Operations Research HW3

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Question 1

Question 2

We set x_1, x_2, x_3 to be production of product 1,2,3 respectively, and let s_1 be a binary predictor of if product 3 is produced. It is clear that $x_3 \leq 100$ so by setting $x_3 \leq 100s_1$ this ensures $x_3 = 0$ if $s_1 = 0$, and x_3 is otherwise unaffected.

$$\begin{aligned} \max\ & 25x_1 + 30x_2 + 45x_3 \\ \text{s.t.} \quad & 3x_1 + 4x_2 + 5x_3 \leq 100 \\ & 4x_1 + 3x_2 + 6x_3 \leq 100 \\ & x_3 - 5s_1 \geq 0 \\ & x_3 - 100s_1 \leq 0 \\ & x_1, x_2, x_3 \in \mathbb{R}_+, s_1 \in \{0, 1\} \end{aligned}$$

Branching on $s_1 = 0$ and $s_1 = 1$

Case: $s_1 = 1$

$$\begin{aligned} \max\ & 25x_1 + 30x_2 + 45x_3 \\ \text{s.t.} \quad & 3x_1 + 4x_2 + 5x_3 \leq 100 \\ & 4x_1 + 3x_2 + 6x_3 \leq 100 \\ & x_3 \geq 5 \\ & x_3 \leq 100 \\ & x_1, x_2, x_3 \in \mathbb{R}_+ \end{aligned}$$

This has a maximal value of: $\frac{2500}{3}$ at a maximizer of: $x^* = (0, \frac{100}{9}, \frac{100}{9})$

Case: $s_1 = 0$

$$\begin{aligned} \max\ & 25x_1 + 30x_2 + 45x_3 \\ \text{s.t.} \quad & 3x_1 + 4x_2 + 5x_3 \leq 100 \\ & 4x_1 + 3x_2 + 6x_3 \leq 100 \\ & x_3 \geq 0 \\ & x_3 \leq 0 \end{aligned}$$

This has maximal value of: $\frac{5500}{7}$ at a maximizer of: $x^* = (\frac{100}{7}, \frac{100}{7}, 0)$

Since $\frac{2500}{3} > \frac{5500}{7}$ We choose to utilize x_3 and produce: $\frac{2500}{3}$ at a maximizer of: $x^* = (0, \frac{100}{9}, \frac{100}{9})$

Question 3

Question 5