Information economics

Lecture 1 – adverse selection

Games of asymmetric information; may lead to market failure. An extended car insurance example (JR, Chapter 8):

- Consumers
  - Identical customers except for the exogenous probability they are in an accident.
  - Accident probability of consumer $i \in \{1, 2, ..., m\}$ is $\pi_i \in [0, 1]$, and the occurrence of accidents is independent across consumers.
  - Consumers know their type.
  - Each has initial wealth w and suffers a loss of L if an accident occurs.
  - Consumers have a VNM utility function u that is continuous, with $u' > 0$ and $u'' < 0$.
- Insurers
  - Numerous, identical, risk neutral.
  - For any, offers full insurance only (pay L if an accident, 0 otherwise).
  - The only costs are the expected losses: a policy sold to consumer i for p gives expected profits $p - \pi_i L$.

Symmetric information

- As a benchmark, suppose insurer's observe consumers' π too. Outcome?
- Insurers will want to adjust price based on risk.

- Let p_i be the price of a policy paying L to consumer i when there is an accident; i^{th} policy. Equilibrium price: p_i^*
- Supply of policy i
  - none if $p_i < \pi_i L$
  - infinite if $p_i > \pi_i L$
  - any number if $p_i = \pi_i L$
- Demand (and recall, no fractional policies allowed)
  - at least one if $p_i < \pi_i L$ (consumer is risk averse)
  - at most one if $p_i > \pi_i L$ (risk aversion again)
- Putting S&D together, only possible equilibrium is $p_i = \pi_i L$.
  - Each consumer i demands exactly one policy
  - One insurer provides each policy (any one)
  - Other insurers are content with 0 profits from not selling any policies as $p_i = \pi_i L$ leads to 0 profits anyway.

Conclusion 1 *When information is freely available to all, there is a unique competitive equilibrium, with $p_i^* = \pi_i L$ for each i. All insurance companies earn 0 expected profits and all consumers are fully insured.*

Claim 1 *This competitive outcome is Pareto efficient, and the gains from trade are fully exhausted.*

Proof More detailed look on p331-2. Sketch: we have eliminated all risk for risk-averse consumers and transferred it to risk-neutral players (exhausted all gains from trade). Any other proposal would fall short of this. Changing p_i either: (i) results in consumer i not purchasing insurance, making him worse off, or (ii) is a transfer from consumer i to an insurer, and so i is worse off. ■

Asymmetric information

- Suppose insurers don't know π_i.
- In reality insurers obtain info about consumers to partially learn their accident probabilities.
- For simplicity: insurers only know the distribution (consumers know realization).
  - Let each π_i be drawn from $[\underline{\pi}, \overline{\pi}] \subseteq [0, 1]$ with CDF F. (full support)
- Again, for now, only full insurance can be sold.

Claim 2 *There is a single equilibrium price of the full insurance policy for all consumers.*

Proof Suppose not, that the price paid by i exceeds that paid by j. The expected profits from selling each must be ≥ 0 in any equilibrium. But since consumer i and j are identical from the insurer's perspective, it must be the policy sold to i earns strictly positive profits. But then insurers would want to provide an infinite amount of such a policy, which cannot occur in equilibrium. ■

- Let p denote this price. What's p^*?
  - Positive expected profits imply infinite supply.
  - Negative....zero supply.
  - How about $p^* = E[\pi]L = L\int_{\underline{\pi}}^{\overline{\pi}} \pi dF(\pi)$.

- Are profits 0 in this case?

- Perhaps not: if p is too high, only consumers with relatively high π choose to buy insurance.

  - Don't look at the *unconditional* distribution of π
  - Must look at the distribution *conditional on those consumers being willing to purchase the policy.*
  - By not factoring this in, profits would be negative when $p = E[\pi]L$.
  - Given π, a consumer purchases a policy of price p iff $u(w-p) \geq \pi u(w-L) + (1-\pi)u(w)$, which upon rearranging becomes

$$\pi \geq \frac{u(w) - u(w-p)}{u(w) - u(w-L)} \equiv h(p). \tag{1}$$

  - So p^* gives a competitive equilibrium if price = expected losses given the accident probabilities of consumers who actually purchase the policies:

$$\begin{aligned} p^* &= E[\pi | \pi \geq h(p^*)]L \quad (2) \\ &= L\frac{\int_{h(p^*)}^{\overline{\pi}} \pi dF(\pi)}{1 - F(h(p^*))}. \end{aligned}$$

  - This is the equilibrium condition: a consumer will buy if $\pi \geq h(p)$, and insurers earn 0 expected profits when such consumers buy (line 2). So supply will be equated to demand.

Claim 3 *Such an equilibrium exists. In fact, multiple equilibria may exist.*

Proof We must find p such that line (2) holds. [Draw graph for following argument; p on horizontal axis; looking for a fixed point]. By inspection of line (1), $h(p)$ increases in p. Next, $E[\pi|\pi \geq h(p)] > 0$ for $p = 0$, and so the RHS of line (2) > 0 too. Finally, the set of p we are considering is bounded since the highest possible p is the p such that $h(p) = \overline{\pi}$ (i.e., for which only the highest risk type buys the policy). Denote this p by $\overline{p}$. So the RHS of line (2) is also bounded above, at $E[\pi|\pi \geq \overline{\pi}]L = \overline{\pi}L < \overline{p}$ (inequality follows from risk aversion). Thus there must be at least one fixed point. In fact, there can be many [both obvious from graph]. ■

Lecture 2

Example 1 *Suppose $F \sim U[0,1]$. Then*

$$E[\pi|\pi \geq h(p)]L = \frac{1+h(p)}{2}L$$

is strictly increasing and convex because $h(p)$ is. This implies there are at most two equilibrium prices.[1] *Any equilibrium price satisfies $p^* = \frac{1+h(p^*)}{2}L$. Note that $h(L) = 1$, and thus $p^* = \frac{1+h(p^*)}{2}L = L$. Thus $p^* = L$ is always an equilibrium. In this case notice from line (2),*

$$\begin{aligned} L &= E[\pi|\pi \geq h(L)]L \\ &\iff \\ E[\pi|\pi &\geq h(L)] = 1, \end{aligned}$$

i.e., the expected probability of a loss for those who buy insurance is 1. So only the consumer who will have a loss for sure buys insurance; no one else does. And type 1 consumer only formally has insurance– no risk transfer. [There may or may not be another equilibrium].

Remark 1 *In the last example, we have the least efficient outcome occuring. No trade takes place and therefore opportunities for Pareto improvements go unrealized. Opposite of the symmetric information case.*

- Prices seems unable to produce an efficient equilibrium.
- Why?
  - Consider a price at which expected profits are negative.
  - All else equal, does raising the price increase profits?

[1] You will show this in JR exercise 8.5 p421 (3rd ed).

- Problem: all else won't be equal.
- Whenever the price of insurance increases, a consumer's utility from buying insurance decreases, but utility from not buying remains the same.
- For some consumers, buying is no longer worth it, so they don't buy.
- Who continues to buy? Those who suffer the most from not having insurance– i.e., those with the highest accident probabilities.
- Thus when p rises, the average risk of customer who continue to buy goes up.

- **Adverse selection**.
  - Negative impact on profits
  - Could be (as in the example above), the negative impact of adverse selection on profits outweights the positive impact of higher prices. And so there's no equilibrium.

Exercise 1 *Find a (simple?) distribution on* $[0,1]$ *such that a non-trivial equilibrium exists for the insurance example we have considered.*

Exercise 2 *JR, p421, 8.1, 8.3, 8.5*

Signaling

- Recall our last example; suppose F is distributed so that a non-trivial eq exists.
- Further suppose you are the low type. Is there any way out for you?
- Credibly communicate you are low?
  - First suppose each consumer can *credibly disclose* his type. What happens?

- Now suppose disclosure isn't possible. Communicate your type by *signaling*: distinguish yourself from others by buying a different type of policy.
- (Note the meanings of 'signal' vs 'disclose' as they are understood in economics).

Insurance signaling game

- Suppose now there are just two accident types: $0 < \underline{\pi} < \overline{\pi} < 1$, and suppose $\alpha \in (0,1)$ of consumers are type $\underline{\pi}$ ("low risk").
- Order:

1. Nature determines whether the consumer is low risk (with probability α) or high risk. The insurer doesn't observe this; the consumer does.
2. Then the consumer chooses a policy (B, p), consisting of a benefit $B \geq 0$ the insurer pays and a premium $0 \leq p \leq w$ (wealth).
3. The insurer accepts or rejects the consumer's proposal. (Doesn't observe his type, does observe the proposal).
   (a) Interpret: insurer is one of many competing companies, and the consumer is randomly selected from the population in which α are low risk.

- See figure 8.1 on p386, JR
- Pure strategy for L is a policy $\psi_l = (B_l, p_l)$ and for H is $\psi_h = (B_h, p_h)$.
- Pure strategy for the insurer specifies Accept or Reject, for any possible policy proposed: a function $\sigma(B, p) \in \{A, R\}$ for each policy (B, p).
  - Note σ does not depend on risk (unobserved)

- Let $\beta(B,p)$ denote the insurer's beliefs that the consumer who proposed (B,p) is low.
- Find pure strategy sequential equilibrium; i.e., a pair $(\psi_l, \psi_h, \sigma(), \beta())$ such that:
  - given σ, proposing ψ_l is utility maximizing for L and ψ_h is maximizing for H.
  - the insurer's beliefs satisfy Bayes' rule; i.e., (i) $\beta(\psi) \in [0,1]$ of all ψ, (ii) if $\psi_l \neq \psi_h$ then $\beta(\psi_l) = 1$ and $\beta(\psi_h) = 0$, and (iii) if $\psi_l = \psi_h$ then $\beta(\psi_l) = \beta(\psi_h) = \alpha$.
  - for every policy $\psi = (B,p)$, the reaction $\sigma(\psi)$ maximizes the insurers profits given its beliefs $\beta(B,p)$.
- Can L distinguish himself from H?
  - Not clear.
  - Note: buying less insurance doesn't reduce the probability of a loss occurring; in this sense the signal used here is *unproductive*.
  - **Idea: L proves who he is by being willing to accept a decreased B in return for a smaller compensating premium reduction than would H.**
  - Crucially, for this to work the types need different MRS's between B and p.

Analysing the game

- Define the expected utility from policy (B,p):

$$
\begin{aligned}
u_l(B,p) &= \underline{\pi} u(w - L + B - p) + (1 - \underline{\pi}) u(w - p) \\
u_h(B,p) &= \overline{\pi} u(w - L + B - p) + (1 - \overline{\pi}) u(w - p)
\end{aligned}
$$

Claims:

1. (a) $u_l(B,p)$ and $u_h(B,p)$ are continuous, differentiable, strictly concave in B and p, strictly increasing in B, and stricting decreasing in p.

   (b) If B is $<, =, > L$, then $MRS_l(B,p) >, = , < \underline{\pi}$.

   If B is $<, =, > L$, then $MRS_h(B,p) >, = , < \overline{\pi}$.

   (c) $MRS_l(B,p) < MRS_h(B,p)$ for all (B,p).

- Explain, prove each.
  - (a) should be straightforward
  - (b): draw a sample $MRS(B,p)$ – upward sloping.
  - (c): holds since h has higher risk
  - (c): called the **single-crossing property**:
    * the indifference curves of the 2 types intersect at most once.
    * here, different types have different MRSs for the same policy
  - See / recreate figure 8.2, p389. Shows (a) and (c).
- Find where the indiff curves cross.
  - here, L will accept a decrease in B in exchange for a smaller compensating premium reduction than H.
  - reducing the benefit is less costly to L since he is less likely to have an accident.
- Insurer maximizes expected profits
  - If it knew the consumer were L, it accepts (B,p) when $p > \underline{\pi}B$, rejects when $<$, and indifferent when $=$.

- If...H, it accepts (B, p) when $p > \overline{\pi} B$, rejects when $<$, and indifferent when $=$.
- Graph 0-profit lines: fig 8.3, p390. Note the slope of each.

- Recall the competitive outcome **with full information**: each is fully insured and pays expected cost. See Fig 8.4, p390.

- **Claim**: that won't be an equilibrium here.

- What is the equilibrium? First establish lower bounds on each consumers' utilities, conditional on having been chosen by nature.

Lemma 1 *[JR, p390] Let* $(\psi_l, \psi_h, \sigma(), \beta())$ *be a sequential eq and let* u_l^* *and* u_h^* *denote the eq utility of the L and H consumer, respectively, given he has been chosen by nature. Then: (1)* $u_l^* \geq \widetilde{u}_l$*, and (2)* $u_h^* \geq u_h^c$*, where* $\widetilde{u}_l \equiv \max_{(B,p)} u_l(B, p)$ *such that* $p = \overline{\pi} B \leq w$*, and* $u_h^c \equiv u_h(L, \overline{\pi} L)$ *denotes H's utility in the competitive eq with full information.*

- *The lemma says*: since the most pessemistic belief the insurer could have is $\beta = \overline{\pi}$, both consumers' utilities are bounded below by the max utility obtainable given this belief.

Proof of Lemma First, the insurer will accept all policies (B, p) above the high-risk zero-profit line $(p > \overline{\pi} B)$, since this must lead to positive expected profits. But in fact the insurer is willing to accept when $p = \overline{\pi} B$ as well; thus any consumer should choose along the line $p = \overline{\pi} B$ (given the insurer's most pessemistic beliefs). So L wants to $\max_{(B,p)} u_l(B, p)$ such that $p = \overline{\pi} B \leq w$, and similarly for H, though given an actuarially fair premium we know H wants to fully insure. ■

- See JR, p 391, Fig 8.5

– **Claim**: lemma implies H must buy some insurance in eq. Pf: $u_h(0,0) < u_h^c$ since he is strictly risk averse. But u_h^c is a lower bound on H's utility, so he must be buying some insurance.

– **Claim**: lemma doesn't imply anything about L buying insurance. Could be: $MRS_l(0,0) > \overline{\pi}$ (in which case $u_l(0,0) < \widetilde{u}_l$), so that some insurance will be bought. (as drawn in the fig). But could be $MRS_l(0,0) < \overline{\pi}$ (so that $u_l(0,0) > \widetilde{u}_l$). In that case the consumer will choose a proposal that's rejected without violating the lemma.

Lecture 3 – continuing with adverse selection

- From before: A solution to Exercise 1: try $\pi \in [0,1]$ distributed with pdf $f(x) = 2 - 2x$, and $u = \surd$.
- Proceed by defining 2 types of eq:

Definition 1 *A pure strategy sequential eq* $(\psi_l, \psi_h, \sigma(), \beta())$ *is separating if* $\psi_l \neq \psi_h$, *while it is pooling otherwise.*

- Later we'll see another type (semi-separating), but with only 2 types here (L or H), the only possibilities are separating or pooling.

Separating equilibria

- In a separating eq, the 2 types choose different policies, and the insurer thus can tell the types apart.
- It's possible a type feigns being the other type.
  - must make sure this doesn't happen
  - "...in a separating equilibrium, *it must not be in the interest of either type to mimic the behavior of the other.*"

Theorem 1 *[JR,p342]. Characterization of sep eq. The policies* $\psi_l = (B_l, p_l)$ *and* $\psi_h = (B_h, p_h)$ *are proposed by the L and H risk consumers, respectively, and accepted by the insurer in some separating eq iff*

1. $\psi_l \neq \psi_h = (L, \overline{\pi}L)$
2. $p_l \geq \underline{\pi}B_l$
3. $u_l(\psi_l) \geq \widetilde{u}_l \equiv \max_{(B,p)} u_l(B,p)$ *such that* $p = \overline{\pi}B \leq w$

4. $u_h^c \equiv u_h(\psi_h) \geq u_h(\psi_l)$.

- Digest this. What does it say? Does it sound true? (mostly rehashing the definition and using the lemma)

- Where's α?

Proof of Theorem First, suppse (1) – (4) are true and show there is a separating eq. So it would remain to find a strategy σ and beliefs β for the insurer such that $(\psi_l, \psi_h, \sigma(), \beta())$ is a seq eq.

It is clearly separating.

Verify this specification works:

$$\begin{aligned}
\beta(B,p) &= \begin{cases} 1 & \text{if } (B,p) = \psi_l \\ 0 & \text{if } (B,p) \neq \psi_l \end{cases} \\
\sigma(B,p) &= \begin{cases} A & \text{if } (B,p) = \psi_l, \text{ or } p \geq \overline{\pi} B \\ R & \text{otherwise} \end{cases}.
\end{aligned}$$

You can see the beliefs are consistent with Bayes' rule (when applicable; when not, off-the-eq-path beliefs are unrestricted), and that the actions maximize expected profits given beliefs. [explain]

For this part of the proof it remains to show L can't find a better policy than ψ_l, nor H find better than ψ_h. Now, we can restrict our attention to the set of policies accepted by the insurer since rejection by the insurer is equivalent to accepting the policy $(0,0)$ (which the insurer is willing to accept). So we can restrict attention to

$$A = \{\psi_l\} \cup \{(B,p) \mid p \geq \overline{\pi} B\}.$$

But #3 implies ψ_l is the best L can do within the set A, while $\psi_h = (L, \overline{\pi} L)$ is best for H since: (a) by #4 ψ_l is not better, and (b) all other available policies are "no better than fair" policies, the best of which must be on the fair line, and we know the best on that line is $\psi_h = (L, \overline{\pi}L)$ (i.e., when picking among actuarially fair policies, H prefers to fully insure).

Now consider the converse case: suppose $(\psi_l, \psi_h, \sigma(), \beta())$ is a separating seq eq. in which the eq policies are accepted. Show #1–4 hold.

Part 1. By definition we must have $\psi_l \neq \psi_h$. To show that $\psi_h = (B_h, p_h) = (L, \overline{\pi}L)$, note the lemma implies ψ_h is such that $u_h(\psi_h) \geq u_h(L, \overline{\pi}L)$. Also, in a separating eq, the insurer places weight one on the type being H given ψ_h was proposed, and since the insurer accepts ψ_h it must be that ψ_h earns non-negative profits: $p_h \geq \overline{\pi}B_h$. But clearly among such policies H will choose to propose a policy where this binds: $(B_h, \overline{\pi}B_h)$, and we've argued before the best such actuarially fair policy for H is full insurance.

Part 2. Since the insurer's profits must be non-negative.

Part 3. From the lemma

Part 4. Since the insurer accepts ψ_l, it must be that $u_h(\psi_h) \geq u_h(\psi_l)$, for otherwise H would propose ψ_l, and the eq wouldn't be separating. ■

- See Fig 8.6, p394 for an illustration. In any sep eq, H is fully insured and pays the actuarially fair premium, while L's policy lies in the shaded area; his proposal must:
  - be above the low risk zero profit line to induce acceptance
  - be above H's indiff curve so H doesn't mimick L.
  - be below L's drawn in indiff curve (best he can do if believe with certainty to be H), so L has no incentive to mimick H.

Exercise 3 *Show that the shaded region in Fig 8.6 is always nonempty, even when $MRS_l(0,0) \leq \overline{\pi}$ (you will use the fact that $MRS_l(0,) > \underline{\pi}$). Thus a pure strategy separating eq always exists.*

Exercise 4 *JR 8.9 (insurance game), and 8.6 (Akerlof's used car model: seller's know the quality of the car but buyers cannot determine it before purchase; under full inofrmation there are gains from trade, but the seller won't sell if he doesn't get a high enough price).*

- By the way, there's are lots of separating eq. And H always gets the same policy. What's best for L (in Fig 8.6)? See Fig 8.7, p396.
- So, to summarize:
  - The proposed policy acts as a signal, separating L from H.
  - Though welfare need not be much improved. E.g., suppose $MRS_l(0,0) \leq \overline{\pi}$; there is a separating eq in which L gets $(0,0)$ and H is fully insured. Notice this doesn't depend on α! (one bad apple ruins it for everyone)
  - Notice there is a cost to signaling– L receives less insurance than he would like.
  - What about giving up? Just pool with H. Might not be so bad if α (pr of L) is high.

Another example of a separating eq: price signaling

- One period
- Firm (monopolist)
  - Product has quality θ_L with probability p and quality θ_H with probability $1-p$, where $\theta_H > \theta_L > 0$.
  - There are no fixed costs.
  - The marginal cost of producing H and L quality are c_H and c_L, respectively, where $0 < c_L < c_H$, and assume $c_j < \theta_j$ for $j = L, H$.

- Consumers
  - There is a unit mass of consumers. Consumer i's utility from purchasing a good of quality j at price p is $v_i + \theta_j - p$, where $v_i \sim U[0,1]$. Utility from no purchase is 0.
  - However, assume consumers do not observe quality but only know the distribution of outcomes; utility from buying is $v_i + E[\theta|\mu] - p$, where μ are beliefs given all available signals.
  - Notice consumers' *expectations* of quality are what matter; we will show these could possibly depend on price.

Claim 4 *Demand is linear, with slope -1, and vertical intercept equal to $1 + E[\theta|\mu]$. (Domain: $q \in [0,1]$, range: $p \in [E[\theta|\mu], 1 + E[\theta|\mu]]$)*

- Both types would rather be thought of as H. Can we find a separating equilibrium?
- First, full information benchmark.
- L's full information profit is $\left(\frac{1+\theta_L - c_L}{2}\right)^2$
  - Demand is $p = 1 + \theta_L - q$, by the claim above.
  - $\Pi_L = (p - c_L)q = (1 + \theta_L - q - c_L)q$
  - $q_L^* = \frac{1+\theta_L - c_L}{2}$, and thus $p_L^* = \frac{1+\theta_L + c_L}{2}$
  - $\Pi_L^* = (1 + \theta_L - \frac{1+\theta_L - c_L}{2} - c_L)\frac{1+\theta_L - c_L}{2} = \left(\frac{1+\theta_L - c_L}{2}\right)^2$
- Similarly, H's full information profit is $\left(\frac{1+\theta_H - c_H}{2}\right)^2$, and $q_H^* = \frac{1+\theta_H - c_H}{2}$, and $p_H^* = \frac{1+\theta_H + c_H}{2}$.

Claim 5 *In the game with asymmetric information, it is not an equilibrium for H to charge p_H^* and L to charge p_L^*.*

Proof L would mimick H by setting price p_H^*, be believed to be high quality, but actually have low production costs, and so earn higher profits than if it were known to be L. ■

- How to proceed? Idea: specify a very high price for H; doing so restricts quantity, which hurts L more since L has a higher profit margin (when pretending to be H).
- That is, find p' such that

$$(p' - c_L)(1 + \theta_H - p') \leq \Pi_L^*. \tag{3}$$

  In words, if L charged p' and so was believed to be H with probability 1, then its resulting profits are no better than its full information profits.

- Can we find such a p'?
  - Sure: the LHS is a quadratic in p' and, for example could be set to 0, so clearly we can make that inequality hold.
  - Even easier: picture the graph of high demand with low costs; we can make these profits as small as we want.
- In fact, there are many solutions. But a Pareto efficient eq would have line (3) bind with equality, and in this case there are only two solutions (we have a quadratic in p). However, while each solution gives the same revenue to H, the solution with higher p gives lower q and so lower costs, and thus higher proftis (show this). So perhaps select that one.

Exercise 5 *Verify that H has no incentive to mimick L.*

Conclusion 2 *There exists a price separating equilibrium in which $p_L = p_L^*$ while p_H is the larger of the two solutions that solve line (3) with equality.*

Exercise 6 *Keep the model the same except let $c_L = c_H = 0$, and let $\alpha \in (0,1)$ consumers be informed of quality while the rest are uninformed. Further, the informeds are equally distributed on $[0,1]$. Show there exists a price signaling equilibrium that achieves separation.*

Lecture 4

Give quiz: March 1, 2018.

Pooling equilibria in the insurance example

- Recall in a pooling equilibrium all types propose the same policy, and so the insurer learns nothing from this. Just uses prior beliefs; accepting proposal (B, p) yields profits $p - E[\pi]B = p - (\alpha\underline{\pi} + (1-\alpha)\overline{\pi})\, B$.
- Define $\widehat{\pi} = \alpha\underline{\pi} + (1-\alpha)\overline{\pi}$
- Accept iff $p \geq \widehat{\pi}B$ (assume accepts when indifferent)
- Consider the pooling zero-profit line $p = \widehat{\pi}B$. Fig 8.8, p397.
- Consider a pooling equilibrium proposal (B, p)
  - Recall from our lemma that in a sequential equilibrium: (1) $u_l^* \geq \widetilde{u}_l$, and (2) $u_h^* \geq u_h^c$, where $\widetilde{u}_l \equiv \max_{(B,p)} u_l(B, p)$ such that $p = \overline{\pi}B \leq w$, and $u_h^c \equiv u_h(L, \overline{\pi}L)$ denotes H's utility in the competitive eq with full information.
  - And we showed that in a sequential equilibrium the policy will be accepted, so $p \geq \widehat{\pi}B$.
  - See Fig 8.9, p 398 for the intersection of these 3 inequalities.

Theorem 2 *The policy $\psi' = (B', p')$ is the outcome in some pooling equilibrium iff it satisfies $u_l^* \geq \widetilde{u}_l$, $u_h^* \geq u_h^c$, and $p \geq \widehat{\pi}B$.*

Proof Already showed the inequalities are necessary. Now show it is sufficient. That is, suppose $\psi' = (B', p')$ satisfies the inequalities. We must define beliefs and a strategy for the insurer such that $(\psi_l, \psi_h, \sigma\left(\right), \beta\left(\right))$ is a seq eq. Try these:

$$\beta(B,p) = \begin{cases} \alpha & \text{if } (B,p) = \psi' \\ 0 & \text{if } (B,p) \neq \psi' \end{cases}$$

$$\sigma(B,p) = \begin{cases} A & \text{if } (B,p) = \psi', \text{ or } p \geq \overline{\pi} B \\ R & \text{otherwise} \end{cases}.$$

So the insurer beliefs any deviation comes from the high risk type, and thus it's profit maximizing to accept $(B,p) \neq \psi'$ only if $p \geq \overline{\pi} B$. Next, notice beliefs satisfy Bayes' rule when ψ' is proposed (i.e., they don't change). And given these beliefs it is profit maximizing to accept since $p \geq \widehat{\pi} B$. For off the path proposals $(B,p) \neq \psi'$ Bayes' rule imposes no restriction and so the specified beliefs are acceptable, and further the decision rule is best given those beliefs (accept only if $p \geq \overline{\pi} B$).

Now show the two types are utility maximizing, given the insurer's strategy. If either type proposes ψ', it is accepted. By deviating to $(B,p) \neq \psi'$, the consumer gets $(0,0)$ if rejected (i.e., $p < \overline{\pi} B$) and (B,p) if accepted (i.e., $p \geq \overline{\pi} B$). Thus proposing policy ψ' is optimal for type $i = l, h$ if $u_i(\psi') \geq u_i(0,0)$, and $u_i(\psi') \geq u_i(B,p)$ for all $\overline{\pi} B \leq p \leq w$. But these follow from $u_l^* \geq \widetilde{u}_l$ and $u_h^* \geq u_h^c$ (see figure 8.9, p398). ■

- There are potentially many pooling eq, as seen in Fig 8.9, p398.
- As α decreases (proportion of L's decreases), the pooling zero-profit line's slope increases while everything else in the figure remains the same.
- I.e., fewer pooling eq exist.
- Eventually, the shaded region is gone: there is no pooling equilibrium since pr(H) is too high.
- As α gets larger, the shaded region grows, and it is possible to find a pooling eq which Pareto dominates any separating eq

- clearly H is better off in any pooling eq
- L can be better off since it is costly to signal (and that cost was invariant to α), so as $\alpha \to 1$ the pooled quality is close to the true quality of L, so pooling isn't too distorting for L.

- Now let's reconsider the extreme beliefs we imposed in the separating and pooling eq

Refinement of beliefs

- In our prior examples we had to specify beliefs for actions off the equilibrium path.
  - That is, according to the equilibrium strategies certain actions were not supposed to happen.
  - What if they do anyway? What should be believed?
  - Bayes' rule is inapplicable.
  - So we were free to specify what we wanted, and usually used "convenient" beliefs: if you deviate from the equilibrium I will think very bad things about you (thus providing deterence from deviating, and helping support the equilibrium).
- While our beliefs satisfied the definition of a sequential equilibrium, we ask now whether we ought to impose an additional requirement on the equilibrium.
- Are the beliefs in the insurance example 'reasonable'? Perhaps not.
  - Consider the pooling equilibria again.

- Consider a deviation: See Fig 8.9, p398, again (Look southwest of the intersection of the two indifference curves). (more correct: look at Figure 8.11, p400)

- Who could have proposed such a policy? The high type is worse off if he proposed that policy and it was accepted; also worse off if it's rejected and he ends up with (0,0). (Worse off than his equilibrium payoff).

- And the low type is better off if he proposes it and it is accepted.

- Note though our specified beliefs in the pooling equilibrium are that the insurer is certain the deviant policy came from H.

- Perhaps we should require the insurer to assign probability 1 to the deviant policy being from L

Definition 2 *[JR, p401] A sequential equilibrium $(\psi_l, \psi_h, \sigma(), \beta())$, yielding equilibrium utilities u_l^* and u_h^* to the low and high risk consumer, respectively, satisfies the intuitive criterion if the following condition is satisfied for every policy $\psi \neq \psi_l$ or ψ_h: If $u_i(\psi) > u_i^*$ and $u_j(\psi) < u_j^*$, then $\beta(\psi)$ places probability 1 on risk type i.*

Theorem 3 *[JR, p351] There is a unique policy pair (ψ_l, ψ_h) that can be supported by a sequential equilibrium satisfying the intuitive criterion. Moreover, this equilibrium is the best separating for the low-risk (i.e., $\psi_l = \overline{\psi}_l$ and $\psi_h = \psi_h^c$ in Fig 8.7, p396).*

Proof sketch There are 3 parts. First, looking at Fig 8.7 (p396), any point in the shaded region (which is a policy proposed by L that is part of a sequential eq that is separating) will cause the eq to fail the intuitive criterion. (Other that the point indicated in the theorem). This is because there is always a deviation south and/or east, still within the shaded region, that gives L higher utility (and which H has no incentive to do, since we are in the shaded region).

Second, the point indicated in the theorem does satisfy the intuitive criterion. I claim it is $\psi_l = \overline{\psi}_l$ and $\psi_h = \psi_h^c$. First, we already know this is a sequential eq. Next, construct the set of policies that only the low-risk type prefers to his equilibrium policy

$$A = \left\{ \psi \mid u_l(\psi) > u_l\left(\overline{\psi}_l\right) \text{ and } u_h(\psi) < u_h\left(\psi_h^c\right) \right\}.$$

But these are policies the insurer would not accept (see Fig 8.13, p403), and so no type would deviate to them.

Third and finally, there does not exist a pooling equilibrium that satisfies the intuitive criterion. This was largely explained before. See Fig 8.9, p398: for any pooling eq there are policies L prefers to deviate to, H doesn't, and that lie above the low-risk zero-profit line. Thus if a deviation by L to such a point occured, to satisfy the intuitive criterion the insurer would have to believe it was L that did that, and by sequential rationality it must accept the proposal. But then L can improve from his 'equilibrium' payoff, so the eq fails. ■

Lecture 5

Job-market signaling example (Gibbons, p190)

Consider a signaling game (due to Spence, 1973) with the following timing:

1. Nature determines a worker's productive ability, η, which can be either high (H) or low (L). The probability that $\eta = H$ is q.

2. The worker learns his ability and then chooses a level of education, $e \geq 0$.

3. Two firms observe the worker's education (but not the worker's ability) and then simultaneously make wage offers to the worker.

4. The worker accepts the higher of the two wage offers, flipping a coin in the case of a tie. Let w denote the wage the worker accepts.

The payoffs are:

- $w - c(\eta, e)$ to the worker, where $c(\eta, e)$ is the cost to a worker with ability η obtaining education e.

- $y(\eta, e) - w$ to the firm that employs the worker, where $y(\eta, e)$ is the output of a worker with ability η who has obtained education e.

- 0 to the firm that does not employ the worker.

We wish to see if there is an equilibrium in which firms interpret education as a signal of ability and so offer a higher wage to a worker with more education.

- In fact Spence showed that even if education had no affect on productivity, it could nonetheless be useful to signal ability.

- More generally, education may also increase productivity, in which case it can serve both purposes. In that case if wages increase in education it will be due in part to increased productivity and signaling of type.

- You can think of e as the difficulty level of classes taken by students at a given college. You could also loosely interpret it as years of schooling, but this makes the model more complicated (at every new period you make sure I don't want to defect from my proposed e (i.e., cut my schooling short)).

A crucial assumption in Spence's model is that L's find signaling more costly than do H's; that is, for every e:

$$c_e(L, e) > c_e(H, e)$$

where c_e is the marginal cost of education.

- Draw this condition graphically, in (e, w) space.

-
  - Suppose H and L start at an initial point (e_1, w_1). Draw the indifference curve for both through this point.
  - How much compensation would a worker need to receive to be willing to do $e_2 > e_1$?
  - Answer: depends on L or H. L needs more than what H would need.

- Thus, L's indifference curves are steeper than H's.

Spence assumed that competition among firms will drive expected profits to 0. So he just asserted that workers would be paid their expected output.

- This is 'reduced form'.

- Instead we will arrive at the same conclusion (workers paid their expected output) with competition between our two firms.

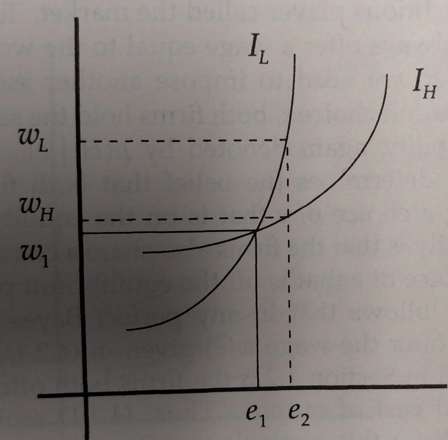

Figure 4.2.3.

from e_1 to e_2. The answer depends on the worker's ability: low-

- Confirm this is true. Actually, we need to assume that the two firms have the same off the equilibrium path beliefs.
- This induces Bertrand competition, so that

$$w(e) = \mu(H|e)y(H,e) + (1-\mu(H|e))y(L,e).$$

Benchmark: IF this were a full information game, what would happen?

- As argued above, a worker with ability η and education e earns the output he produces: $w(e) = y(\eta, e)$.
- Thus a worker's problem is

$$\max_e y(\eta, e) - c(\eta, e)$$

  and denote the solution by $e^*(\eta)$. And let $y(\eta, e^*(\eta)) = w^*(\eta)$.
- Fig 4.2.4, p195: draws in (e, w) space both y and indifference curves for type η

END of benchmark; the worker's type is private information.

The low type might try to mimick the high type. There are two cases here:

1. It could be that acquiring education $e^*(H)$ is so costly for L that it isn't even worth it:

$$w^*(L) - c(L, e^*(L)) > w^*(H) - c(L, e^*(H)).$$

   In other words, the full information education level $e^*(H)$ achieves separation. See Fig 4.2.5, p195.
2. It could be that inequality doesn't hold, so L would like to try to imitate H. More interesting case, more realistic. See Fig 4.2.6, p196.

Let's first look for a pooling eq:

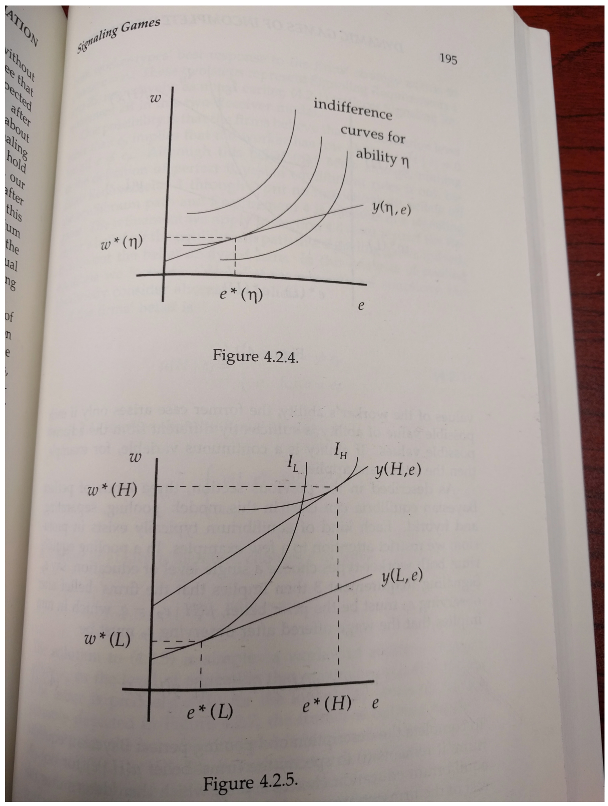

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Figure 4.2.4.

Figure 4.2.5.

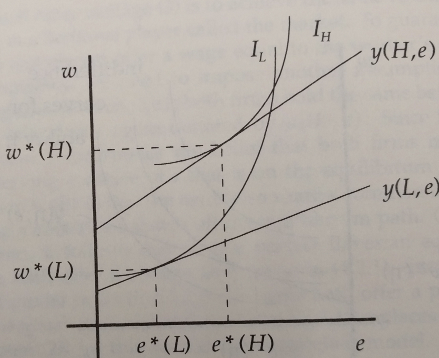

Figure 4.2.6.

values of the worker's ability, the former case arises only possible value of ability is sufficiently different from the a possible values. If ability is a continuous variable, for ex then the latter case applies.

As described in the previous section, three kinds of

- If both pool on e_p, then no updating is possible upon observing this level of education: $\mu(H|e_p) = q$, and thus the wage must be

$$w_p = qy(H, e_p) + (1 - q)y(L, e_p)$$

- What are off the equilibrium path beliefs? I.e., what is $\mu(H|e)$ when $e \neq e_p$?

- Knowing that, we can determine the rest of the firms' strategy $w(e)$, and then show both worker types are best responding by picking $e = e_p$.

- Specify $\mu(H|e) = 0$ for all $e \neq e_p$. (and it $= q$ for $e = e_p$).

- Thus firms' best response given their beliefs is to

$$w(e) = \begin{cases} y(L, e) & \text{for } e \neq e_p \\ w_p & \text{for } e = e_p \end{cases}.$$

- So worker of type η chooses e so that $\max_e w(e) - c(\eta, e)$, the solution to which is either e_p or e that maximizes $y(L, e) - c(\eta, e)$. (for L the $e^*(L)$ is the maximizer of the latter). (Fig 4.2.7 p198 is relevant here, but a bit too messy so omit).

- So for the pooling eq to exist e_p must be the best response for both.

- There are other pooling eq: e.g., specify a slightly higher level of pooled e.[2]

Exercise 7 *Do the beliefs specified above satisfy the intuitive criterion? If not, can you find beliefs for a pooling equilibrium that do?*

Now let's find a separating eq

[2] This is analogous to the price signaling example in which an even higher price than the one we found would constitute an equilibrium. However, this is Pareto dominated (for the workers).

- For case 1 above, the proposed separating equilibrium is clear. But to be complete we do need OEP beliefs. Verify that this works:

$$\mu(H|e) = \begin{cases} 0 & \text{for } e < e^*(H) \\ 1 & \text{for } e \geq e^*(H) \end{cases} .$$

- Now case 2. Type H can't choose $e^*(H)$ and earn $y(H, e^*(H))$ since L would copy.

- Must have $e_s > e^*(H)$ to signal ability, since otherwise the L type would mimick any value of $e \in (e^*(H), e_s)$ if doing so tricks the firms into believing that the worker is H.

- Need to pick $e_s > e^*(H)$ high enough so that this isn't true.

- If you make it just bind: see Fig 4.2.8, p201, but reproduce it in steps on the board.

- So have actions $e(L) = e^*(L)$, $e(H) = e_s$, and beliefs $\mu(H|e^*(L)) = 0$, $\mu(H|e_s) = 1$ and wages $w(e^*(L)) = w^*(L)$, $w(e_s) = y(H, e_s)$.

- OEP beliefs; try this:

$$\mu(H|e) = \begin{cases} 0 & \text{for } e < e_s \\ 1 & \text{for } e \geq e_s \end{cases} .$$

- and thus the firms' strategy is then

$$w(e) = \begin{cases} y(L, e) & \text{for } e < e_s \\ y(H, e) & \text{for } e \geq e_s \end{cases} .$$

- If e_s is as depicted in the figure, then L has two best responses: choose $e^*(L)$ and earn $w^*(L)$ or choose e_s and earn $y(H, e_s)$.

  - Assume L picks the former.
  - If you don't like that, you can increase e_s by an arbitrarily small amount.

- For type H, recall $e^*(H)$ is best given he is known to be H.
  - Thus $e_s > e^*(H)$ implies any $e > e_s$ are worse than e_s (even if H were believed to by H; in which case H is choosing a point along the $y(H, e)$ curve).[3]
  - For $e < e_s$, note that H's indifference curve through the point $(e_s, y(H, e_s))$ is above L's through this region, and L's is tangent to the wage function $w = y(L, e)$ (defecting = choosing a point along this curve). Thus for any of these e H would be on a lower indifference curve.
  - Thus for H, e_s is optimal.

- There are other separating eq
  - For example, specify e a bit larger than e_s; as long as H doesn't want to deviate to $e^*(L)$, this works.
  - Could ease up a bit on the OEP beliefs as well.

Exercise 8 *Find a "hybrid" equilibrium in which H chooses e_h while L randomizes between e_h and e_L. Minor hint: remember beliefs ought to be formed by Bayes' rule*

[3] In response to a comment. The argument is that since $e_s > e^*(H)$, choices of $e > e_s$ are inferior to e_s. A sufficient condition for this argument is the assumption that $y(H, e) - c(H, e)$ is single-peaked in e (i.e., there is a unique maximizer and the payoff is monotonically lower as the distance from this maximizer increases), a sufficient condition for which is

$$\frac{\partial^2}{\partial e^2}\left(y(H, e) - c(H, e)\right) < 0 \Longleftrightarrow y''(H, e) < c''(H, e).$$

and be consistent with the workers' strategies, and show that $e_L = e^*(L)$ *(from the full information case).*

Lecture 6

Cheap talk models

Go over Crawford and Sobel's classic 1982 paper "Strategic information transmission."

- One sender that privately observes the state of the world θ and sends a costless, non-binding, non-verifiable message m (this is called "cheap talk").
- A receiver who knows the prior distribution of the state receives the message, updates beliefs, and then takes an action y.
- Note the payoffs in the prior signaling model would not allow for cheap talk to be persuasive: if there has a cheap talk message that induces firms to pay more, then every type, irrespective of the truth, would like to send that message. But then the receiver (employer) doesn't learn anything from the message, and so wouldn't pay more given it.
- So we need different sender-types to have different preferences over the receiver's actions.
- Also need the receiver to prefer different actions depending on the sender's type.
- Payoffs of both players depend on the state and the action taken:

$$U_r(\theta, y)$$
$$U_s(\theta, y)$$

  - There must be some commonality of interest; if interests were completely opposed cheap talk messages could never influence beliefs (e.g., in the Spence job market signalling model). (in such cases the sender always wants to induce to maximal belief and so will not be heeded).

  - The message has no direct effect on payoffs (unlike education in the signaling model). The message can only matter through its informational content.

- The message space: there are many ways to say the same thing.

  - Gibbons: "Since the same information can be communicated in different languages, different message spaces can achieve the same results."
  - For example, if $\Theta = [0, 1]$, then perhaps $M = [0, 1]$ and the message $m = x$ is interpreted as "$\theta = x$." But it could also be that $M = [1, 2]$ and the message $m = x$ is interpreted as "$\theta = x - 1$." We'll have more to say about this later.

- What does a pure strategy perfect Bayesian equilibrium look like here? A pair of strategies $m(\theta)$, $y(m)$, and beliefs $\mu(\theta|m)$ such that

  - Beliefs are consistent with $m(\theta)$ and formed using Bayes' rule when possible.
  - Action $y(m)$ is optimal given beliefs.
  - Message $m(\theta)$ that is a best response given $y(m)$.

Claim 6 *There always exists a pooling equilibrium.*

Proof If all sender types pool on the same message, then the receiver learns nothing. Thus it is a best-response to ignore any messages and take an action given the prior. But given the receiver is ignoring all messages, it is a best response of the sender to do anything at all, including pooling on a single message. ■

Remark 2 *This is referred to as a "babbling equilibrium."*

Remark 3 *Babbling can also occur in different ways: for example, suppose the sender randomly chooses a message, independent of the state.*

- Do there exist non-pooling equilibria?
- Let's look at a special case of Crawford and Sobel's paper.

Crawford and Sobel's uniform-quadratic example

- The sender privately observes $\theta \sim U[0,1]$ and sends a message $m \in M[0,1]$.
- The receiver updates beliefs $\mu(\theta|m)$ given the message and takes action $y \in [0,1]$.
- Payoffs are

$$\begin{aligned} U_r(\theta, y) &= -(\theta - y)^2 \\ U_s(\theta, y, b) &= -(\theta + b - y)^2 \end{aligned}$$

  where $b > 0$ is a bias parameter.

- Notice these are quadratic loss functions, with ideal actions
  - $y_s^*(\theta) = \theta + b$ for the sender and
  - $y_r^*(\theta) = \theta$ for the receiver.
- Thus the b measures the similarity of interests between the players; they are not perfectly aligned since $b > 0$.

Claim 7 *[Crawford and Sobel] Any perfect Bayesian equilibrium of this model is equivalent to a partially pooling equilibrium in which the type space is divided into n intervals $[0, x_1)$, $[x_1, x_2)$, ..., $[x_{n-1}, 1]$. All types in a given interval send the same message, but types in different intervals send different messages.*

- For a given partition of this form, there are trivially infinitely many outcome-equivalent equilibria. Explain.

- We already know a pooling equilibrium ($n = 1$) always exists.

- We will show that given b,

  - there is a maximum number of intervals $n^*(b)$ that can occur in equilibrium.

  - And that a decrease in b increases $n^*(b)$. That is, more communication occurs when interests are more closely aligned.

  - Also $n^*(b) < \infty$ for all $b > 0$ but approaches ∞ as $b \to 0$.

The $n = 2$ case

- Let's start by looking for an equilibrium in which $n = 2$.

- That is, suppose there is an x_1 such that all types in $[0, x_1)$ send one message while types in $[x_1, 1]$ send another.

- After receiving the low message the receiver believes $\theta \sim U[0, x_1)$ while after receiving the high message $\theta \sim U[x_1, 1]$.

- Thus the receiver's optimal action given low is $\frac{x_1}{2}$ while after high it is $\frac{1+x_1}{2}$.

- For the sender types in $[0, x_1)$ to send the low message they must prefer action $\frac{x_1}{2}$ instead of $\frac{1+x_1}{2}$.

- Similarly, types in $[x_1, 1]$ send the high message iff they prefer $\frac{1+x_1}{2}$ instead of $\frac{x_1}{2}$.

- The sender's preferences are symmetric around his optimal action $y_s^*(\theta)$, so

  - The type-θ sender prefers $\frac{x_1}{2}$ instead of $\frac{1+x_1}{2}$ if $y_s^* = \theta + b$ is closer to $\frac{x_1}{2}$ than $\frac{1+x_1}{2}$.

- The type-θ sender prefers $\frac{1+x_1}{2}$ instead of $\frac{x_1}{2}$ if $y_s^* = \theta + b$ is closer to $\frac{1+x_1}{2}$ than $\frac{x_1}{2}$.

- For the $n = 2$ equilibrium to exist, the type x_1 must be indifferent between the two actions.
  - That is, the distance between $y_s^*(x_1)$ and $\frac{1+x_1}{2}$ must be the same as the distance between $y_s^*(x_1)$ and $\frac{x_1}{2}$:

$$\begin{aligned} x_1 + b - \frac{x_1}{2} &= \frac{1+x_1}{2} - (x_1 + b) \\ &\iff x_1 = \frac{1}{2} - 2b. \end{aligned}$$

  - This will imply all types $\theta < x_1$ prefer the low to high action, and conversely for types $\theta > x_1$.

- Since $\Theta = [0, 1]$, $x_1 > 0 \iff b < \frac{1}{4}$.
- For $b \geq \frac{1}{4}$ the players' preferences are too dissimilar to allow even this limited communication.
- To complete the discussion, we must address off the equilibrium path messages.
  - Kill this issue, like CS did, by assuming the sender mixes with full support on $[0, x_1)$ and on $[x_1, 1]$.
  - Alternatively, specify a pure strategy, say: all types $\theta < x_1$ say 0 while all types $\theta \geq x_1$ say x_1. Specify OEP beliefs of $U[0, x_1)$ for messages in $(0, x_1)$ and $U[x_1, 1]$ for messages in $(x_1, 1]$.

The general case

Claim 8 *Each interval must be $4b$ larger than the preceding interval.*

Proof Consider the intervals $[x_{k-1}, x_k)$ and $[x_k, x_{k+1})$. Let the length of the first be denoted by $c = x_k - x_{k-1}$. The induced action on the lower interval is...This is ___ lower than the sender-threshold type's preferred action. So the induced action on the higher interval must be ____ higher than the sender-threshold type's preferred action to make the threshold sender type indifferent. Thus given x_{k-1} and x_k, x_{k+1} is determined. ■

- Thus in an n-step equilibrium, if the first step is of length d, the second is of length $d + 4b$, the third $d + 8b$, and so on.

- The n^{th} step must end at $\theta = 1$, so we must have

$$d + (d + 4b) + ... + [d + (n - 1)4b] = 1.$$

- Using the fact that $1 + 2 + ... + (n - 1) = n(n - 1)/2$, we have

$$nd + n(n - 1)2b = 1.$$

- Given any n such that $n(n-1)2b < 1$, there exists a d which solves the previous equation.

- Since $d > 0$, the largest possible number of steps in an equilibrium, $n^*(b)$, is the largest value of n such that $n(n - 1)2b < 1$.

- Applying the quadratic formula shows $n^*(b)$ is the largest integer less than

$$\frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{b}}\right).$$

- Consistent with what we previously found, when $b \geq \frac{1}{4}$ the formula gives $n^*(b) = 1$.

- Also, the other claims about $n^*(b)$ also follow from the expression.

- Note: we found the finest partition of the state space possible. But when b is quite small, the $n = 2$ equilibrium still exists.
  - Verify this.
  - Which equilibrium should we select?

Exercise 9 *Show the equilibria in the CS model are Pareto ordered. In fact, show the equilibrium that has $n^*(b)$ intervals is Pareto dominant.*

Exercise 10 *Look through the construction of the equilibrium to see how the argument would proceed if θ wasn't uniformly distributed.*

Lecture 7

Screening

- Back to JR, p353 and the insurance example.
- Let's switch the roles of who makes an offer.
- Suppose the insurer offers a menu of policies, and allows the consumers to select one from that list.
- Screening: choose the menu so that H consumer pick one policy and L's pick another.
- Take the signaling model as before, but change it slightly
  - 2 insurers (not needed, but this will introduce more interesting things to consider)
  - The 2 insurers simultaneously choose a finite list of policies.
  - Nature determines the consumer is L with probability α.
  - The consumer chooses a single policy from one of the insurance companies' lists.

Claim 9 *We may restrict the insurance companies to lists with at most two policies.*

- (since there are only two types)
- Denote the insurers by $j = A, B$

Strategies?

- A pure strategy for j is a pair of policies $\Psi^j = (\psi_l^j, \psi_h^j)$

- ψ_l^j is interpretted as the policy intended for type L (though L is free to choose any policy). Similarly for ψ_h^j.
- A pure strategy for consumer $i = l, h$ is a choice function $c_i()$ for each pair of policy pairs, (Ψ^A, Ψ^B), an insurance company and one of its policies, or the null policy.
- Thus $c_i(\Psi^A, \Psi^B) = (j, \psi)$, where $j = A$ or B, and where $\psi = \psi_l^j, \psi_h^j$, or $(0, 0)$.

Equilibria

- Pooling vs separating
- **Cream skimming**– one insurer takes strategic advantage of the set of policies offered by the other by offering a policy that would attract away *only the low-risk consumers* from the competing company.
  - The "raiding" insurer takes the best customers and leaves the worse.
  - In eq, both companies must ensure the other cannot do this.

Lemma 2 *[JR, 8.2, p406] Both insurers earn 0 expected profits in every pure strategy subgame perfect equilibrium.*

- The proof is a Bertrand competition argument.

Theorem 4 *[JR, 8.4, p408] There are no pure strategy* ***pooling eq*** *in the insurance screening game.*

Proof By contradiction. Suppose $\psi^* = (B^*, p^*)$ is chosen by both consumers. By the lemma, profits are 0:

$$\alpha(p^* - \underline{\pi}B^*) + (1 - \alpha)(p^* - \overline{\pi}B^*) = 0.$$

Case 1: $B^* > 0$. Since $p^* - \underline{\pi}B^* > p^* - \overline{\pi}B^*$, and profits are a weighted average of the two, which equals 0, it must be that $p^* - \underline{\pi}B^* > 0$. Thus $p^* > 0$. Thus we do not have the null policy (0,0) being pooled on. The policy is somewhere off the axes on (B,p) space (see Fig 8.16, p408). By the single-crossing property there is a region R such that ψ^* is the limit of the policies in R (see the Fig). Let ψ' be a policy in R close to ψ^*. Then if insurer A offers ψ^*, B would do better offering ψ', since H won't accept while L will. If ψ' is close enough to ψ^* B will earn positive profits, a contradiction of the lemma.

Case 2: $B^* = 0$. But then the lemma implies $p^* = 0$. The null policy. But now either company can defect to offering a single policy $(L, \overline{\pi}L + \epsilon)$ for $\epsilon > 0$ sufficiently small. The insurer would earn strictly positive profits off both types (if they buy), and the high-risk type has incentive to buy. ■

- Notice we have used creak-skimming arguments in these proofs.

Theorem 5 *[JR, 8.5, p409] Suppose that ψ_l^* and ψ_h^* are policies chosen by the low- and high-risk consumers, respectively, in a pure strategy separating equilibrium. Then $\psi_l^* = \overline{\psi}_l$ and $\psi_h^* = \psi_h^c$.*

- Recall ψ_h^c was the outcome for H under full information and perfect competition, while $\overline{\psi}_l$ was the outcome for L in the best separating equilibrium for consumers in the insurance signaling game (which also uniquely satisfied the intuitive criterion).

- This, with the prior theorem, implies the eq identified is the only eq of the game.

Proof Established by a series of claims.

Claim 1: H must obtain at least utility u_h^c. By the lemma, insurers earn 0. Thus it can't be that H prefers $(L, \overline{\pi}L + \epsilon)$ for $\epsilon > 0$ to ψ_h^*. If so, one insurer could offer only that policy and earn positive profits (both L and H give positive profits from that policy). But insurers can't earn positive profits. Thus we must have

$$u_h(\psi_h^*) \geq u_h(L, \overline{\pi}L + \epsilon)$$

for all $\epsilon > 0$. Taking the limit as $\epsilon \to 0$ gives the result.

Claim 2: ψ_l^* must lie on the low-risk zero-profit line. First, ψ_l^* must lie on or above the low-risk zero-profit line since by Claim 1 ψ_h^* is on or below the high-risk 0-profit line, and aggregate profits are 0. Now, by way of contradiction, suppose $\psi_l^* = (B_l^*, p_l^*)$ lies above the low-risk 0-profit line. Then $p_l^* > 0$. But then $B^* > 0$ too or else L would take the null policy. Thus ψ_l^* is not on the vertical axis (see Fig 8.19, p411). Now, a cream skimming argument.

Claim 3: $\psi_h^* = \psi_h^c$. By claim 2, L's generate no profits, and by the lemma there are no overall profits, and thus H's must be on the H-risk 0-profit line. But by claim 1, H gets at least as much utility as offered by ψ_h^c. Since there is no other policy on the 0-profit line that satisfies this inequality, it must be that $\psi_h^* = \psi_h^c$.

Claim 4: $\psi_l^* = \overline{\psi}_l$. By claim 2, the policy in on the L-risk 0-profit line. See Fig 8.20, p411. If the policy is to the right of $\overline{\psi}_l$, H will defect to it. If to the left, a cream skimming argument generating positive profits is available. ■

- Note the theorem doesn't say a separating screening eq exists; it just says if it exists then it looks like...

- Cream skimming is a useful way to rule out eq in this model.

- It could rule them all out! For example, this occurs when H's are rare in the population (a pooling defection from the separating policies in the theorem provides positive profits and strict incentive for both L and H to accept). See Fig 8.21, p412.

0.1 A bit about mechanism design[4]

- "A mechanism is a set of rules that one player constructs and another freely accepts in order to convey information from the second player to the first."
- "A mechanism contains an information report by the second player and a mapping from each possible report to some action by the first."
- For example, in the **insurance screening game**:
  - the insurer offered two contracts and the consumer picked one.
  - This can equivalently be viewed as the consumer reporting "I am H" or "I am L" and the insurer committing to offer full or partial insurance as a consequence (the H policy and price to whoever says "I am H", and similarly for L.)
  - The contract offers a mechanism for the players to truthfully report their type.

0.2 Example: Rasmusen's production game VIII (p277–9)

- Players: a principal and an agent
- Order of play:
  - The P offers A a wage contract of the form $w(q, m)$ where q is output and m is a message to be sent by the agent.
  - The A accepts or rejects the P's offer.
  - Nature chooses the state s: with probability 0.5 it is good and 0.5 it is bad. A observes s but P doesn't; the distribution is commonly known.

[4] Part of this discussion comes from Rasmusen, Chapter 10.

- If A accepted, he exerts effort e unobserved by P, and sends message $m \in \{good, bad\}$ to P.
- Output is $q(e, s)$, where $q(e, good) = 3e$ and $q(e, bad) = e$, and the wage is paid.

- Payoffs:
  - If A rejects, $\pi_A = 0 = \pi_P$.
  - If A accepts, $\pi_A = w - e^2$ and $\pi_P = q - w$.
  - (notice m is cheap talk)

- First best: $e_g = 1.5$ resulting in $q_g = 4.5$, and $e_b = 0.5$ resulting in $q_b = 0.5$.

- But if P doesn't observe s he doesn't know which effort level is appropriate.

- Assumed (see payoffs) that A won't just report $m = s$ all the time.

- So P implements a mechanism to extract A's information.

- P's problem is

$$Max_{q_g,q_b,w_g,w_b} 0.5(q_g - w_g) + 0.5(q_b - w_b)$$

  where contract (q_g, w_g) is implemented if $m = good$ and (q_b, w_b) is implemented if $m = bad$, and producing the wrong output for a given contact results in being boiled in oil.

- The contracts must induce participation and self-selection.

- Since $q(e, good) = 3e$ and $q(e, bad) = e$, when $s = good$ and A must produce q he exerts effort $e = \frac{q}{3}$. When it is bad he exerts $e = q$.

- Self-selection constraints. In the good state

$$\pi_A(q_g, w_g|good) = w_g - \left(\frac{q_g}{3}\right)^2 \geq \pi_A(q_b, w_b|good) = w_b - \left(\frac{q_b}{3}\right)^2$$

and in the bad state

$$\pi_A(q_b, w_b|bad) = w_b - q_b^2 \geq \pi_A(q_g, w_g|bad) = w_g - q_g^2$$

- The participation constraint is

$$0.5\pi_A(q_g, w_g|good) + 0.5\pi_A(q_b, w_b|bad) = 0.5\left(w_g - \left(\frac{q_g}{3}\right)^2\right) + 0.5\left(w_b - q_b^2\right) \geq 0$$

- This PC will be binding, and the good state's IC constraint will bind (since when good the A is tempted to take the easier contract, so:

$$\begin{aligned} 0.5\left(w_g - \left(\frac{q_g}{3}\right)^2\right) + 0.5\left(w_b - q_b^2\right) &= 0 \\ w_g - \left(\frac{q_g}{3}\right)^2 &= w_b - \left(\frac{q_b}{3}\right)^2 \end{aligned}$$

- Solving these yields $w_b = \frac{5}{9}q_b^2$ and $w_g = \frac{1}{9}q_g^2 + \frac{4}{9}q_b^2$.

- Substituting these into the profit max problem gives

$$\max_{q_g, q_b} 0.5\left(q_g - \frac{1}{9}q_g^2 - \frac{4}{9}q_b^2\right) + 0.5\left(q_b - \frac{5}{9}q_b^2\right)$$

with no constraints.

- FOCs are

$$\begin{aligned} \frac{\partial \pi_P}{\partial q_g} &= 0.5\left(1 - \frac{2}{9}q_g\right) = 0 \Longrightarrow q_g = 4.5 \\ \frac{\partial \pi_P}{\partial q_b} &= 0.5\left(-\frac{8}{9}q_b\right) + 0.5\left(1 - \frac{10q_b}{9}\right) = 0 \Longrightarrow q_b = 0.5 \end{aligned}$$

- With these, substitute into $w_b = \frac{5}{9}q_b^2$ and $w_g = \frac{1}{9}q_g^2 + \frac{4}{9}q_b^2$ to find $w_b \approx 0.14$ and $w_g \approx 2.36$.

Direct and indirect mechanisms

- Consider CS's cheap talk game.
- Suppose there's an eq with 3 intervals and thus 3 actions (y_1, y_2, y_3) taken in eq.
- Go through formally what happens here.
- Now consider an alternative: at the beginning of the game the principal commits to a *delegation set*; i.e., a set of actions the sender (agent) can choose from. So the timing is:
  - The principal commits to a delegation set $D \subseteq A$ (A is the action space)
  - The agent privately observes the state, then picks an action from D that he suggest to the principal.
  - The principal must take the suggested action. Equivalently, the authority to take action is delegated to the agent (subject to that action being in D).
- Question: what's the difference between the CS model mentioned above and the game I just described in which $D = \{y_1, y_2, y_3\}$?
- Any eq of a game without commitment (like CS) must be ex-post incentive compatible.
- Example of other type of sender-receiver game: veto-based delegation.
- Or any other thing you can think up.
- Each must be ex-post IC.

- These are called indirect mechanisms. A message is sent, the receiver updates, then takes an action that is optimal given his posterior.
- A direct mechanism can always be found that is outcome equivalent: just choose a delegation set that contains only those actions that would have appeared in the indirect mechanism game.
  - In this case, commitment doesn't matter.

Theorem 6 *[Gibbons, p165] (The revelation principal). Any Bayesian Nash equilibrium of any Bayesian game can be represented by an incentive-compatible direct mechanism.*

- But think more generally of the problem of a mechanism design: how can I pick the elements of D so as to maximize the ex-ante payoff of the principal?
  - Note in general this may result in delegated actions that *are not* ex-post IC; that is, given the sender recommended an action the receiver has updated such that he no longer wants that action to occur. But if the receiver can reneg on his promise (to do whatever the sender selected from D), then the sender wouldn't have made that choice and divulged the information in the first place. So commitment is necessary.
  - In general, the principal can do better with this commitment power than without.
- Revelation principal: Who cares?
- It is typically much easier to look at the set of all direct mechnisms, than all indirect mechanisms (so many possibilities).

- For example, in auction settings it has been shown what the best direct mechanism for the seller is, and the expected revenue this raises. It has also been shown that some known auction format (an indirect mechanism) achieves this same level of expected revenue for the seller. There that does not exist a better auction format for the seller. Neat!

0.3 Price discrimination

Just point this out– it is a mechanism design problem. Here, a participation constraint too.

Lecture 8

0.4 The Groves mechanism (Rasmusen p293–6)

- Hidden knowledge is important in public economics, particularly government spending and taxation.
- In Mirrlees (1971) (optimal taxation), citizens differ in the income-producing ability and the government wishes to demand higher taxes from more able citizens.
  - the government can't observe ability directly.
- Or think about the optimal spending on public goods, which requires knowing the preferences of all citizens.
- Our principal here is altruistic– prefers to maximize the sum of agents' payoffs.

Players: the mayor and 5 households.

Order:

1. Nature chooses values v_i that household i places on having a streetlight installed, using distribution $f_i(v_i)$. Only i observes v_i.
2. The mayor announces a mechanism, M, which requires a household who reports m to pay $w(m)$ if the streetlight is installed and installs the streetlight if $g(m_1, ..., m_5) \geq 0$.
3. Household i reports value m_i simultaneously with all other households.
4. If $g(m_1, ..., m_5) \geq 0$, the streetlight is built and household i pays $w(m_i)$.

Payoffs: $\pi_{mayor} = \left(\sum_{i=1}^{5} v_i\right) - 100$ if the streetlight is built, 0 if not. Subject to $\sum_{i=1}^{5} w(m_i) \geq 100$ so it can be funded. Household i has payoff $\pi_i(m_1, ..., m_5) = v_i - w(m_i)$.

A fews ideas:

$$M_1 : w(m_i) = 20, \text{ build iff } \sum_{i=1}^{5} m_i \geq 100$$

that is, each resident pays \$20 if the light is built, and it is built iff the sume of the reports exceeds 100.

Problem:

- a pro-light household might inflate his valuation so it gets built
- an anti-light household might claim to have a negative valuation so it won't get built.
- Talk is cheap– it's dominant to over- or under-report.

Try this idea:

$$M_2 : w(m_i) = \max\{m_i, 0\}, \text{ build iff } \sum_{i=1}^{5} m_i \geq 100$$

So each resident pays his report iff it is built, and it is built iff the sum exceeds 100.

- There is no dominant strategy here.
- Household i would annount $m_i = 0$ if he thought the project would go through without his support, based on his estimates of the others' values
- But i would announce up to his true value if necessary.
- If the households knew each others' values perfectly, there would be a continuum of equilibria.

- E.g., suppose the values were known to be $(10, 30, 30, 30, 80)$.
- One equilibrium would be $(0, 25, 25, 25, 25)$.
- (anything that exactly sums to 100, and has no one paying more than his value, is an eq).
- However, since most such equilibria are asymmetric, it's hard to imagine they all figure out which equilibrium to play.

Now consider this mechanism, in which it is *dominant to tell the truth.*

$$M_3 : w(m_i) = 100 - \sum_{j \neq i} m_j, \text{ build iff } \sum_{i=1}^{5} m_i \geq 100$$

- Under this mechanism i's message does not affect his tax bill except by its effect on whether or not the light is installed.
- Consider household i
  - light is built iff $\sum_{i=1}^{5} m_i \geq 100 \Longleftrightarrow m_i + \sum_{j \neq i} m_j \geq 100$
  - if built, his payoff is $v_i - 100 + \sum_{j \neq i} m_j$
  - not built if $m_i + \sum_{j \neq i} m_j < 100$, giving a payoff of 0
  - He prefers it to be built iff

$$\begin{aligned} v_i - 100 + \sum_{j \neq i} m_j &> 0 \\ v_i &> 100 - \sum_{j \neq i} m_j \end{aligned}$$

and it is built iff

$$\begin{aligned} m_i + \sum_{j \neq i} m_j &\geq 100 \\ m_i &\geq 100 - \sum_{j \neq i} m_j \end{aligned}$$

so report your true value.

- One potential problem: this scheme need not be budget-balanced.
  - The tax is $100 - \sum_{j \neq i} m_j$ for each
  - The first part raises 500, which after paying for the light is reduced to 400
  - The second part minus 4 times everyone's message: $-4\left(\sum_{i=1}^{5} m_i\right)$
  - However, if the project goes through, we know $\sum_{i=1}^{5} m_i \geq 100$ and thus $4\left(\sum_{i=1}^{5} m_i\right) \geq 400$
- In fact, total taxes could be negative!
  - Suppose $v = 60$ for all players.
  - Then $m = 60$ for each.
  - Tax revenue per household is

$$100 - \sum_{j \neq i} m_j = 100 - 4(60) = -140$$

and thus total tax revenue is -140×5!

Principal-agent models

- The principal's payoff depends on the agent's action, but that can't be observed.
- The principal seeks to design an incentive scheme so the agent takes an appropriate action.

Insurance example

- Single insurer, single consumer.
- There are L levels of losses: \$1, ..., \$L.
- No accident = loss of \$0.

- Let $\pi_l(e) > 0$ be the probability of loss $l \in \{0, 1, ..., L\}$ where e is the amount of effort exerted to reduce the loss.
- For any e, $\sum_{l=0}^{L} \pi_l(e) = 1$.
- Suppose there are two effort levels, low and high: $e = 0$ and $e = 1$.
- We want higher effort to mean a lower likelihood of an expensive accident:
- **ASSUMPTION**: $\frac{\pi_l(0)}{\pi_l(1)}$ is strictly increasing in $l \in \{0, 1, ..., L\}$. *Monotone likelihood ratio property*
- That is, conditional on observing the accident loss l, the relative probability that L versus H effort was exerted increases with l.
- I.e., you are more confident that $e = 0$ when the observed accident is higher.
- Increasing, concave VNM utility function over wealth $u(w)$, where $w > L$, and $u() - d(e)$ where d is the cost of effort; $d(1) > d(0)$.
- The insurer observes l but not e.
  - The contract has depend on the former but not the latter.
  - A tuple $(p, B_0, B_1, ..., B_L)$, where p is the premium and B_l is the insurance payout when the loss is l.
  - FACT FOR LATER: it is wlog that $B_0 = 0$. If $B_0 > 0$ and the policy is $(p, B_0, B_1, ..., B_L)$, consider the alternative policy that offers the same utility: $(p - B_0, 0, B_1 - B_0, ..., B_L - B_0)$.
- What kind of policy will the insurer offer?

Symmetric information

- Benchmark case: suppose the insurer *can* observe e.
- Since the insurer can offer a policy that pays only if the desired level of e was exerted, it is as if the insurer selects e (subject to a participation constraint).
- The maximization problem is.... (see 8.7, p364), where $\overline{u}$ is the consumer's reservation utility (either calculate expected utility without insurance, or from some unmodeled competing insurance company's offer of insurance).
- The max problem is: choose an insurance policy and effort level to max profits subject to the consumer being willing to purchase.
- Solve 8.7 by assuming $e \in \{0, 1\}$ is fixed and form the Lagrangian considered as a function of $p, B_0, ..., B_L$ only. This gives:
- [lines 8.8, 8.9, and 8.10 from p 364]
- Line 8.8 is implied by the $L + 1$ equations in 8.9.
- The equalities in 8.9 imply $\lambda > 0$, and that $u'(w - p - l + B_l) = \frac{1}{\lambda}$.
  - Thus $B_l - l$ must be constant for all l.
- Line 8.10 binds with equality, so it becomes line 8.11:

$$u(w - p - l + B_l) = d(e) + \overline{u}, \text{ for all } l \geq 0$$

- Wlog, $B_0 = 0$. Consider line 8.11 for the case $l = 0$.
  - Then it reduces to a function of p alone.
  - Further, since $B_l - l$ is constant for all l (established above), and since $B_0 - 0 = 0$, we have $B_l = l$ for all l.

- Thus for either effort level, it is best to provide full insurance for every level of loss.
  - Of course: the consumer is risk-averse, the insurer is risk-neutral. Efficient risk sharing.
  - The price p equates the consumer's utility at the required effort level to his reservation utility. Makes sense.
- We know the best policy for each e. Now find the best e.
- Using 8.11 and the fact that $B_l = l$ for each l, we get

$$u(w - p(e)) = d(e) + \overline{u}.$$

  (call this line 8.12)

- Thus the insurer chooses $e \in \{0, 1\}$ to max $p(e) - \sum_{l=0}^{L} \pi_l(e)l$.
- Tradeoff seen from 8.12: requiring higher effort necessitates a lower premium but reduces the expected losses from an accident. Show that last part:

Exercise 11 *JR, Ex 8.11, p371. The MLRP implies expected losses are lower with higher effort.*

- So, just perform the calculation and find which e is best.

Conclusion 3 *Full insurance. Pareto efficient.*

Asymmetric information

- e is unobserved by the insurer.
- Pick a policy to max profits.

- a desired effort level in mind
- provide incentive for that effort to be exerted (it's not observed)

- This adds another constraint to the insurer's maximization problem. The policy and effort must
  - induce participation
  - induce the desired effort (incentive compatability)
- The max problem is: lines 8.13, 8.14, 8.15.
  - First 2 are the same as before.
  - Last is the IC constraint: the e the insurer has in mind when setting the policy is utility maximizing given that policy.
- Like last time, solve the problem for fixed e, then compare which of the 2 is best.

The optimal policy for $e = 0$

- Among all policies that induce low effort, which is best for the insurer (and would be accepted)?
- Simpler method than the Lagrangian.
- Absent the IC constraint, the optimal policy when $e = 0$ is to choose $p, B_0, ...B_L$ so that $u(w - p) = d(0) + \overline{u}$ and $B_l = l$ for all l.
- Adding the IC constraint can't increase profits, so if this old solution satisfies the IC constaint, it is the optimizer here as well.
- It does:

- Let $e = 0$ in the IC reduces to $d(0) \leq d(1)$, which holds strictly by assumption.
- Or in words: the consumer is fully insured in all states, and so doesn't gain at all from exerting effort, but does incur the cost from it. So $e = 0$ would be best for the consumer.

The optimal policy for $e = 1$

- Write out the Lagrangian. Line 8.17, p 367.
- FOCs, 8.18 – 8.21, p367.
- As before, 8.18 is implied by the $L + 1$ conditions in 8.19.
- As before, wlog $B_0 = 0$.
- Rewrite 8.19 as 8.22, p368.
- Claim: λ and $\beta \neq 0$.
- Suppose $\beta = 0$.
  - Then 8.22's LHS is constant in l, implying $w - p + B_l - l$ is constant in l.
  - But then 8.21 reduces to $d(0) - d(1) \geq 0$, which is false.
  - Contradiction.
- Claim $\lambda \neq 0$
  - MLRP implies there exists an l such that $\pi_l(0) \neq \pi_l(1)$.
  - Since $\sum_l \pi_l(0) = 1 = \sum_l \pi_l(1)$, there exists an l and l' such that $\pi_l(0) > \pi_l(1)$ and $\pi_{l'}(0) < \pi_{l'}(1)$.
  - Thus the term in brackets in 8.22 takes on positive and negative values.

- If $\lambda = 0$ since $\beta \neq 0$, the RHS takes positive and negative values, but that can't happen: LHS >0 always. So $\lambda \neq 0$, and in fact this shows $\lambda > 0$.

- Since λ and $\beta \neq 0$, both constraints (8.20 and 8.21) are binding: the consumer is held to his reservation utility, and he is indifferent between high and low effort.

- Let's now show in fact $\beta > 0$.

  - Suppose $\beta < 0$.
  - MLRP implies RHS of 8.22 is strictly decreasing (in l), thus $u'(w-p+B_l-l)$ is strictly increasing (in l).
  - But $u'' < 0$, so the argument of u' must be decreasing (in l); i.e., $B_l - l$ is decreasing (in l); i.e., $l - B_l$ is increasing (in l).
  - THUS the optimal high effort policy doesn't provide full insurance; and it imposes a larger deductible for larger losses.

The optimal policy and efficiency

- So overall, just compare whether $e = 0$ or $e = 1$ leads to highest profits.

- Suppose that under symmetric info, $e = 0$ was best. Then it also is under asymm:

  - Asymm with $e = 0$ yields the same profits as symm.
  - Symm with $e = 0 >$ symm $e = 1$ by hypoth.
  - Asymm with $e = 1 \leq$ symm with $e = 1$ since the former case has another constraint.

- Thus the insurer earns the same profits with symm or asymm, and we reach a Pareto efficient outcome.

- Suppose $e = 1$ is best under symm.
  - This can lead to lower profits for the insurer with asymm.
  - Recall profits and outcomes for $e = 0$ are the same in asym and symm.
  - But it's costly to get $e = 1$ in asymm.
  - So it's possible it's best for the insurer to pick $e = 0$.
  - But that is inefficient by hypothesis.
  - Consumer still gets his reservation utility, but the insurer's profits are lower.

Exercise 12 *JR, Ex 8.13, p372*