Question 1

a

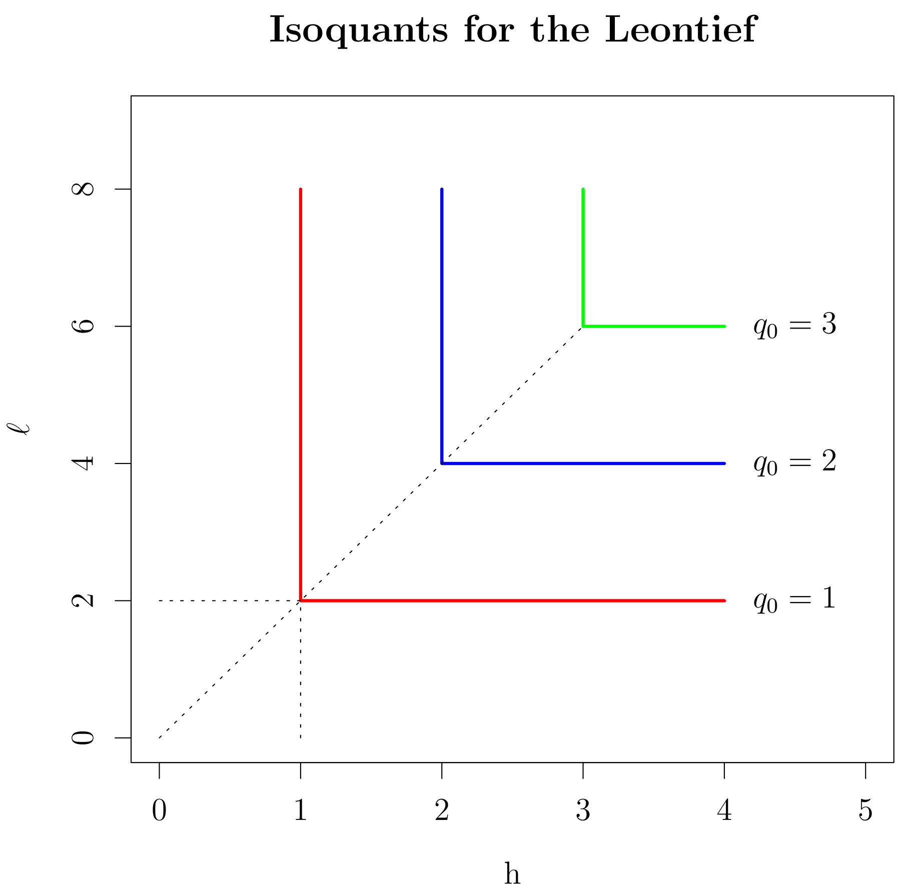

Fig. 1: Level Sets of a Leontief Production Function

α and β are technology parameters that control how much of each input to reach a certain amount of output. To reach q level of output, one needs $\frac{q}{\alpha}h$ and $\frac{q}{\beta}\ell$. We may note that this production function has constant returns to scale, so it has a linear scale effect.

b

$$C(q, w, s, \alpha, \beta) = \min_{h,\ell} hw + \ell s \quad \text{s.t.} \;\; \min\{\alpha h, \beta \ell\} \geq q$$

Since w and s are input prices, we will assume they are positive, so the constraint will bind. Since the constraint binds, we know that: $\min(\alpha h, \beta \ell) = q$ and that $\alpha h = \beta \ell = q$. If it were otherwise, the firm could lower the usage of either h or ℓ and minimize the objective function without effecting the constraint. Therefore: $h = \frac{q}{\alpha}$ and $\ell = \frac{q}{\beta}$. The cost function is therefore:

$$C(\alpha, \beta, w, s, q) = \frac{wq\beta + sq\alpha}{\alpha\beta} = q\frac{w\beta + s\alpha}{\alpha\beta}$$

The marginal cost of this function is the derivative with respect to q, and is:

$$m(\alpha, \beta, w, s) = \frac{w\beta + s\alpha}{\alpha\beta}$$

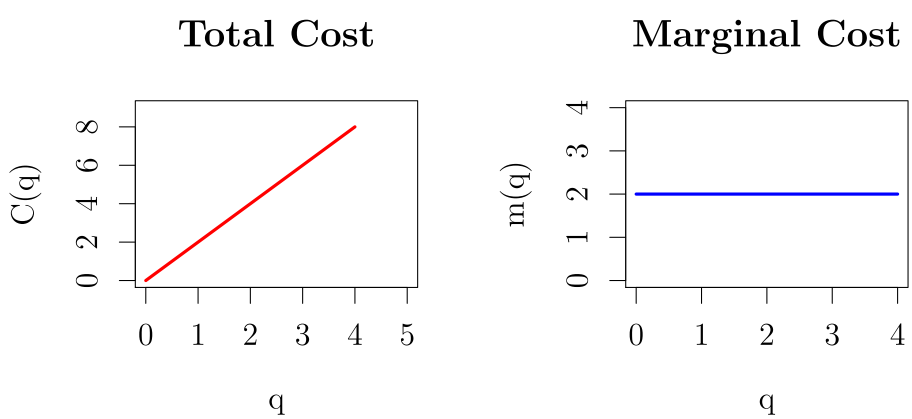

Fig. 2: Total and Marginal Cost

Question 2

The purpose of game theory is to be able to study strategic interaction between agents when all agents have some power. The two main branches of Game Theory are strategic and coalitional games. In Strategic Games player's available moves are specified, and optimal behavior is derived. In Coalitional games, optimal behavior is described. All games feature:

- Players: P
- Rules: R
- Information structure stating what each player knows: I
- Set of Strategies available to players: S_i
- Payoff functions for strategies: U_i
- Some Notion of equilibrium

Two notions of equilibrium used to solve games are Nash-Equilibrium, where each player is playing the best-response to opponents' best response functions; and Dominant equilibrium, where a player plays a strategy irrespective of the actions taken by the other players.

One example of a game is a student who wishes to ask questions to a certain professor, but if he asks a dumb question, the professor will chastise him in front of his colleagues. He must strategically estimate how the professor will react to his question based on prior knowledge to decide whether or not he should ask it, while the professor must decide whether or not to harshly respond in order to either minimize or maximize questions depending on his preferences of class. In this case $P = \{\text{Student, Professor}\}$, the Rules are that the student may ask any question, and the professor may respond in any way. The student may continue to ask questions as long as class is still in session.

We assume that the professor has full knowledge of the subject, so he has perfect information about if the question is stupid or not, while the student only has partial information, but with each question successfully asked gains information. The student has no knowledge of how the professor will respond, and the professor knows that a certain amount of belittling will dissuade the student from asking questions, but he does not know the threshold.

Therefore the set of strategies for the student is a distribution of questions where the more informative questions are more likely to be viewed by the professor as stupid, and no questions, for when the student has been sufficiently chastised. This is crossed with itself for a countably infinite number of times. The professor's strategy is a continuum of how well to answer the question (encouraging questions and learning at the cost of time) and how much to berate the student (discouraging questions and increasing time available for lecture.)

The student faces an increasing utility for information gained, and some negative quantity for when he is berated by the professor. The professor may have an increasing utility for increasing student knowledge, or any other utility function depending on his desires while teaching.
It is unclear whether or not this game would lead to a Bayes-Nash equilibrium because of very little being stated about the payoffs of the Professor, and more structure may need to be imposed to enforce an equilibrium arises.

Another example would be deciding where to go to lunch. The players in the game are all the other people seeking lunch, and everyone dislikes waiting in a line, so there is a negative effect caused by others going to the same restaurant as you. The set of players is everyone who is seeking lunch.
The rules are that they must wait in line when they arrive behind everyone else who has already arrived.
The information structure is such that each person knows when they leave for lunch, and the distribution of when everyone is getting lunch, but not specific times when each person has gone for lunch.
The payoff functions are such that each person has preferences over restaurants and in waiting in line, and the combination of this forms their utility for the lunch.

Question 3

a.

By inspection, we may note that child A finds confess to be a dominant strategy, since $-1 > -2$ and $0 > -2$. By the same logic, child B finds confession to be a dominant strategy, so the equilibrium will be both confess and face payoffs of -1 apiece. This equilibrium is a Dominant Equilibrium, as both chose strategies that were best for them regardless of the other's choice.

b

If child A knew B would confess, his best response would be to confess as well, and if he knew that B would be silent, his best response is to be silent. By symmetry, the best response for B is the same. We can clearly see that both children have a best response of being silent when the other is silent, and to confess when the other confesses. Thus there are two Nash Equilibria, both confessing, and both being silent.

Question 4

Extensive form games are games that allow for sequential play, leading to dynamics. This element allows them to accommodate incomplete information. This is managed through the use of the decision tree, which allows for an easy representation of not only their available moves at each stage, but what information they have been exposed to and can therefore act upon.

One phenomena involving incomplete information is a private-values auction, where the buyers all have private valuations of an object that they are very unlikely to disclose, especially to a seller. This can cause the participants to lie about their valuations, particularly if they have high private valuations and they are forced to pay their bid.

Another phenomena is a game of poker, where the cards held by each player are private information, and the only information given is by bets placed at each round as public information (flop turn river) is given out. Players have an incentive to lie in order to give the false idea that they have a better or worse hand than they do.