Quiz 2

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7.5

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Approach by Contradiction. Claim that there is a pure strategy that strictly dominates another pure strategy, that is the payoff of this strategy is strictly better than the payoff of that strategy for all strategy profiles played by other players. Let $N \geq 3$. Let X be the pure strategy that dominates all others.

For some pure strategy y, if all other players play 100, the average will be given by: $\frac{100(N-1)+y}{3N} = \frac{100}{3} + \frac{y-100}{3N}$ Let Y be equal to the highest number that can still win if all other players are playing 100. So $Y = argmax_y\{y| \left|\frac{100}{3} + \frac{y-100}{3N} - y\right| < \left|\frac{100}{3} + \frac{y-100}{3N} - 100\right|\}$

Let $1 \leq X \leq Y$ If all other players play 100, the average will be: $\frac{100(N-1)+X}{3N} = \frac{100}{3} + \frac{X-100}{3N}$. For all strategies in this range, there is the same payout to the player, therefore none of these strategies can strictly dominate the others.

If $X > Y$, and all other players play 1, the average will be given by: $\frac{N-1+X}{3N} = \frac{1}{3} + \frac{X-1}{3N}$. This is closer to one than $X \quad \forall X > Y$, this means that for all pure strategies $> Y$ the player earns the same payout when all other players play 1, so none of these strategies can be strictly dominate the other strategies in this case.

Consider two pure strategies, X,W where $X \leq Y, W > Y$, If $X \geq 2$ and all other players play the pure strategy of 1, X returns the same payoff as W, so X cannot strictly dominate W, and W cannot strictly dominate X. If $X = 1$ and all other players play 100, except one other who plays the number that will win, which for N ≥ 3 is guaranteed to not be equal to 1, then one does just as well as any number greater than Y, so neither can strictly dominate each other.

This exhausts all cases, so therefore it is impossible for any pure strategy to strictly dominate any pure strategy.