University of Chicago Economics
Department

Writing Sample

An Application of Cumulative Prospect Theory to Online lotteries in Counter-Strike: Global Offensive

Timothy Schwieg

Originally Submitted as a term paper for ECO 6315 Current Seminar in Economic Topics

April 27, 2018

1 Introduction

A common criticism of neo-classical economics is the structure that is placed on actions taken by agents under risk. Often times economists assume that agents act to optimize expected utility, where the utility is characterized by a Von-Neumann Morgenstern utility function. This has many attractive properties, but can lead to situations where the structure is too rigid to allow for commonly observed behavior.

One such example is lotteries. People who are generally risk-averse in areas such as insurance act as if they are risk-loving in playing a lottery. One model that is able to account for this is cumulative prospect theory. It incorporates several important concepts not seen in classical decision making under risk.

It incorporates reference dependence, which implies that people view things in the context of losses and gains rather than changes to their overall wealth. This is attractive for computational reasons. It also utilizes loss aversion, the notion that losses are relatively more costly than gains. The model also incorporates diminishing sensitivity, the notion that valuations are concave in losses and convex in gains. The final concept is probability weighting, the notion that consumers act as if they were facing different probabilities than what they encounter.

The nature of probability weighting leads to an overweighting of the tails of the distributions. This can cause behavior such as preferring a 0.001 chance of $5000 to a certain gain of $5, and simultaneously preferring a certain loss of $5 to a .001 chance of losing $5000. As noted by Barberis (2013), there have been more sophisticated approaches to the application, however this investigation will be limited to the relatively simple functional forms suggested first by Tversky and Kahneman (1992). This was chosen due to the computationally arduous estimation method utilized.

In this paper I apply cumulative prospect theory to small lotteries that take place within the video game *Counter Strike: Global Offensive*. In this game players receive "loot boxes", which are cases that can be opened with the purchase of a key and return a random reward based on some well-known probabilities. There is a well defined market for cosmetic items, taking the form of a continuous double auction where players can buy and sell virtually all items in the game. There is a large number of items sold daily at non-trivial prices, so the data is a powerful framework for investigating consumer behavior under lotteries.

A very attractive feature of this data is that both the contents of the lottery and the lotteries themselves are both traded on the Steam Community Market. While the API does not provide flawless data, there is still a large amount of data. This comes in two forms: Market history given to the last hour, and Buy and Sell Orders that have not currently been fulfilled.

I will test if the data gathered from the steam community market is consistent with the predictions of cumulative prospect theory, as well as look at how the different structures of the lotteries affect the way in which consumers treat the lotteries. In particular, I am interested in testing whether or not loss aversion is present in this data, and whether or not several of the lotteries which have many more low probability but very high value items have different valuation functions compared to the other lotteries.

2 Model

2.1 General Approach

Because of the nature of the data, as well as the complicated structure that cumulative prospect theory places upon the valuation function, it is difficult to implant it within the structure used commonly in double auction literature. Almost all models of auctions specify a structure between the valuation and the price that is the valuation for the item minus the price paid for the item at the auction. This difference cannot be reconciled with the framing effects required by cumulative prospect theory.

This forces us to take a relatively agnostic take on the structure of the model. Since the valuations of every consumer are not observed, we only observe purchases made, we know that the valuation of the buyer must be at or above the price of the item. One powerful result shown by Cripps and Swinkels (2004) is that will drive all the computations of the model is that a continuous double auction converges in order $\mathcal{O}(N^2)$ to a competitive market. From this, I can consider that in large double auctions, such as the case for the data present, that we are working in a competitive market. From this we can conduct our analysis as if a buyer of a lottery is at a competitive market, sees the price given by the market exogenously, and makes a decision choice based upon his valuation as a function of the price. While there is not nearly enough information given to be able to identify supply and demand within this market, we are only interested in the valuations of the purchasers of the lotteries, and we may approach this by considering a single buyer who has independently formed a private valuation of the lottery and enters the market.

Since there is a large number of consumers in the market, we may take price as exogenous. Since the order of the covariance between the price and the valuations is $\mathcal{O}(\frac{1}{N})$. The differences between the competitive market and the double auction become meaningless since the double auction is converging at a much faster rate to the competitive market. That is, even if the order of the covariance was higher for a double auction, we are converging to a competitive market so quickly we can effectively consider the market fixed at a competitive market and then consider the covariance between price and valuations. Due to the large magnitude of the data being examined, the price is effectively independent of an individuals valuation, so the assumption of exogenous prices is reasonable, and will drive much of the computation throughout the paper.

This leads us to one of two outcomes in the market, either there is a purchase made, indicating a valuation above the price seen by the consumer, or he leaves a buy order, indicating a price he would be willing to pay. The structure of the double auctions is such that the buyer pays the seller's price, not his valuation. Since in a double auction where the buyer pays the seller's price, Li (2015) has shown that it is dominant to tell the truth. As a result I will consider these buy orders as the actual valuations of the consumer. This means that while the data taken at market is censored by the market price, the data taken from the buy orders is not, and our model will have to encompass both forms.

2.2 Discrete Choice

There is no data on the consumers purchasing the items, and we only have data on a single item being purchased rather than consumers facing bundles of possible items in $\mathbb{R}^n$ such as consumer theory would ordinarily predict. These two results mean that we must work within a framework of making a single decision to purchase the lottery or not, and with relatively little structure so as to support the complex structure of cumulative prospect theory. One such model is Discrete Choice theory.

By Following the discrete choice model, we assert that decisions made by the individual are made such that if the valuations of lottery plus ξ_l is greater than the valuation of not purchasing which is given by ξ_n where $\xi_\mathrm{j} \sim$ Gumbel. This means that the purchase is dictated by if: $V(f) + \xi_b - \xi_n > 0$. Since the difference of two Gumbel distribution is distribution logistically, and V(f) is not random. Knowing that the Logistic Distribution is in the location scale family, we may consider the entire expression as: $V_i(f) \sim logit(V(f), s)$.

However, we observe some censored realizations of V. That is, we observe valuations based on the buy orders which are non-censored realizations, but we also observe purchases which are censored. Denote the censored purchases by $d_n = 1$ and uncensored by: $d_n = 0$. The logarithm of the likelihood function for censored data as shown by Paarsch (2018) is:

$$\sum_{j=1}^{J} \sum_{n=1}^{N_j} d_{n,j} \log(\frac{1}{4s} sech^2(\frac{x_{n,j} - V_j}{2s})) + (1 - d_{n,j}) \log(\frac{\exp(\frac{x_{n,j}-V_j}{s})}{1 + \exp(\frac{x_{n,j}-V_j}{s})}) \tag{1}$$

Where V_j is the valuation for the box of type j, $\mathrm{x_{n,j}}$ is the price paid by observation n of type j, and $\mathrm{d_{n,j}}$ is whether or not the data at observation n of type j was censored.

We may maximize this log-likelihood to find the parameters that best fit. What remains to be decided is the form of the function V. We will have to assume a structural form for the function, and while we will be able to compare between different structural forms, we cannot apply any statistical tests between our different assumptions.

2.3 Cumulative Prospect Theory

Since one key aspect of cumulative prospect theory is that we are more strongly motivated by losses than by gains, it is immediately obvious that the value function of the contents of the lottery will not be symmetric. The easiest way to handle this asymmetry will be to estimate two separate functions, one of which is used to evaluate losses, and one of which estimates gains. By applying a piecewise function, we can still arrive at a continuous function (where both are zero at zero) which we can use. The question of whether or not loss aversion will be displayed however, is one for the data to answer.

Since I am following the Discrete Choice model, the decision to purchase a create will be driven by the valuations of the crate against the alternative which is buying nothing. Since Cumulative prospect theory functions by looking at deviations from a reference point, which we will use as the price of the box combined with the costs of opening it (the key). Later in the analysis, this will be relaxed, and compared against several different reference points. As noted in Barberis (2013), the question of the proper reference point is actively debated, so we will examine different possibilities.

Following the Notation of Tversky and Kahneman (1992), we represent the valuation of the lottery as $V(f) = \sum_{i=-m}^{n} \pi_i v(x_i)$ where v is a function that is convex in losses and concave in gains. π is the weights applied to each item in the lottery, and will not necessarily be their actual probabilities.

In this paper, the effects of dynamics will not be considered. There is a common pattern of the price of lotteries decreasing over time, and there are few covariates available. Because of these two concerns, I will not consider purchases of the boxes intended as an investment rather than to be opened. This simplifying assumption will mean that the valuations of the contents of the lottery drive the value, and I believe it is reasonable based upon the price histories available. Without this, there is not enough data available to attempt to explain valuations of the boxes, and the dynamic nature of the problem would increase the complexity immensely.

2.3.1 Valuations

The question of how do we measure the gains of the lottery now looms. Since each box contains an item that has a particular value to the consumer who opened it, and is unobserved, the only thing that can be observed is the market price of the item over time. Identification of buyer and seller valuations, even in a simplified situation where there is only one buyer and one seller, still requires more information than we are given. As shown in Li and Liu (2015), for their identification strategy, all the bids in the auction are required, and for the strategy shown in Li (2015), there must exist exclusive covariates that shift only one trader's value distribution, which are not given by the data. Absent an ability to identify the valuations of the specific losses and gains in the lottery, I will have to assume that the market value of each item represents its value to the consumer. Any item obtained in the box could in theory be sold at the market value of the item, so we will consider this the value for each item.

The valuations of the i^{th} possible element of the lottery will be given by: v($\mathrm{p_i}$ - $\mathrm{p_l}$ - $\mathrm{p_{key}}$), where $\mathrm{p_i}$ is the average price of the i^{th} element at market, $\mathrm{p_l}$ is the price paid for at market for the lottery, and $\mathrm{p_{key}}$ is the price of the key required to open the lottery. Depending on whether or not this difference is positive or negative will result in different functions being used to evaluate the valuation. Using the specification suggested by Tversky and Kahneman (1992):

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\alpha} & x < 0 \end{cases} \tag{2}$$

2.3.2 Probability Weighting Function

In cumulative prospect theory, the cumulative mass (distribution) function is weighted such that individuals overweight the tail probabilities. This is especially important in this model, as there are many high valued rare items, that if this part of the theory is correct, heavily influence the valuation of the box, despite their extremely low probability of occurrence.

Again, I will use the specification suggested by Tversky and Kahneman (1992), and use the cumulative transformation function of:

$$w(P) = \frac{P^\delta}{(P^\delta + (1-P)^\delta)^{\frac{1}{\delta}}} \tag{3}$$

To define the decision weights π_i, we must first order the prospects of the lottery in ascending order of gains. the weight π_i then is defined by:

$$\pi_i = w(\sum_{j=-m}^{i} P(x_j)) - w(\sum_{j=-m}^{i-1} P(x_j)) \tag{4}$$

2.3.3 The Valuation Function

From these, we can create a valuation function for an individual facing a particular lottery:

$$V_i = \begin{cases} (w(\sum_{j=-m}^{i} P(x_j)) - w(\sum_{j=-m}^{i-1} P(x_j)))(p_i - p_l - p_{key})^\alpha & (p_i - p_l - p_{key}) \geq 0 \\ -\lambda(w(\sum_{j=-m}^{i} P(x_j)) - w(\sum_{j=-m}^{i-1} P(x_j)))(p_l + p_{key} - p_i)^\alpha & (p_i - p_l - p_{key}) < 0 \end{cases} \tag{5}$$

3 Data

3.1 The Data

The data is price data of items on the Steam Community Market for the game *Counter Strike: Global Offensive.* Players in game earn items random that they can sell on the market or open themselves. However most rare items are earned via opening of dropped "loot boxes" that are then opened by players via purchasing of a key. These boxes can be earned by playing or received randomly from players who are watching games of professionals play. The probabilities of the drops are not known or even estimated well, as they change depending on many factors including time.

However, once a box has been obtained, the probability of receiving an item is well documented as required by Chinese Law. Each item has a certain grade of rarity, for example the Ak-47 Redline has a rarity level of Classified which means that there is a 3.2% chance of receiving a Classified item in the crate. All Classified items contained in the crate have the same probability of being dropped by the crate.

However there are many variants of each item. Each item has a quality ascribed to it, the float of the item. This describes the wear on the item, and is distributed uniformly on the interval 0-1. On the market the items are split into intervals: Battle-scared, well-worn, field-tested, minimal wear and factory new. Each quality is a separate listing on the market with a separate price. In addition to each item having a quality type there is also a 10% chance of each item being labeled as StatTrak, which also distinguishes the value of a weapon. This means that each item has 10 possible different variations all with different probabilities of being obtained. Some rare items, usually knives and gloves may have more or less variants, but the amount and probabilities are known, and can easily be determined by checking if there is a market history for the item.

The probabilities for each condition are as follows:

Float	Condition
0.00 - 0.07	Factory New
0.07 - 0.15	Minimal Wear
0.15 - 0.38	Field-Tested
0.38 - 0.45	Well-Worn
0.45 - 1.00	Battle-Scarred

Each item has a 10% chance of being StatTrak if that item has statTrak enabled. float values are distributed uniformly, making the probability calculations simple.

However the rarity of a skin also controls its probability of being dropped in a particular lottery. These rarities are set by Valve, and are specified for each crate. They rank from gold (very rare) to blue (not very rare) The probabilities of getting an item of a rarity is given as follows:

Probability	Rarity
.0026	Special (Gold)
.0064	Covert (Red)
.032	Pink (Classified)
.1598	Purple (Restricted)
.7992	Blue (Mil-spec)

In each box there are several items of each rarity, each one is equally likely to be found when the lottery is explored.

Each box contains some subset of these items that is known, and the market value of each item at a particular time period is also known, so the expected value, or any other modified version of a valuation of the lottery can easily be calculated.

3.2 Sources of the Data

The data has been mined from the Steam Community Market API, which provides a purchase history for every item on the market, down to the hour for the last thirty days and daily for the rest of the lifetime of the item. It does not provide a record of every purchase, just the quantity sold in that time period as well as the median price they were sold at. Obviously this is less than ideal, but I believe it will cause less problems than the inaccuracies introduced by the market only working in one cent intervals.

Also available is current buy and sell orders for each item. If a potential buyer wishes to buy on this market, he may either select a box directly and purchase from a particular seller, or he may put forward a buy order, which he stipulates a price, and as soon as a seller puts an item up for sale below that price, it is sold to the buyer and he is charged the seller's price. This gives the valuations of people who have not yet obtained the item directly. However, it does not appear that there is a history available for these items. In some ways this is beneficial because it would impossible to determine the differences between buy orders that were fulfilled and buy orders that were removed because of changes in the prices of underlying assets. I have decided to treat all outstanding buy orders as valuations in the final time period that simply are below the market price. I will not consider the case that there are buy orders placed and forgotten about.

3.3 Treatment of the data

There are approximately 11,000 items mined through the procedure followed by the script `BuildData.py`. This data must be organized so that it can be used effectively. First a hierarchical file structure was created by `MoveFiles.py`, this sorted each item by its type, skin, and finally quality. However, To this end, I created text files that contained the contents of each lottery that was to be examined, and then aggregated this data using `rarity.py`. This aggregation included the different varieties of items included in each box (condition and StatTrak), controlling for availability of items by searching for them in the file structure.

Once each individual lottery had the available items, the actual price data that had been mined by `BuildData.py` could be applied. Using the script `CreateData.py`, for each transaction of the lottery that was recorded during the period where there is hourly data, the price and quantity was noted, as well as the most current price of each item contained in the lottery. This data was combined with the probabilities of each item being drawn, as well an indicator variable for whether or not this data was censored or not. All data that was drawn from the market purchases is considered censored, and buy order data was considered uncensored. This data is finally saved in csv format for each lottery, containing probability data, price history and censor information of the box as well as the probability of obtaining each item in the lottery.

4 Computations

4.1 Problems

There are two problems that prevent easy calculation. The first problem is the presence of censored data, which make estimation difficult by providing a non-convex likelihood function. This problem is further complicated by the structure imposed by cumulative prospect theory, which specifies that consumers valuations are convex in gains, and concave in losses, and a weighted linear combination of these valuations which is in general not a convex optimization problem. Since we are attempting to estimate the shape of this function, there is no manner in which this problem can be couched as a convex optimization problem. This means that our problem lies in the NP-Hard class, and it will be very difficult to arrive at a maxima, and especially difficult to guarantee that it is a global maximum of the likelihood function. In particular, if a strong stance is not taken on the structure of the underlying distribution that these valuations are drawn from. By assuming that it is a logistic distribution, and that the shape parameter is 1, this forces the variations observed to be driven by the differing valuations.

4.2 The approach

Since there is such a computation load, the programming language Julia will be used for its speed. I applied conjugate gradient descent using the library Optim.jl to conduct my estimation. In order to allow for deviations to be caused by the shape of the functions describing the valuations as well as for computational ease, the shape parameter of the logistic distribution was set equal to one. This allows for all of the changes in the valuations to be driven by changes in the relative values of the lotteries rather than the shape of the distribution and the shape of the functions within the lottery. Without this assumption I was unable to compute values for any of the parameters in the estimation process due to its non-convex nature.

The constraints on the parameters are that each is strictly positive, as the weight function is not even defined for $\delta = 0$, and positive values for α and λ are required to maintain continuity for the valuation function. To this end I parametrized each of the variables by taking the exponential of them. That is the choice of λ, α and δ are determined by three auxiliary variables that are the natural logarithms of the parameters of interest. This ensures that while the axillary variables are completely unconstrained, the actual parameters of interest are constrained to being strictly positive. However, the actual mathematical program of estimation is one of unconstrained optimization and is therefore more easily estimated.

4.3 Calculations

The next step in calculating the maximum likelihood estimators is to calculate the gradient of the log-likelihood function. Since our likelihood function is broken into many parts, but the parameters of interest only appear in the Valuation function rather than in the density and distribution. The s only appears in the density and distribution function, and is held constant throughout the calculations, so we need only concern ourselves with the α, λ, δ gradients in the valuaton function. To simplify this procedure we will break it into smaller steps using the chain rule. We can see that:

$$\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial V}\frac{\partial V}{\partial \alpha} \quad \frac{\partial L}{\partial \lambda} = \frac{\partial L}{\partial V}\frac{\partial V}{\partial \lambda} \quad \frac{\partial L}{\partial \delta} = \frac{\partial L}{\partial V}\frac{\partial V}{\partial \delta}$$

4.3.1 Likelihood Function Derivatives

Our likelihood function is given by:

$$\sum_{j=1}^{J}\sum_{n=1}^{N_j} d_{n,j}\log(\frac{1}{4s}sech^2(\frac{x_{n,j}-V_j}{2s})) + (1-d_{n,j})\log(\frac{\exp(\frac{x_{n,j}-V_j}{s})}{1+\exp(\frac{x_{n,j}-V_j}{s})})$$

Taking the derivative with respect to s gives us:

$$\frac{\partial L}{\partial s} = \sum_{j=1}^{J}\sum_{n=1}^{N_j} d_{n,j}\frac{-1}{s^2}\left(2s - \frac{1}{2}(V_j - x_{n,j})\tanh\left(\frac{V_j - x_{n,j}}{2s}\right)\right) + (1-d_{n,j})\frac{(V_j - x_{n,j})\, e^{\frac{1}{s}(V_j - x_{n,j})}}{s^2\left(e^{\frac{1}{s}(V_j - x_{n,j})}+1\right)}$$

Taking the derivative with respect to v gives us:

$$\frac{\partial L}{\partial V} = \sum_{j=1}^{J}\sum_{n=1}^{N_j} d_{n,j} - \frac{1}{2s}\tanh\left(\frac{V_j - x_{n,j}}{2s}\right) - (1-d_{n,j})\frac{e^{\frac{1}{s}(V_j - x_{n,j})}}{s\left(e^{\frac{1}{s}(V_j - x_{n,j})}+1\right)}$$

4.3.2 Valuation Function Derivatives

As we can see from equation 5, the valuation function is a piecewise function, and it is not immediately obvious that it is differentiable. However it can be rewritten by partitioning the x values based upon whether or not they are gains or losses. That is whether or not $p_i - p_l - p_{key}$ is positive or negative. We will consider the set S to be the set of x values for which $p_i - p_l - p_{key}$ is negative, and the set $\bar{S}$ to be the set of x values for which $p_i - p_l - p_{key}$ is weakly positive.

We may now write our valuation function as

$$V_i = \sum_{i \in S} \pi_i (p_i - p_l - p_{key})^\alpha - \sum_{i \in \bar{S}} \pi_i \lambda (p_l + p_{key} - p_i)^\alpha \tag{6}$$

Since $p_i - p_l - p_{key}$ is independent of α, δ, λ the sign does not change at any point during the calculations, so this summation will be along the same sets for all iterations, and this is a continuously differentiable function in α, δ, λ.

We may now take the derivatives with respect to the parameters:

$$\frac{\partial V_i}{\partial \alpha} = \sum_{i \in S} \pi_i (p_i - p_l - p_{key})^\alpha \log(p_i - p_l - p_{key}) - \sum_{i \in \bar{S}} \pi_i \lambda (p_l + p_{key} - p_i)^\alpha \log(p_l + p_{key} - p_i)$$

$$\frac{\partial V_i}{\partial \lambda} = - \sum_{i \in \bar{S}} \pi_i (p_l + p_{key} - p_i)^\alpha$$

$$\frac{\partial V_i}{\partial \delta} = \sum_{i \in S} \frac{\partial \pi_i}{\partial \delta} (p_i - p_l - p_{key})^\alpha - \sum_{i \in \bar{S}} \frac{\partial \pi_i}{\partial \delta} \lambda (p_l + p_{key} - p_i)^\alpha$$

4.3.3 Probability Weighting Function Derivatives

Since it is known that $\pi_i(x, \delta) = w(\sum_{j=-m}^{i} P(x_j), \delta) - w(\sum_{j=-m}^{i-1} P(x_j), \delta)$, then

$$\frac{\partial \pi_i}{\partial \delta}(x, \delta) = \frac{\partial w}{\partial \delta}(\sum_{j=-m}^{i} P(x_j)) - \frac{\partial w}{\partial \delta}(\sum_{j=-m}^{i-1} P(x_j))$$

$$\frac{\partial w}{\partial \delta}(p, \delta) = \frac{p^d}{\delta^2} \left(p^\delta + (1-p)^\delta\right)^{-(1+\frac{2}{\delta})} \Bigg(\delta^2 \left(p^\delta + (1-p)^\delta\right)^{\left(1+\frac{1}{\delta}\right)} \log(p) + \left(p^\delta + (1-p)^\delta\right)^{\frac{1}{\delta}}$$
$$\left(-\delta \left(p^\delta \log(p) + (1-p)^\delta \log(1-p)\right) + \left(p^\delta + (1-p)^\delta\right) \log\left(p^\delta + (1-p)^\delta\right)\right) \Bigg)$$

As we can see, the complexity of calculating these derivatives quickly becomes staggering and difficult to optimize by hand, so as a result I have utilized automatic differentiation from the package `ForwardDiff.jl` Revels et al. (2016). This allows for a substantial reduction in computational time required and allows for more robust accommodation of parametrization of the variables of interest. It is also amenable to internal gains of speed by vectorization that are required to be able to solve this problem in reasonable time.

5 Results

The first estimation of the model was under the assumption that for all the boxes, consumers had the same valuation function and probability weighting functions. Each box was just a different lottery that was valued the same. The results are summarized in the table and graphs below:

Likelihood:	-2.3889668179405564 x 10^6
λ	6.556362963666051 x 10^{-11}
α	1.2170030277917575 x 10^{-14}
δ	0.7917631380628156

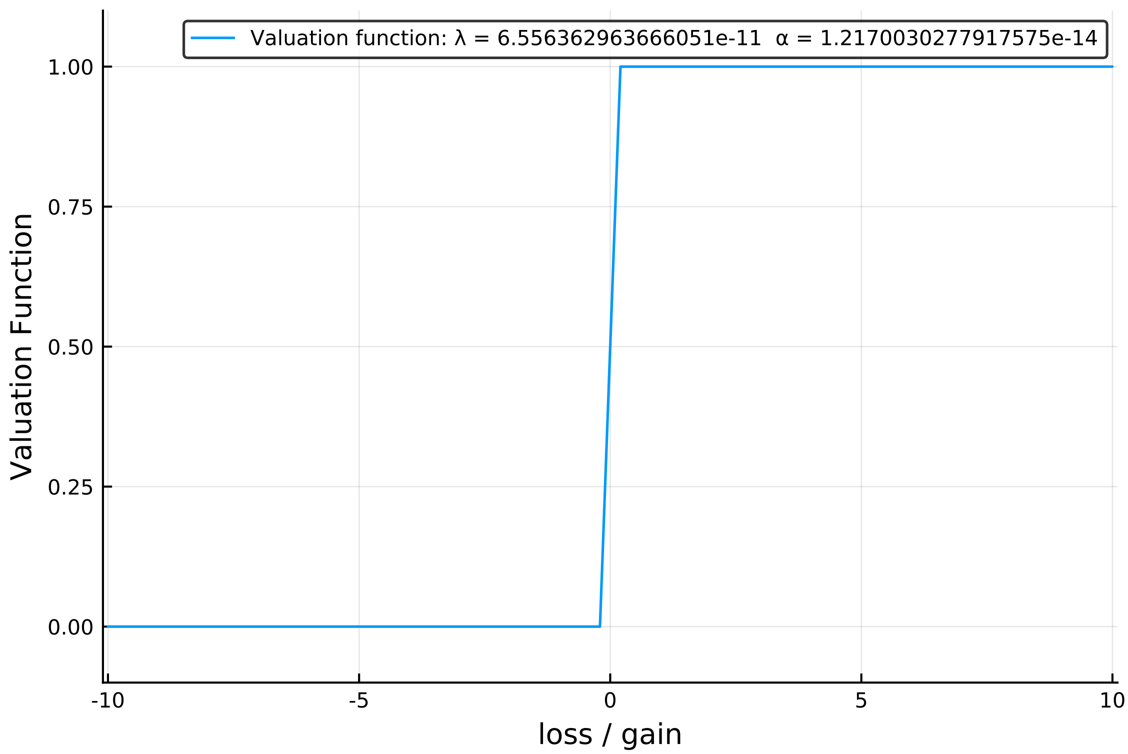


Fig. 1: Valuation function for all lotteries

The most striking results from this is the shape of the valuation function, which appears to approximate an indicator function. The notion of loss-aversion that is usually assumed in the literature on cumulative prospect theory is not apparent here, in fact, the opposite is present. Consumers appear to not be affected at all by the magnitude of losses, nor from the magnitude of the gains seen, only seeming to care about the presence of a gain.

The Probability weighting function resembles the shape found by Kahnmemann and Tversky, which had a δ of .65, the chief difference being that it crosses the identity line at a higher cumulative probability. This means that low-probability high-value items are actually given less weight than Kahneman and Tversky suggest, which goes contrary to my prior beliefs. It appears that the probabilities are not that severely distorted, and although

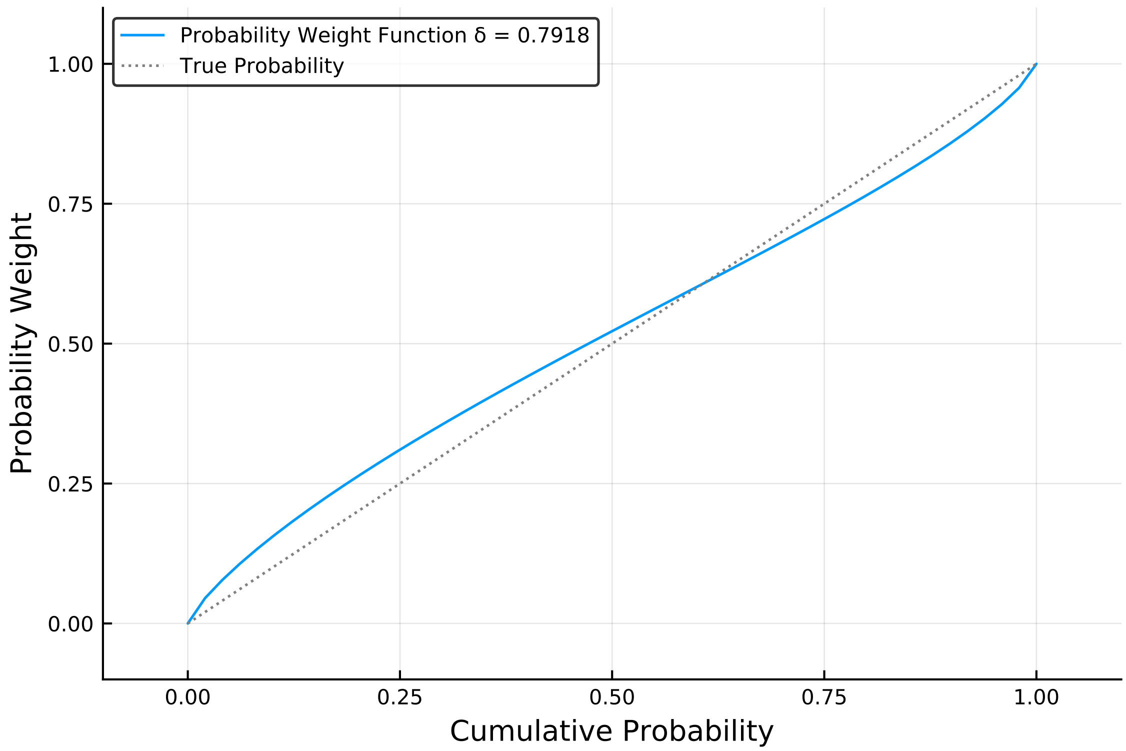


Fig. 2: Probability Weighting Function for all lotteries

the relatively rare probabilities are in fact overstated, the magnitude is not as immense as one might expect in a lottery of this form.

One possible explanation for the lack of loss-aversion shown in this data is the magnitude of the losses seen by the consumers. The current price of many of the boxes is $0.03, which indicates that many of the losses can never have a magnitude of less than $2.50. This is caused by the fact that the items themselves can at smallest be valued at $0.03. In contrast the gains can be many magnitudes higher with some items selling at upward of $100.00. Since there is such a large difference between possible gains and losses, consumers who are particularly susceptible to confirmation bias may not value the losses as they are low magnitude.

5.1 Structural Differences between different lotteries

However, some boxes contain many more rare items than others. In particular the cases titled `Chroma Case` and `Chroma Case 2` contain many different rare items in many unique variants. These items are quite valuable on the market, and there are many available in the box. I tested to see if consumers behaved differently between these boxes and the remaining boxes. The results are summarized below:

	Chroma Case	Remaining Cases
Likelihood:		-2.388950505905629 x 10^{6}
λ	8.93682 x 10^{-55}	1.76366 x 10^{-14}
α	0.486737	2.57372 x 10^{-10}
δ	1.28239	0.774166

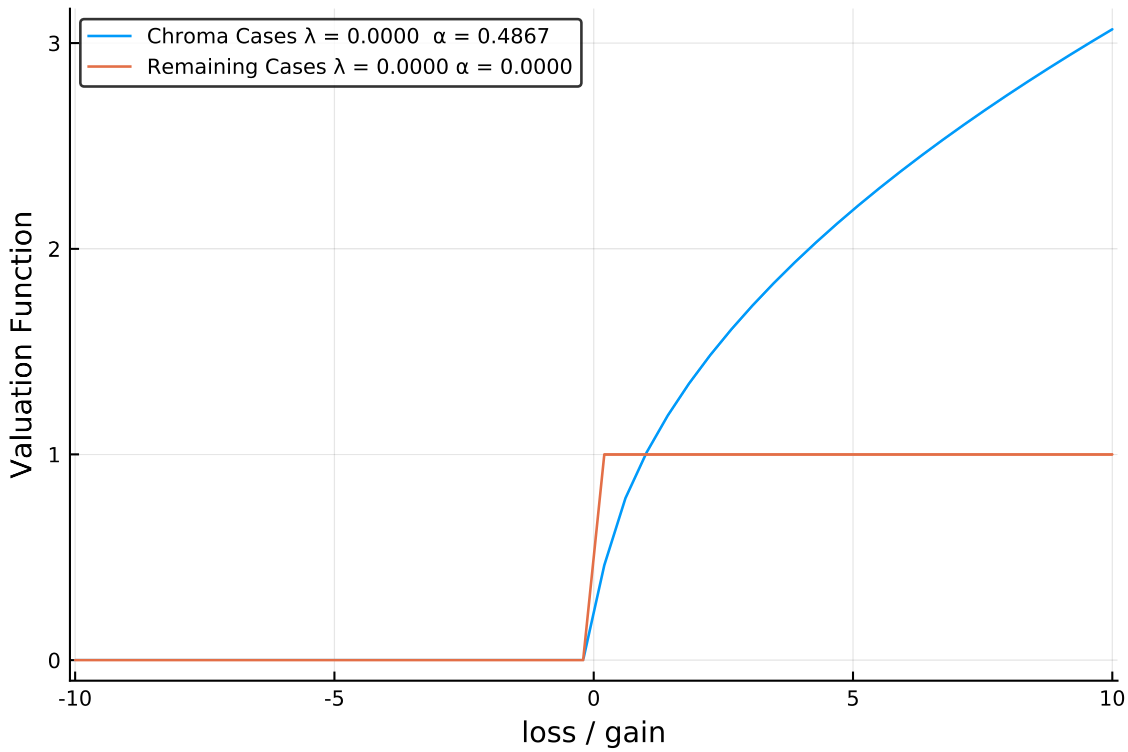


Fig. 3: Valuation function for different lotteries

The first question to ask is whether or not these models are significantly different from each other. Specifically, I am testing if there is a difference between individual behaviors under the cases containing many different items, and the cases containing many more rare items. To this end I applied the likelihood ratio test against the null hypothesis that there is no structural difference between the two models. This led to a p-value of: 3.866 x 10^{-7} . Under this structure, there is a significant difference between the behavior of individuals opening the chroma cases and the remaining cases.

Under both groups of data, loss aversion is simply not present. Using a Wald statistic on the hypothesis that $\lambda_c = 0$ and $\lambda_r = 0$ gives a probability that this did not occur from random noise of 7.107 x 10^{-52}, effectively a p-value of 1. This implies that under the assumed structure of the model, there is simply no loss aversion present. Whether or not this is caused by the structure placed upon the model is not established, but I believe that the framework is robust enough to preclude that possibility.

An interesting difference between these two models is the differences in the α between the two structures. Since there are many more high valuation items in the chroma cases, consumers are more sensitive to higher valuations, as shown by the magnitude of the α. This

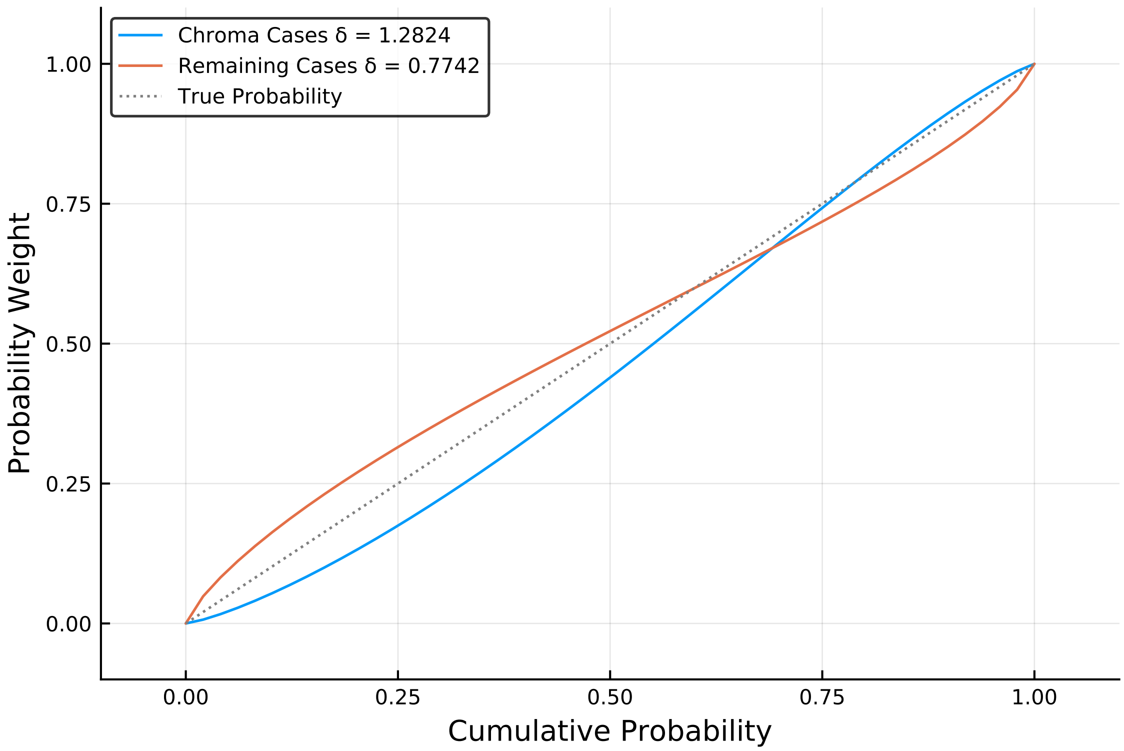


Fig. 4: Probability Weighting function for different lotteries

implies that consumers are less convex in their valuations for the chroma cases, as there are more high value items available, despite the same probability of getting a very rare item.

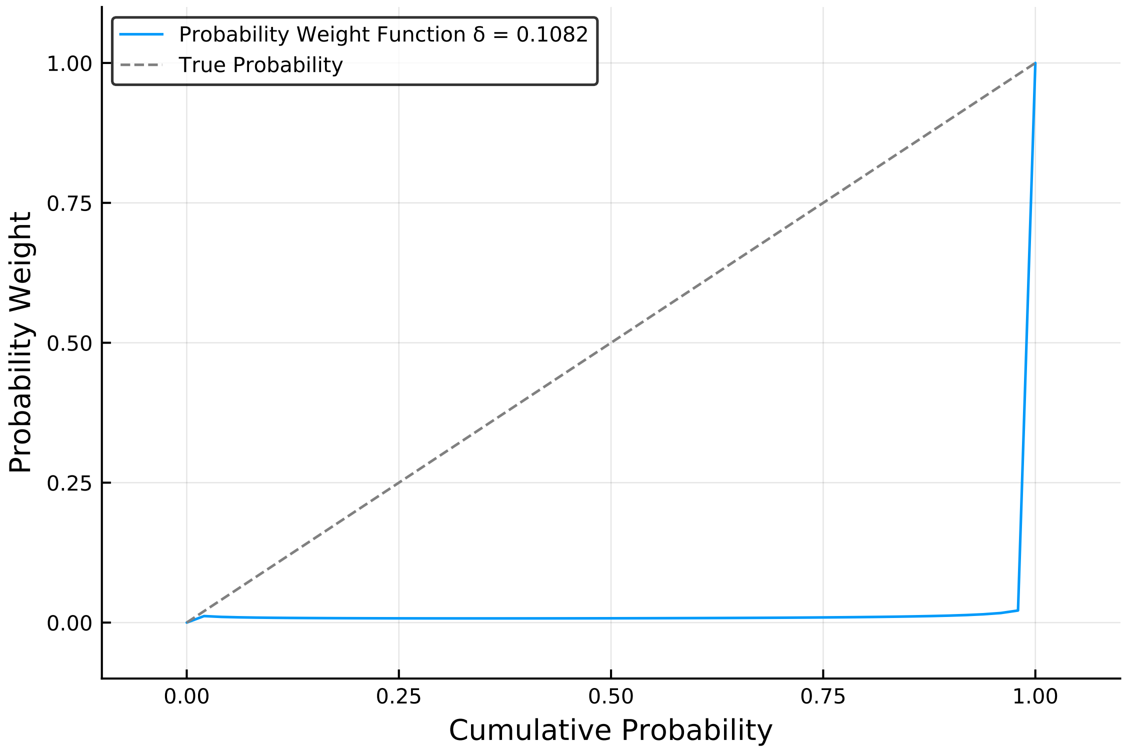
The result that I found most perplexing however, was the value for δ. This value goes completely contrary to what cumulative prospect theory predicts. As there are more high-value rare items available it appears that less weight is applied to their probabilities. Applying a Wald Test against the null hypothesis that $\delta < 1$ yields a p-value of 0, implying that individuals are really not acting as predicted as being risk seeking in low probability gains and risk averse in higher probability gains. They appear to not weight the probabilities very much at higher cumulative probabilities, overweighting the probabilities of the middle-valued items, and under weighting the probabilities of the very rare either in low or high value. One might have expected that when there are more rare items, the probabilities of these rare items would be overweighted rather than under weighted, since the consumers are more sensitive to higher valuations, as shown by the α.

5.2 Different reference dependence

In this section I consider the model if individuals consider gains and losses as relative to the expected value of the crates and the price they paid to open the crates. I changed the reference point to be the price of the box, the price of the key, and the expected value of the contents of the crate at the particular time. This changes the viewpoint of the consumer: instead of viewing gains and losses as relative to the initial income of the opener, we consider them as gains and losses relative to what is expected to gain. Within this paradigm a much different picture emerges.

Likelihood:	-2.389256828231863 x 10^{6}
λ	1.48236 x 10^{-11}
α	0.0
δ	0.108213

First we notice that the same results for λ and α are present, that is there is no loss aversion, and the valuation shape is approaching an indicator function on whether or not it is a gain. However, the probability weighting function has a much different value. This has distorted its shape massively, lowering the function except for the very high-value low-probability items.



This begins to explain buying patterns in these crates, as it seems to imply that the low value items do not matter at all, and that the very rare items that have extremely low probabilities are massively overweighted. This is consistent with the notion that only the very rare high value items are driving the prices for the crates. It would imply that losses opened from the crates are simply consolation prizes, and their value is irrelevant. However I am reluctant to say that this model fully explains the patterns of valuations because of the lack of convexity of the valuation function, whose shape has not changed. I believe that an extreme distortion of the probabilities is occurring, as shown much more by this intrepretation of the structure of the reference point. There are still aspects of the valuation not explained by this particular form, nor by any form that I was able to fit while maintaining the computational properties needed for calculation.

5.3 Further thoughts

While there is no way to investigate whether or not the data exhibits reference dependence, as it is assumed under the structural form. However, the fact that all specifications exhibit zero loss aversion may be a powerful indicator that looking at total wealth values may be the correct method to proceed in this context. If consumers that are purchasing these items have a high level of wealth, then small decreases in their income would have very little effect, as represented in the data, and gains could have more of an effect, especially if individuals engaging in these lotteries are risk-loving. However, due to the lack of any covariate data available, this is not something that can be tested in the current framework and data, and is outside the scope of this paper.

It is difficult to test the predictive power of this model, since it predicts valuations, and the vast majority of the data points are censored. While in theory a test set of untested boxes could be formed, and valuations for the box at each price level formed, and then compared to the market price at that time, evaluating how close to correct the data is would be difficult. Since a purchase only tells us that the valuation was above the price, all that could be tested was whether or not the valuation generated was above the price. This is a poor metric for testing fit, and is a very difficult hurdle to overcome. Looking explicitly at the buy orders is also not a random sample of the valuations, as it is only the valuations that are above the market price. As a result, I do not believe examining the predictive power of this model on a test set is an effective way to determine how well it describes valuations.

6 Conclusions

The lack of more detailed data precludes many forms of complex analysis, such as accounting for unobserved heterogeneity between purchasers, or examining the extent at which current levels of income affect behavior. Despite this, the data clearly shows that under the structural assumptions of cumulative prospect theory, loss aversion simply is not present. However it is impossible to say whether the data is better described by this model than traditional theory. One possibility for testing this would require data that is simply not available for privacy reasons, such as covariates of wealth or correlated variables. One alternative for further study is to data mine the transaction history in the Steam Community Market, and view the total value of the inventory of users who purchased. This would allow for estimating wealth of the users; which would in turn be used to test the assumption that lotteries are viewed in the context of losses or gains rather than changes in wealth. However this would require explicit permission from Valve as I believe it would be invasive of users' privacy, and controlling for users that did not allow for it would be pose problems with random sampling.

What the lack of loss aversion implies about consumer behavior is interesting, as it either implies that the reference point of consumers opening these boxes is not the price they pay, or that they simply are not interested in instances when they incur a loss. There is still a very small α value for many of the boxes, and changing the reference point to be the price of the box, key and the expected value of the crate does little to change this. These combined imply that the lack of loss aversion is not explained by the reference point. The valuation of the box seems to be driven by simply whether it makes a loss or a gain rather than the degree. While there is an exception in the cases that contain many rare items, whose value is driven by the values of these rare items, the vast majority of the crates are not driven by the individual item valuations, only whether or not there is a gain. But even in the exception, since the probability weighting function applies less weight to these rare events, it may be that this structure explains the behavior of individuals facing lotteries poorly, and exploring the avenues of expected utility theory would give more insight into the valuations of consumers. This combined with more powerful computation and structure may allow for a more robust model of the valuations.

I do believe that a more robust model is required for properly explaining the valuations seen in the data. While the computational requirements of any more complicated model can be very high, allowing for much more variation in the valuation function of individual contents of the lotteries may better explain what is driving the valuations of the entire lotteries. I find it hard to accept that the valuation is driven simply by distortions of the probabilities and the number of items that have positive valuations. Even if the distortions are as extreme as predicted by the inclusion of the expected value in the reference dependence, I do not believe that the an indicator function properly describes the valuations. By requiring that the valuations are viewed in the context of gains and losses, and imposing convexity or concavity upon all gains and losses, virtually all of the descriptive power of the model relies upon a proper border between losses and gains being identified exogenously and correctly. However, if it is chosen endogenously, possibly as expected value under the probability weighting function's measure instead of the true probabilities, it becomes extremely difficult to estimate the model. A model that allowed for this change may have much more descriptive power in a framework such as this.

7

References

Counter-strike weapon cases. `http://liquipedia.net/counterstrike/Portal:Containers/Weapon_cases`. Accessed: 2018-03-10.

Steam community market. `https://steamcommunity.com/market/`. Accessed: 2018-03-10.

Barberis, N. C. (2013). Thirty years of prospect theory in economics: A review and assessment. *Journal of Economic Prespectives 27*, 173–196.

Cripps, M. W. and J. M. Swinkels (2004). Efficiency of large double auctions.

Li, H. (2015). Nonparametric identification of k-double auctions using price data. Job Market Paper.

Li, H. and N. Liu (2015). Nonparametric identification and estimation of double auctions with bargaining. *Shanghai University of Finance and Economics*.

Paarsch, H. J. (2018). The foundations of empirical intuition. Version: Sunday 7th January, 2018.

Parsons, S., M. Macinkiewicz, J. Niu, and S. Phelps (2006). Everything you wanted to know about double auctions, but were afraid to (bid or) ask. Unpublished Paper.

Revels, J., M. Lubin, and T. Papamarkou (2016). Forward-mode automatic differentiation in julia. *arXiv:1607.07892 [cs.MS]*.

Tversky, A. and D. Kahneman (1992). Advances in prospect theory: Cumulative reperesentation of uncertainty. *Journal of Risk and Uncertainty 5*, 297–323.