Behavioral HW4

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1

a

Since the amount that each bidder has to pay is unrelated to his bid, each bidder will attempt to maximize the chance that he wins provided that he does not have to pay above his valuation.

If he bids above his valuation and lost, he gains nothing and loses nothing, if he bids above his valuation and wins, there is a chance that he pays more than his valuation and is strictly worse off than if he had lost. So bidding zero weakly dominates bidding above his valuation.

If he bids below his valuation and wins, he is no better off than if he bid anything above his bid, as he does not pay his bid. If he bids below his valuation and loses, but he could have won by bidding his valuation, he is strictly worse off than bidding his valuation. If he bids below his valuation and loses, but could not have won by bidding his valuation, he is no better off than if he bid at his valuation.

Clearly bidding at the valuation weakly dominates the other strategies, and is the dominant-strategy equilibrium-bid function for the English Auction. $\beta(v) = v$

b

Each student seeks to maximize:

$$\begin{aligned}
\max_s \quad U(v-s)P(s=\max\{s_1,...s_N\}) \\
\max_s \quad \eta(v-s)^{\frac{1}{\eta}}F_V(\sigma^{-1}(s_i))^{N-1} \\
-(v-s)^{\frac{1-\eta}{\eta}}F_V(\sigma^{-1}(s_i))^{N-1}+\eta(v-s)^{\frac{1}{\eta}}(N-1)F_v(\sigma^{-1}(s_i))^{N-2}f_v(\sigma^{-1}(s_i))\frac{1}{\sigma'(s_i)}=0 \\
-(v-s)^{\frac{1-\eta}{\eta}}\sigma^{-1}(s)^{N-1}+\eta(v-s)^{\frac{1}{\eta}}(N-1)\sigma^{-1}(s)^{N-2}\frac{1}{\sigma'(v)}=0 \\
(v-s)^{\frac{1-\eta}{\eta}}\sigma^{-1}(s)=\eta(v-s)^{\frac{1}{\eta}}(N-1)\frac{1}{\sigma'(v)} \\
v\sigma'(v)=\eta(v-s\sigma(v))(N-1) \\
\sigma'(v)+\frac{\eta(N-1)}{v}\sigma(v)=\eta(N-1) \\
(\sigma(v)v^{\eta(N-1)})'=\eta(N-1)v^{\eta(N-1)} \\
\sigma(v)v^{\eta(N-1)}=\frac{(N-1)v^{\eta(N-1)+1}}{\eta(N-1)+1}+C \\
\sigma(v)=\frac{\eta(N-1)v}{\eta(N-1)+1}+Cv^{\eta(1-N)} \\
\sigma(v)=\frac{\eta(N-1)v}{\eta(N-1)+1}
\end{aligned}$$

c

Since $\sigma(v)$ is a monotone function, we may apply the method of transformations to find the PDF of W.

$$\begin{aligned}
\frac{s\,(N\eta-\eta+1)}{\eta\,(N-1)}=\sigma^{-1}(s) \\
\frac{\partial}{\partial s}\sigma^{-1}(s)=\frac{(N\eta-\eta+1)}{\eta\,(N-1)}
\end{aligned}$$

$$f_w(w) = f_{V_{(1:N)}}(\sigma^{-1}(w)) \left| \frac{\partial \sigma^{-1}(w)}{\partial s} \right|$$
$$f_w(w) = \frac{N}{n\left(N-1\right)} \left(\frac{w\left(N\eta - \eta + 1\right)}{\eta\left(N-1\right)} \right)^{N-1} \left(\eta(N-1)+1\right)$$
$$f_w(w) = \frac{N}{w} \left(\frac{w\left(\eta(N-1)+1\right)}{\eta\left(N-1\right)} \right)^{N}$$

d

$$\mathbb{E}[W] = \int_0^1 w f_w = \int_0^1 w \frac{N}{w} \left(\frac{w\left(\eta(N-1)+1\right)}{\eta\left(N-1\right)} \right)^{N} dw$$
$$\mathbb{E}[W] = N \left(\frac{\eta(N-1)+1}{\eta(N-1)} \right)^{N} \int_0^{\frac{\eta(N-1)}{\eta(N-1)+1}} w^N dw$$
$$\mathbb{E}[W] = \frac{N\eta(N-1)}{(N+1)(\eta(N-1)+1)}$$

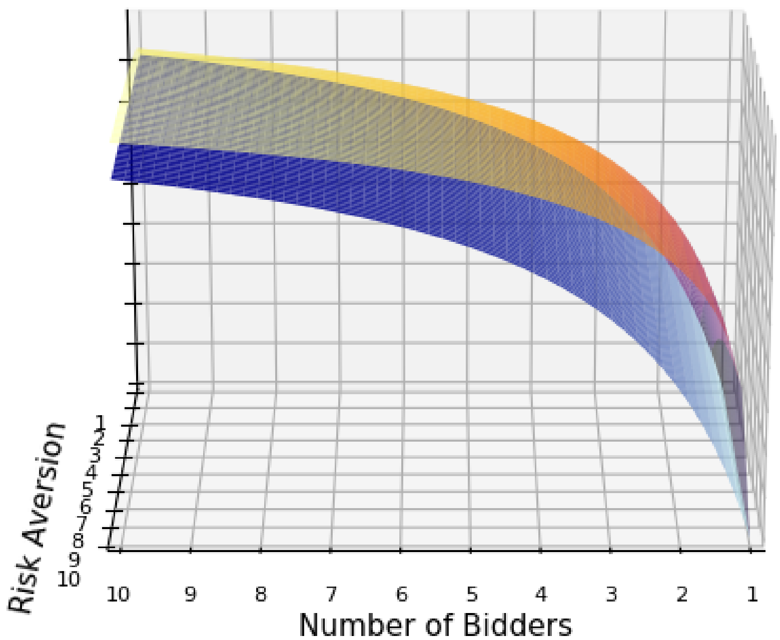
e

$$\mathbb{E}[W^2] = \int_0^1 w^2 f_w = \int_0^1 w^2 \frac{N}{w} \left(\frac{w\left(\eta(N-1)+1\right)}{\eta\left(N-1\right)} \right)^{N} dw$$
$$\mathbb{E}[W^2] = N \left(\frac{\eta(N-1)+1}{\eta(N-1)} \right)^{N} \int_0^{\frac{\eta(N-1)}{\eta(N-1)+1}} w^{N+1} dw$$
$$\mathbb{E}[W^2] = \frac{N(\eta(N-1))^2}{(N+1)(\eta(N-1)+1)^2}$$

$$\mathbb{V}(W) = \mathbb{E}[W^2] - \mathbb{E}[W]^2$$
$$\mathbb{V}(W) = -\frac{N^2\eta^2\left(N-1\right)^2}{\left(N+1\right)^2\left(n\left(N-1\right)+1\right)^2} + \frac{N\eta^2\left(N-1\right)^2}{\left(N+2\right)\left(n\left(N-1\right)+1\right)^2}$$
$$\mathbb{V}(W) = \frac{N\eta^2\left(N-1\right)^2\left(-N\left(N+2\right)+\left(N+1\right)^2\right)}{\left(N+1\right)^2\left(N+2\right)\left(\eta\left(N-1\right)+1\right)^2}$$
$$\mathbb{V}(W) = \frac{N\eta^2(N-1)^2}{(N+1)^2(N+2)(\eta(N-1)+1)^2}$$

f

If $\eta = 1$ There is revenue equivalence. However, if we increase η we can see how the expected revenue changes as people become more risk averse.



This leads us to believe that a risk-neutral seller would prefer a sealed pay-your-bid auction if he believes that the people attending are risk averse, as they will bid higher.

2

a

Since every entrant has to pay what he bids, the pay-off function will always have the bid subtracted from it, and he will receive his valuation of the object if he wins the auction. This means that with some probability, depending on the bid he will receive his valuation, and he will always lose his bid.

$$\mathbb{E}[\text{Payoff}] = vP(\text{win}|s) - s$$

b

$$\max_s \quad vP(win|s) - s$$
$$\max_s \quad vF_V(\sigma^{-1}(s))^{N-1} - s$$
$$v(N-1)F_V(\sigma^{-1}(s))^{N-2} f_v(\sigma^{-1}(s))\frac{1}{\sigma'(v)} - 1 = 0$$

c

$$\sigma'(v) = v(N-1)f_v(\sigma^{-1}(s))F_V(\sigma^{-1}(s))^{N-2}$$
$$\sigma'(v) = v(N-1)f_v(v)F_V(v)^{N-2}$$
$$\sigma'(v) = v(N-1)v^{N-2} = (N-1)v^{N-1}$$
$$\sigma(v) = \frac{N-1}{N}v^N$$

d

The seller collects $s = \sigma(v)$ from every entrant. This means his expected revenue is: $N\mathbb{E}[\sigma(v)]$.

$$\sigma^{-1}(s) = \left(\frac{N}{N-1}s\right)^{\frac{1}{N}}$$
$$\frac{\partial}{\partial s}\sigma^{-1}(s) = \frac{\left(\frac{Ns}{N-1}\right)^{\frac{1}{N}}}{Ns}$$
$$f_{\sigma(v)} = f_v(\sigma^{-1}(s))\left|\frac{\partial}{\partial s}\sigma^{-1}(s)\right|$$
$$f_{\sigma(v)} = \frac{\left(\frac{Ns}{N-1}\right)^{\frac{1}{N}}}{Ns}$$
$$\mathbb{E}[\sigma(v)] = \int_0^{\frac{N-1}{N}} s\frac{\left(\frac{Ns}{N-1}\right)^{\frac{1}{N}}}{Ns}ds$$
$$\frac{N-1}{N(N+1)}$$

So his revenue to collect is:

$$\frac{N-1}{N+1}$$

This is the same revenue as if the seller were to have a first-price sealed bid auction, so he is, on average, just as happy as if he hosted an all-pay auction or the first price sealed bid auction.