Micro Theory 1 Problem Set 2

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Question 1

The three conditions that must be verified are: $p^T x = y$, S is symmetric and S is negative semi-definite.

$$p_1 q_1 + p_2 q_2 = y$$
$$\frac{2p_1 y}{2p_1+p_2} + \frac{p_2 y}{2p_1+p_2} = y\frac{2p_1+p_2}{2p_1+p_2} = y$$

Symmetry: Note first that:

$$\frac{\partial q_1}{\partial p_1} = \frac{-4y}{(2p_1+p_2)^2} \quad \frac{\partial q_1}{\partial p_2} = \frac{-2y}{(2p_1+p_2)^2} \quad \frac{\partial q_1}{\partial y} = \frac{2}{2p_1+p_2}$$
$$\frac{\partial q_2}{\partial p_1} = \frac{-y}{(2p_1+p_2)^2} \quad \frac{\partial q_2}{\partial p_2} = \frac{-y}{(2p_1+p_2)^2} \quad \frac{\partial q_2}{\partial y} = \frac{1}{2p_1+p_2}$$

$$S_{1,2} = \frac{\partial q_1}{\partial p_2} + q_2 \frac{\partial q_1}{\partial y} = \frac{-2y}{(2p_1+p_2)^2} + \frac{2y}{(2p_1+p_2)^2} = 0$$
$$S_{2,1} = \frac{\partial q_2}{\partial p_1} + q_1 \frac{\partial q_2}{\partial y} = \frac{-2y}{(2p_1+p_2)^2} + \frac{2y}{(2p_1+p_2)^2} = 0$$

Thus we can see that S is symmetric.

Negative Definite: First complete the slutsky matrix.

$$S_{1,1} = \frac{\partial q_1}{\partial p_1} + q_1 \frac{\partial q_1}{\partial y} = \frac{-4y}{(2p_1+p_2)^2} + \frac{4y}{(2p_1+p_2)^2} = 0$$
$$S_{2,2} = \frac{\partial q_2}{\partial p_2} + q_2 \frac{\partial q_2}{\partial y} = \frac{-y}{(2p_1+p_2)^2} + \frac{y}{(2p_1+p_2)^2} = 0$$

Thus we can plainly see that: $S = 0$ and $x^T S x = 0 \leq 0 \quad \forall x \in \mathbb{R}$

While we cannot construct the exact utility function of the consumer, we can produce the indirect utility function that would produce these demand functions.

$$\frac{\partial e}{\partial p_1} = \frac{2e}{2p_1 + p_2} \qquad \frac{\partial e}{\partial p_2} = \frac{e}{2p_1 + p_2}$$
$$\frac{\partial \log e}{\partial p_1} = \frac{2}{2p_1 + p_2} \qquad \frac{\partial \log e}{\partial p_2} = \frac{1}{2p_1 + p_2}$$

$$\log e = \log(2p_1 + p_2) + C(p_2, u)$$
$$\frac{\partial \log e}{p_2} = \frac{1}{2p_1 + p_2} + C_{p_2}(p_2, u) = \frac{1}{2p_1 + p_2}$$
Clearly: $C_{p_2}(p_2, u) = 0$ and $C(p_2, u) = C_1(u)$
$$\log e = \log(2p_1 + p_2) + C_1(u)$$
$$e = C_2(u)(2p_1 + p_2)$$
$$e(p_1, p_2, u) = u(2p_1 + p_2)$$
Note that: $U(x) = max\{u \geq 0 | p^T x \geq e(p, u) \quad \forall p >> 0\}$
$$U(x) = max\{u \geq 0 | p_1 x_1 + p_2 x_2 \geq u(2p_1 + p_2) \quad \forall p >> 0\}$$
$$U(x) = max\,\{u \geq 0 | p_1(x_1 - 2u) + p_2(x_2 - u) \geq 0 \quad \forall p >> 0\}$$
$p_1, p_2 \geq 0 \quad \forall p >> 0$ so: $(x_1 - 2u), (x_2 - u) \geq 0$
$$u \leq \frac{x_1}{2}, u \leq x_2$$
$$U(x) = max\left\{u \geq 0 | u \leq \frac{x_1}{2}, u \leq x_2\right\}$$
$$U(x) = min(\frac{x_1}{2}, x_2)$$

Question 2.

Part a.

Let us begin by examining the prices of the different bundles. Let $x_0 = (3,1,7), x_1 = (7,3,1), x_2 = (1,7,3)$ and $p_0 = (2,3,3), p_1 = (3,2,3), p_2 = (3,3,2)$

$p_0 x_0 = 30$	$p_1 x_0 = 32$	$p_2 x_0 = 26$
$p_0 x_1 = 26$	$p_1 x_1 = 30$	$p_2 x_1 = 32$
$p_0 x_2 = 32$	$p_1 x_2 = 26$	$p_2 x_2 = 30$
x_0 Preferred	x_1 Preferred	x_2 Preferred

Weak Axiom of Revealed Preference: if $p_i x_j \leq p_i x_i \implies p_j x_i > p_j x_j$
This leaves three cases to check.
$p_0 x_1 \leq p_0 x_0$ and $p_1 x_0 > p_1 x_1$ ✓
$p_2 x_0 \leq p_2 x_2$ and $p_0 x_2 > p_0 x_0$ ✓
$p_2 x_1 \leq p_2 x_2$ and $p_1 x_2 > p_1 x_1$ ✓
So the Weak Axiom of Revealed Preference is satisfied by these choices.

Part b.

Note: x_0 is revealed preferred to x_1 at price p_0, and x_1 is revealed preferred to x_2 at price p_1 and x_2 is revealed preferred to x_0 at price p_2. This leads to the conclusion that x_0 is preferred to x_1 and x_1 is preferred to x_2 and x_2 is preferred to x_0. This is a violated of the Strong Axiom of Revealed Preference, so this behavior is not consistent.

Question 4.

Since f has constant returns to scale, it is HOD 1, and by Euler's theorem: $f_1(x_1,x_2)x_1 + f_2(x_1,x_2)x_2 = f(x_1,x_2)$ so: $f_1(x_1,x_2) - f(x_1,x_2) = -f_2(x_1,x_2)$
We may define Average product with respect to a good as: $AP_{x_1} = \frac{f(x_1,x_2)}{x_1}$
If Average product with respect to x_1 is increasing then: $\frac{\partial AP_{x_1}}{\partial x_1} > 0$ and $\frac{x_1 f_1(x_1,x_2) - f(x_1,x_2)}{x_1^2} > 0$
Thus: $\frac{-f_2(x_1,x_2)x_2}{x_1^2} > 0$ and $-f_2(x_1,x_2)x_2 > 0$ so: $f_2(x_1,x_2) < 0$
We can see that marginal product of good 2 is negative.

Question 5.

a.

Let $Y \subset \mathbb{R}^3 = \{(x_1,x_2,x_3)|x_1,x_2 \le 0, 0 \le x_3 \le A(-x_1)^{\alpha_1}(-x_2)^{\alpha_2}, A, \alpha_i > 0\}$
$\frac{\partial f}{\partial x_1} = \alpha_1 A x_1^{\alpha_1 - 1} x_2^{\alpha_2}$, $\frac{\partial f}{\partial x_2} = \alpha_2 A x_1^{\alpha_1} x_2^{\alpha_2 - 1}$ MRTS $= \frac{\alpha_1 x_2}{\alpha_2 x_1}$

b.

$Y \subset \mathbb{R}^3 = \{(x_1,x_2,x_3)|x_1,x_2 \le 0, 0 \le x_3 \le min\{-a_1x_1, -a_2x_2\}, a_1, a_2 > 0\}$
Since this function is not differentiable everywhere, its MRTS is not defined everywhere, and does not make sense, as perfect compliments have no substitution.

c.

Let $Y \subset \mathbb{R}^3 = \{(x_1,x_2,x_3)|x_1,x_2 \le 0, 0 \le x_3 \le -a_1x_1 - a_2x_2\}$
$\frac{\partial f}{\partial x_1} = a_1$, $\frac{\partial f}{\partial x_2} = a_2$ MRTS $= \frac{a_1}{a_2}$

d.

Let $Y \subset \mathbb{R}^3 = \{(x_1,x_2,x_3)|x_1,x_2 \le 0, 0 \le x_3 \le (a_1(-x_1)^\rho + a_2(-x_2)^\rho)^{\frac{\epsilon}{\rho}}\}$
$\frac{\partial f}{\partial x_1} = \frac{\epsilon}{\rho}(a_1x_1^\rho + a_2x_2^\rho)^{\frac{\epsilon}{\rho}-1}\rho a_1 x_1^{\rho-1}$, $\frac{\partial f}{\partial x_2} = \frac{\epsilon}{\rho}(a_1x_1^\rho + a_2x_2^\rho)^{\frac{\epsilon}{\rho}-1}\rho a_2 x_2^{\rho-1}$ MRTS $= \frac{a_1 x_1^{\rho-1}}{a_2 x_2^{\rho-1}}$

As ρ tends to 1, the constant elasticity of substitution function tends to: $(a_1x_1 + a_2x_2)^\epsilon$ for $\epsilon = 1$ this is perfect substitutes, but in general it does not simplify to another known production function.