# Valuations of Items in Counter-Strike: Global Offensive

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# The Problem

- People are randomly distributed items in the game.
- They have private valuations for each item that are not known to the designers
- A market is created in order to ensure an efficient outcome.
- Takes the form of a double auction - converging to competitive equilibrium

# Matching

- One context to think of the problem as one of matching individuals in order to maximize the total surplus.
- We know from Micro2 that this is equivalent to thinking about a decentralized market.
- The Objective function is valuation of the buyers and the sellers

# Who Gets What

- Both buyers and sellers have the same distribution of valuations
- However, the masses of the buyers and sellers are not equal.
- Only some percentage are endowed with the item
- Market is efficient - highest valuations end up with the item.

# The Planner's Problem

$$\max_{\alpha_{i,j}} \sum_{i=1}^{I} \sum_{j=1}^{J} (V_i - V_j)\alpha_{i,j}$$

$$\text{subject to: } \forall j, 1 \leq j \leq J \quad \sum_{i=1}^{I} \alpha_{i,j} \leq 1$$

$$\forall i, 1 \leq i \leq I \quad \sum_{j=1}^{J} \alpha_{i,j} \leq 1$$

# Planner's Problem (cont)

- The solution to this is not unique.
- The difference in valuations is both sub and super-modular. This implies that both PAM and NAM are supported, and all permutations between the sellers and buyers selected are supported.
- This means we know who is matched but not with whom.

# The dual

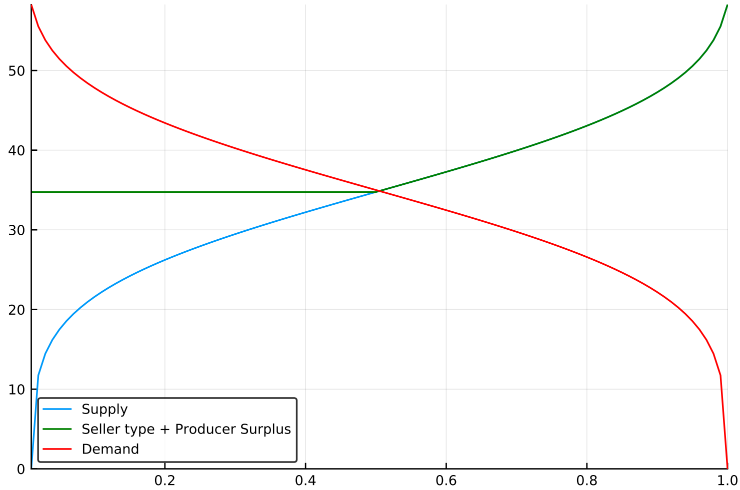
$$\min_{x,j} \sum_{i=1}^{I} x_i + \sum_{j=1}^{J} y_j$$

$$\text{subject to: } \forall i,j; \quad 1 \leq j \leq J, \quad 1 \leq i \leq I$$

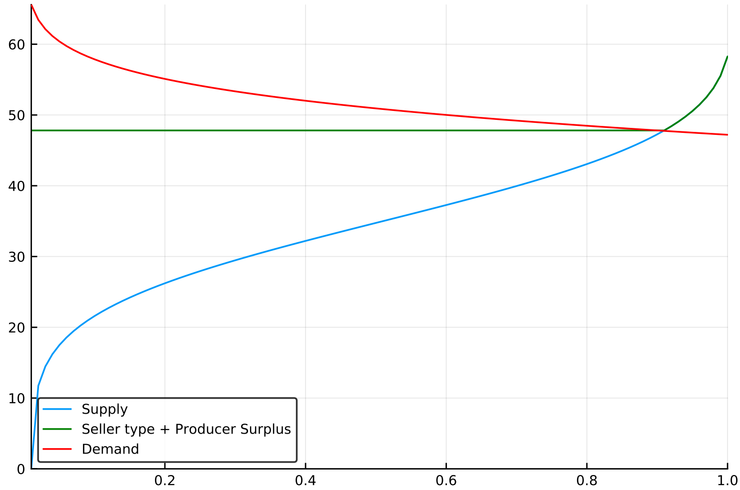
$$x_i + y_j \geq V_i - V_j$$

- This has a unique solution - for each buyer and seller it gives the shadow price: the surplus that each commands.
- Because the function is modular, the valuation plus the surplus for all sellers is equal - this is the price the market supports.

# What it looks like



# Unequal Buyers and Sellers



# Equilibrium

- Let the proportion of the population that received the item be denoted $\xi$.
- For normally distributed valuations, the price is defined by:

$$\Phi\left(\frac{p^* - \mu}{\sigma}\right) = \frac{1-\xi}{\xi}\left[1 - \Phi\left(\frac{p^* - \mu}{\sigma}\right)\right]$$
$$p^* = \mu + \sigma\Phi^{-1}(1-\xi)$$

# Known $\xi$

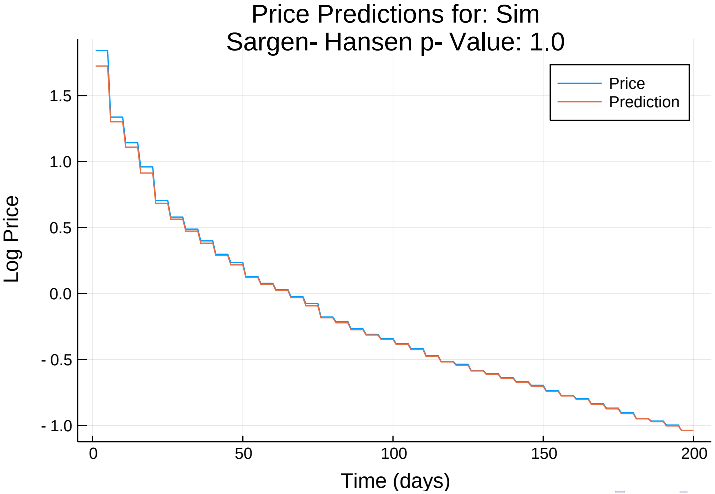
- If we knew $\xi$, this model could be estimated via linear regression
- Can handle even if there is measurement error in calculating $\xi$.
- However, even if we know the quantity of sales, and the number of people playing, no idea of people engaging in the market.
- Need to use the price to endogenize $\xi$.

# Dynamic Approach

- Let this process repeat over many time intervals.
- Assume no entry into the market.
- Since this market is efficient, the top portion of the buyers always purchases the item, and the price slowly falls
- This can only support a decreasing price.

# A Simulation

- $\mu = 0, \sigma = 1, \xi = .05, N = 1000, T = 40$



# Specification

$$\mathbb{E}[q_s] = N \prod_{t=0}^{T-1} (1-\xi_t) \xi_T \frac{\Phi\left(\frac{\log(p_T^*)-\mu}{\sigma}\right)}{\prod_{t=0}^{T-1}(1-\xi_t)}$$

$$\mathbb{E}[q_d] = N \prod_{t=0}^{T} (1-\xi_t) \left[1 - \frac{\Phi\left(\frac{\log(p_T^*)-\mu}{\sigma}\right)}{\prod_{t=0}^{T-1}(1-\xi_t)}\right]$$

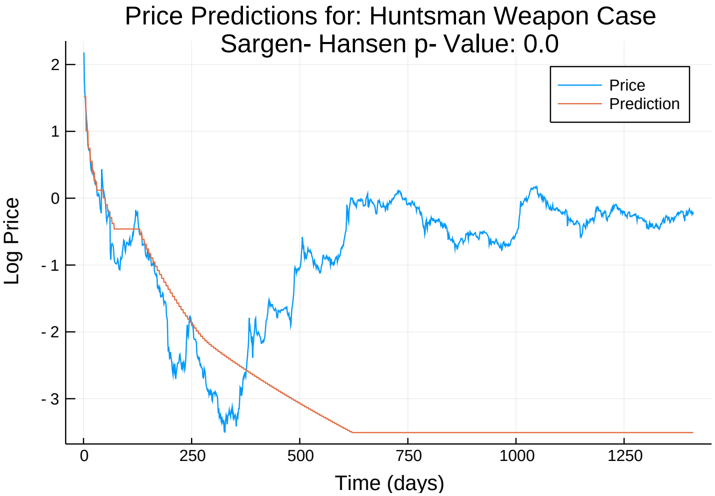
$$\log(p_T^*) = \mu + \sigma \Phi^{-1} \left[\prod_{t=0}^{T}(1-\xi_t)\right]$$

$$q_T^* = N \prod_{t=0}^{T} (1-\xi_t) \xi_T$$

# Problems with Data

- This model cannot support the prices increasing.
- One possibility is to add white noise, which increases the variance on all observations, and can explain some jumps in prices.
- This cannot explain trends in prices that are observed in some items.
- Worse yet, it predicts price to eventually fall to zero, which is not represented by some of the cases

# Failure in Prediction



# What can we predict?

- We are predicting the price to eventually drop to zero, but we do not have an equilibrium specification. So for data where the price is driven on a downward trend, we can estimate the data.
- We choose to group together data in periods of 5 days. Assume model is in equilibrium in each of those days. This generates moments for estimation

# Generalized Method of Moments

- Function g(Y$_t$ ,$\mu$,$\sigma$,$\xi$) which gives the moment condition for each time period

$$\mathbb{E}[g(Y_t, \mu, \sigma, \xi)] = 0$$

- Sample Analog: $\hat{m}(\mu, \sigma, \xi) = \frac{1}{M} \sum_{m=1}^{M} g(Y_m, \mu, \sigma, \xi)$

$$\hat{\theta} = \arg\min_{\theta} \hat{m}(\theta)' W \hat{m}(\theta)$$

# Generalized Method of Moments

- What is this W matrix? How do we get it?
- Using Iterated GMM Estimator

$$\hat{W}_i = \left[ \frac{1}{M} \sum_{m=1}^{M} g(Y_m, \hat{\theta_{i-1}}) g(Y_m, \hat{\theta_{i-1}})' \right]^{-1}$$

$$\hat{\theta}_i = \arg\min_{\theta} \hat{m}(\theta_i)' \hat{W}_i \hat{m}(\theta_i)$$

# Complication?

- Forming W this way involves inverting a matrix that may not be of full rank.
- Add some positive number times the identity matrix in order to obtain full rank as well as positive definiteness.
- One advantage of the Iterated Method is that the W matrix formed is invariant to the scale of the data, which is especially important for this data

# Monte Carlo

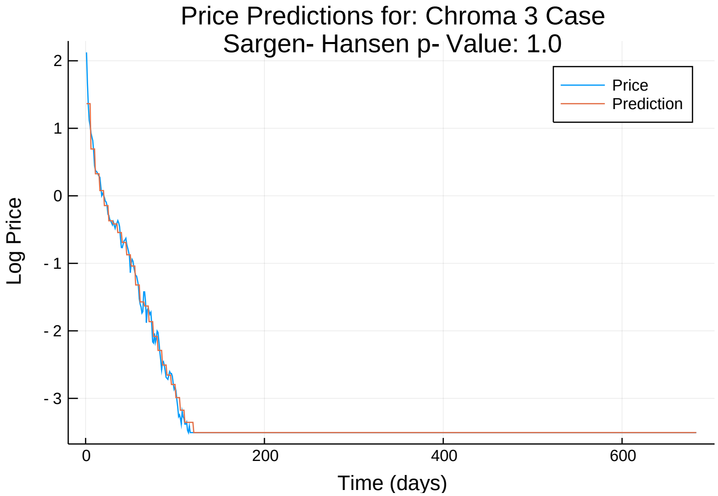
- However, we are still estimating a dynamic system, and that is notoriously difficult.
- This is especially the case in our model since early estimated values of $\xi$ have a large impact on the later values.
- These tests were not conducted near the magnitude of the data collected, as solving LPs of that size ( $10^{13}$ ) is not feasible
- These simulations may overstate the role of random noise.

# Monte Carlo

- I ran 1000 simulations of this model, all with N = 1000, $\mu = 0$, $\sigma = 1$, $\xi = 0.05$, T = 50.
- Tested: Sargan Hansen Test, LR Test for $\xi$ constant, LR Test for $\mu$ =0, $\sigma = 1$, $\xi = 0.05$
- Rejected with $\alpha = 0.05$

| | Sargan Hansen | $\xi$ constant | Simulation Primitives |
|---|---|---|---|
| Reject % | 3.7 | 44.0 | 100.0 |

# Some Predictions



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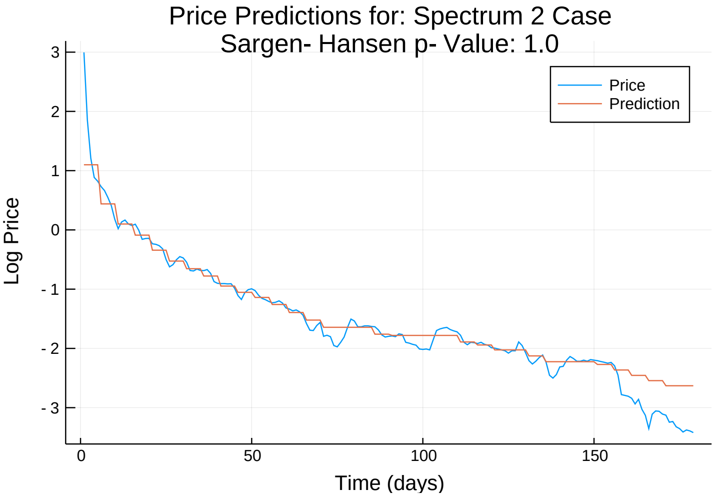

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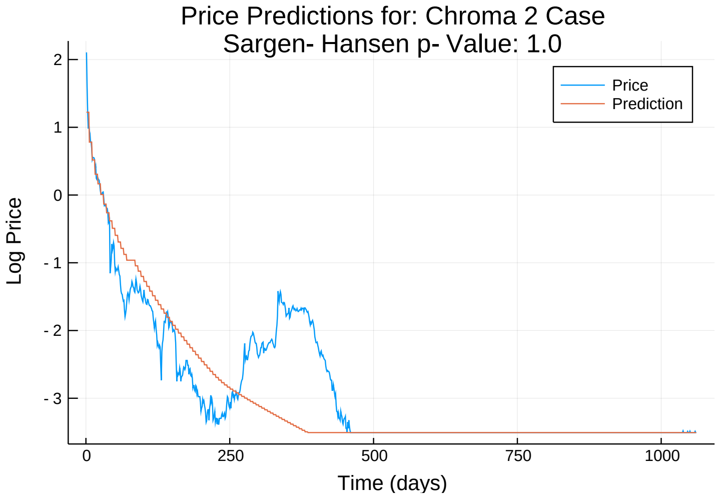

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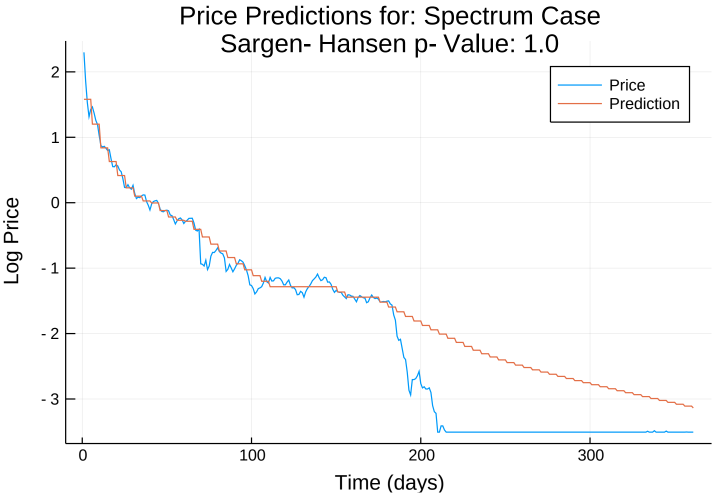


# Market Entry

- For the price to be able to increase, there must be new people entering the market.
- Let $\lambda_t$ denote the percent of new entrants into the market.
- Since each new entrant has the original valuations, we must consider all owners of the item, even past owners.
- This leads to both buyers and sellers having a mixing distribution of valuations

# Masses of Buyers and Sellers

$$M_B(T) = N(1-\xi_T)\prod_{t=0}^{T-1}(1-\xi_t+\lambda_t)$$

$$M_S(T) = N\sum_{i=0}^{T}\xi_i\prod_{t=0}^{i-1}(1-\xi_t+\lambda_t)$$

$$M_B(T) = NB_T(p_T)$$

$$M_S(T) = N\left(1 - B_T(p_T) + \sum_{t=1}^{T-1} R_t(\lambda, p)\right)$$

$$R_i(\lambda, p) = \lambda_i\left[B_{i-1}(p_{i-1}) + R_{i-1}(\lambda, p)\right]$$

$$R_0(\lambda, p) = \lambda_0$$

# Valuations of Buyers and Sellers

$$
\begin{aligned}
B_T(p) &= \frac{B_{T-1}(p_{T-1})}{B_{T-1}(p_{T-1}) + \lambda_1} \min\left\{1, \frac{B_{T-1}(p)}{B_{T-1}(p_{T-1})}\right\} \\
&\quad + \frac{\lambda_1}{B_{T-1}(p_{T-1}) + \lambda_1} B_0(p) \\
S_T(p) &= \frac{M_S(T-1)}{M_S(T)} \max\left\{0, \frac{B_{T-1}(p) - B_{T-1}(p_{T-1})}{1 - B_{T-1}(p_{T-1})}\right\} \\
&\quad + \frac{M_S(T) - M_S(T-1)}{M_S(T)} B_T(p)
\end{aligned}
$$

- $B_t(p)$ and $S_t(p)$ are strictly increasing functions of p, so the intersection between $q_d, q_s$ is uniquely defined.

# Problems

- There are some serious identification problems with this

model

- 2T Moments, but 2T+2 Primitives in the model.
- Assuming $\xi$ constant over the lifetime is one possible identification strategy.
- However, a bigger problem with the estimation presents itself.

# Behavior Over Time

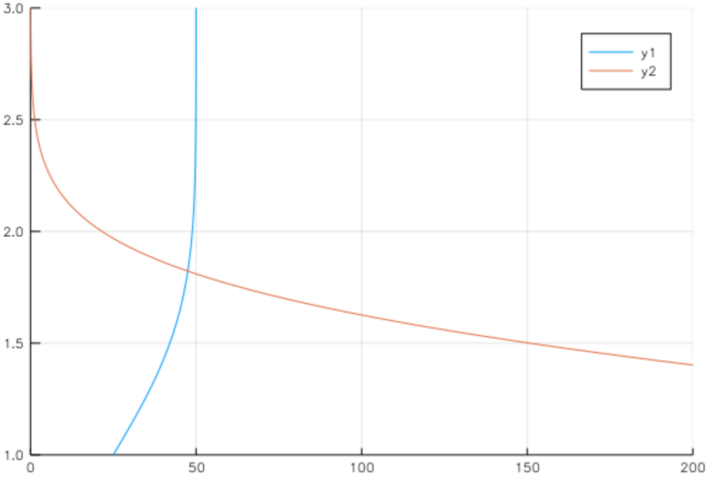



Figure: Time 1

# Behavior Over Time

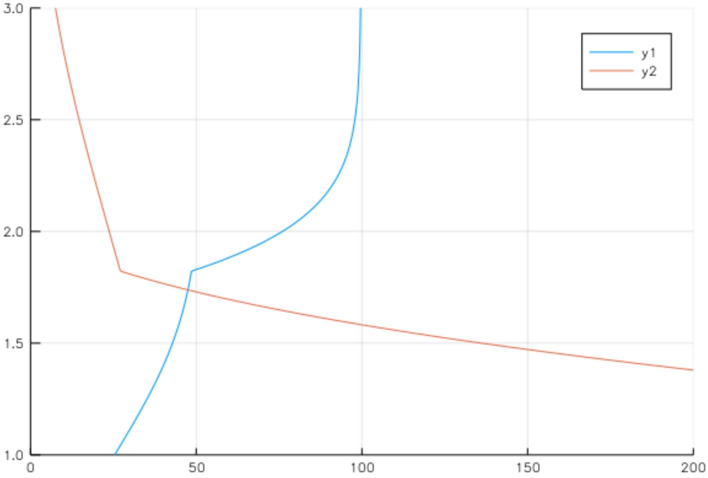



Figure: Time 2

# Behavior Over Time

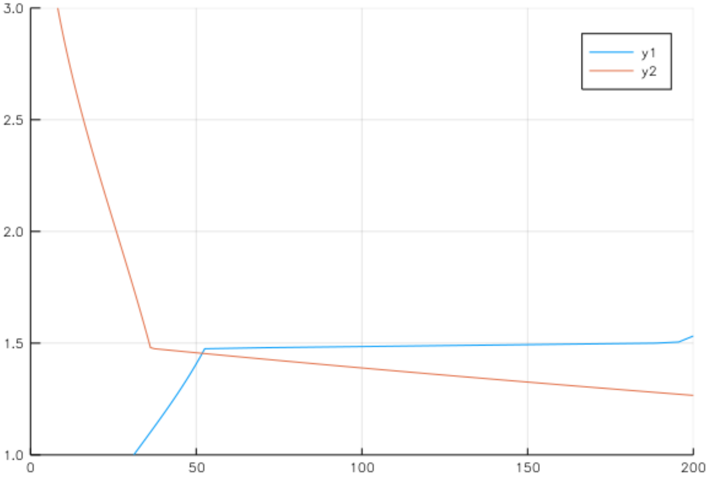



Figure: Time 10

# Behavior Over Time

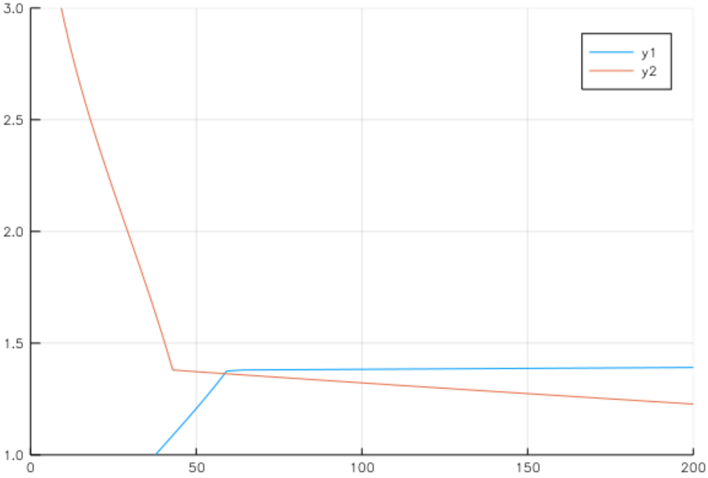



Figure: Time 15

# Behavior Over Time

- As we can see, the supply becomes extremely elastic above the previous equilibrium price
- The demand also becomes very elastic below the equilibrium price of last period.
- This means that the quantity sold will be extremely volatile, and the price can be for large increases/decreases.

# Non-Constant Valuations

- While the valuation of some items in the game might remain constant
- Items of interest such as the loot boxes have their values influenced by the prices as well as rarity of the items contained.
- Of interest is the magnitude of this over the lifetime of the item
- Use the fact that the distribution of the items reveals the quantiles of the distribution

# Quantile Regression

- In the model without any growth:

$$\prod_{t=0}^{T}(1-\xi_t) = F_V\left(p^*\right)$$

- The proportion of people given the item reveals quantiles of the true valuations.

# Quantile Regression

- If we want to remain agnostic about the percent of people given the item, the only choice we have is to examine how different quantiles of the pricing distribution are affected.
- This involves quantile regression, and abandoning many of the structural results hoped for.
- One approach is to estimate many different quantiles and plot them

# Multivariate Quantile Regression

- However, each loot box is drawn from a different distribution, so the quantile regression becomes a question of vector optimization.
- Following some fun in Convex Optimization, the scalarization where each box is given equal weight reduces to the simple weighted quantile regression problem.

# Formulation

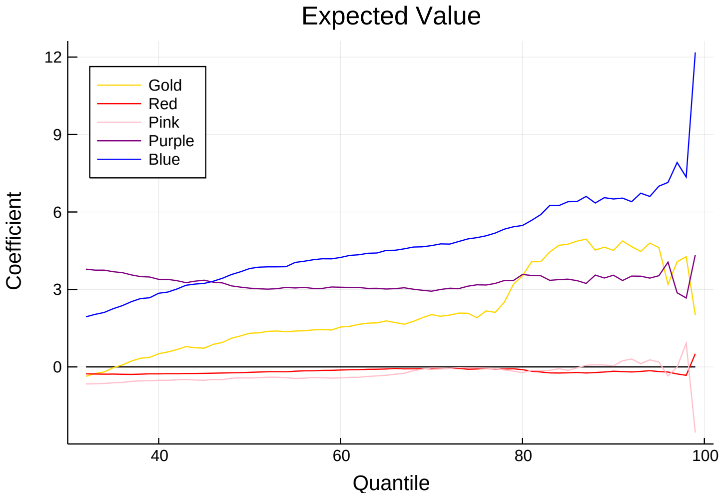
$$\min \sum_{j=1}^{J} \tau q_j^T u_j + (1-\tau) q_j^T v_j$$

$$X_j \beta + Z_j \delta_j + u_j - v_j = Y_j \quad \forall j$$

$$u, v \geq 0$$

$\delta_j$ can be equivalently treated as indicators contained in $X_j$, and the problem treated as quantile regression over the entire data set, weighted by the quantities sold.

# Loot box Averages



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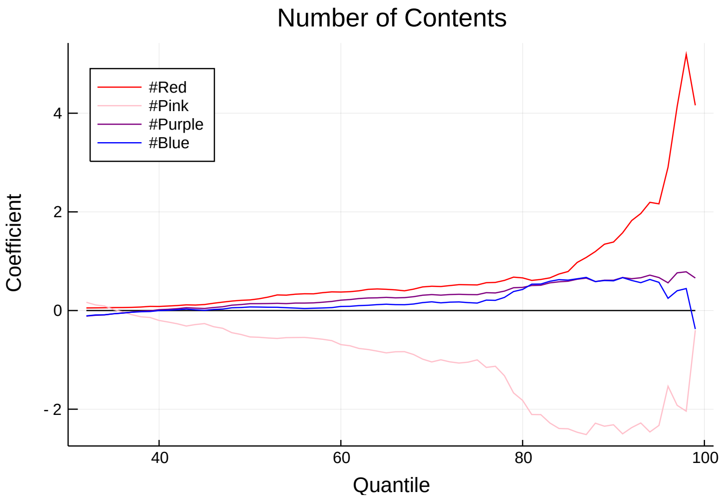

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