THE UNIVERSITY OF CHICAGO
HARRIS SCHOOL
OF PUBLIC POLICY

Risk Reporting and an Open-Source Python System

Thomas Coleman, Kunyi Du, Dana Jiaying Shen

July 2023

Risk reporting goes beyond Value-at-Risk. In Chapter 10 of my book *Quantitative Risk Management* I laid out various risk reports: Expected Volatility and Value-at-Risk, Contribution to Risk and Top Contributors, and Best Replicating Portfolios. These reports provide insight into the size and scope of risks that are embedded within a complex portfolio. The published reports were produced for a sample portfolio with risks in multiple asset classes (yields, equities, options, credit) and currencies (USD, GBP, EUR), and based on historical risk factor volatilities and correlations. Working with two students from the University of Chicago Harris School of Public Policy - Kunyi Du and Dana JiaYing Shen - I have released on GitHub (https://github.com/tscoleman/RiskManagement-Public) an open-source Python version of the code that produces the risk reports.

To illustrate and explain the techniques for analyzing portfolio risk the python risk system builds a small portfolio with diverse positions and risks, and focuses on a sample risk report that includes the marginal contribution, best hedges, etc. (The portfolio securities and holdings can be seen in the image just below.) The intention is to explain what the measures are and how to calculate them, to provide insight into how to use them and why they are valuable, and to provide code that users can work with for aid in understanding the underlying calculations.

Security	Amount (mn local)	Curr	Subportfolio
10yr UST	20	$	Government
5yr Bond Opt	−20	$	Government
10yr UST	30	$	Swaps
10yr Swap	−20	$	Swaps
10yr UKG	25	£	Government
FTE CDS	55	£	Credit
CAC Index Futures	7	€	Equity
FTE Equity	5	€	Equity
GBPCash	−10	£	Government

Portfolio Securities and Holdings

Risk and reporting in this system is all based on delta-normal or parametric estimation of the volatility and VaR. There are four broad components needed:

1. Definitions of security characteristics (maturity, currency, etc.) and mapping of securities to risk factors. These definitions are all in the module "SamplePort_define.py" where the dataframe "df_sec" is defined.
2. Translation of security holdings into risk factor delta sensitivities. This is accomplished by calculating numerical approximations to the delta (derivative or sensitivity) for each security using valuation functions defined in the module "RiskMgmtFunctions.py".
3. Estimation of risk factor variance-covariance matrix – the correlations and volatilities. The current release of the system does not independently estimate these, but simply takes a set of values (based on volatilities from 2011).
4. Calculation of the portfolio statistics (expected volatility, VaR, contribution to risk, etc.) based on the risk factor sensitivities and risk factor variance-covariance matrix.

Summary Risk Report

Table 10.18 from /Quantitative Risk Management/ shows a sample portfolio risk report for this portfolio or trading operation. This is based on delta-normal or parametric estimation of the volatility and VaR. The report is the top-level report for the portfolio and summarizes the over-all exposure and major sources of risk. A good risk-reporting programme, however, is a little like an onion or a set of Russian dolls– each layer when peeled off exhibits the next layer and shows more detail. This is the top layer; in a following section I discuss more detailed reports, which parallel Table 10.18 but zero-in on a specific sub-portfolio.

TABLE 10.18 Sample Portfolio Risk Report—Summary Report

Panel A—Expected Volatility by Asset Class

	Exp Vol ($)	Contribution	Correlation with portfolio
Overall	616,900	100.0	
FI—rates	345,800	39.2	0.700
FI—swap spreads	38,760	–0.4	–0.071
Credit	65,190	2.8	0.265
Equity	296,400	35.8	0.746
FX	236,100	21.8	0.571
Volatility	8,678	0.7	0.510

Panel B—Expected Volatility by Subportfolio

	Exp Vol ($)	Contribution	Correlation with portfolio
Overall	616,900	100	
Credit	65,540	2.5	0.237
Equity	312,100	39.0	0.771
Government	376,000	51.6	0.847
Swaps	75,350	6.9	0.562

Panel C—Volatility and 1-out-of-255 VaR

Volatility	616,900
VaR Normal	–1,640,000
VaR Student t (6df)	–1,972,000
VaR Normal Mix (α = 1% β = 5)	–1,680,000
VaR 4-sigma rule-of-thumb	–2,468,000

Top-Level Portfolio Risk: Functions "reportFunction1", "volbySubPortfolio", and "varsumm"

The first panel of Table 10.18 is produced by the function "reportFunction1" (first element of the list); the second panel by "volbySubPortfolio"; the third by "varsumm".

One note before turning to the analysis of the portfolio: The measures and statistics in any report such as this are based on estimates and past history. They are good and reasonable estimates, but anybody who has spent time in markets knows that uncertainty abounds and one should always treat such reports, measures, and statistics carefully.

They provide a view into what happened in the past and what might happen in the future, but the markets always provide new and unexpected ways to make and lose money.

Volatility

The first thing to note is the overall volatility: The daily or expected volatility is around $616,900. By this we mean that the standard deviation of the daily P&L distribution is roughly $616,900. When considering the daily volatility we are examining every-day trading activity and not tail events, and so we can have some confidence that assuming normality is probably reasonable. Using this we can infer that the daily losses or profits should be more than +/- $616,900 about one day out of three, based on the fact that the probability of a normally-distributed variable being below –1*sigma* or above +1*sigma* is roughly 30%.

The observation on likely P&L immediately provides a scale for the portfolio. For example, if this were a real-money portfolio with capital of $10 million we would expect gains or losses roughly 6.2% or more of capital every three days – a hugely volatile and risky undertaking. On the other hand if the capital were $500 million we would expect a mere 0.1% or more every three days or roughly 2% per year (multiplying by sqrt(255) to annualize) – a ridiculously low-risk venture with probably correspondingly low returns.

The daily volatility gives a scale for the portfolio at a point in time, but even more importantly provides a reasonably consistent comparison across time. Were the daily volatility to rise to $1.2 million next week we could be pretty confident that the risk of the portfolio, at least the risk under standard day-by-day trading conditions, had roughly doubled.

The volatility also provides a reasonably consistent comparison across asset classes and trading desks. The report shows that the daily volatility for fixed income products (bonds and swaps) is about $346,000 and equity is about $296,000. These statistics are the daily volatility of these products considered in isolation: the P&L distribution of fixed income products alone has a volatility of about $346,000. The similar scale of risk in these two products is valuable information, because there is no way to know this directly from the raw nominal positions: the notional in fixed income ($20 million in US Treasuries, ₤25 million in UK Gilts, $20 million in swap spreads) is many times that in equities (€7 million in CAC futures, €5 million in France Telecom stock).

Volatility by asset class naturally does not sum to the overall volatility: the sum by asset class of $990,000 versus the overall of $616,900 shows the effect of diversification.

VaR

The next item to note is the daily VaR. The VaR is calculated at a 0.4%-level. This means the probability of a worse loss should be 0.4% or 1-out-of-250. The probability level for VaR is always somewhat arbitrary. In this case 0.4% was chosen because it corresponds to roughly one trading day per year (1-out-of-255). Such a value should not be considered an unusual event; in Litterman's words (1996, p. 74): “think of this not as a ‘worst case,’ but rather as a regularly occurring event with which [one] should be comfortable.”

As with the volatility, the VaR provides a scale, in this case the minimum loss one should expect from the worst day in a year. It is important to remember that this is the minimum daily loss one should expect from the worst trading day in the year. Purely due to random fluctuations the actual loss may of course be worse (or possible better) and there could be more than one day in a year with losses this bad or worse.

Five values for the VaR are shown. The first is derived from the normality assumption and is just 2.652x the daily volatility – the probability that a normal variable will be 2.652sigma -times below the mean is 0.4%. The second is based on an assumption that the overall P&L distribution is student-t with six degrees of freedom. This is allows for fat tails – the student-t has the same volatility but fatter tails than the normal. The third is based on an assumption that each asset’s P&L distribution is a mixture of normals (99% probability volatility=1x*sigma*, 1% probability volatility=5x*sigma*), and again allows for fatter tails relative to normal. The fourth is based on Litterman’s rule of thumb that a 4-*sigma* event occurs roughly once per year, so that the VaR is just four times the volatility.These four alternate values for VaR are useful and adjust for the possibility that the distribution of market risk factors may have fat tails.

These VaR values should be used with care, more care indeed than the volatility above. In particular one might want to know whether assets such as those in this portfolio have exhibited fat tails in the past, and whether and to what extent assets in the portfolio have generated skewed or fat-tailed distributions. The estimates here are based on assumptions of normality for risk factors and linearity for asset sensitivities (the estimates are delta-normal or parametric). The portfolio contains a put option which is non-linear and will generate a skewed P&L distribution. The delicate nature of estimating and using VaR estimates really argues for a separate report and more detailed analysis.

In the end, I think the common-sense approach said to be used at Goldman (Litterman 1996, p.54) has much to recommend it: "Given the non-normality of daily returns that we find in the financial markets, we use as a rule of thumb the assumption that four-standard deviation events in financial markets happen approximately once per year." Under normality once per year events are only 2.65-standard deviations so a 4-*sigma* rule of thumb is substantially higher, as seen from the report.

Marginal Contribution to Volatility and Correlation

The marginal contribution to volatility is one of the most useful tools for decomposing and understanding volatility and risk. Table10.19 shows the MCP – proportional (or percentage) marginal contribution – so terms add to 100%. The marginal contribution by asset class shows that fixed income and equities are the biggest contributors, each contributing roughly one third of the risk. Marginal contribution shows, however, that in a portfolio context fixed income and equities contribute roughly the same (39% versus 36%). Because portfolio effects are paramount but often difficult to intuit, the marginal contribution is a better guide to understanding portfolio risk than is the stand-alone volatility. In this simple portfolio fixed income and equities have roughly the same stand-alone volatility and roughly the same contribution but for more complex portfolios this will not always be the case.

The tables show a break-down of marginal contribution by asset class and sub-portfolio. Depending on the institutional structure different classifications and break-downs may be more useful. The table by asset class shows the risk for fixed income instruments independent of where they are held. The swaps desk holds some outright rate risk, as we shall see, so that the volatility and contribution for swap spread and for the swaps desk itself are different. Examining the contribution by sub-portfolio shows that the government desk contributes most to the overall portfolio volatility. Much of the FX risk is held by the government desk (in the form of a partially hedged UK bond) and this leads to the large contribution by the government desk.

Swap spreads actually show a small but negative contribution to the overall volatility. The negative contribution does not mean that there is no "risk" in to swap spreads – on a particular day swaps spreads may move in the same direction as the rest of the portfolio thus leading to larger gains or losses, but it does give a reasonable expectation that across time the exposure to swap spreads will not add very much to the overall portfolio volatility.

The correlation of the swap rates with the full portfolio helps elucidate why swaps has a negative contribution. The correlation is slightly negative, and so the swaps position hedges (slightly) the overall portfolio, and for small increases the swaps hedges the overall portfolio. Turning back to the contribution and correlation by asset class, we see that equities are the most highly correlated with the portfolio, which explains why equities contribute so much to the volatility even though the stand-alone volatility is less than for fixed income.

Depending on the size and complexity of the portfolio, examining contribution to risk by individual assets may be useful. For a large and diverse portfolio there will generally be many assets and contribution by individual assets should be left to a more detailed next reporting level, below the top-level summary. For a smaller portfolio examination of all assets is valuable.

For most any portfolio, however, the top contributors provide useful insight into the portfolio. For this sample portfolio the top three contributors give a succinct summary of the major risks faced by the portfolio: equity index (CAC) and US and UK yields. The top negative contributor shows those assets that reduce risk or hedge the portfolio. For this sample portfolio there is only one asset – 5 year US yields –that has a negative contribution.

Best Single Hedges and Replicating Portfolios

The marginal contributions show the contribution to risk for the existing portfolio and provides a guide to how the volatility will likely change for small changes in holdings. But the marginal contributions are not the best guide to the likely effect of large changes in asset holdings, or what the best hedging assets might be. For this the best hedges and replicating portfolios are useful.

For any particular asset the best hedge position is that position which minimizes the expected volatility. This involves a finite, possibly large, change in position. The top best hedge will often differ from the top marginal contributor; for this sample portfolio shown in Table 10.19 the Equity Index (CAC) is the largest marginal contributor but the second-top best hedge.

TABLE 10.19 Sample Portfolio Risk Report—Top Contributors and Replicating Portfolios Report

Panel A—Top Contributors to Risk (volatility)

	Exp Vol (1-sig P&L)	Contribution	Curr Position (M eqv)	Trade to Best Hedge (eqv)	% Reduction in Volatility to Best Hedge	Zero Position
CACEqIndex	346,200	37.1	10.5	–12.4	25.0	24.4
GBPYld10yr	187,600	20.8	25.0	–56.2	27.1	17.8
USDYld10yr	202,900	20.6	31.0	–59.0	21.9	16.5

Top 1 negative

	Exp Vol (1-sig P&L)	Contribution	Curr Position (M eqv)	Trade to Best Hedge (eqv)	% Reduction in Volatility to Best Hedge	Zero Pos'n
USDYld5yr	21,430	–2.1	–6.3	–107.1	19.3	–2.1

Top 3 Best Single Hedges

	Exp Vol (1-sig P&L)	Contribution	Curr Position (M eqv)	Trade to Best Hedge (eqv)	% Reduction in Volatility to Best Hedge	Zero Position
GBPYld10yr	187,600	20.8	25.0	–56.2	27.1	17.8
CACEqIndex	346,200	37.1	10.5	–12.4	25.0	24.4
GBPYld5yr	548	–0.1	0.0	0.0	22.6	–0.1

Panel B—Best Replicating Portfolios

	One Asset		Three Assets		Five Assets	
	% Var	% Vol	% Var	% Vol	% Var	% Vol
%Var/%Vol Explained	46.8	27.1	86.7	63.6	98.4	87.5
	Asset	Eqv Pos'n	Asset	Eqv Pos'n	Asset	Eqv Pos'n
Asset/Eqv Pos'n	GBPYld10yr	56.2	GBPYld10yr	43.2	GBPYld10yr	26.1
			CACEqIndex	8.9	CACEqIndex	11.9
			GBPFX	18.6	GBPFX	19.4
					FTEEqSpecific	6.1
					USDYld10yr	24.1

Top Contributors and Best Hedges: Function "reportFunction1"

The four panels in Table 10.19 are from the elements 1, 2, 3, 4 (2nd, 3rd, 4th, 5th) from the output of the function "reportFunction1".

The top contributors and the top single hedges measure different characteristics of the portfolio. The top contributor to risk is the top contributor given the current positions. It tells us something about the composition of the current portfolio. The best single hedge, in contrast, is that asset that would give the largest reduction in volatility if we bought or sold some large amount. It tells us what would happen for alternate positions. We can also treat the best hedge as a mirror or replicating portfolio.

For the sample portfolio in Table 10.19 the CAC Equity Index is the top contributor, but GBP 10-year yields is the top best hedge. The GBP 10-year yields is the best hedge because it is highly correlated with USD 10-year yields and together these contribute 27% of the risk. A hedge using GBP 10-year will hedge both the existing GBP10-year and the USD 10-year position.

The top best hedge can be thought of as a replicating portfolio, in the sense that it is the single asset that best replicates the portfolio. For the GBP 10-year yield the trade from the current holding to the best hedge is a sale of 56 million pounds-worth, which means that a buy of £56.2 million would be the best single-asset replicating portfolio. Such a replicating portfolio would explain 27.1% of the volatility.

Replicating portfolios can provide a useful proxy or summary of the actual portfolio but the single-asset portfolio is often too simple. The three-asset and five-asset portfolios provide a much richer summary, and explain far more of the portfolio volatility. The five-asset portfolio explains 87.5% of the volatility and provides a valuable summary of the portfolio: it largely behaves like:

1. Long GBP 10-year yields (long 10-year bond, £26 million)
2. Long CAC Equity index (€11.9 million)
3. Long GBP FX (£19.4 million-worth of FX exposure due to holding foreign currency bonds and equities)
4. Long company-specific equity exposure (€6.1 million)
5. Long US 10 year yields ($24.1 million equivalent)

The reports in Tables 10.18 and 10.19 show the top-level summary for the full portfolio. According to those reports, the government portfolio contributes almost half the risk to the overall portfolio. The subportfolios can be examined in more detail, simply by applying the same reporting to the subportfolios on their own.

Python Tables and Code Snippets

Following is a list of the tables produced by the python code, with code snippets (from "Tables_Ch10.py")

First, create list of various tables and put into variable "formatted_xx":
formatted_xx = fns.reportFunction1(seclist, holdings, settledate,nav=1)

Table10.18 Summary Report - Panel A - Expected volatility – 1-day 1-sigma move:
print(tabulate(formatted_xx[0], headers='firstrow', tablefmt='grid'))

	Exp Vol ($)	Contribution	Correlation
Overall	616,900.0	100	with portfolio
FI - rates	345,801.0	39.2	0.7
FI - swap spreads	38,755.0	-0.4	-0.071
Credit	65,188.0	2.8	0.265
Equity	296,408.0	35.8	0.746
Commodity	0.0	0	
FX	236,070.0	21.8	0.571
Volatility	8,677.0	0.7	0.51

Table 10.18 Panel A - Expected Volatility by Asset Class

Table10.18 Summary Report - Panel B - Expected volatility by Subportfolio :

yy = fns.volbySubPortfolio(seclist, holdings, settledate, subportfolio)

print(tabulate(yy, headers='firstrow', tablefmt='grid'))

	Exp Vol ($)	Contribution	Correlation
Overall	616901	100	with portfolio
Government	376015	51.6	0.847
Credit	65536	2.5	0.237
Swaps	75351	6.9	0.562
Equity	312096	39	0.771

Table 10.18 Panel B - Expected Volatility by Subportfolio

Table 10.18 Panel C - Volatility and 1-out-of-255 VaR

table = fns.varsumm(seclist, holdings, settledate)

print(tabulate(table, headers='firstrow', tablefmt='grid'))

VOLATILITY and 1-ot-of 255 VaR	None
Volatility	616,900.9643018151
VaR Normal	-1,640,184.8158174886
VaR Student-t (6df)	-1,972,193.974127802
VaR Normal Mix (alpha=1% beta=5)	-1,531,846.623968077
VaR 4-sigma rule-of-thumb	-2,467,603.8572072606

Table 10.18 Panel C - Volatility and 1-out-of-255 VaR

Table10.18 Part2: Top Contributors and Best Hedges Report Panel A

print(tabulate(formatted_xx[1], headers='firstrow', tablefmt='grid'))

	Exp Vol		Curr Pos'n	Trade to best	% Reduction in Vol to	...
	(1-sig P&L)	Contribution	(mn eqv)	Hedge (eqv)	Best Hedge	Zero Pos'n
CACEqIndex	346,231.9748863345	37.1	10.5	-12.4	25.0	24.4
GBPYld10yr	187,597.2670679061	20.8	25.0	-56.2	27.1	17.8
USDYld10yr	202,936.7814220231	20.6	31.0	-59.0	21.9	16.5

Table 10.19 Panel A Top Contributors

print(tabulate(formatted_xx[2], headers='firstrow', tablefmt='grid'))

	Exp Vol		Curr Pos'n	Trade to best	% Reduction in Vol to	...
	(1-sig P&L)	Contribution	(mn eqv)	Hedge (eqv)	Best Hedge	Zero Pos'n
USDYld5yr	21,428.099686491325	-2.1	-6.3	-107.1	19.3	-2.1

Table 10.19 Panel A Top Negative Contributor

print(tabulate(formatted_xx[3], headers='firstrow', tablefmt='grid'))

	Exp Vol		Curr Pos'n	Trade to best	% Reduction in Vol to	...
	(1-sig P&L)	Contribution	(mn eqv)	Hedge (eqv)	Best Hedge	Zero Pos'n
GBPYld10yr	187,597.2670679061	20.8	25.0	-56.2	27.1	17.8
CACEqIndex	346,231.9748863345	37.1	10.5	-12.4	25.0	24.4
GBPYld5yr	547.5336397684166	-0.1	-0.1	-74.7	22.6	-0.1

Table 10.19 Panel A Best Hedges

print(tabulate(formatted_xx[34, headers='firstrow', tablefmt='grid'))

	One Asset	One Asset	Three Assets	Three Assets	Five Assets	Five Assets
	% Var	% Vol	% Var	% Vol	% Var	% Vol
%Var / %Vol Explained	46.8	27.1	86.7	63.6	98.4	87.5
	Asset	Eqv Pos'n	Asset	Eqv Pos'n	Asset	Eqv Pos'n
Asset / Eqv Pos'n	GBPYld10yr	56.2	GBPYld10yr	43.2	GBPYld10yr	26.1
			CACEqIndex	8.9	CACEqIndex	11.9
			GBPFX	18.6	GBPFX	19.4
					FTEEqSpecific	6.1
					USDYld10yr	24.1

Table 10.19 Panel B Best Replicating Portfolios