HW1

Timothy Schwieg

Question 1

a

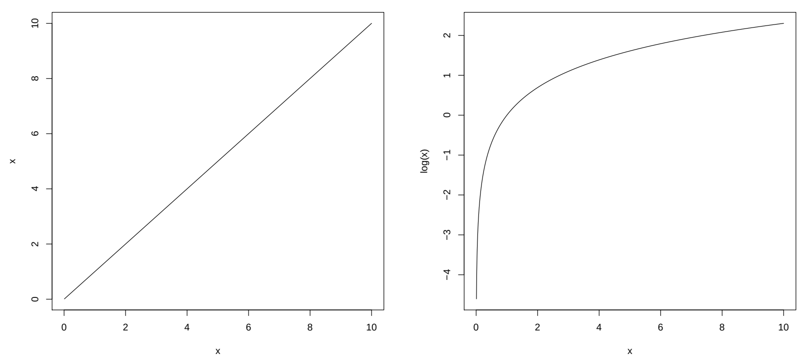

Fig. 1:

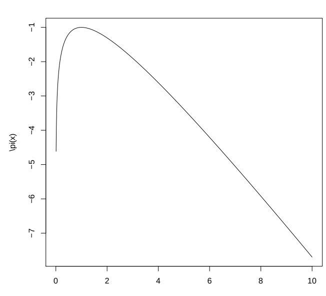

Fig. 2:

b

$$\begin{aligned}\frac{d}{dx}\pi(x) = \frac{1}{x} - 1 = 0\\ \frac{1}{x} = 1\\ 1 = x\end{aligned}$$

Question 2

a

$$\begin{aligned}\int_a^b y\frac{d}{dy}e^{-y}dy = -ye^{-y} + \int_a^b e^{-y}dy\\ -ye^{-y} - e^{-y}\Big|_a^b\\ -be^{-b} - e^{-b} + ae^{-a} + e^{-a}\end{aligned}$$

b

$$\begin{aligned}\lim_{x\to\infty}\int_0^x y\frac{d}{dy}e^{-y}dy =\\ \lim_{x\to\infty} -xe^{-x} - e^{-x} + 0e^0 + e^0 =\\ \lim_{x\to\infty} -xe^{-x} + 1 =\\ \lim_{x\to\infty}\frac{1}{e^x} + 1 = 1\end{aligned}$$

Question 3

a

$$\begin{aligned}\frac{dy}{dx} &= x\\ dy &= xdx\\ y &= \frac{1}{2}x^2 + C\end{aligned}$$

b

$$\begin{aligned}1 &= \frac{1}{2}0^2 + C\\ 1 &= C\\ y &= \frac{1}{2}x^2 + 1\end{aligned}$$

Question 4

CDF

$$F_V(v) = \begin{cases} 0 \text{ for } v < 0 \\ \int_0^v 1dv \text{ for } 0 \leq v \leq 1 \\ 1 \text{ for } v > 1 \end{cases}$$

$$F_V(v) = \begin{cases} 0 \text{ for } v < 0 \\ v \text{ for } 0 \leq v \leq 1 \\ 1 \text{ for } v > 1 \end{cases}$$

Mean

$$\mathbb{E}[X] = \int_0^1 vdv$$
$$\frac{1}{2}v^2|_0^1 = \frac{1}{2}$$

Variance

$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$
$$\mathbb{E}[X^2] = \int_0^1 v^2 dv = \frac{1}{3}v^3|_0^1 = \frac{1}{3}$$
$$Var(X) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

Question 5

a

It is well understood that the distribution of a maximum of iid random variables is given by:

$$F_Z(z) = [F_V(z)]^N = \begin{cases} 0 \text{ for } z < 0 \\ [\int_0^z dv]^N \text{ for } 0 \leq z \leq 1 \\ 1 \text{ for } z > 1 \end{cases}$$

$$F_Z(z) = \begin{cases} 0 \text{ for } z < 0 \\ z^N \text{ for } 0 \leq z \leq 1 \\ 1 \text{ for } z > 1 \end{cases}$$

We may find the pdf of Z by taking the derivative of $F_Z(z)$ with respect to z.

$$f_Z(z) = \frac{d}{dz}F_Z(z) = \begin{cases} 0 \text{ for } z \notin [0,1] \\ Nz^{N-1} \text{ for } z \in [0,1] \end{cases}$$

b

$$\mathbb{E}[X] = \int_0^1 Nz^N = \frac{N}{N+1}z^{N+1}\big|_0^1 = \frac{N}{N+1}$$

$$\mathbb{E}[X^2] = \int_0^1 Nz^{N+1} = \frac{N}{N+2}z^{N+2}\big|_0^1 = \frac{N}{N+2}$$

$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{N}{N+2} - (\frac{N}{N+1})^2 =$$

$$\frac{N(N+1)^2}{(N+2)(N+1)^2} - \frac{N^2(N+2)}{(N+2)(N+1)^2} = \frac{N}{(N+2)(N+1)^2}$$