Risk, Ambiguity, and the Savage Axioms

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How much uncertainty is there?

- Even though Knight claims uncertainty dominates, people tend to behave "as if" they have numerical probabilities assigned to events.
- Shackle's Cricket Match
- Can we provide odds we are willing to back?

- "The race is not always to the swift nor the battle to the strong, but that's the way to bet." - Damon Runyon
- Do these bets reveal anything about our "hidden probabilities?"
- If this were possible, we could turn these uncertainties that people were willing to bet on, and transform them into something alike risk.

Enter Savage

- Savage claims that for someone who is "rational" all uncertainties can be reduced into risks.
- A willingness to bet on 2:1 odds means a belief in a probability of $\frac{1}{3}$
- Must still somehow untangle probabilities from preferences, but in a controlled setting this is feasible.
- What does "rational" mean?

What can we infer from behavior?

- What does indifference between two gambles imply?
- Equal probabilities, given you assigned probabilities.
- May just be attempting to mimimax - this only reflects they have the same worst-case.
- Seek an operator $\geq$ - Qualitative Probability

What the operator gives us:

- $\textcircled{\geq}$ is a complete ordering among events
- If an event α is more probable than β, then $\bar{\alpha}$ is less probable than $\bar{\beta}$.
- If α and γ are mutually exclusive and so are β and γ, then if α is more probable than β, $\alpha \cup \gamma$ is more probable than $\beta \cup \gamma$
- Under the Savage axioms, $\textcircled{\geq}$ holds these properties.

The Savage axioms

- Gambles have a complete ordering
- If two gambles have the same payoff, its value is irrelevant.
- Dominated Actions are always rejected - "noncontroversial"
- Choice in a gamble is independent of the relative magnitudes of the rewards, only the ranking. Size of the prize doesn't affect choice.

What happens when these are not fulfilled?

- We cannot infer probabilities from actions
- There is no VNM Utility function that we can apply.
- These are not met in a particular class of situations
- These situations are where ambiguity rules.

Experiment Time

- Which would you prefer to bet on?

Box I	10 Red M&Ms, 10 Green M&Ms
Box II	20 M&Ms All of which are Red or Green

- Red M&M or Green M&M from Box I?
- Red M&M or Green M&M from Box II?
- Red M&M from Box I or Red from BoxII?
- Green from Box I or Green from BoxII?

Honey I broke the axioms

- Preferring Box I to Box II in both violates the savage axioms.
- If you prefer Box I Red, then you must view it as more probable, and therefore Box I Green as less probable than Box II Green.
- However you also prefer Box II Green to Box I Green, and we have a contradiction.

Do we really have total ignorance?

- Lets take a few samples out of Box II.
- Have your preferences for betting changed?
- This is typical, and it only changes willingness to bet slightly.

Another Experiment

- There are 20 known blue M&Ms, and forty M&Ms that are all red or green.

Bet on Blue	Bet on Red
Bet on Blue or Green	Bet on Red or Green

- When the ambiguity is removed by Red or Green being an option, it is suddenly the preferred case.
- Sophisticated individuals still continue to violate the axioms even on reflection with the notion they are violating them.

Standard uncertainty behaviors

- Individuals are not minimaxing purely.
- Nor are they maximizing any weighted average of the best and worst case.
- They aren't even minimaxing regret, as these examples are designed so that their regret would be identical.
- Yet these choices are fairly obvious and intuitive.

Ambiguity Aversion

- We have some information about the problem, but we just aren't sure how good our information is.
- However we aren't "completely ignorant" so common techniques for handling uncertainty don't apply either.
- Limited to distributions $(\frac{1}{3}, \lambda, \frac{2}{3} - \lambda)$.
- No real knowledge of which of these distributions is more "likely"
- This is different from the uninformed prior!

Further Problems

- Even if an individual could assign relative weights to each possible distribution and apply a prior, he does not know how useful the data is.
- Information can still be ambiguous, for example hearing about a players' FG% in basketball may inform you on their chances in the playoffs this year more or less depending on your knowledge.
- This cannot be expressed in terms of likelihoods.

Where does ambiguity apply?

- Where information can often be unreliable.
- New processes. (Vegas Games rely on the same stochastic devices).
- Returning to the Cricket Game:
  You would be certain to bet on who bats first
  How certain are you on betting on who wins the game?

A possible Computational Device

- It is possible that maybe there is an expected distribution (prior), but since the individual is not certain that he is correct, he weights it against the worst possible case.
- Ellsberg uses a linear combination for simplicity, but it could be more complex.
- Provides nothing more than a heuristic.
- No formal model presented.

How does this predict behavior?

- When we are less sure of our predictions, we become more conservative.
- A bet on known things is preferred to a bet on unknown odds.
- This doesn't mean that people are not acting optimally, nor are they being "irrational"