



Decision-making under uncertainty – A field study of cumulative prospect theory[☆]

Gregory Gurevich^a, Doron Kliger^{a,*}, Ori Levy^b

^a Department of Economics, University of Haifa, Mount Carmel, Haifa 31905, Israel

^b Coral Capital Management, United States

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ABSTRACT

The presented research tests cumulative prospect theory (CPT, [Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. *Econometrica* 47, 263–291; Tversky, A., Kahneman, D., 1981. The framing of decisions and the psychology of choice. *Science* 211, 453–480]) in the financial market, using US stock option data. Option prices possess information about actual investors' preferences in such a way that an exploitation of conventional option analysis, along with theoretical relationships, makes it possible to elicit investor preferences. The option data in this study serve for estimating the two essential elements of the CPT, namely, the value function and the probability weighting function. The main part of the work focuses on the functions' simultaneous estimation under CPT original parametric specification. The shape of the estimated functions is found to be in line with theory. Comparing to results of laboratory experiments, the estimated functions are closer to linearity and loss aversion is less pronounced.

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1. Introduction

The solution of various problems in economics, as well as in other social sciences, requires understanding of agents' behavior under risk and uncertainty. Expected utility theory (EUT) served for this purpose for a long time as a normative model of rational choice. However, actual choices often exhibit systematic deviations from this widely accepted theory, as has been reported by a range of studies.¹ To resolve this discrepancy, an alternative model, cumulative prospect theory (CPT), was developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). The purpose of this descriptive model is to explain agents' behavior in uncertain environments, which remained unexplained by EUT.²

Laboratory experiments and field studies are two possible ways to check the explanatory power of CPT. The laboratory provides the experimenter with a controlled environment for measuring agents' utility and event probabilities, the two essential elements at issue.

Laboratory results, however, may have limited applicability to real life situations, being based either on decisions regarding imaginary choices, or on small-scale artificial gambling situations. The problem was clearly pointed out by Kahneman and Tversky in their pioneering paper on prospect theory (Kahneman and Tversky, 1979, p. 265): "The reliance on hypothetical choices raises obvious questions regarding the validity of the method and the generalizability of the results. . . Laboratory experiments have been designed to obtain precise measures of utility and probability from actual choices, but these experimental studies typically involve contrived gambles for small stakes, and a large number of repetitions of very similar problems. These features of laboratory gambling complicate the interpretation of the results and restrict their generality". So, while accumulated laboratory results generally support CPT (cf. Edwards, 1996), the question of the model's validity out of the lab remains open. The purpose of the present research is to fill this gap by conducting a field study of the theory.

CPT may be viewed as a modification of EUT which keeps the model's bilinear form. One modification regards the weights assigned to the possible outcomes. CPT replaces the expectation principle with a more general rule, according to which the utility of each possible outcome is multiplied by a corresponding decision weight obtained by a specific probability weighting function (PWF), which represents a non-linear transformation of the physical probabilities. The PWF has the following characteristic features: it is regressive, i.e., it overweights low probabilities and underweights high probabilities; it is inverse-S-shaped, exhibiting

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* Corresponding author.

E-mail addresses: ggurev01@study.haifa.ac.il (G. Gurevich), kliger@econ.haifa.ac.il (D. Kliger), Ori@coralcm.com (O. Levy).

¹ The question of investor behavior and its market-wide implications has long been and remains a source of much controversy; for a recent example, see Deng (2006).

² Assessing investors' behavioral features currently comprises an area of intensive research; e.g., Venezia and Shapira (2007) analyze widely observed "weekend effect" and its extent among professional and amateur investors.

diminishing marginal sensitivity when moving away from the boundary probability levels of 0 and 1; and it is asymmetric, having a fixed point at a probability level of about 1/3. The theory allows for different PWFs for gains and losses with the above-mentioned basic properties.

Another modification is the definition of the utility function on wealth changes, rather than on wealth levels. According to CPT, the value (utility) function has a reference point, representing a status quo point such as a current wealth level. Outcomes are recorded as either losses or gains relatively to the reference point, and serve as arguments for the value function (VF), an S-shaped curve, exhibiting diminishing marginal sensitivity when moving away from the reference point, and manifesting “loss aversion”, by having a higher slope for losses than for gains. The VF and the PWFs rely on a range of empirical examinations of agents’ behavior in uncertain environments (see, e.g., Tversky and Kahneman 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996; Bleichrodt and Pinto 2000). For instance, the two functions described above yield a four-fold pattern of risk attitudes: risk seeking for gains and risk aversion for losses of low probabilities, and risk aversion for gains and risk seeking for losses of high probabilities.³

The presented research tests CPT in the financial market, using US stock option data. Option data enable us to access implicit investor preferences by applying established theoretical relationships to the observed market prices. Similarly to other studies using option prices to shed light on investors’ preferences, we extract aggregate functions.⁴ The options data in current study serve for estimating the two essential elements of the CPT, namely, the VF and the PWF. The technique employed here was also used in Kliger and Levy (2007, forthcoming) in S&P500-based tests of CPT which yielded results qualitatively close to those achieved in the current work; moreover, a technique similar to the one described below was applied by Kliger and Levy (2002) in S&P500-based elicitation of investors’ risk aversion functions.⁵ A number of related studies, such as those above-mentioned, were performed using index options. Our work sheds light on investors’ behavior using options written on individual stocks, rather than on the index. Our analysis requires the elicitation of risk-neutral and physical probabilities. In the next Section, the former are derived using option prices, and the latter are reconstructed from historical return data. The main part of the work focuses on the VF and the PWF simultaneous estimation under the original parametric specifications. The obtained results are then analyzed. Qualitatively, the results support the central principles of the model: the shapes and the properties of the estimated functions are in line with the theory. Quantitatively, the estimated functions are both more linear in comparison to those acquired in laboratory experiments and the utility function exhibits less loss aversion than was obtained in the laboratory. Overall, the empirical results suggest confirmative evidence for

the effects predicted by CPT, while the strength of the effects may be lower than that reported by lab experiments.

The rest of the paper is organized as follows: Section 2 introduces theoretical relations that are used to test CPT and describes our method for arriving at estimation equations, Section 3 presents the raw data and the method for extracting the information required for model estimation, Section 4 presents the results obtained by the empirical testing of the CPT, and Section 5 concludes.

2. A field study of CPT – Method and application

In this section we introduce theoretical relations required to test the CPT and describe our method for arriving at estimation equations.⁶ In addition, we present an econometric procedure for estimation of parameters that cannot be directly observed and measured.

2.1. A theoretical relation between marginal utilities and stochastic discount factors

The subjective stochastic discount factor (SDF) between times t and $t + \tau$ is defined (see, e.g., Ait-Sahalia and Lo, 2000; also cf. Rubinstein, 1976) as follows:

$$M_{t,t+\tau} \equiv U'(W_{t+\tau})/U'(W_t), \quad (1)$$

where W_t is the wealth level at time t and $U'(W_t)$ is the marginal utility. According to this definition, the SDF is the marginal rate of substitution of time t and $t + \tau$ utility.

According to one of the results achieved by Ait-Sahalia and Lo (2000), the marginal utilities are cross-sectionally proportional to the SDFs, i.e.:

$$U'_{t+\tau,s} = A_t M_{t+\tau,s}, \quad s = 1, \dots, S, \quad (2)$$

where $U'_{t+\tau,s}$ is the marginal utility at time $t + \tau$ under state of nature s , $A_t > 0$ is a state-independent constant and S is the total number of possible states of nature.

2.2. A relation between state prices and marginal utilities

The state price, $Q_{t,s}$, is defined as the price of a contract that guaranties its owner one \$US if state s occurs and nothing otherwise. The subjective SDF of state s can be represented (see Kliger and Levy, 2002) as a ratio of the state s price and the decision weight (DW) assigned by the subject to that state:

$$M_{t+\tau,s} = Q_{t,s}/DW(s; p_t), \quad s = 1, \dots, S, \quad (3)$$

where $DW(s; p_t)$ is the subjective decision weight of state s and p_t is the physical probability distribution function (i.e., $p_t \equiv (p_{t,1}, \dots, p_{t,S})$, where $p_{t,s}$ is the physical probability of state s).⁷

Substituting $M_{t+\tau,s}$ from (3) into (2), the following relation between state prices and marginal utilities is established:

$$U'_{t+\tau,s} = A_t Q_{t,s}/DW(s; p_t), \quad s = 1, \dots, S. \quad (4)$$

2.3. A relation between option prices and state prices

We employ the option-pricing model developed by Cox and Ross (1976). According to their risk-neutrality argument, the call option price is determined as follows:

³ An extensive psychological foundation explaining the characteristic features of the utility function and the PWFs may be found in Tversky and Kahneman (1981).

⁴ The empirical analysis in Jackwerth (2000), for instance, employs a model of the economy implying the existence of a representative investor Constantinides (1982) to derive risk aversion functions across wealth. Note that in a reference dependent model, such as CPT, the wealth distribution problem raised by Constantinides is of less concern.

⁵ However, in contrast with Kliger and Levy (2002, 2007, forthcoming), our paper implicitly tests the relevance of the narrow framing assumption at the firm level, an assumption consistent with results obtained by, e.g., Abbink and Rockenbach (2006) and Kroll et al. (1988). The former study compared performance of students and professional traders in an option pricing experiment and found, inter alia, that investors overlook arbitrage considerations; the latter study experimentally tested basic assumptions underlying the capital asset pricing model and found that investors disregard return covariances. Both results corroborate the narrow framing assumption by showing that investors actually focus on individual assets’ performance in forming their decisions.

⁶ Leland (1980) presents a closely related model linking physical and risk-neutral probabilities. However, as the present work considers a more general case, for instance requiring conversion of probabilities into decision weights, the derivation of the appropriate model is outlined below.

⁷ Recall that in CPT framework the decision weights are determined by the PWF, while according to EUT they are equal to the physical probabilities of the relevant states of nature.

$$C_t = \exp(-R_t^F \tau) \sum_{S_{t+\tau,j}=-\infty}^{\infty} q_{t,j} \max(0, S_{t+\tau,j} - K), \quad (5)$$

where C_t is the call option price at time t , R_t^F the risk-free rate of return till the option's expiration, τ is the time left to the option's expiration, $q_{t,j}$ is the expiration date risk-neutral probability of state j , $S_{t+\tau,j}$ is the price of the underlying asset at the expiration date in the case of state j occurrence and K is the option's strike price. Differentiating (5) twice with respect to K yields the state prices (cf. Breeden and Litzenberger, 1978):

$$Q_{t,j} = \exp(-R_t^F \tau) q_{t,j} = \partial^2 C_t / \partial K^2 | K = S_{t+\tau,j}. \quad (6)$$

Thus the state price equals risk-neutral probability multiplied by a discount factor.

2.4. The estimation equation

Eq. (6) determines the price of state j at time t . Let $s = j$ and substitute $Q_{t,s}$ from (6) into (4), to obtain $U'_{t+\tau,s} = A_t \exp(-R_t^F \tau) q_{t,s} / DW(s; p_t)$, $s = 1, \dots, S$. Rearranging the terms yields:

$$q_{t,s} = \frac{U'_{t+\tau,s} DW(s; p_t)}{A_t \exp(-R_t^F \tau)}, \quad s = 1, \dots, S. \quad (7)$$

To simplify the notation, substitute $B_t = \frac{1}{A_t \exp(-R_t^F \tau)}$ into (7) to obtain the estimation equation:

$$q_{t,s} = B_t U'_{t+\tau,s} DW(s; p_t), \quad s = 1, \dots, S, \quad (8)$$

where $U'_{t+\tau,s}$ and $DW(s; p_t)$ are determined by the VF and PWF, respectively, so that estimating (8) by employing multiple regression analysis yields values for CPT parameters α and λ (as characterizing the VF according to the specification presented below), and γ and δ (as characterizing PWFs according to the specification presented below).⁸

Evidently, the explicit form of (8) depends on the functional form of the VF and the PWF, enabling thereby testing CPT under alternative functional specifications. The risk-neutral probabilities, $q_{t,s}$, in (8) may be reconstructed from (6), while the state price $Q_{t,s}$ is approximated by pricing a “butterfly spread”, a position achieved by short selling two call options with strike prices $K = S_{t+\tau,s}$ at a prices of $C_t(K = S_{t+\tau,s})$ each, and purchasing two call options with strike prices $K = S_{t+\tau,s} + \Delta$ and $K = S_{t+\tau,s} - \Delta$ at prices $C_t(K = S_{t+\tau,s} + \Delta)$ and $C_t(K = S_{t+\tau,s} - \Delta)$ respectively. In case state $S_{t+\tau,s}$ occurs at the expiration date, the position holder receives a payoff of Δ . The price of state s is obtained by dividing the butterfly spread price by Δ , which normalizes the payoff:

$$Q_{t,s} \approx \frac{C_t(K = S_{t+\tau,s} + \Delta) - C_t(K = S_{t+\tau,s})}{\Delta} - \frac{C_t(K = S_{t+\tau,s}) - C_t(K = S_{t+\tau,s} - \Delta)}{\Delta}. \quad (9)$$

Noting that $Q = \frac{\partial^2 C}{\partial K^2}$ from (6) is approximated by (9) with $\Delta \rightarrow 0$, the above state price is used to reconstruct a probability $q_{t,s}$, which is required to estimate (8).

In the next section we present an explicit form of (8), using the original specifications proposed by Tversky and Kahneman (1992).

2.5. The model's estimation under the original specification

In general, an empirical test of CPT refers to lotteries of the form $XP \equiv (x_1, p_1; \dots; x_S, p_S)$, where $x_1 \leq \dots \leq x_K \leq 0 \leq x_{K+1} \leq \dots \leq x_S$, i.e., the states numbered 1 to K are loss states and the states $K+1$ to S are gain states.

We focus on the VF and PWF specifications of Tversky and Kahneman (1992). The reflective form of the VF is⁹:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0, \\ -\lambda(-x)^\alpha & \text{if } x < 0. \end{cases} \quad (10)$$

The respective PWFs for gains and losses are:

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \quad \text{and} \quad w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}. \quad (11)$$

Using (10) and (11), we get the following marginal utility:

$$U'_{t+\tau,s} = \begin{cases} \alpha x_s^{\alpha-1} & \text{if } x_s \geq 0, \\ -\lambda \alpha (-x_s)^{\alpha-1} & \text{if } x_s < 0, \end{cases} \quad (12)$$

and decision weights:

$$DW^+(s, p) \equiv w^+\left(\sum_{i=s}^S p_i\right) - w^+\left(\sum_{i=s+1}^S p_i\right), \quad (13)$$

and

$$DW^-(s, p) \equiv w^-\left(\sum_{i=1}^s p_i\right) - w^-\left(\sum_{i=1}^{s-1} p_i\right). \quad (14)$$

The explicit form of (13) and (14) makes use of the PWF specification in (11).

An estimation of Eq. (8) under the specification presented by (12)–(14) should provide estimates of the parameters characterizing the VF and PWF, namely, α , λ , δ and γ (note that in the special case $\delta = \gamma = 1$ decision weights are equal to physical probabilities). The testable hypothesis and its alternative are as follows:

$$\begin{aligned} H_0 : & \quad \lambda = \delta = \gamma = 1, \\ H_1 : & \quad \lambda > 1; \delta < 1; \gamma < 1. \end{aligned} \quad (15)$$

To wit, H_0 epitomizes the case of EUT while H_1 generalizes to CPT; rejecting the null would corroborate loss aversion or non-linear transformation of physical probabilities.

2.6. A reconstruction of the physical probabilities

In this Section, the physical probabilities, i.e., $p_{t,s}$ that constitute p_t in (8), are estimated, by the following two-stages: (i) the stock price evolution is approximated by GARCH(1,1), and (ii) the physical probabilities of the underlying stock's prices are bootstrapped.

2.6.1. Stage (i)

A GARCH(1,1) process estimated for the natural logarithm of the underlying stock empirical return is used to represent the stock price evolution.¹⁰ A fundamental element of the estimation process is the time t estimation of the model's day $t+1$ error term, e_{t+1} , which determines the return evolution.

2.6.2. Stage (ii)

The variance of e_{t+1} is determined according to the GARCH(1,1) structure. Particularly, the conditional variance at $(t+1)$ is given by $\sigma_{t+1}^2 = c_3 + c_4 e_t^2 + c_5 \sigma_t^2$, where all of the right-hand side values are known at time t . The random variable e_{t+1} is distributed normally with mean 0. A random selection from this distribution provides a single realization of e_{t+1} , and thereby a forecast of $\ln(R_{t+1})$ accord-

⁸ Testable hypothesis concerning these parameters is presented below, after the introduction of the explicit form of (8).

⁹ The general form is $v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases}$. VFs that satisfy $\alpha = \beta$ are dubbed “reflective”, and are homogeneous in the payoff x . As investment levels are unobservable, only reflective VFs are empirically identifiable.

¹⁰ Appendix A presents a detailed description of the process.

Table 1

General description of the stocks.

S&P100 weight	Stock	Ticker	Run no.
2.32	American Int'l	AIG	01
1.24	Ameritech	AIT	02
1.09	Amoco	AN	03
1.27	American Express	AXP	04
1.27	Boeing Company	BA	05
1.53	Bank America Co.	BAC	06
1.90	Bell Atlantic	BEL	07
2.85	Bristol-Myers	BMJ	08
1.82	Citicorp	CCI	09
2.33	Du Pont	DD	10
2.04	Walt Disney Co.	DIS	11
1.66	Ford Motor	F	12
7.29	General Electric	GE	13
1.36	General Motors	GM	14
1.73	Hewlett-Packard	HWP	15
3.05	Int. Bus. Mach.	IBM	16
2.48	Johnson & Johnson	JNJ	17
5.18	Coca Cola Co.	KO	18
1.21	McDonald's Co.	MCD	19
0.99	MCI Comm.	MCQ	20
1.01	Minn Mining	MMM	21
1.63	Mobil Corp.	MOB	22
3.75	Merck & Co.	MRK	23
0.89	Northern Telecom	NT	24
1.65	PepsiCo Inc.	PEP	25
1.04	Schlumberger Ltd	SLB	26
2.64	AT&T Co.	T	27
3.31	Wal-Mart Stores	WMT	28
4.64	Exxon Co.	XON	29
0.89	Xerox Co.	XRJ	30
66.06	Total	Total	01–30

Table 2

VF and PWF estimation results.

Estimated parameter	Mean value	Median value	Standard deviation
α	0.98	0.98	0.03
λ	1.10	1.04	0.43
γ	0.92	0.84	0.36
δ	0.77	0.76	0.13

Table 1 presents the options' underlying stocks. For each of the 30 stocks, the Table reports the ticker, the firm's name and the weight of the stock in the S&P100 index (as of May 1998). The stocks are numbered according to the alphabetical order of the tickers. The last row reports the aggregate weight of the 30 stocks in the S&P100 index.

4. Empirical testing

We use the functional forms given by Eqs. (10) and (11) to estimate Eq. (8) for each one of the 30 stocks.¹⁵ Table 2 presents the estimation results.

Recall that γ and δ determine the PWFs' fixed points. According to the mean (median in parentheses) estimates, the fixed points are at probabilities of 0.47 (0.44) and 0.41 (0.41), for gains and losses, respectively.

Table 3 provides a descriptive statistics of the results, manifesting the heterogeneity characterizing the data. For each parameter of the original specification, the Table reports the minimal and maximal estimated values among the 30 stocks; the theoretical range of each parameter according to CPT; and the number estimates (out of 30) deviating from that range.

Fig. 1 depicts the estimated PWFs for gains (thick curve) and losses (fine curve), according to the median values presented in Table 2.¹⁶

Table 4 shows the results of an additional analysis, performed on the pooled dataset of the 30 stocks and 1686 observations. These results comprise a sort of an "average" outcome, which ignores possible differences between individual stocks.

According to the estimated values of γ and δ , the PWFs' fixed points are at probabilities of 0.47 and 0.41 for gains and losses, respectively. Evidently, the results of this test are fairly close to those achieved for 30 separate samples (see Table 2).

4.2. Quantitative analysis

Quantitatively, the estimated parameters' ranges conform with theory. Specifically:

1. $\alpha < 1$: This result corresponds to a CPT assertion, according to which the VF exhibits diminishing marginal sensitivity, i.e., the marginal gain or loss value deteriorates when moving away from the reference point.
2. $\lambda > 1$: This result points at the loss aversion, resulting in steeper VF slope for losses than for gains.

¹⁵ For the empirical testing, we use the status quo level as the reference point, as done also in related literature, e.g., Kliger and Levy (forthcoming); more on reference point can be found in Kliger and Kudryavtsev (2008). For robustness, we conducted additional analysis with two alternative reference points, namely: (i) the risk-free rate on the day of each butterfly spread evaluation, and (ii) the historical US nominal risk-free short return (cf. Siegel, 1992, p. 230, Table 1, M–P period). The results using these alternative reference points hardly change.

¹⁶ As the mean and median results are close to each other, so is their graphical depiction. The graphical depiction of the VF according to the parameters presented in Table 2 is highly linear, thus not presented.

ing to $\ln(R_{t+1}) = c_1 + c_2 \ln(R_t) + e_{t+1}$. After conducting a random selection for all trading days until the option's expiration, summing up all the logarithmic daily returns provides a single realization of the logarithmic return of the stock over the chosen period. This process may be repeated any number of times to get a desirable number of different return realizations.¹¹

3. Data

The empirical test we present here relies on American¹² equity-options traded at Chicago Board Options Exchange written on the stocks of 30 companies leading the S&P100 market index, from 01/01/1991 to 12/31/1995.¹³ The options data, taken from Berkeley Options Database, were provided to us by Gurdip Bakshi, Dilip Madan and Nikunj Kapadia.¹⁴

Apart from the options data, we use time series of the rates of return of the underlying stocks, taken from The Center for Research in Security Prices (CRSP) database. As a proxy for the risk-free interest rates we use returns on 3-months Treasury Bills (TB03), which were taken from www.frbchi.org (the link is no longer available; Treasury Bills data can now be found at www.federalreserve.gov).

¹¹ Appendix B shows how the physical probabilities of possible expiration date returns are reconstructed and presents a detailed description for the relevant state characteristics.

¹² While American options entail the possibility of early exercise, there are no European options on individual stocks. The problem introduced by employing American options is attenuated, however, as their implied volatilities are rather similar to European and early exercise is not very often executed (cf. Bakshi et al., 2003, pp. 119–121, Table 2).

¹³ A comprehensive description of the raw data and its manipulation are presented in Appendix C.

¹⁴ More detail on the data used here can be found in Bakshi et al. (2003). We are grateful to them for providing us with these data.

Table 3
Estimation results; descriptive statistics.

Estimated parameter	Minimal value	Maximal value	CPT theoretical area	Deviations (out of 30)
α	0.91	1.06	Between 0 and 1	6
λ	0.45	2.29	Higher than 1	13
γ	0.49	2.21	between 0 and 1	6
δ	0.46	1.10	Between 0 and 1	1

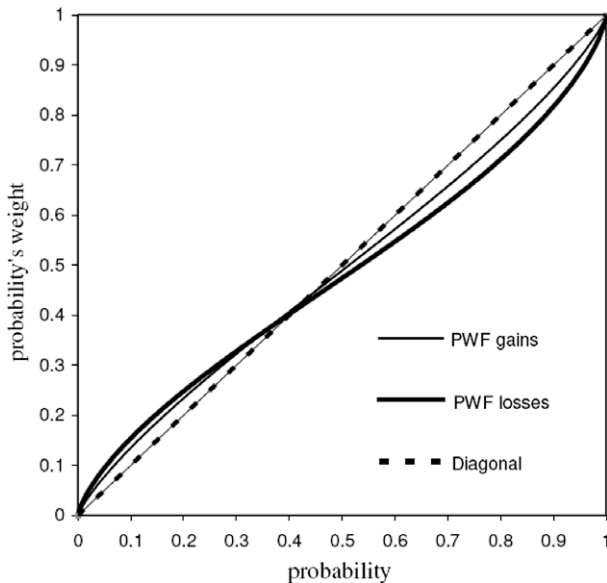


Fig. 1. PWF for gains and losses according to median estimation results.

Table 4
The results of estimation for the combined sample.

Estimated parameter	Estimated value	Standard deviation
α	0.98	0.005
λ	1.12	0.044
γ	0.91	0.033
δ	0.77	0.018

3. $\gamma < 1$, $\delta < 1$: According to this result, the PWFs for gains and losses also exhibit diminishing marginal sensitivity, i.e. the marginal impact of probability change deteriorates when moving away from any one of the natural boundaries, the 0 and 1 probability levels.

4.3. Qualitative analysis

As a qualitative evaluation of the results, Table 5 compares them to the experimental results of Tversky and Kahneman (1992).

The quantitative comparison reveals some differences, comprising, as we show below, a certain trend or tendency.

1. Our estimated value of α is higher than that its experimental value, the former being numerically close to 1. The meaning of this difference is that our estimated VF is more linear than the estimated VF in Tversky and Kahneman (1992), which in turn means that the marginal sensitivity of the former diminishes “more slowly” than that of the latter.

2. Our estimated value of λ is lower than that in Tversky and Kahneman (1992), the former being numerically closer to 1. The meaning of this is that our results also exhibit the loss aversion effect, but it is less prominent than that in Tversky and Kahneman (1992).

Our estimated values of γ and δ are both higher than those in Tversky and Kahneman (1992) (respectively), the former being numerically closer to 1.¹⁷ I.e., our PWFs for gains and losses are generally of the form suggested by the CPT, but their form is more linear than that estimated in the laboratory experiment.

To sum up, the comparison between our results and those obtained in a laboratory test reveals that the estimated VF and PWFs in our data are more linear, and the VF exhibits loss aversion that is less pronounced. Perhaps, it should be emphasized that these conclusions refer to the average results only. The separate results for 30 stocks reveal considerable heterogeneity, and the general conclusion may not apply to all of the separate samples.¹⁸ In general, despite the data heterogeneity and even the observed deviations from the theoretical assertions (see Table 3), most of the obtained results fit the CPT framework, as well as the mean and median results presented in Table 2. Occasional deviations from the theoretical values (and therefore functional shapes) are observed and reported by related experimental studies as well (e.g., Bleichrodt and Pinto, 2000, pp. 1492–1492; Abdellaoui, 2000, pp. 1508–1509; Camerer and Ho, 1994, pp. 188–190, especially Fig. 7). As we note earlier, the estimated functions are “more linear” than those obtained in the laboratory, but differ from strictly linear functions in a statistically significant manner.

This conclusion also applies to a number of additional possible comparisons which go beyond the now classical work by Tversky and Kahneman. To complete the picture, we present a brief summary of some other results of laboratory experiments conducted in different areas of decision making. The results obtained in these experiments can be meaningfully compared to ours, which served as obvious criterion of their selection out of otherwise vast number of similar works in this area.

1. Laboratory experiment by Camerer and Ho (1994): $\gamma = 0.56$, PWF fixed point (for gains) at 0.3.
2. Laboratory experiment by Wu and Gonzalez (1996): $\alpha = 0.52$, $\gamma = 0.71$, PWF fixed point (for gains) between 0.3 and 0.4.
3. Laboratory experiment by Bleichrodt and Pinto (2000): $\alpha = 0.779$, $\gamma = 0.674$.
4. Laboratory experiment by Abdellaoui (2000): $\delta = 0.70$, $\gamma = 0.60$.

Evidently, a comparison between these results to those presented in our Tables 2 and 4 leads to aforementioned conclusions concerning general forms and quantitative characteristics of the estimated functions. Regarding the values of the estimated CPT parameters, the evident disagreement with results achieved in other studies may at least partially stem from our work’s non-laboratory nature. Real market environment may differ in important ways from experimentally designed situations, and subjects may apply different decision rules when confronted with real as opposed to laboratory decision tasks. Consequently, field study results may differ considerably from laboratory outcomes, as is the case in our analysis. As already mentioned, results qualitatively close to reported here were also achieved by Kliger and Levy

¹⁷ It may be worth noting that in the combined sample estimation this result is replicated (see Table 4), while, being close to 1, γ and δ however differ from 1 in a statistically significant manner (for each one of $\gamma = 1$ and $\delta = 1$ hypotheses we get $P < 0.01$ in Wald Test). The same is right for α and λ in clauses 1 and 2 above.

¹⁸ The same phenomenon is recognized by Tversky and Kahneman in connection to their laboratory test (cf. Tversky and Kahneman, 1992).

Table 5

A comparison with Tversky and Kahneman (1992).

Estimated parameter	Median value among laboratory experiment participants	Median value across the 30 stock samples
α	0.88	0.98
λ	2.25	1.04
γ	0.61	0.84
δ	0.69	0.76

(2007, forthcoming) in S&P500-based tests of CPT. The magnitudes of the estimated parameters obtained by these latter studies are different from the estimates obtained in our empirical analysis; likewise, Ait-Sahalia and Lo (2000) report less linear preferences and a risk aversion pattern different from that in our study. These differences may be at least partially related to the fact that the data we employ are different; for instance, we use individual stocks' options, rather than index options, to estimate the CPT parameters. The narrow framing assumption implicit in our analysis (consistent with aforementioned results by Abbink and Rockenbach (2006) and Kroll et al. (1988)), as well as higher noise due to the relatively low trading volume and relatively high volatility, may also affect the estimation results.

The mean and median values obtained for the PWFs suggest non-linearity of the decision weights, in line with CPT (see Table 2). Regarding the low estimated (although statistically significant) loss aversion, we have tried to discern possible differences among underlying stocks, which may be related to the wide range of λ 's obtained by separate estimation (see Table 3). For instance, we have tested for correlations with individual stock characteristics such as firms' CAPM Betas, weight in the S&P 100 index, and monthly implied volatility.¹⁹ Out of these possible explanatory variables, only the latter is correlated with the estimated value of loss aversion (λ), with a correlation coefficient of 0.43 suggesting that higher loss aversion may be exhibited when investors deal with more volatile stocks (Table 6).²⁰

Having presented the general conclusions, we henceforth perform sensitivity analyses to examine the robustness of the VF and PWF parameter estimates, by imposing values for some of the parameters and conducting restricted estimation for the remaining ones. Overall, the results suggest that forcing the PWF parameters, γ and δ , to deviate from the unrestricted estimation values introduces estimation difficulties, while restricting the VF parameters, α and λ , is inconsequential, as long as the values define a VF which is sufficiently close to linear and manifests low loss aversion.

Specifically, restricting the PWFs to linearity ($\gamma = \delta = 1$), seems to be unjustified by the data, as 11 out of 30 cases result in an estimation problem (near singular matrix in seven cases, and no convergence in the others). The 19 cases which converged resulted in the mean estimated values of 0.99 and 0.98 for α and λ , respectively (the median values, as well as those obtained using combined sample of all stocks, are essentially the same).

Moreover, assuming γ and δ values closer to the experimental results, for instance $\gamma = \delta = 0.6$ (cf. Tversky and Kahneman, 1992), the respective estimated values for α and λ are 0.96 and 0.94.

We turn now to imposing restrictions on the VF. Assuming $\alpha = \lambda = 1$, the estimated values for γ and δ are 0.81 and 0.86, respectively; assuming $\alpha = 0.95$ and $\lambda = 1.5$, the respective values are 0.66 and 1.04; and imposing $\alpha = 0.88$ and $\lambda = 2.25$, as in Tversky and Kahneman (1992), the mean estimated values for γ and δ are 0.52 and 1.05, respectively (the median values, as well as those obtained using combined sample of all stocks, are essentially the same).

Overall, therefore, up to a certain point the results suggest sub-additive PWFs, while from a certain point on, imposing a more concave VF and higher loss aversion forces the estimated PWFs into a region whereby the estimated parameters are not immediately interpretable ($\delta > 1$).

5. Summary and conclusions

The presented research tests CPT in the financial market, using US stock option data. Option prices contain information about actual investors' preferences, which are possible to elicit using conventional option analysis and theoretical relationships. The options data in this study are used for estimating the two essential elements of CPT, namely, the VF and the PWF. The main part of the work focuses on the functions' simultaneous estimation under the original parametric specifications used in Tversky and Kahneman (1992). Qualitatively, the results support the central principles of the model: the shapes and the properties of the estimated functions are in line with the theory. Quantitatively, the estimated functions are both more linear in comparison to those acquired in laboratory experiments, and the utility function exhibits less loss aversion than was obtained in the laboratory. It is worth pointing out that the above-mentioned conclusions are reached while ignoring a distinction between different stocks. When estimated separately for the individual stocks, the obtained functions manifest a certain degree of heterogeneity and the general conclusions do not necessarily apply to each one of them. The reasons for this heterogeneity may lie in some differences between the underlying stocks.²¹ The estimated functions may vary depending on the investors' attitude toward a specific stock, which in turn may depend on such factors as company size and trading volume.

Appendix A. The GARCH(1,1) process

The GARCH(1,1) process used here to represent the stock price evolution is as follows. The (natural log of the) stock return depends linearly on its first lag, and the conditional variance depends linearly on its own lag and the lagged squared model error²²:

$$\ln(R_t) = c_1 + c_2 \ln(R_{t-1}) + e_t, \quad (16)$$

and

$$\sigma_t^2 = c_3 + c_4 e_{t-1}^2 + c_5 \sigma_{t-1}^2, \quad (17)$$

where R_t is the stock's day t return, e_t is the model's day t error term and σ_t^2 is the day t conditional variance. Estimation of the system (16) and (17) provides values for the parameters c_1, c_2, c_3, c_4 and c_5 . The estimated values enable forecasting the stock price evolution from a given date on. A fundamental element of the estimation process is the time t estimation of e_{t+1} , which determines the return evolution.

¹⁹ A related issue concerned with the observed connection between implied volatility and stock returns has been a subject of a range of studies; for instance, Hibbert et al. (2008) suggest a behavioral explanation for the aforementioned phenomenon.

²⁰ Linear regression of loss aversion on the logarithm of the implied volatility reveals statistically significant ($p < 0.05$) positive correlation (regarding the involved variables distribution, the probability of Jarque–Bera value for both variables is higher than 0.1, suggesting that the normality hypothesis cannot be rejected). Also, nonparametric estimation performed while removing from 5 to 10 extreme observations invariably reveals strong positive correlation ($p < 0.05$). Overall, the results suggest that the magnitude of loss aversion may be positively correlated with the implied volatility of the underlying stock.

²¹ Cremers et al. (2008) suggest an analysis of informational content of individual equity-option prices.

²² The symbol σ_t^2 denotes a conditional variance because it refers to a forecasted variance that is based on a previous periods' information.

Table 6

Description of the data used for estimation.

Run no.	Ticker	Sample size	Number of monthly cross-sections	Average number of observations in monthly cross-sections	Average implied volatility in monthly cross-sections	Average time to expiration in monthly cross-sections
01	AIG	100	57	1.75 (0.89)	0.233 (0.034)	0.070 (0.010)
02	AIT	20	19	1.05 (0.23)	0.230 (0.037)	0.069 (0.011)
03	AN	9	9	1.00 (0.00)	0.251 (0.031)	0.071 (0.011)
04	AXP	24	21	1.14 (0.36)	0.347 (0.070)	0.072 (0.011)
05	BA	38	34	1.12 (0.33)	0.299 (0.043)	0.070 (0.010)
06	BAC	42	40	1.05 (0.22)	0.339 (0.062)	0.071 (0.010)
07	BEL	25	24	1.04 (0.20)	0.252 (0.030)	0.072 (0.010)
08	BMJ	57	46	1.24 (0.52)	0.233 (0.027)	0.069 (0.010)
09	CCI	40	36	1.11 (0.32)	0.378 (0.104)	0.072 (0.010)
10	DD	29	28	1.03 (1.19)	0.256 (0.034)	0.073 (0.009)
11	DIS	139	51	2.72 (2.47)	0.302 (0.041)	0.071 (0.009)
12	F	50	46	1.09 (0.28)	0.325 (0.049)	0.070 (0.010)
13	GE	93	52	1.79 (0.89)	0.229 (0.039)	0.070 (0.009)
14	GM	55	49	1.12 (0.33)	0.326 (0.053)	0.070 (0.010)
15	HWP	156	60	2.60 (1.34)	0.339 (0.055)	0.070 (0.009)
16	IBM	171	60	2.85 (1.47)	0.297 (0.061)	0.070 (0.009)
17	JNJ	65	41	1.58 (0.97)	0.263 (0.047)	0.071 (0.010)
18	KO	44	32	1.37 (0.75)	0.268 (0.042)	0.071 (0.010)
19	MCD	26	26	1.00 (0.00)	0.286 (0.052)	0.074 (0.010)
20	MCQ	28	27	1.04 (0.19)	0.406 (0.071)	0.071 (0.011)
21	MMM	80	47	1.70 (0.66)	0.211 (0.026)	0.070 (0.010)
22	MOB	34	31	1.10 (0.40)	0.203 (0.029)	0.072 (0.010)
23	MRK	104	39	2.67 (2.18)	0.295 (0.057)	0.069 (0.009)
24	NT	7	7	1.00 (0.00)	0.438 (0.072)	0.074 (0.011)
25	PEP	20	18	1.11 (0.32)	0.294 (0.056)	0.075 (0.011)
26	SLB	47	40	1.17 (0.45)	0.274 (0.029)	0.070 (0.010)
27	T	28	27	1.04 (0.19)	0.244 (0.043)	0.072 (0.010)
28	WMT	38	31	1.22 (0.50)	0.318 (0.056)	0.070 (0.010)
29	XON	19	18	1.05 (0.23)	0.202 (0.028)	0.069 (0.009)
30	XRX	114	60	1.90 (1.07)	0.269 (0.042)	0.070 (0.009)
01–30	AIG–XRX	1702	1076	1.58 (1.15)	0.284 (0.070)	0.071 (0.010)

Appendix B. Reconstruction of physical probabilities

The physical probabilities of possible expiration date returns are reconstructed by bootstrapping 100,000 possible realizations of the stock's return over a period until the option's expiration. The required probabilities are reconstructed then according to the following formula:

$$p_{a,b} = \left[\sum_{i=1}^{100,000} I(a \leq R_i \leq b) \right] / 100,000, \quad (18)$$

where $p_{a,b}$ is the physical probability of return to fall between values a and b , R_i is the i th realization out of 100,000 simulation results and $I(\cdot)$ is a binary indicator receiving value 1 if the condition in the parentheses is satisfied and 0 otherwise. To obtain precise numerical values for the decision weights in (14) and (15), the high resolution of 10^{-7} was used in calculations of the physical probabilities $P_{a,b}$.

The characteristics of the relevant state are approximated by butterfly spread's pricing. Fig. 2 facilitates an explicit identification of the considered state's characteristics.

The horizontal axis in Fig. 2 is the stock price on the option's expiration date, and the vertical axis is the value of a butterfly spread on this date. The stock prices characterizing the butterfly spread are as follows:

K_h – the high strike price among the options included in the butterfly spread (in terms of Eq. (9), $K_h = S_{t+\tau,S} + \Delta$), K_m – the middle strike price among the options included in the butterfly spread (in terms of Eq. (9), $K_m = S_{t+\tau,S}$), K_l – the low strike price among the options included in the butterfly spread (in terms of Eq. (9), $K_l = S_{t+\tau,S} - \Delta$), K_{hm} – the middle point between K_h and K_m , K_{lm} – the middle point between K_l and K_m .

The butterfly spread presented by the triangle in Fig. 2 comprises an approximation for the rectangle in Fig. 2. This position guaranties to its owner a payoff Δ conditional on the underlying stock price's appearance between K_{lm} and K_{hm} on the option's expiration date.²³ Let $R(K_{lm})$ be a whole-period return of the stock which price on the option's expiration date is K_{lm} . Similarly, let $R(K_{hm})$ be a whole-period return of the stock which price on the option's expiration date is K_{hm} . The physical probabilities required for the model's estimation are the probability for the stock's return being lower than $R(K_{lm})$, the probability for the stock's return being between $R(K_{lm})$ and $R(K_{hm})$, and the probability for the stock's return being higher than $R(K_{hm})$. Aforementioned probabilities comprise the full characterization of a relevant state. The two-stage method described earlier serves for the estimation of the corresponding values.

Appendix C. Extended data description

C.1. Comprehensive description of the raw data

The raw option data are comprised of the following components: time to expiration (in days); strike price; type (call or

²³ In a case that the butterfly spread priced by the available options is not symmetric, i.e. $K_h - K_m \neq K_m - K_l$, the required position is achieved by the following actions:

1. Writing down $(1 + (K_m - K_l)/(K_h - K_m))$ options with a middle strike price,
2. Purchasing $(K_m - K_l)/(K_h - K_m)$ options with a high strike price, and
3. Purchasing an option with a low strike price. The resulting position guaranties to its owner a payoff $\Delta = K_m - K_l$ if the underlying stock price will be K_m at the option's expiration date. The approximation presented by Fig. 2 determines a payoff Δ conditional on the underlying stock price's appearance between K_{lm} and K_{hm} on the option's expiration date, so the case is identical to that with a symmetric butterfly spread.

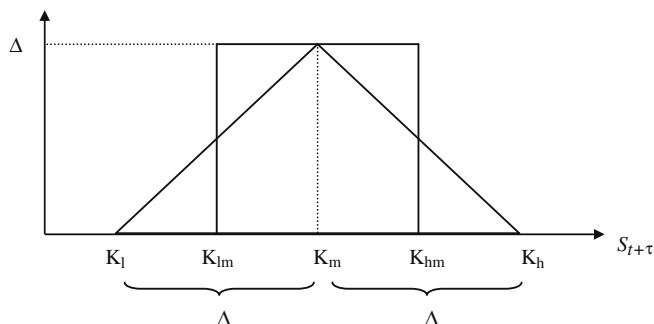


Fig. 2. The characteristics of state approximated by butterfly spread's pricing.

put); underlying stock's price (at the time of transaction); implied volatility (according to Black and Scholes (1973)). The data are given at daily frequency, while for each date the options are sorted by expiration dates and strike prices. The initial processing of the raw data is summed up in the following actions: (a) reconstruction of the options' prices by substituting the implied volatility in the Black and Scholes formula for options pricing (Black and Scholes (1973)); (b) calculation, for each given date, of the average stock price in this date's transactions²⁴; and (c) conversion of the time to expiration to annual basis, by dividing the number of days by 365.

C.2. Extracting the data used for model estimation out of the primary data array

The primary data array includes all of the daily trading data from 01/01/1991 to 12/31/1995. The data we use for model estimation are extracted from this general collection according to the algorithm described below. The purpose of the algorithm is to collect, for each stock, the data for non-overlapping options having one month to expiration. The algorithm's stages are as follows:

C.2.1. Stage 1

For each option series we find an expiration date. In most cases the expiration is on third Friday of the month. If this date is not trading day (e.g., due to holiday), the expiration falls on adjacent Thursday (i.e., one day earlier).

C.2.2. Stage 2

Out of the primary data array we extract the data for Wednesday after each expiration date. The data we choose is for the series that is most close to expiration, i.e., that with time to expiration of about month. The dates of the Wednesdays are determined as *primary source dates*.

C.2.3. Stage 3

At this point we check, for each *primary source date* (which is set at the previous stage), the number of strike prices (in the relevant series) that were traded at this date. The data extraction is conducted according to one of the two possibilities: if the number of relevant strike prices at *primary source date* is 3 or more (i.e., it is possible to build at least one butterfly spread), then the date is determined as *final source date* and its data are used for model estimation; and if the number of relevant strike prices at *primary source date* is 2 or less (including the cases when the relevant series was not traded at this date or when the date was not trading day), then we conduct a search for a date containing at least 3 strike

prices in the relevant series. The search is conducted within the two days preceding the *primary source date* and the two days following it. Out of these four days we choose (if it exists) the day containing at least 3 relevant strike prices. If we find more than one such day, we choose the one that is most close to the *primary source date*. The chosen day is determined as *final source date*, its data replace the *primary source date*'s data and we use it for model estimation. If the search does not yield any results (i.e., no days with at least 3 relevant strike prices are found), then we do not make use of the *primary source date*'s data (and there is no *final source date* in this "lack of data" case).

As a result of applying the described algorithm, the data for non-overlapping options with about month to expiration are collected. For each one of the 30 stocks we determine up to 60 dates to extract the data from.²⁵ Each of these dates contains at least one observation comprised of 3 strike prices. The data collected for one specific stock can be viewed as a separate sample, in which case all of the data extracted for model estimation constitute 30 samples comprised of specific stocks' observations. The number of observations is different across samples and averages 56.

Table 6 presents a description of the data chosen for model estimation. The description contains the following information for each stock: sample size – the number of observations extracted out of primary data array, while each observation is comprised of 3 strike prices used for butterfly spread construction; number of monthly cross-sections – the number of *final source dates* determined by the algorithm (each cross-section contains at least one observation, while there are up to 60 cross-sections overall); average number of observations in monthly cross-sections – the arithmetical average of observations across the monthly cross-sections (the standard deviation presented in parentheses); average implied volatility in monthly cross-sections – the arithmetical average of the implied volatility across the monthly cross-sections, while the implied volatility in each cross-section is calculated as arithmetical average of the implied volatility in all relevant transactions of the day as reported in the raw data (the standard deviation presented in parentheses); average time to expiration in monthly cross-sections – the average time (in years) until option's expiration in the cross-sections chosen for model estimation²⁶ (the standard deviation presented in parentheses). The last raw reports the information for the combined sample comprised of all 30 stocks.

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²⁵ The overall period is 5 years, and the data are extracted once a month, except for the "lack of data" cases explained in the algorithm description. In these cases we have less than 60 dates to extract the data from.

²⁶ As we say earlier, the need for 3 strike prices for observation sometimes required a search around the *primary source date*, which leads to differences in times to expiration across different cross-sections. Yet, as can be seen from Table 6, the data used for model estimation are fairly homogenous in this aspect.

²⁴ Henceforth this price is used in all calculations as the underlying stock price of the day. The above-mentioned calculation of the Black and Scholes formula also makes use of this average stock price.

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