Micro Quiz Corrections

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1 Question 2

1.1 Part b

The insurance company wishes to ensure that the consumer will still participate, so his utility under e = 1 must be higher than not having insurance.

His utility for having insurance is given by: $\frac{2}{3}\sqrt{100-p}+\frac{1}{3}\sqrt{100-p-L+B}-\frac{1}{3}$ and his utility if he does not purchase insurance is: $\frac{1}{3}\sqrt{100-L}+\frac{2}{3}\sqrt{100}-\frac{1}{3}$

His condition for ensuring participation for the consumer must then be:

$$\begin{aligned}
\frac{2}{3}\sqrt{100-p}-\frac{1}{3}\sqrt{100-p-L+B} &\geq \frac{1}{3}\sqrt{100-L}+\frac{2}{3}\sqrt{100} \\
\sqrt{100-p} &\geq \frac{1}{3}\sqrt{49}+\frac{2}{3}\sqrt{100} \\
\sqrt{100-p} &\geq \frac{7}{3}+\frac{20}{3} \\
(100-p) &\geq 81 \\
p &\leq 19
\end{aligned}$$

2 Question 3

The consumer receives marginal benefit of $P(n-1)-P(n)$ with each additional search, and faces marginal cost of c. The consumer wishes to continue searching as long as his marginal benefit is greater than his marginal cost. So he will choose to search

$$n^* = \max_{n\in\mathbb{Z}^+} n \text{ such that: } P(n-1)-P(n) \leq c$$

To prove that this search rule is in fact optimal, first we will establish monotonicity then approach via Contradiction.

Monotonicity: We wish to show that $P(n-1) - P(n)$ is an decreasing function of n.

$$P(n) = K \int_0^1 (1 - F(p))^n dp$$

Let $L(n) = P(n-1) - P(n)$. We wish to show that $L(n)$ is an decreasing function:

$$L(n+1) - L(n) = P(n) - P(n+1) - P(n-1) + P(n) =$$
$$K \left(\int_0^1 (1 - F(p))^n - (1 - F(p))^{n+1} - (1 - F(p))^{n-1} + (1 - F(p))^n dp \right)$$
$$K \left(\int_0^1 (1 - F(p))^{n-1} \big((1 - F(p)) - (1 - F(p))^2 - 1 + (1 - F(p)) \big) dp \right)$$
$$K \left(\int_0^1 (1 - F(p))^{n-1} \big(1 - F(p) - 1 + 2F(p) - F(p)^2 - 1 + 1 - F(p) \big) dp \right)$$
$$K \left(\int_0^1 (1 - F(p))^{n-1} (-F(p)^2) dp \right) < 0$$

Assume that n^* is not optimal. Let $n' \neq n^*$ be the any optimal number of searches to make that minimizes the costs faced by the agent. Either $n' < n^*$ or $n' > n^*$.

Case: $n' < n^*$ Consider the costs faced by searching $n' + 1$ times. The difference of costs between this and n' is given by: $(n'+1)c + P(n'+1) - n'c - P(n') = c + P(n'+1) - P(n')$. Since it is known that $n' < n^*$, $P(n') - P(n'+1) \leq c$ so $c + P(n'+1) - P(n') \leq 0$. This contradicts n' being the minimum of costs, as $n' + 1$ has lower costs.

Case: $n' > n^*$. Since n* is the maximum n such that $P(n-1) - P(n) \leq c$, it must be true that $P(n'-1) - P(n') > c$. Consider the difference between the cost of n' and n' - 1 given by: $(n'-1)c - n'c + P(n'-1) - P(n')$. This is positive since it reduces to have $P(n'-1) - P(n') - c$ Since it is positive, n' cannot be the minimum of the costs, as the cost of $n' - 1$ is less. Therefore it is not possible for $n' > n^*$.

This is a contradiction to $n' \neq n^*$, so there cannot be a minimum to costs that is not equal to n^*