Valuations of Items in Counter-Strike: Global Offensive

Timothy Schwieg

July 9, 2018

The Problem

- People are randomly distributed items in the game.
- They have private valuations for each item that are not known to the designers
- A market is created in order to ensure an efficient outcome.
- Takes the form of a double auction - converging to competitive equilibrium

Matching

- One context to think of the problem as one of matching individuals in order to maximize the total surplus.
- We know from Micro2 that this is equivalent to thinking about a decentralized market.
- The Objective function is valuation of the buyers and the sellers

Who Gets What

- Both buyers and sellers have the same distribution of valuations
- However, the masses of the buyers and sellers are not equal.
- Only some percentage are endowed with the item
- Market is efficient - highest valuations end up with the item.

The Planner's Problem

$$\max_{\alpha_{i,j}} \sum_{i=1}^{I} \sum_{j=1}^{J} (V_i - V_j)\alpha_{i,j}$$

$$\text{subject to: } \forall j, 1 \leq j \leq J \quad \sum_{i=1}^{I} \alpha_{i,j} \leq 1$$

$$\forall i, 1 \leq i \leq I \quad \sum_{j=1}^{J} \alpha_{i,j} \leq 1$$

Planner's Problem (cont)

- The solution to this is not unique.
- The difference in valuations is both sub and super-modular. This implies that both PAM and NAM are supported, and all permutations between the sellers and buyers selected are supported.
- This means we know who is matched but not with whom.

The dual

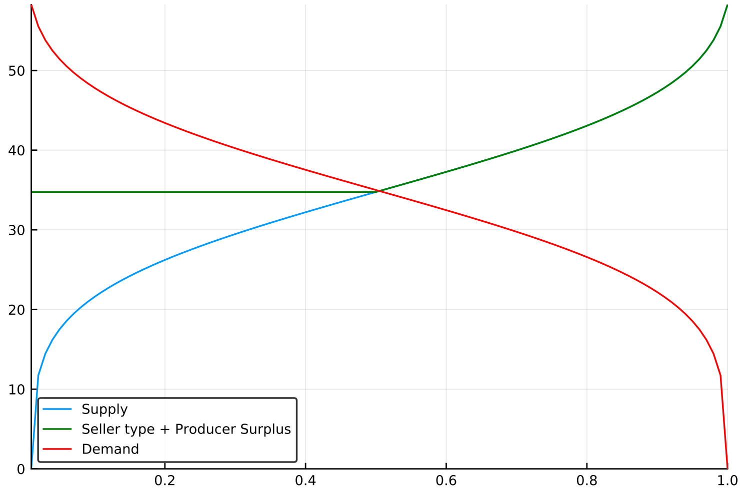
$$\min_{x,j} \sum_{i=1}^{I} x_i + \sum_{j=1}^{J} y_j$$

$$\text{subject to: } \forall i, j; \quad 1 \leq j \leq J, \quad 1 \leq i \leq I$$

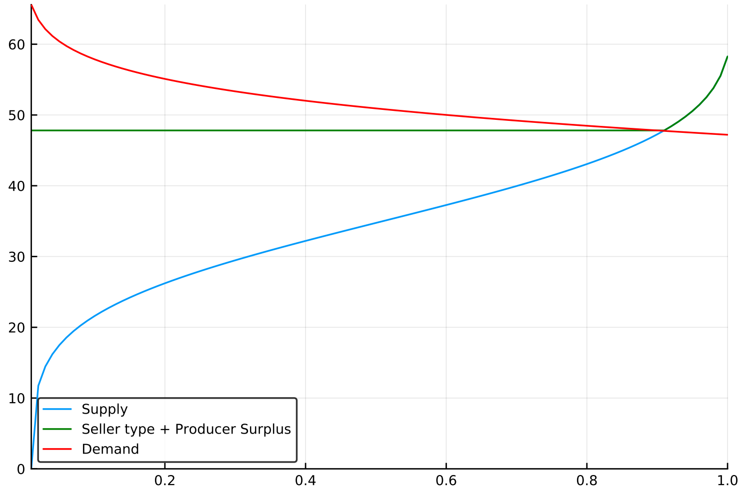
$$x_i + y_j \geq V_i - V_j$$

- This has a unique solution - for each buyer and seller it gives the shadow price: the surplus that each commands.
- Becuase the function is modular, the valuation plus the surplus for all sellers is equal - this is the price the market supports.

What it looks like



Unequal Buyers and Sellers



Equilibrium

- Let the proportion of the population that recieved the item be denoted ξ.
- For normally distributed valuations, the price is defined by:

$$\Phi\left(\frac{p^* - \mu}{\sigma}\right) = \frac{1-\xi}{\xi}\left[1 - \Phi\left(\frac{p^* - \mu}{\sigma}\right)\right]$$
$$p^* = \mu + \sigma\Phi^{-1}(1-\xi)$$

Known ξ

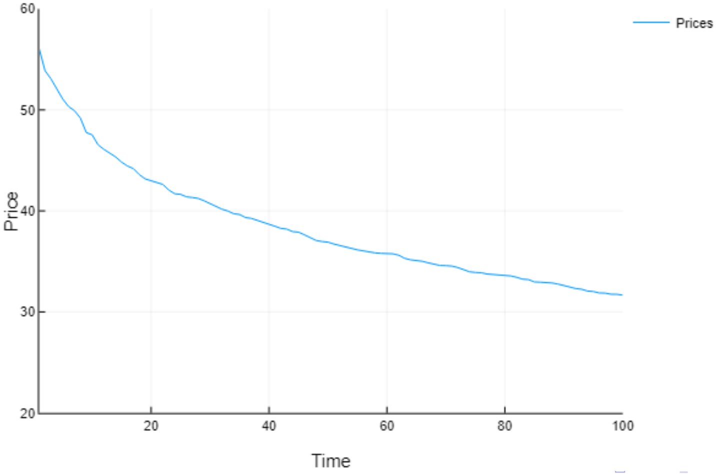
- If we knew ξ, this model could be estimated via linear regression
- Can handle even if there is measurement error in calculating ξ.
- However, even if we know the quantity of sales, and the number of people playing, no idea of people engaging in the market.
- Need to use the price to endogenize ξ.

Dynamic Approach

- Let this process repeat over many time intervals.
- Assume no entry into the market.
- Since this market is efficient, the top portion of the buyers always purchases the item, and the price slowly falls
- This can only support a decreasing price.

A Simulation

- $\mu = 35, \sigma = 10, \xi = .01, N = 1000$



Specification

$$q_s = N \prod_{t=0}^{T-1} (1-\xi_t)\xi_T \frac{\Phi\left(\frac{p-\mu}{\sigma}\right)}{\prod_{t=0}^{T-1}(1-\xi_t)}$$

$$q_d = N \prod_{t=0}^{T} (1-\xi_t) \left[1 - \frac{\Phi\left(\frac{p-\mu}{\sigma}\right)}{\prod_{t=0}^{T-1}(1-\xi_t)}\right]$$

$$\log(p_T^*) = \mu + \sigma\Phi^{-1}\left[\prod_{t=0}^{T}(1-\xi_t)\right]$$

$$q_T^* = N \prod_{t=0}^{T} (1-\xi_t)\xi_T$$

$$\log(p^*) = \mu + \sigma\Phi^{-1}\left[\frac{q^*}{N\xi_T}\right]$$

Problems with Data

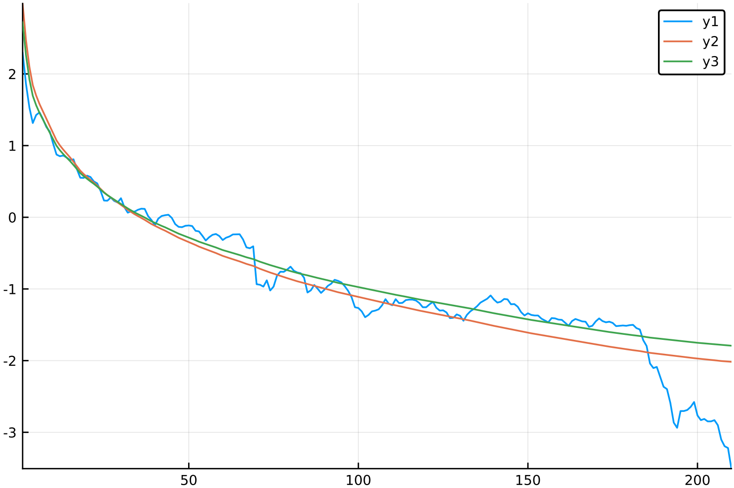
- This model cannot support the prices increasing.
- One possibility is to add white noise, which increases the variance on all observations, and can explain some jumps in prices
- This cannot explain trends in prices that are observed in some items.
- Worse yet, it predicts price to eventually fall to zero, which is not represented by most items

What can we predict?

- We are predicting the price to eventually drop to zero, but we do not have an equilibirum specification. So for data where the price is driven on a downward trend, we can estimate the data.
- We still need some sort of identifying assumption on ξ.
- Choose to hold it constant over over a month.
- Then estimate the values of μ and σ using Linear Regression or Least Absolute Deviations.

Some Predictions

Case.pdf Case.bb



Market Entry

- For the price to be able to increase, there must be new people entering the market.
- Let λ_t denote the percent of new entrants into the market.
- Since each new entrant has the original valuations, we must consider all owners of the item, even past owners.
- This leads to both buyers and sellers having a mixing distribution of valuations

Masses of Buyers and Sellers

$$M_B(T) = N(1-\xi_T)\prod_{t=0}^{T-1}(1-\xi_t+\lambda_t)$$

$$M_S(T) = N\sum_{i=0}^{T}\xi_i\prod_{t=0}^{i-1}(1-\xi_t+\lambda_t)$$

$$M_B(T) = NB_T(p_T)$$

$$M_S(T) = N\left(1-B_T(p_T)+\sum_{t=1}^{T-1}R_t(\lambda,p)\right)$$

$$R_i(\lambda,p) = \lambda_i\left[B_{i-1}(p_{i-1})+R_{i-1}(\lambda,p)\right]$$

$$R_0(\lambda,p) = \lambda_0$$

Valuations of Buyers and Sellers

$$
\begin{aligned}
B_T(p) &= \frac{B_{T-1}(p_{T-1})}{B_{T-1}(p_{T-1}) + \lambda_1} \min\left\{1, \frac{B_{T-1}(p)}{B_{T-1}(p_{T-1})}\right\} \\
&\quad + \frac{\lambda_1}{B_{T-1}(p_{T-1}) + \lambda_1} B_0(p) \\
S_T(p) &= \frac{M_S(T-1)}{M_S(T)} \max\left\{0, \frac{B_{T-1}(p) - B_{T-1}(p_{T-1})}{1 - B_{T-1}(p_{T-1})}\right\} \\
&\quad + \frac{M_S(T) - M_S(T-1)}{M_S(T)} B_T(p)
\end{aligned}
$$

- $B_t(p)$ and $S_t(p)$ are strictly increasing functions of p, so the intersection between q_d, q_s is uniquely defined.

Problems

- There are some serious identificaiton problems with this

model

- What changes are caused by ξ, and what by λ?
- Assumptions such as holding each fixed within a month are ineffective
- Worse yet, all attempts seem to drive the estimated variance to infinty.

Non-Constant Valuations

- While the valuation of some items in the game might remain constant
- Items of interest such as the loot boxes have their values influenced by the prices of the items contained.
- Of interest is the magnitude of this over the lifetime of the item
- Use the fact that the distribution of the items reveals the quantiles of the distribution

Quantile Regression

- In the model without any growth:

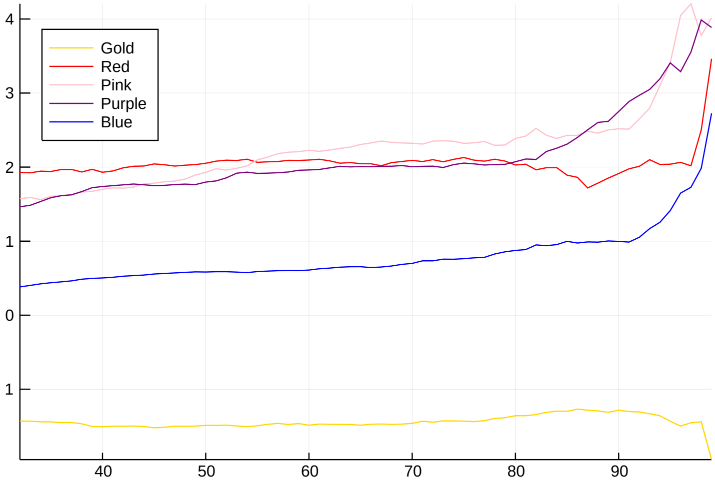
$$\prod_{t=0}^{T}(1-\xi_t) = F_V\left(p^*\right)$$

- The proportion of people given the item reveals quantiles of the true valuations.

Quantile Regression

- If we want to remain agnostic about the percent of people given the item, the only choice we have is to examine how different quantiles of the pricing distribution are affected.
- This involves quantile regression, and abandoning many of the structural results hoped for.
- One approach is to estimate many different quantiles and plot them

Loot box Averages



A Slightly more Sophisticated Approach

- Multiple Quantile Regression can allow for non-parametric estimates of the effects, or for more efficient estimates of the quantiles affects.
- Mutlivariate Quantile Regression can allow for shared effects between boxes, as applying quantile regression to the price data for the boxes combined is not reasonable.
- Wish to fix the effect of the presence of items across the boxes, while allowing the other affects to change over quantiles

A Specification

- Let β be the shared effects, and δ be the non-shared effects.

$$
\begin{aligned}
\min \sum_{i=0}^{I} & \tau 1^T u_i + (1-\tau) 1^T v_i \\
\text{s.t. } & X(\beta + \delta_i) + u_i - v_i = Y_i \quad \forall i \in I \\
& u_i \geq 0, v_i \geq 0
\end{aligned}
$$