Let us believe that individuals have a valuation for loot boxes as suggested by Cumulative Prospect Theory. There is also an additive error shock that is distributed type-1 extreme value.

Therefore we can say that the utility of a consumer i for loot box j is given by:

$$u_{ijt} = V(x_{jt}, p_{jt}) + \epsilon_{ij} \quad \epsilon_{ij} \sim \mathit{Gumbel}$$

The demand for this good then is given by the probability that it has the maximum utility. This can be computed using the properties of the Type-1 extreme value distribution. The maximum follows a logistic distribution, and the probability is given by:

$$\Pr(i \to j) = \frac{\exp(V(x_{jt}, p_{jt})}{\sum_{k \in \mathcal{F}} \exp(V(x_{jt}, p_{jt}))}$$

In this sense, demand is non-random, and the Econometrician observes the price of the box, the covariates of the box, as well as the equilibrium quantity q_{jt}. All facets here observed, save the fact that the price and quantity are equilibrium prices and quantity rather than various points along the same demand curve.

Assume that there exists some outside output, and if we take the log of the shares and subtract the outside option we arrive at:

$$\log s_{jt} - \log s_{0t} = V(x_{jt}, p_{jt})$$

Let the observed equilibrium shares be given by the true demand plus some unobserved zero mean error U_D that is exogenous to the valuations $V(x_{jt}, p_{jt})$. We therefore have $\mathbb{E}\left[V(x_{jt}, p_{jt})U_D\right] = 0$

The supply of these lotteries is players who are playing the game and randomly receive the item as a reward for playing. Let us believe that there is a group of players that simply do not open these loot boxes upon receiving them. These players then immediately sell their lotteries on the steam community market. This essentially means that there is a perfectly inelastic supply of these boxes each day. However, to complicate matters there is a binding minimum exchange price of \$0.03. At this price, we treat supply as perfectly elastic, indicating that the supply curve is a corner.

The supply curve can then be given as $q^S_{jt} = \xi_j N_t$ when $p_j > .03$ and $p = .03$ otherwise. There is no randomness in the quantity supplied in the case where the price floor is binding. So endogeneity in prices only occurs when the price is non-binding. Price is also not endogenous for the active buy orders. It is only in the case when the price is determined by the intersection of supply and demand that the price is endogenous. We divide the supply by the players that are searching for weapons denoted N^*. This is the total number of players that demand weapons, and $\frac{q^S_{jt}}{N^*} = s_{jt}$. Therefore we can write supply as $s_{jt} = \xi'_j N_t$ where both terms have been dividing by N^*. Taking logs gives us:

$$\log s_{jt} = \log \xi_j + \log N_t - \log N^*$$

We can note immediately that since neither ξ_j or N^* are observed, we cannot identify these parameters, however since N^* is fixed across goods, we could identify differences in the drop rates: $\xi_j - \xi_k$.

We can formulate this problem as a GMM estimation procedure.

$$\begin{aligned}
\min \quad & (e_{jt} \quad m_{jt})' W (e_{jt} \quad m_{jt}) \\
\text{subject to: } & e_{jt} = V(x_{jt}, p_{jt}) - \log s_{jt} + \log s_{0t} \\
& m_{jt} = \log \delta_j + \log N_t - \log s_{jt} \text{ when } p_{jt} > .03
\end{aligned}$$