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A Structural Approach to Estimation of Valuations Randomly Distributed in Counter-Strike: Global Offensive

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1 Introduction

In the first-person shooter game *Counter-Strike: Global Offensive*, players play online games against each other featuring various versions of modern guns. Players can buy or earn skins for these weapons, a recoloring of the weapon that is visible to both themselves and anyone currently in the game. There are skins for virtually every element of the game: guns, knives and gloves, with many being very rare, and thus very expensive. These skins vary from recoloring to artistic designs. They are purely cosmetic and have no influence on the outcome of the game.

As an incentive for playing, players can randomly receive an item as a reward at the end of a match. These items can come in two possible forms: common skins and weapon cases. The weapon cases are lotteries that contain various weapons with fixed and known probabilities. If one believes that each individual has fixed preferences between each of the items contained in the boxes, a utility function over lotteries and there exists unobserved heterogeneity between individuals in their risk preferences, then this induces a distribution of valuations for the lotteries. A large number of potential items contained in each of the boxes exist, as there are many varieties to a single item.

These items are issued randomly over time to players, and the process with which they are distributed is of interest. When a new case is introduced to the market, one would like to see if there is a change in the probability of receiving the older cases, as well as if there are adjustments made over time by Valve, the company running the game. Since these items are distributed randomly, there is no reason to believe that they are in any way efficiently allocated. In order to allow for an efficient outcome, these items are traded at market. This market allows those with the highest valuation to end up with the item, reaching an efficient allocation. The market is available to all players with a steam account and is public.

I seek to impose a strong structure on how these items are distributed to players, and determine the nature of the valuations as well as how many players are active in the market and how many enter or leave the market over time. These primitives can then be extracted to determine future policy decisions made about the implementation of newer items into the game.

2 Model

For any given item, assume that there is a mass of consumers who have valuations based on some distribution F_V. With some exogenous probability, some consumers are endowed with an item with probability $\boldsymbol{\xi}$. Consequently, the same distribution of valuations in those endowed as well as those who are not endowed, even if there are different numbers of people who are endowed.

Because the process of granting items is random, it is in no way efficient. The individuals who value the good most do not necessarily receive it under the current function. In order to achieve this efficiency, a market is implemented, taking the form of a double auction.

It is known that double auctions converge rapidly to a competitive environment; see Cripps and Swinkels (2006). The data pulled are relatively poor for extracting valuations from the bids, as for much of the data the bids are not observed; as a result attempting to identify valuations from the bids would not work well. Even though the result is certainly possible for double auctions under conditions such as sealed-bid, and one buyer and seller, there has been no identification, based on a dominant or equilibrium argument in the continuous double auction. See: Parsons, Macinkiewicz, Niu, and Phelps (2006)

Consequently, I shall abstract from the dynamics and the mechanism of the double auction, and due to the large amount of traffic, focus on its convergence into a competitive market. If the market is efficient, then a matching between buyers and sellers obtains after the trades where those with the highest valuations have the items. Since there is large amount of buyers in each time period, we will use he rapid convergence to assume a competitive equilibrium without much ado.

2.1 Matching Problem

I examine the competitive market in the context of a matching problem, Following the model by Shapley and Shubik of perfectly transferable utility. Chade, Eeckhout, and Smith (2017) Since it is known that the planner's problem of maximizing total welfare, and the decentralized market are equivalent for this problem, one can examine either interchangeably to provide motivation for the problem.

The surplus generated by any exchange between a buyer and a seller is given by the valuation of the buyer minus the valuation of the seller. A central planner, who wishes to maximize the total surplus then faces the question of finding a (partial) matching between buyers and sellers such that the surplus generated is maximized. For some arbitrary I buyers and J sellers:

$$\begin{aligned} &\max_{\alpha_{i,j}} \sum_{i=1}^{I} \sum_{j=1}^{J} (V_i - V_j)\, \alpha_{i,j} \\ \text{subject to: } \forall j, 1 \leq j \leq J \quad &\sum_{i=1}^{I} \alpha_{i,j} \leq 1 \\ \forall i, 1 \leq i \leq I \quad &\sum_{j=1}^{J} \alpha_{i,j} \leq 1 \end{aligned}$$

The constraints serve to require that each individual make at most one exchange. One desirable result is that this linear program is always maximized at integer values of α. This ensures that the solution contains no partial matching. Of interest as well is the dual of the problem, specified below.

$$\begin{gathered} \min_{x,j} \sum_{i=1}^{I} x_i + \sum_{j=1}^{J} y_j \\ \text{subject to: } \forall i, j; \quad 1 \leq j \leq J, \quad 1 \leq i \leq I \\ x_i + y_j \geq V_i - V_j \end{gathered}$$

The solution to this problem form the shadow prices of the exchange, or the amount of surplus that a buyer or seller takes based on their type. These allow for the price to be computed directly.

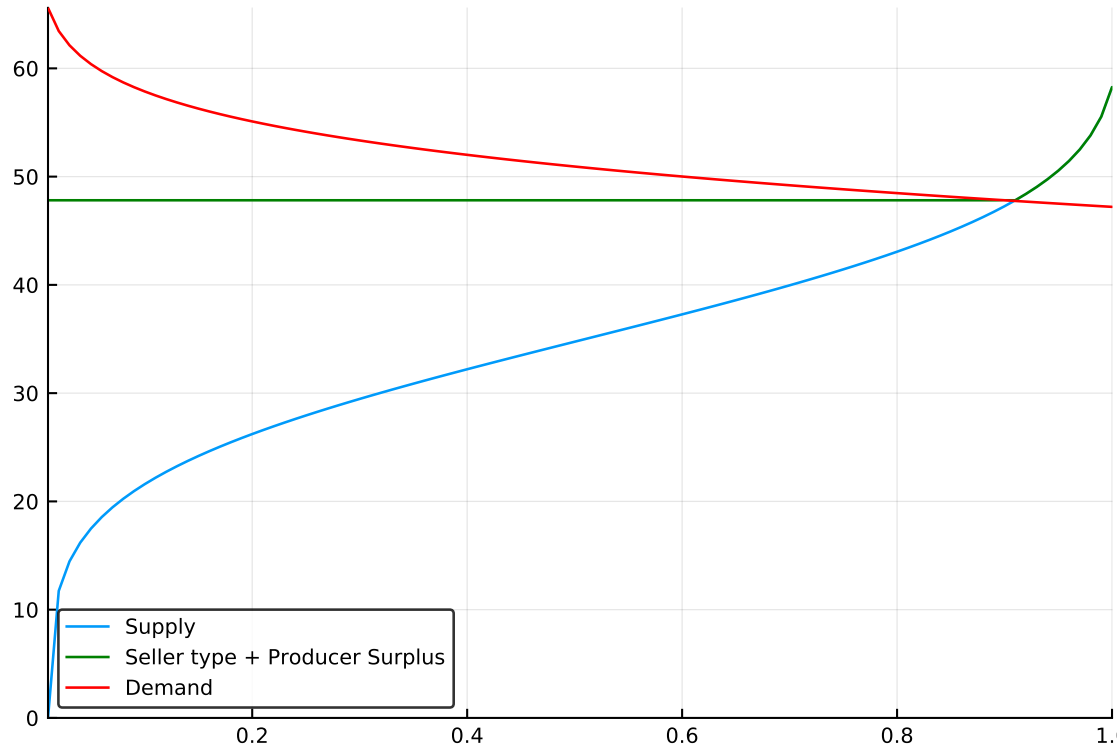
2.1.1 Results

One important thing to note about the objective function is that it is the buyer's valuation less the seller's valuation. At the time of the matching, V_i, V_j are known and fixed, and not random. The objective function is both super-modular and sub-modular. For this matching problem, this implies that both positive assortative mating and negative assortative mating are supported. Chade et al. (2017) After some inspection, one can see that even though the process will determine which of the sellers and buyers match, any permutation of the matches is just as optimal. This means that any maximizer of the primal is not unique.

That said, the dual of the problem does have a unique solution, as it is the shadow price for the type of the seller and the buyer. These values are the producer and consumer surplus for each type. Since it is a competitive equilibrium, there is one price supported, as the good is homogeneous, and the matching is occurring between valuations for the good. The seller's valuation plus his shadow price will be equal to the competitive price for all sellers who do exchange.

For equal-sized buyer and seller valuations, this gives the intuitive result that the lower half of the distribution of sellers will sell to the upper half of the distribution of buyers, and we will have the efficient result. As the size of the seller's mass shrinks, with the rarity of the item increasing, a higher proportion of the sellers choose to sell, and the receiving end of the distribution of buyers shrinks, as the price increases. This is demonstrated below for valuations that are distributed normally, with mean 35, and standard deviation of 10. One tenth of the population is endowed with the item. The equilibrium price is calculated by taking the seller's valuation plus his shadow price.

If one considers the decentralized market version of the problem, all buyers are indifferent between the sellers they choose, as they must give up the producer surplus to the seller, and as a result face a constant price to buy from any seller type.



The distribution of buyers has become truncated by the difference in the number of buyers and sellers. To maintain the efficient outcome, only the top 10 percent of the buyers are able to purchase, and 90 percent of the sellers are now selling.

Within the context of this matching model, the change in the relative sizes of the population of suppliers acts to truncate the buyers rather than lower the supply. It is important to note that these are not exactly supply and demand in the normal sense, as instead of quantity, the x-axis is the proportion of the sellers that exchange.

2.1.2 Equilibrium

As a result of the lens in which this market is viewed, a slightly different sort of equilibrium obtains. Although all the desirable properties of an equilibrium hold, notably efficiency, and being in the core, we are only examining exchanges in one good, so it remains a partial equilibrium. Chade et al. (2017)

Assume that the valuations of the players are distributed normally, as in the examples above. Then the supply function can be written as $q^s = N\xi\Phi\left(\frac{p-\mu}{\sigma}\right)$ and the demand function can be written as: $q^d = N\left(1-\xi\right)\left[1-\Phi\left(\frac{p-\mu}{\sigma}\right)\right]$, ξ is the percent of people endowed with the item. In equilibrium, the quantity of buyers and sellers are equal:

$$\xi\Phi\left(\frac{p^*-\mu}{\sigma}\right) = (1-\xi)\left[1-\Phi\left(\frac{p^*-\mu}{\sigma}\right)\right]$$
$$p^* = \mu + \sigma\Phi^{-1}(1-\xi)$$

The price supported by the market is the average valuation plus a component that depends on the rarity of the item. Essentially, the price is controlled by some universal notion of value, such as the design of the skin, as well as a rarity element that drives price up or down depending on how easy it is to obtain.

2.2 Dynamics

The data are ordered according to time intervals, so the process must be estimated dynamically. Consider a series of time intervals, in which there is the matching device described above. In each interval, a percentage of the population is awarded the item, and the matching device functions to distribute efficiently.

Firstly, consider the model with no entrants. After the initial exchange, those that do not have the item are random attributed the item again, but their distribution is no longer the initial distribution, it has been conditioned on losing the top portion of its mass. Therefore the distribution of those that are possible sellers is a mixture of this truncated distribution, and the top portion that left the potential buyers. In this model, the top portion of those that have the item will never sell it, as the valuations of those that do not are all strictly below them: consider the seller distribution to be a percentage of the buyers. The process then repeats, albeit with a slightly truncated portion of the valuation function.

This model also more captures more elements of the market than the original, as it can explain the behavior observed of a high initial price, slowly dropping to some equilibrium level. With an explanation of the dynamics of the process in place, we can look at the entire lifetime of the item, and only have to control for the truncation of the valuations for the demand. As long as there is no entrance of individuals into the model, the price will necessarily decrease.

2.2.1 Specification

For each time period t, the drop rate to individuals estimated is given by: ξ_{t}. The price observed in that period is p_t. In the first time period, everything proceeds according to the previous model. In the second time period however, allow the top ξ_0 percent to exit the model. There are $N(1-\xi_0)$ people remaining, of which ξ_1 have received the item, so the mass of suppliers is: $\xi_1(1-\xi_0)N$. The mass of the buyers is: $(1-\xi_1)(1-\xi_0)N$. It should be noted that the distribution of both the supply and demand is binomial, with its mass, and probability of purchase at each price.

$$\Pr\left[V_2 < v | V_1 < F_V^{-1}(1-\xi_0)\right] = \frac{F_V(v)}{F_V(F_V^{-1}(1-\xi_0))} = \frac{F_V(v)}{1-\xi_0}$$

$$\mathbb{E}\left[q_2^s\right] = N(1-\xi_0)\xi_1\left[\frac{\Phi\left(\frac{\log(p_t)-\mu}{\sigma}\right)}{1-\xi_0}\right]$$

$$\mathbb{E}\left[q_2^d\right] = N(1-\xi_0)(1-\xi_1)\left[1-\frac{\Phi\left(\frac{\log(p_T)-\mu}{\sigma}\right)}{1-\xi_0}\right]$$

One may continue the process, noting that with each truncation, there is a multiplication of $(1-\xi_t)$ in the denominator of the valuation function.

$$
\begin{aligned}
\mathbb{E}\left[q_T^s\right] &= N\prod_{t=1}^{T-1}(1-\xi_t)\xi_T\frac{\Phi\left(\frac{\log(p_T)-\mu}{\sigma}\right)}{\prod_{t=1}^{T-1}(1-\xi_t)}\\
\mathbb{E}\left[q_T^d\right] &= N\prod_{t=1}^{T}(1-\xi_t)\left[1-\frac{\Phi\left(\frac{\log(p_T)-\mu}{\sigma}\right)}{\prod_{t=1}^{T-1}(1-\xi_t)}\right]\\
\log\left(p_T^*\right) &= \mu+\sigma\Phi^{-1}\left[\prod_{t=1}^{T}(1-\xi_t)\right]\\
q_T^* &= N\xi_T\prod_{t=1}^{T}(1-\xi_t)
\end{aligned}
$$

2.3 Market Entry

Although the percentage endowment model does describe several of the price processes quite well, it struggles to rationalize the nearly constant quantity of items sold in each period. One way to explain that is to allow for market entry over time.

Consider the case in which the number of entrants in the market is not held constant, and instead new entrants to the market have a different distribution from older ones. The distribution of the buyers in the following period is now a mixture distribution. Since one can now find a buyer of the highest valuation, it is possible that sellers who had previously bought might be willing to sell again. Consequently, the entire seller's distribution must be considered as well, as a mixture of the highest valuations, and the individuals that have just become endowed.

After the first exchange of items, λ_0 percent of N people enter the market, drawing their valuations from a potentially different distribution E_t. Then the endowment process is repeated, and exchange occurs. After this process, λ_1 percent of the $N(1+\lambda_0)$ people enter the market. That is, λ_t is the proportion of the inhabitants of the market that enter the market in time period t. However, they enter the market after the exchange has occurred. This ensures that there is no entrance in the first time period where it would be indistinguishable from N.

The distribution of buyers and sellers remains binomial. Since all sellers are possible sellers now, the distribution and mass of the buyers and sellers has become noticeably more complex. The mass of the sellers is now the sum of the mass of the buyers times the percent of people endowed in each time interval. That is, in time period one, the sellers received $N\xi_0$ mass, and the mass of the buyers was: $N(1-\xi_0)$. However, then $N\lambda_0$ people arrived, and for time period one the buyers had mass: $N(1-\xi_0+\lambda_0)(1-\xi_1)$, and the sellers had mass: $N\xi_0+N(1-\xi_0+\lambda_0)\xi_1$.

The mass of the buyers and the sellers continues on this trend and is given by:

$$M_B(T) = (1-\xi_T)\left[M_B(T-1) + \lambda_{T-1}\prod_{t=1}^{T-2}(1+\lambda_t)N\right]$$
$$M_S(T) = N\prod_{t=1}^{T-1}(1+\lambda_t) - M_B(T)$$

In each time period, it is assumed that the market clears and, therefore, the price observed in each time period determines the percent of people that choose to purchase. All buyers with valuations above the price choose to purchase, and all sellers with valuations below the price choose to sell. Since there is entry into the market, the distribution will no longer simply be truncated. The truncated distribution of people who chose not to buy will be mixed with an untruncated distribution of individuals who enter the market. The incoming distribution may be significant enough to drive the price up; thus it is possible that those who bought the item in a previous time period may wish to sell it as the price is driven upward. Thus, one must consider everyone who has received the item in the supply, rather than just the individuals who received it in the current time period.

If the price increases from time T to time $T+1$, we know therefore that all purchasers must be entrants to the market, as everyone who could have bought it in the previous time period would have. If the price decreases from time T to time $T+1$, then the sellers distribution must only contain people who received it during this time period. These two facts imply that our supply and demand functions will be kinked, and analytic representations of price and quantity will be impossible to obtain except for trivial cases.

It is assumed that at each time period, the market is in equilibrium, so the previous valuation function evaluated at the previous equilibrium price signals the percent of the buyer's mass that purchased the item. Let $E_T(p)$ be the distribution function for the valuations of a person entering the market at time T. Let $B_0 = S_0$ be the initial valuation functions.

$$B_T(p) = \frac{B_{T-1}(p_{T-1})}{B_{T-1}(p_{T-1})+\lambda_T}\min\left\{1, \frac{B_{T-1}(p)}{B_{T-1}(p_{T-1})}\right\} + \frac{\lambda_1}{B_{T-1}(p_{T-1})+\lambda_T}E_T(p)$$
$$S_T(p) = \frac{M_S(T-1)}{M_S(T)}\max\left\{0, \frac{B_{T-1}(p)-B_{T-1}(p_{T-1})}{1-B_{T-1}(p_{T-1})}\right\} + \frac{M_S(T)-M_S(T-1)}{M_S(T)}B_T(p)$$

$B_T(p)$ and $S_T(p)$ are strictly increasing functions of p, so the intersection between $B_T(p), S_T(p)$ is uniquely defined. In the case when $\lambda_t = 0$ this is the dynamic model covered previously. The moment conditions we shall invoke are implied by the competitive equilibrium: Namely, expected values be equal and equal to quantity sold.

$$M_B(T)B_T(p_T^*) = M_S(T)S_T(p_T^*)$$
$$M_B(T)B_T(p_T^*) = q_T^*$$

3 Source of Data

The data has been mined from the Steam Community Market API, which provides a purchase history for every item on the market, down to the hour for the last thirty days and daily for the rest of the lifetime of the item. It does not provide a record of every purchase, just the quantity sold in that time period as well as the median price they were sold at. The price data is only accurate to the cent, and features a price floor of $0.03. The selling mechanism is a K-double auction where the seller always receives his bid. Buy and sell orders are placed, and then matched based on price and time spent waiting in queue.

The data is price and quantity sold data of items on the Steam Community Market for the game *Counter Strike: Global Offensive*. Players in game earn items random that they can sell on the market or open themselves. Most rare items are earned via opening of dropped *loot boxes* that are then opened by players via purchasing of a key. These boxes can be earned by playing or received randomly from players who are watching professional games. The probabilities of the drops are not known or even estimated well, as they change depending on many factors including time. They are one primitive that we seek to estimate.

Once a box has been obtained, the probability of receiving an item is well documented as required by Chinese Law. Each item has a certain grade of rarity, for example the AK-47 Redline has a rarity level of Classified which means that there is a 3.2 percent chance of receiving a Classified item in the crate. All items of equal quality contained in the crate have the same probability of being dropped by the crate.

Many variants of each item exist. Each item has a quality ascribed to it, the float of the item. This describes the wear on the item, and is distributed uniformly on the interval 0-1. On the market the items are split into intervals: Battle-scared, well-worn, field-tested, minimal wear and factory new. Each quality is a separate listing on the market with a separate price. In addition to each item having a quality type there is also a ten percent chance of each item being labeled as StatTrak, which also distinguishes the value of a weapon. This means that each item has ten possible different variations all with different probabilities of being obtained. Some rare items, usually knives and gloves may have more or less quality variants, but the amount and probabilities are known, and can easily be determined by checking if there is a market history for the item.

The probabilities for each condition are as follows:

Float	Condition
0.00 - 0.07	Factory New
0.07 - 0.15	Minimal Wear
0.15 - 0.38	Field-Tested
0.38 - 0.45	Well-Worn
0.45 - 1.00	Battle-Scarred

Each item has a 10 percent chance of being StatTrak if that item has StatTrak enabled. float values are distributed uniformly, making the probability calculations simple.

The rarity of a skin also controls its probability of being dropped in a particular lottery. These rarities are set by Valve, and are specified for each crate. They rank from gold (very rare) to blue (common). The probabilities of getting an item of a rarity is given as follows:

Probability	Rarity
0.0026	Special (Gold)
0.0064	Covert (Red)
0.032	Classified (Pink)
0.1598	Restricted (Purple)
0.7992	Mil-spec (Blue)

In each box there are several items of each rarity, each one is equally likely to be found when the lottery is realized.

Each box contains some subset of these items that is known, and the market value of each item at a particular time period is also known, so the expected value, or any other modified version of a valuation of the lottery can easily be calculated.

4 Estimation

Initially, consider estimates of the model without entry. In each time period, the distribution of supply and demand is binomial. These distributions are independent, and the difference between two binomial distributions that are not independent is difficult to estimate using likelihood methods. Consequently, the generalized method of moments will be utilized. Since the price is uniquely defined in each time period, as is the quantity supplied, the question of estimation is feasible.

For period T, there exists two moment conditions specified, one for price, and one for quantity. Under the specification for the model:

$$F_V\left(p_T^*\right) = \prod_{t=1}^{T}(1-\xi_t)$$

$$q_T^* = \xi_T \prod_{t=1}^{T}(1-\xi_t)$$

This provides us with $2T$ moment restrictions on the model, and allows for estimation of up to $2T$ parameters.

A distinction must be made between observations and time periods. The data are divided into the median price and quantity sold in each day, and the question of how many data points are in a time period must be answered. For the purposes of the estimation in this paper, I shall use five observations per time period. If there are N observations, then there are $T = \lceil \frac{N}{5} \rceil$ time periods. This arises from the need for no serial correlation between the data points in order to exploit the statistical properties of the method.

For the model specified with T time periods, and for a distribution of prices of log-normal, there are two parameters for the distribution, and T parameters for $\boldsymbol{\xi}$. There are $2T$ moment restrictions, so the model is in fact over-identified. This allows us to test the specification for our model using the Sargan-Hansen J-test.

4.1 Complications

One important complication is that there exists a price-floor in the market. No item is able to be sold at less than \$0.03, this means that for all data points where the price is at this floor, the equilibrium condition is not binding. All that one can say is that the equilibrium price must be below the price floor. This censored condition may be written as $p^* < p_T$. This is equivalent to: $\max\{p^* - p_T, 0\} = 0$. The equilibrium quantity is also not represented by the data when the price is censored. The only condition that is binding is that the quantity demanded at the price is the given quantity when the price floor is binding. One can rewrite the two conditions using the primitives of the model to give the binding price-floor moment conditions:

$$\max\left\{\prod_{t=1}^{T}(1-\xi_t) - \Phi\left(\frac{p_T^* - \mu}{\sigma}\right), 0\right\} = 0$$
$$N\prod_{t=1}^{T}(1-\xi_t)\left[1 - \frac{F_V(p_T^*)}{\prod_{t=1}^{T-1}(1-\xi_t)}\right] - q_T^d = 0$$

The maximum function is not differentiable at zero, where the optimal value of our function is located. To overcome this, a differentiable alternative to the maximum function will be used:

$$\max\{x, y\} \approx \frac{1}{\rho}\log\left[\exp(\rho x) + \exp(\rho y)\right] \qquad \text{for some } \rho > 0$$

One important aspect of the data that the theory does not represent well is the path of the equilibrium quantity. Under the currently defined model, the equilibrium quantity is a necessarily decreasing function. Quality paths in the data follow a nearly constant path. One way to handle this is to seek a different setting where the equilibrium quantity can be viewed as a percent of the starting number of people in the market. This can be rationalized by imagining that after each transaction, the sellers re-enter the market with the truncated valuation distribution. This means that there is always N people available at each iteration of the market, but the distribution continues to be truncated as before. This leaves the price moment unaffected.

$$q_T^* = N\xi_T(1-\xi_T)$$

This specification permits estimation of quantities sold that all remain on the same order of magnitude, as opposed to the strictly decreasing quantity path specified in the original specification.

4.2 Implementation

Consider a function $g(Y_t, \mu, \sigma, \boldsymbol{\xi})$ which gives the moment condition for each time period, evaluated at the t^{th} element in that time period. Under the Null Hypothesis that this model fits the data, then the expected value of this function is zero.

$$\mathbb{E}[g(Y_t, \mu, \sigma, \boldsymbol{\xi})] = 0$$

I sought to estimate the parameters μ, σ, $\boldsymbol{\xi}$ by minimizing the sample analog of this with respect to a quadratic form of weighting matrix W. The sample analog is formed by averaging the data found contained in each time period. $\hat{\boldsymbol{m}}(\mu, \sigma, \boldsymbol{\xi}) = \frac{1}{M}\sum_{m=1}^{M} g(Y_m, \mu, \sigma, \boldsymbol{\xi})$. Combine the parameters of the model into a vector $\boldsymbol{\theta}$. The goal then becomes to estimate a value of $\hat{\boldsymbol{\theta}}$ by minimizing the quadratic form of $\hat{\boldsymbol{m}}(\hat{\boldsymbol{\theta}})$ with respect to matrix $\boldsymbol{W}$.

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \quad \hat{\boldsymbol{m}}(\boldsymbol{\theta})'\boldsymbol{W}\hat{\boldsymbol{m}}(\boldsymbol{\theta})$$

The choice of $\boldsymbol{W}$ is selected by first choosing a positive definite matrix $\boldsymbol{W}$, and estimating the model, and then estimating the matrix by the following method:

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} = \arg\min_{\boldsymbol{\theta}} \quad \hat{\boldsymbol{m}}(\boldsymbol{\theta})'\widehat{\boldsymbol{W}}_{\boldsymbol{i-1}}\hat{\boldsymbol{m}}(\boldsymbol{\theta})$$

$$\widehat{\boldsymbol{W}}_{\boldsymbol{i}} = \left[\frac{1}{M}\sum_{m=1}^{M} g(Y_m, \hat{\boldsymbol{\theta}}_{i-1})g(Y_m, \hat{\boldsymbol{\theta}}_{i-1})'\right]^{-1}$$

This process is then continued until the value of θ_{i-1} is a minimizer for $\widehat{\boldsymbol{W}}_{\boldsymbol{i}}$. This iterated GMM estimator is invariant to the scale of the data, which is important in this model, as the price and the quantity data are of wildly different magnitudes Hall (2005). This method is also asymptotically equivalent to the Continuous Updating Efficient GMM, but does not have as many numerical instabilities.

This process is complicated by $\widehat{\boldsymbol{W}}_{\boldsymbol{i}}$ being of rank M. If the matrix is not of full rank, then it is not invertible, and one cannot estimate the model. In order to ensure that it has full rank, the process is regularized by adding a positive number times the identity matrix to ensure that $\widehat{\boldsymbol{W}}_{\boldsymbol{i}}$ is both positive definite and invertible. However, the asymptotic properties of the J-test require that the matrix $\widehat{\boldsymbol{W}}_{\boldsymbol{i}}$ be converging in probability to the true variance matrix. As a result, we divide our positive number added by the number of observations in each time period, allowing for our change to converge to 0 in probability.

The model was estimated using the code found in the file `dataTest2.jl` using the programming language Julia. Utilizing the package `Optim.jl`, the objective function was minimized using the BFGS algorithm. This ensured that numerical problems that could arise out of calculations of inverting a small hessian were avoided. Several of the fits are shown below. Gradient calculations were made using Forward Automatic Differentiation, see: Revels, Lubin, and Papamarkou (2016).

One important note on the price predictions is that the model is able to predict the price path relatively well at the start, but begins to struggle with matching the price in the latter half of the model. This is due in part to the specification which gives undue control over the price path to the earliest values of $\boldsymbol{\xi}$, causing the early under-estimation of the price in the Spectrum 2 Case. Dominating the latter part of the model is numerical problems. Even using the exponential of sums of logarithms, the value of $\prod_{t=1}^{T}(1-\xi_t)$ still becomes ill-behaved numerically.

When there is lots of variability, leading itself to upward trends in the price, the model can struggle trying to be able to incorporate the data into its structure.

Even though the model struggles greatly with large increases in price, and has no way of incorporating them into the primitives, sharp decreases in the price are also difficult for the model to handle. In this model, large decreases in the price require a large number of individuals endowed with the item in that time period, something which must also be supported by the quantity data.

4.2.1 Numerical Complications

Two problems exist with the model as currently estimated: The sensitivity to the initial value as well as the magnitude of the zero condition on the moments.

The core issue with estimation in this dynamic system is that each time period involves parameters from all the previous time periods. This gives the estimated values of the endowment in the first time periods enormous effect on the fit of the model. This also means that the relationship between the different values of $\boldsymbol{\xi}$ is highly complex and nonlinear. As a result of the nonlinearity, there are many saddle points contained in the geometry.

To complicate matters is the form of $\widehat{\boldsymbol{W}}$. Since there are so few data points in each time period, $\widehat{\boldsymbol{W}}$ relies on the regularization to be positive definite and invertible, the optimization problem is not well formed. One alternative would be to follow a simulation sampling process to arrive at the minimum value. The process followed in this paper is to test based on several different initial values of μ and σ, and the minimum of those will be treated as the global minimum. This does not guarantee that the process will converge to the global minimum. This creates problem using the iterative method of forming $\widehat{\boldsymbol{W}}$, as optimization mishaps in the first instance are compounded into an improperly formed covariance matrix that does not need to even be positive definite.

These problems stem from the magnitude differences between the two types of moment conditions. The price moment condition, which means on the order of magnitude between negative three and three, and the quantity moment which is on the magnitude of 30000. While the final iteration of $\widehat{\boldsymbol{W}}$ is invariant to differences in magnitudes of the moments, problems formed in the initial optimization problem can manifest themselves, preventing the routine from reaching the global minimum.

Since it is known that the limit of $\widehat{\boldsymbol{W}}_{\boldsymbol{i}}$ is invariant to differences in magnitudes between the components of g_t, that means that one can adjust the magnitudes of the moments to allow for the optimization routine to converge to the true minimum in the early instances. As a result, the quantity moment is divided by N in order to place it on a magnitude with the price moment. This allows for the price moments to impact the optimization routine in the first few instances.

As a result, the moments used in estimating the procedure in each time period are as given:

$$\begin{aligned}\exp\left[\sum_{t=1}^{T}\log(1-\xi_t)\right] &-\Phi\left[\frac{\log(p_T^*)-\mu}{\sigma}\right]=0\\ \exp\left[\sum_{t=1}^{T}\log(1-\xi_t)\right] &- \qquad\qquad \frac{q_T^*}{N}=0\end{aligned}$$

For numerical stability, $\prod_{t=1}^{T}(1-\xi_t)$ has been replaced by $\exp\left[\sum_{t=1}^{T}\log(1-\xi_t)\right]$, which is much more stable when dealing with small and large values of $\boldsymbol{\xi}$. On a numerical note, since full identification requires that $\xi \in (0, \frac{1}{2})$. $\boldsymbol{\xi}$ will be parametrized using a logistic function. In order to maintain that σ will always be strictly positive, it will be parametrized according to an exponential function, and μ will be parametrized by a logistic function simply to reduce saddle points caused by large μ and σ. N will not be parametrized throughout the model.

Using these specifications, estimation of the model remains a question of unconstrained optimization, and though it is poorly specified and difficult to minimize globally, the problem is, in principle, solvable. Only one serious numerical concern remains.

The order of price constraint is not very representative of the magnitude of the error in the predictions in price. Currently, the moment requires a sufficiently small difference between the cdf of the valuations of the buyers that have ramined in the market and the product of the endowments. This creates problems when relatively large differences in prices create relatively small differences in this moment. This can lead to solutions where the price does not tend to zero in the limit. This is caused by the model fitting the quantity moment well but the relatively large difference in the price moment having a small effect on the price moment condition. Sadly, applying the inverse cdf transform to the function eliminates many of the ideal properties required for optimization. The inverse cdf is not defined analytically, and while its derivative is given by the composition of the reciprocal of the derivative and the inverse function, this formulation performed extremely poorly numerically, preventing converge of any kind. The problem remains, causing there to be higher than reasonable prices when the model is attempting to fit the quantity evenly. This can cause problems such as in the Spectrum Case, pictured below, where a large difference in the price is taken as a small difference between the cdf and $\prod_{t=1}^{T}(1-\boldsymbol{\xi})$

4.3 Monte Carlo Analysis

With these numerical problems in mind, the question of is this estimation feasible remains. To this end, I simulated the process one thousand times, and tested the specifications of the simulations under the model. Since the early values of $\boldsymbol{\xi}$ influence the quantity and price in the later time periods greatly, this allows for noise within the early stages of the model to propagate down the time intervals.

One thousand simulations were run consisting of one thousand people, with Log-Normal distributions of $\mu = 0$, $\sigma = 1$. In each time period, five percent of the participants were endowed with the item, and this continued on for fifty time periods. After each simulation, a J-test for fitting the model was conducted, as well as a LR test for $\boldsymbol{\xi}$ being constant, and a LR test for the simulation primitives being equal to what they were. These tests were run at a significance level of $\alpha = 0.05$.

	Sargan Hansen	$\boldsymbol{\xi}$ constant	Simulation Primitives
Reject %	3.7	44.0	100.0

As one can see, the J-test rejected in an acceptable percent of the simulations, but the likelihood ratio tests were rejected at a far higher rate. This simulation is not quite on the order of magnitude of N as the data is because the linear program required to solve it scales at a size proportional to the square of the number of participants. This meant that to simulate at the order of magnitude for N required would require solving fifty linear programs at a magnitude of 10^{14}. This means that the simulations are overstating the role of the random noise compared to the data used, and the LR test for $\boldsymbol{\xi}$ may be slightly more powerful.

4.4 Testing

Since the model is over-identified, one can test for model-fit using the J-test for model fit. Hall (2005). Formally, this entails testing the hypothesis that $M\hat{\boldsymbol{m}}(\hat{\boldsymbol{\theta}})'\widehat{\boldsymbol{W}}\hat{\boldsymbol{m}}(\hat{\boldsymbol{\theta}}) = 0$. Since there are $2T$ moments in the model, and $T+3$ primitives in the model, the J-statistic is distributed $\chi^2(T-3)$.

Of interest is the question of whether or not there has been a constant drop rate of an item to users in the game over time. This can be written in the form of: $\boldsymbol{\xi} = \mathbf{1}\xi^c$. That is, $\boldsymbol{\xi}$ is constant over the entire lifetime of the model. This hypothesis can be tested with a Likelihood-Ratio test. We estimate the model under the null and compare it to the unconstrained model with the difference in the J-statistic distributed $\chi^2(T-1)$ as shown in: Hall (2005).

Case	J-test p-Value	LR Test	μ	σ	N
Clutch	0.95853	0.00140	1.47814	0.85317	1.0 x 10^{7}
Glove	5.02424 x 10^{-7}	1.0	0.83034	1.03382	810801.569
Gamma	6.95436 x 10^{-55}	0.0	1.42149	0.42634	169083
Spectrum 2	9.47595 x 10^{-14}	1.00000	0.50699	0.61172	278163.43674
Operation Hydra	1.00000	1.00000	0.79300	0.00000	9.99999 x 10^{6}
Glove	1.98425 x 10^{-8}	0.97342	0.76676	0.97756	899619.98944
Spectrum	1.52588 x 10^{-22}	1.00000	1.09140	0.50655	9.99999 x 10^{6}
Operation Wildfire	1.27968 x 10^{-38}	1.84931 x 10^{-8}	0.59941	1.27118	658963.25133
Revolver	1.66667 x 10^{-42}	0.00044	0.94046	1.79520	896235.25688
Gamma 2	1.55873 x 10^{-46}	1.76041 x 10^{-54}	0.49284	0.59216	464143.69810
Huntsman	3.86569 x 10^{-5}	1.62886 x 10^{-38}	1.01952	0.39789	4.30935 x 10^{6}
Chroma 2	3.02951 x 10^{-100}	2.96312 x 10^{-43}	1.00659	0.51421	3.38519 x 10^{6}
Winter Offensive	1.00000	1.48174 x 10^{-120}	0.94375	0.23719	2.45622 x 10^{6}
Chroma 3	4.82901 x 10^{-65}	9.96035 x 10^{-37}	0.31676	0.55150	9.48558 x 10^{6}
Falchion	9.50759 x 10^{-48}	4.52701 x 10^{-10}	0.81208	0.84943	9.99997 x 10^{6}
Shadow	1.78457 x 10^{-82}	4.06310 x 10^{-16}	0.58131	0.71472	9.99996 x 10^{6}
Operation Bravo	1.00000	0.00000	5.00000	0.00000	9.99999 x 10^{6}
Chroma	3.12200 x 10^{-107}	1.34974 x 10^{-25}	1.22655	0.61548	9.93782 x 10^{6}
Operation Vanguard	1.00000	0.00000	0.00000	0.00000	2.57984 x 10^{8}

One striking result is that in many of the boxes, the LR-test does reject the possibility that there is an equal endowment process in many of the boxes. Only in three of the examined boxes, Glove, Spectrum and Spectrum 2 does the process reject the Null. The other rejections arise from a poor J-test fit, and no information about the uniformity of $\boldsymbol{\xi}$ can be deduced. While the Glove case contains the rarest and most expensive items that are obtainable from the cases, the Spectrum Cases do not. It may be the case that the LR-test rejected a fit that was actually true, as seen during the Monte-Carlo simulations, so I believe that there may not be enough information to deduce a pattern in the boxes that rejected the null hypothesis. Almost all of the other boxes accept this hypothesis at virtually all significance levels, determining that random noise is not causing the differences. Whether the structure imposed has forced there to be a near equal drop rate is not testable under this model, but the question remains open.

Almost all the boxes that were rejected at any significance level reached nearly the same minimizer. For all of them, the mean was driven as high or low as possible, and the standard deviation as low as possible. This allowed for the price moment to be as close to matched as possible, as the path the price took could not be well described by the structure of the model. At first I believed that fits such as that were the result of poor initialization of the routine, but I was unable to find a starting point that did not converge to the same minimizer. I believe that the price path described by the data simply cannot be rationalized by the model, as they feature rapidly increasing prices, and prices rising above their initial price level, behavior that is completely inexplicable under this model. As a result, the model fixes the valuations as constant, and attempts to fit the quantity as best as possible. This is possible because the distribution of valuations plays no part in the quantity sold.

Fig. 1: Initial Time Period

Fig. 2: Second Time Period

4.5 Market Entry

I now consider the possibility of estimation of the market entry process.

4.5.1 Identification

This new model is significantly more complicated and features many new primitives. For each time period, there are now four primitives. Since there are only two moments defined per time period, there will need to be identifying assumptions in order to ensure that there is identification. The easiest of these assumptions to make is that the entry distributions are all the same distribution. This assumption is relatively innocuous and greatly reduces the number of primitives. However, there remains the 2 primitives of the initial distribution, as well as the two for the entry distribution, as well as T for $\boldsymbol{\xi}$ and T for $\boldsymbol{\lambda}$. One further reduction is to claim that the distribution of the item has remained constant over time. That is, $\boldsymbol{\xi} = \mathbf{1}\xi^c$. This reduces our model to one of having $T+5$ primitives. However, a great deal of structure is imposed in exchange.

The price floor is still present, and is handled in the same way that it was handled in the model without any entry. For time periods in which the price floor is binding, the equilibrium condition is not binding, and the only condition that binds is that the quantity demanded must be equal to the mass of buyers times the buyer's distribution function. The same moment is applied.

As long as $T > 5$, then there are enough moments for identification, so all that must be verified are the conditions for GMM Estimation. The single crossing property of the competitive equilibria ensure that $\mathbb{E}[\boldsymbol{m}(Y, \boldsymbol{\theta})] \neq 0$ for $\boldsymbol{\theta} \neq \boldsymbol{\theta_0}$. The only other property required is that the expected value of the total derivative of $\boldsymbol{m}(Y, \boldsymbol{\theta})$ has full rank. This is assumed under the specification of the model.

4.5.2 Monte Carlo

A serious problem arises with the supply and demand functions when there is entry allowed within the model. As people re-enter, a small portion have mass above the previous allowed price. This extends the mass out near the first price, and in just a few iterations, the supply and demand curves become extremely flat near the equilibrium price and quantity. This creates a serious problem for estimation, as tiny shocks in the distribution can move the price and quantity a large amount. The numerical problems present in the model without growth are only exacerbated by the mixing distribution induced in the growth model.

A Monte-Carlo simulation was set up with values of $T = 20, \mu_1 = 1, \sigma_1 = 0.5, \lambda_i = 0.05, \xi_i = 0.5, \mu_2 = 2, \sigma_2 = 1$. The buyer and seller distributions for several of the time periods are shown below:

As one can see, after the first transaction, the distributions become kinked, but since there are new arrivals with valuations above the price, the demand distribution does not become truncated as before. This leads to the recursive definition of the demand and supply given in the theory. It also complicates the estimation process greatly, as there are now two

Fig. 3: After 10 Time Periods

Fig. 4: Time Period 15

variables in the early time instances that have a great impact upon the model in later time periods. This means that the estimation process can be very unstable in the latter time periods, which manifests itself even in a simulation over 20 time periods.

As one can see, the equilibrium price and quantity are determined by the intersection between a very flat area of both supply and demand. This area is extremely sensitive to changes in the price, as well as noise in the data. For a simulation with $N = 10000$, I was unable to obtain convergence to any reasonable estimates of the distribution.

The moments used in estimation were:

$$M_B(T)B_T(p_T^*) = M_S(T)S_T(p_T^*)$$
$$M_B B_T(p_T^*) = q_T^*$$

The same care was taken to avoid problems with the binding price floor, and numerical problems. In all the simulations, the numerical problems dominated the process, and even when initialized at the starting values for the distribution, minimization was always obtained by setting the standard deviation to infinity, allowing for a fit in the price moment that way. Because of the numerical issues, as well as the extreme elasticity in the demand and supply near the equilibrium price, I do not believe that the estimation of this model is a possibility. Thus, estimation of other elements of the process will have to be undertaken in a different direction.

4.6 Quantile Regression

One initial goal of this paper was to aggregate the cases together, and introduce covariates into the model. Even with simple covariates, the optimization procedure would break down with as little as three or four cases being optimized together through a scalarization process. Most importantly, due to the structure of this model, it would not be possible to consider the case where the mean of the distribution had changed universally among all participants. For the purposes of estimation, the distribution must be fixed across time, and each data point must be serially uncorrelated with the other data points in its moment.

Due to these problems, another approach to the apparent instability in prices would be useful for policy predictions. In both models, the price is used to signal the cutoff in the valuations deciding who obtains an item. Everyone above this quantile receives the item, either by being endowed, or by trading for it.

The items in question are random lotteries containing other items of interest. Instead of believing that each person is identical ex ante a different draw from an urn of distributions, we could instead believe that those with different valuations value a different aspect of the contents of these lotteries. One way to examine how these affects change over the distribution of prices is to use quantile regression. The higher quantiles of the price show the behavior of consumers who have the highest valuations, and bought the item the soonest.

Using the fact that quantiles of our model are revealed over time, one estimate of interest is the conditional quantile. At high quantiles, when the item is in high demand, we may notice different behavior as buyers of rare items value different elements.

4.6.1 Multiple Quantile Regression

To this end I employed quantile regression. Even though one can easily apply quantile regression to a single loot box, and estimate the conditional quantiles, there is reason to believe that each box does not follow the same distribution. Thus, it is not immediately possible to simply combine all the different loot boxes into one data set, using indicators in the X and estimate the different quantiles.

First consider the optimization problem at the heart of quantile regression. We seek the quantile function such that: $Q_Y(\tau) = F_Y^{-1}(\tau)$. As shown by Koenker and Bassett (1978) it can be found by minimizing the following function:

$$\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \sum_{n=1}^{N} \rho_\tau(\boldsymbol{Y_i} - \boldsymbol{X_i\beta})$$

This is known to be equivalent to solving the following linear program:

$$\begin{aligned} \min \tau \mathbf{1}'\boldsymbol{u} + (1-\tau)\mathbf{1}'\boldsymbol{v} \\ \boldsymbol{X\beta} + \boldsymbol{u} - \boldsymbol{v} = \boldsymbol{Y} \\ \boldsymbol{u}, \boldsymbol{v} \geq 0 \end{aligned}$$

To estimate the quantile regression estimator for multiple boxes, consider a world where each box's quantile can be written in the following form:

$Q_T^i(\tau) = \boldsymbol{X}^i\boldsymbol{\beta} + \boldsymbol{Z}^i\boldsymbol{\delta}^i$. Where $\boldsymbol{X}$ and $\boldsymbol{Z}$ are matrices of covariates. Effectively, some covariates are shared by all boxes, and are contained within the vector $\boldsymbol{\beta}$, and some are unique to each box, and shared by the vector $\boldsymbol{\delta}$. We wish to examine the conditional quantile function of each box, conditioned that each box must contain the same $\boldsymbol{\beta}$. This is a question of multicriterion optimization. Boyd and Vandenberghe (2004)

For the j$^{\text{th}}$ component of the vector to be optimized:

$$\begin{aligned} \min\, [\tau \mathbf{1}'\boldsymbol{u_j} + (1-\tau)\mathbf{1}'\boldsymbol{v_j}] \\ \boldsymbol{X_j\beta} + \boldsymbol{Z_j}\boldsymbol{\delta}_j + \boldsymbol{u_j} - \boldsymbol{v_j} = \boldsymbol{Y_j} \quad \forall j \\ \boldsymbol{u}, \boldsymbol{v} \geq 0 \end{aligned}$$

By doing this, one chooses the $\boldsymbol{\beta}$ that minimizes each of the boxes residuals. Since there is no reason for us to favor any of the boxes over the others, we may consider the scalarization with the unit weight function applied to each of the objectives. Since we are interested in smallest possible sum of residuals between all boxes, the unit weight function makes intuitive sense to form our specification.

This effectively is choosing the Pareto Optimal point that has the smallest magnitude in the u,v space. In English this is the $\boldsymbol{\beta}$ value that allows us the smallest absolute sum of residuals over all the loot boxes. Since our data is presented as prices and quantities, we need to weight each of the residuals by this quantity sold, The scalarization of this problem is readily formed:

$$\begin{aligned} \min \sum_{j=1}^{J} [\tau \mathbf{1}'\boldsymbol{u_j} + (1-\tau)\mathbf{1}'\boldsymbol{v_j}] \\ \boldsymbol{X_j}\boldsymbol{\beta} + \boldsymbol{Z_j}\boldsymbol{\delta}_j + \boldsymbol{u_j} - \boldsymbol{v_j} = \boldsymbol{Y_j} \quad \forall j \\ \boldsymbol{u}, \boldsymbol{v} \geq 0 \end{aligned}$$

This is well-defined optimization problem as it is a Linear Program, and thus can be solved in $\mathcal{O}(N^3)$ run-time. Which is tractable even for large amounts of data, and can be applied to this model.

4.6.2 Application

The first, and most important covariate for determining the price is the expected value of the item. The expected value was broken down into five different components, one for each of the different qualities. Each case clearly indicates which items it contains as well as which quality these items have. Each of the qualities was multiplied by the probability of obtaining that case in order to ensure that the coefficients were of comparable magnitude.

Knowing how the price is influenced by the value of the contents is not particularly useful in designing new cases. Since the price of the contents is typically controlled by factors related to tastes that are simply not observed until the case has been released. What can be controlled easily is the number of items of each quality contained in each box. For almost all of these cases, there are between 3-5 items of each quality. So a linearization of the effect is not creating too large of a misspecification error.

Finally, one important consideration is if the presence of an item will influence the price. Each item contained is a weapon used within the game, and certain weapons are used much more often, and thus their inclusion should influence the price more. Due to the limited number of cases that have been released however, it is impossible to include an indicator for every single item in the game, as the corresponding matrix would not be of full rank. Thus, fifteen of the least popular weapons have been removed from the regression in order to maintain linear independence within the model. The chosen weapons are listed in the table below:

Weapons				
AK-47	AWP	DesertEagle	M4A4	M4A1-S
FAMAS	USP-S	Glock-18	P250	P90
SG553	CZ75-Auto	AUG	SSG08	Five-SeveN
MAC-10	MP7	MP9	UMP-45	

The model is estimated in R using the package `quantreg` Koenker (2018) contained in the file `QuantReg.r`. The entire conditional quantile regression is estimated using multiple quantile regression and using the Barrodale and Robert's alogrithm Koenker and Zhao (1996). A technical problem exists in the R package, preventing large datasets from being transferred to the internal FORTRAN code. As a result, the lowest observations were culled from the data set. This eliminated the need to handle them as censored observations, and allows for the process to be calculated. We estimate each conditional quantile of the distribution to be linear in each of the covariates with a single intercept element. Due to the restrictions in linear independence, no elements are included in our specification of $\boldsymbol{\delta}$, as an indicator for each lottery would be linearly dependent with the indicators of items contained.

4.6.3 Results

The coefficients over time for each of the different qualities, as well as the number of items contained, and the coefficients for the five most popular items used in game are shown:

At the lowest quantiles, the results are unexpected, the value of the rare, but obtainable items (red and pink) weight most heavily on the value of the item. As the quantile increases, the importance of the least valuable items rises. This goes against the intuitive opinion that when the box is least valuable, the value would be determined by the most commonly obtained items as a kind of "consolation prize."

Far more interestingly, is the fact that the coefficient for the red items dips into negatives for most of the quantiles. This implies that increases in the expected value of the rare items drive the price down. One possible interpretation of this result could be that individuals regard increases in the prices of red items as a decrease in the supply, lowering the chances of rare items, and driving the valuation and price down. This requires a strong stance on individuals beliefs on the market that is not substantiated anywhere.

Of interest is how by the highest quantiles, when the case is newest, the value of the least common items dominates the effects of the others. At this time, when the value of the items contained in the box is uncertain, individuals are more likely to consider a min-max approach, and look to the worst case scenarios instead of the rarer and less certain items.

For most sales, the number of items of each rarity in each box seems to have little effect on the price. However, as we reach the top twenty percent of sales, the value of an additional item increases for the pink item rises. However, much of these results are not statistically different from zero, and the behavior at the highest quantiles is extremely noisy as a result. Clearly though, additional items in the rarest categories of the case drive the price up in almost all quantiles. It must be noted that all the data for the number contained in the box lies within the range of two to four items for those rarities. I believe that this pattern would not extend globally, as individuals seeking a specific item lower their probability of obtaining it as there are more choices added.

This plot again suffers from noise and lack of significance at the highest quantiles for each of the items, and the M4A1-S remains insignificant at virtually all quantiles. The most striking result here is that containing the AWP, an item that is very expensive for all of its varieties and very sought after, drives the price down at the lower quantiles. While it must be remembered that the inclusion of the AWP can mean that there is no inclusion of the M4A4 and the AK-47, the two other weapons that are the most popular. These three items are never included together in a case, and at most only two of them are contained within the same case. In the context of the opportunity cost of including the AWP is losing a more popular weapon, the coefficient for the AWP can be explained.

5 Conclusion

From a policy perspective, the quantile regression suggests that the most important thing to drive up the revenue earned by the loot boxes is to drive the value of the common items contained in the box upwards. This goes against the intuitive thought that by making the very rare items as desirable as possible, the value of the box will be driven upward. The inclusion of certain items appears to have as little of an effect as the number of items contained in the boxes.

For further analysis, a model featuring a linear state space would be more numerically and computationally sound, and would allow for better estimation for processes that extend over large time periods. Most importantly, this model is, in the words of Rothschild, a "partial-partial equilibrium." Rothschild (1973) Only the demand side of a single part of the market is considered. A more complete analysis would consider the effects of all the markets, beyond simply their price at the current time period. This model would allow individuals to form beliefs about prices, and consider a sequential equilibrium. No consensus exists among Economists for an equilibrium strategy in a continuous double auction with beliefs about prices, so this approach presents many problems beyond the scope of this paper. Parsons et al. (2006)

An interesting question for further study in this data is whether the current mechanism (randomization with taxed markets for resale) is optimal from the perspective of the seller, and what properties are induced by this mechanism. The uniqueness of this mechanism, as well as the success it has had in the market seems to suggest that it is optimal under some conditions that are unique to this world. The derivation of which would be a very interesting extension.

6 Data Appendix

The Data were gathered through the script `dataPull.py`, and moved into the sub-directory `/Data/CSV/`. This data included the price and quantity history of every single item sold at market in *Counter-Strike: Global Offensive*. The data was then placed into a hierarchical file structure by the script `MoveFiles.py`. This script sorted each item by its type, skin and then quality. Certain files were created by hand: text files that contained the contents of each lottery examined. `CreateData.py` then aggregated the prices of each of the contents of the lottery at each time interval when a lottery was sold with the probability of it being received as well as its quality and an indicator for which type of gun it was. This data was then used by `newDataRead.jl` Where the expected values and other covariates for quantile regression were calculated. The quantile regression was undertaken in the file `quantRegR.r`.

The remaining files were altered by `dateAdjuster.py` to make the date format amenable to julia's `DataFrames` Package. These files were then used by `dataTest2.jl` to estimate the structural model.

A sample of one file pulled using `dataPull.py` is as follows:

Date	Price	Quantity
15-10-16	0.054	42502
16-10-16	0.045	38618
17-10-16	0.051	31563
18-10-16	0.053	32452
19-10-16	0.052	36564
20-10-16	0.049	35290
21-10-16	0.048	43502
22-10-16	0.047	38081
23-10-16	0.04	39843
24-10-16	0.036	32493
25-10-16	0.042	30841

A sample of one row from the files generated by `CreateData.py`, the first row of the Revolver Case:

Case Price:	8.262	Case Quantity:	132808
Price	Probability	Rarity	WeaponID
142.576	0.00043333	1	-1
57.309	0.00043333	1	-1
64.017	0.00043333	1	-1
161.424	0.00043333	1	14
232.763	0.00043333	1	-1
230.35	0.00043333	1	-1
123.516	0.00049778	2	25
77.369	0.00056889	2	25
49.229	0.00163556	2	25
48.541	0.00049778	2	25
105.682	0.000224	2	13
46.822	0.000256	2	13
21.616	0.000736	2	13
20.323	0.000224	2	13
14.833	0.00176	2	13
53.181	0.00074667	3	0
38.945	0.00085333	3	0
26.673	0.00245333	3	0
23.986	0.00074667	3	0
15.222	0.00586667	3	0
16.153	0.00201802	3	23
5.893	0.00663063	3	23
5.231	0.00201802	3	23
35.978	0.00091756	3	8
5.53	0.00263799	3	8
4.775	0.00080287	3	8
4.787	0.00630824	3	8
6.281	0.00186433	4	24
3.21	0.00213067	4	24
1.8	0.00612567	4	24
1.469	0.00186433	4	24
1.345	0.01464833	4	24
2.624	0.00304381	4	19
1.382	0.00266333	4	19
1.371	0.02092619	4	19
8.324	0.00186433	4	7
3.951	0.00213067	4	7
2.211	0.00612567	4	7
1.651	0.00186433	4	7
1.454	0.01464833	4	7
7.802	0.00490614	4	28
3.002	0.00560702	4	28
1.786	0.01612018	4	28
10.971	0.00186433	4	30
5.421	0.00213067	4	30

Price	Probability	Rarity	WeaponID
3.143	0.00612567	4	30
2.614	0.00186433	4	30
1.888	0.01464833	4	30
3.666	0.00242121	4	33
2.552	0.0027671	4	33
1.519	0.00242121	4	33
1.455	0.01902381	4	33
33.066	0.009324	5	25
10.289	0.010656	5	25
4.782	0.030636	5	25
4.42	0.009324	5	25
2.821	0.07326	5	25
1.506	0.009324	5	1
0.657	0.010656	5	1
0.34	0.030636	5	1
0.526	0.009324	5	1
0.285	0.07326	5	1
1.006	0.02072	5	4
0.675	0.02368	5	4
0.481	0.06808	5	4
0.6	0.02072	5	4
0.915	0.02453684	5	21
0.66	0.02804211	5	21
0.563	0.08062105	5	21
0.882	0.01351304	5	26
0.282	0.01351304	5	26
0.276	0.10617391	5	26
0.657	0.01210909	5	27
0.444	0.01383896	5	27
0.297	0.01210909	5	27
0.286	0.09514286	5	27

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