. The drop rates are unknown, and believed to change often. Upon receiving a weapon crate, a player may elect to spend \$2.50 to open it, or sell it on the community market.

These crates display which weapon skins they may contain, and the probability of obtaining each item within the crate is public knowledge, as required by Chinese Law. That is, the contents of the crate follow a known distribution, and can therefore be estimated under theories of risk. The contents of the crate can then be held onto, or sold at market.

3.1 Market

The market that these weapons can be sold at is the *Steam Community Market* which is run by Valve, the same company that makes Counter-Strike: Global Offensive. The market is a continuous time double-auction. Sellers may place sell orders, and buyers buy orders, and the market functions by matching the buyers and sellers, always selling at the seller's price. This is known to converge quickly to a competitive market, and will be treated as such for this project. **?** There are two complications however, there is a 15% tax placed on the market by Valve, which is taken from the seller's earnings. This is complicated by the discrete nature of the selling, and the tax always rounds up in favor of Valve. That is, an item selling for \$0.03 would return \$.02 to Valve rather than 15%. This will not be a large factor in my model as I am primarily interested in calculating demand.

For the past 30 days, there is data on hourly median market price as well as quantity sold. For the remaining time that an item has been at market, there is data for daily median price and quantity sold. The data also contain active buy and sell orders at the time of its mining: (June 7^{th} 2018). No history for these buy and sell orders is available.

3.2 Characteristics

Since the model used will be in the characteristic space rather than the product space, I am especially interested in characteristics of the different weapons in the game. I shall ignore the characteristics that will be used to determine the market for the weapon, detailed in Assumption 1 in the model section. Unique to each weapon is a float value, between 0 and 1, which indicates the wear on the weapon. Wear does not change with use, and is determined when a weapon is un-boxed. This float is distributed uniformly, but based on its value, places the weapon into different brackets for sale. We will consider all weapons in a particular bracket as homogeneous. The contents of the crate are divided into several tiers based on their rarity from being obtained in a box. All of the statistics of a particular item that can be unboxed are summarized in the tables below.

Tab. 1: Condition Probabilities

Float	Condition
0.00 - 0.07	Factory New
0.07 - 0.15	Minimal Wear
0.15 - 0.38	Field-Tested
0.38 - 0.45	Well-Worn
0.45 - 1.00	Battle-Scarred

Tab. 2: Rarity Probabilities

Probability	Rarity
.0026	Special (Gold)
.0064	Covert (Red)
.032	Classified (Pink)
.1598	Restricted (Purple)
.7992	Mil-spec (Blue)

Independent of wear, each item also has a 10% chance of being StatTrak$^{\text{TM}}$, where the gun includes a tracker that counts the number of kills a player has with this weapon. This number is reset on sale, so it can be treated simply as a binary indicator.

Conditioned on the rarirty, there are still many variants of the contents available, and there is substantial hetoegeneity in the amount of contents in each of these brackets. The largest amount of heterogeneity is in the Gold tier. Several of these lotteries have in excess of a hundred possible contents, reaching a maximum of 228 items in the gold tier. This leads to an extremely large amount of items with extremely low probabilities of being unboxed. This provides a distinction between two groups of lotteries, ones with many rare items, and the others where they are distributed evenly among the brackets.

Tab. 3: Lottery Details

	Values		Number of Contents				
Case	$\mathbb{E}[V]$	Price	#Blue	#Purple	#Pink	#Red	#Gold
Operation Wildfire	0.89891	2.5307	26	18	14	9	50
Operation Breakout	0.77011	2.5305	24	15	12	10	56
Falchion Case	0.95072	2.5323	27	24	11	9	59
Shadow Case	0.85299	2.5349	29	17	14	10	59
Huntsman Weapon Case	0.95531	3.3181	25	17	12	8	62
Spectrum Case	0.98146	2.53	34	23	15	9	68
Chroma 2 Case	1.0058	2.53	25	13	13	9	81
Chroma 3 Case	0.66099	2.53	30	19	11	10	81
Chroma Case	0.83215	2.55	23	20	10	4	81
Glove Case	0.84301	2.53	27	26	9	12	89
Operation Hydra	1.5465	4.0827	25	20	14	9	89
Gamma 2 Case	0.68335	2.53	31	22	13	7	128
Gamma Case	0.80717	2.53	31	21	11	10	128
CS:GO Weapon	4.4611	9.3248	7	6	7	2	228
eSports 2013 Case	3.2708	10.354	8	13	7	2	228
eSports 2013 Winter	1.5687	2.6441	18	9	11	3	228
eSports 2014 Summer	1.4136	2.7414	21	19	16	9	228
Operation Bravo	4.3567	12.628	26	15	9	6	228
Operation Phoenix	0.85507	2.5416	15	12	9	7	228
Operation Vanguard	1.038	2.5928	17	13	12	10	228
Revolver Case	1.1045	2.53	24	25	12	9	228
Winter Offensive	1.299	3.5079	14	14	12	6	228

4 Model

4.1 Discrete Choice Demand

Let us believe that individuals have a valuation for loot boxes characterized by some function $V(.)$. Following a discrete choice framework for demand estimation, I assume that the utility of a consumer i for loot box j in time t is given by:

$$u_{ijt} = V(x_{jt}, p_{jt}; \theta) + \xi_{jt} + \epsilon_{ij} \quad \epsilon_{ij} \sim Gumbel$$

Where p_{jt} is the price, x_{jt} are the covariates, ξ_{jt} is some demand shock common to all consumers (this can be rationalized as unobserved benefits), ϵ_{ij} is a type-1 extreme value shock unique to the consumer and good, and θ is the vector of parameters for the valuation function

The demand for this good then is given by the probability that it has the maximum utility. This can be computed using the properties of the Type-1 extreme value distribution. The maximum follows a logistic distribution, and the probability is given by:

$$\Pr(i \to j) = \frac{\exp(V(x_{jt}, p_{jt}; \theta) + \xi_{jt})}{\sum_{k \in \mathcal{F}} \exp(V(x_{jt}, p_{jt}; \theta) + \xi_{kt})}$$

In this sense, demand is non-random, and the Econometrician observes the price of the box, the covariates of the box, as well as the equilibrium quantity q_{jt}. All facets here observed, save the fact that the price and quantity are equilibrium prices and quantity rather than various points along the same demand curve.

Following the structure of **?** we consider an outside option that has some market share. The outside option is simply not partaking in any of the lotteries, and thus the valuation of this is 0. However there is still some unobserved demand ξ_{0t}. Inversion to solve for this parameter is simple, as $\Pr(i \to 0) = \frac{1}{\sum_{k \in \mathcal{F}} \exp(V(x_{jt}, p_{jt}; \theta) + \xi_k)}$. Dividing each demand equation by the outside option and taking logs yields us:

$$\log s_{jt} - \log s_{0t} = V(x_{jt}, p_{jt}; \theta) + \xi_{jt}$$

Since ξ_{jt} is unobserved by the econometrician, it takes the form of the unobserved error in the demand estimation procedure. However, it is sometimes endogenous to price as price is formed by the intersection of both supply and demand shocks. We need valid instruments for the estimation of this demand.

4.2 Instruments

Endogeneity occurs in this model via the simultaneity of supply and demand. Valid instruments for the price therefore must be supply shifters that do not affect the demand. Supply can be thought of as the players who have received a loot box randomly and wish to sell it. To simplify the dynamics of the problem, upon receiving the item individuals plan to sell it or not, so the supply of these loot boxes is heavily dependent on active players in that day and the previous day.

It is assumed that demand is a function of the long-run average number of players, or the amount of “active players” over the period of the month. This number is different from the daily players that play each day, as relatively few people are able to play each day for many reasons. However, loot boxes are given randomly to each player who plays in a day. We wish to use this fact to construct instruments for the demand. Suppose the true number of active players is N. Then daily players is $N + \epsilon_t$, i.e. some shock that determines daily player-base. We wish to use this shock ϵ_t as a cost-shifter that does not affect demand. If we estimate N by the average of all players over a significant time period, we can instrument demand using the daily deviations from this average. We instrument for price with the deviations of the current day of sale as well as the previous days.

Supply can be thought of as upward sloping with an active price floor at a price of .03 which is often binding. In the set of transactions where the price floor is binding, there is no concern of simultaneity, and therefore price is exogenously determined by the existence of the price floor. Since we use multiple instruments for price in the endogenous case, the remaining instruments will be made zero for the exogenous price case.

Demand can then be estimated off of the condition that:

$$\mathbb{E}\left[Z_t(\xi_{tj})\right] = 0 \qquad \mathbb{E}\left[p_{jt}\xi_{jt}\right] = 0 \quad \text{When } p = \$.03$$

Also present in the data is active buy orders, these are orders that there is not yet supply to fulfill. However it is a dominant strategy for place your valuation as the bid. Therefore there is no concerns about shading, and we may treat these orders as true valuations. In the case of these estimates, we should find that demand shock is equal to zero, and uncorrelated with the valuation, or that $\mathbb{E}\left[p_{jt}\xi_{jt}\right] = 0$. I note that the same choice and discrete choice framework holds for those that post unfulfilled buy orders, so there is no different model for valuations under the unfulfilled order framework.

This gives us instruments to identify price effects for each of the possible cases of the data. The other exogenous covariates present in the model

The rest of the covariates are the probabilities of obtaining each of the items, which are obviously exogenous and the last known prices of the contents of the loot boxes. We shall takes these prices as exogenous as they were determined by the supply and demand of the item in previous time periods. In this sense we are completely abstracting the problems from dynamic choices regarding optimal opening of boxes or strategies in continuous time double auctions.

We may combine all of these into a vector x_{jt} along with a constant term and our condition becomes one of $\mathbb{E}\left[x_{jt}\xi_{jt}\right] = 0$. This provides us with $k+3$ moments per data point, when there are k contents, and there are 3 parameters of interest to estimate. We are extremely over-identified, allowing for the possibility of more complicated functional forms such as splines as extensions.

4.3 Cumulative Prospect Theory

We now examine the structure of the Valuation function $V(x_j, p_j; \theta)$. Denote the probabilities of each of the contents of the lotteries by π_i and their values as x_i. We now re-index these by a permutation s_i such that $x_{s_1} < x_{s_2} < ...$ and so on. The cumulative probability Π_i can be written as $\sum_{i=1}^{K} \pi_{s_i}$. From these objects we can construct the value function for the lottery.

Cumulative Prospect Theory includes several important concepts not observe in classical decision making under risk. It incorporates reference dependence, which implies that people view things in the context of losses and gains rather than changes to their overall wealth. This is attractive for computational reasons. It also utilizes loss aversion, the notion that losses are relatively more costly than gains. The model also incorporates diminishing sensitivity, the notion that valuations are concave in losses and convex in gains. The final concept is probability weighting, the notion that consumers act as if they were facing different probabilities than what they encounter.

We note that the price of opening a case is two-fold, first the case must be bought at the market for its price, and then the price of the key, denoted p_{key} must also be paid to open the case. The time required to open the case is trivial, and will not be considered. Since gains are treated differently than losses, let the parameterization of these gains and losses be defined as:

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\alpha} & x < 0 \end{cases}$$

In this sense, α captures the risk-loving or risk-averse nature of the consumer, while λ captures their level of loss-aversion.

In cumulative prospect theory, the cumulative mass (distribution) function is weighted such that individuals overweight the tail probabilities. This is especially important in this model, as there are many high valued rare items. If this is the case, a severely distorted distribution could lead to individuals systematically overvaluing lotteries despite even being risk-averse. Colloquially, it is believed that this concept is what allows for this type of randomization to flourish in a market that is primarily populated by young people.

$$w(P) = \frac{\gamma P^{\delta}}{\gamma P^{\delta} + (1-P)^{\delta}}$$

Following the intuition provided by **?**, δ captures the notion of diminishing sensitivity, and γ captures attractiveness. Diminishing sensitivity captures how individuals discriminate cumulative probabilities away from the endpoints, i.e. the curvature of the weighting function. Attractiveness indexes how over or under-weighted certain levels of cumulative probability are treated. It can be viewed as how attractive a $50 - 50$ bet would be to a risk-neutral individual. It captures the elevation of the probability weighting function.

For some item that is within the contents of the box, a consumer's valuation is transformed by the function:

$$F(x_i) = \left[w(\Pi_{s_i}) - w(\Pi_{s_i - 1})\right] v(x_i - p_j - 2.50)$$

The valuation for the lottery can then be written as the sum of this transformation for all of the contents of the lottery:

$$V(x, \pi, p_j) = \sum_{i=1}^{K} F(x_i)$$

In this manner, this structure nests a the hyperbolic absolute risk aversion utility function of $U(X) = \sum p(x)x^{\alpha}$ often employed for its tractability. This corresponds to $\delta = 1, \lambda = 1$. In this sense, testing for these parameter values can be thought of as a test of whether or not there is any additional information gained through the application of the behavioral model for lottery valuations.

5 Estimation

5.1 Estimation

$$\xi_{jt} = \log s_{jt} - \log s_{0t} - V(x_{jt}, p_{jt}; \theta)$$

Consider a matrix of exogenous variables Z_j defined as above, we wish to estimate the parameters of V based on the condition that this matrix is orthogonal to ξ. This could be accomplished using either Nonlinear Least-Squares or Generalized Method of Moments, I shall employ the latter.

All of the orthogonality conditions combine to:

$$\mathbb{E}\left[Z'_{jt}\xi_{jt}\right] = 0$$

The estimation procedure can be written as:

$$\min_{\boldsymbol{\xi}_{j,t}, \xi_{j,t}, \beta_0, \theta} \sum_{j,t} \boldsymbol{\xi}'_{j,t} \Omega \boldsymbol{\xi}_{j,t} \tag{1}$$

$$\text{subject to: } \xi_{j,t} = \log s_{jt} - \log s_{0t} - \beta_0 - V(x_{jt}, p_{jt}; \theta) \tag{2}$$

$$\boldsymbol{\xi}_{j,t} = \xi_{j,t} \boldsymbol{Z}_{j,t} \tag{3}$$

Where β_0 is an estimated constant that incorporates the expected value of ξ and the normalization utility of the outside option. We will not interpret it as meaningful Economically. For the weighting matrix Ω, we follow the standard of using the two-stage least-squares weighting matrix $Z'Z^{-1}$.

Tab. 4: Results

$\mathbb{E}[V]$ + Price Reference Point			
α	0.56534 (2.03484)	λ	1.36844 (10.8477)
γ	1.0 (6.36280)	δ	1.0 (9.47887)
In Sample RMSE	1.23649	Out Sample RMSE	1.4337
$\bar{R}^2$	0.18880	J-Statistic	825.185
$\mathbb{E}[V]$ + Price Reference Point and Fixed Effects			
α	0.79549 (4.6084)	λ	0.60091 (9.85376)
γ	1.0 (9.79014)	δ	1.0 (21.5814)
In Sample RMSE	1.08121	Out Sample RMSE	1.10642
$\bar{R}^2$	0.51688	J-Statistic	558.41
Price Reference Point			
α	0.47457 (5.4068)	λ	0.54667 (14.56967)
γ	1.0 (10.97014)	δ	1.0 (10.85583)
In Sample RMSE	1.52584	Out Sample RMSE	1.5258
$\bar{R}^2$	0.08117	J-Statistic	860.261
Price Reference Point and Fixed Effects			
α	0.8215 (7.6682)	λ	0.3152 (7.3252)
γ	1.0 (7.6753)	δ	1.0 (15.7306)
In Sample RMSE	1.01432	Out Sample RMSE	1.07900
$\bar{R}^2$	0.54053	J-Statistic	351.73
Rational - CRRA			
α	0.17411 (6093.657)	β	-0.32488 (0.28937)
In Sample RMSE	0.98513	Out Sample RMSE	1.10097
$\bar{R}^2$	0.52163	J-Statistic	570.394