1)

a) $\mathcal{L} = a(\partial_\mu \varphi)^*(\partial^\mu \varphi) - am^2|\varphi|^2$

the normalization $a=1$ gives a propagator

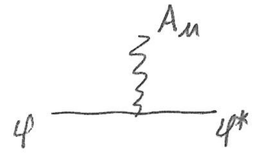
$$\text{———} = \frac{i}{p^2-m^2} \quad \text{w/ residue } i$$

b) $\mathcal{L} = (D_\mu\varphi)^*(D_\mu\varphi) - m^2|\varphi|^2$ w/ $\varphi \to e^{-ie\alpha}\varphi$

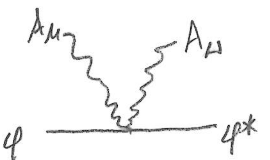
$$D_\mu\varphi = (\partial_\mu + ieA_\mu)\varphi$$

$$|D_\mu\varphi|^2 = (\partial_\mu - ieA_\mu)\varphi^*(\partial_\mu + ieA_\mu)\varphi$$

$$= |\partial_\mu\varphi|^2 - ieA_\mu\varphi^*\partial_\mu\varphi + ie(\partial_\mu\varphi^*)\varphi + e^2|\varphi|^2A^2$$



$\sim e p^\mu$



$\sim e^2 \eta_{\mu\nu}$

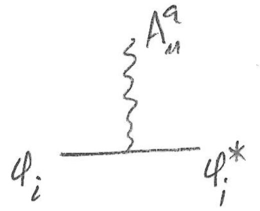
c) $\mathcal{L} = (D_\mu\varphi)_i^\dagger(D_\mu\varphi)_i - m^2\varphi_i^\dagger\varphi_i$ w/ $\varphi_i \to e^{-igT^a\alpha^a}\varphi_i$

$$(D_\mu\varphi)_i = (\partial_\mu\delta_{ij} + igA_\mu^a T_{ij}^a)\varphi_j$$

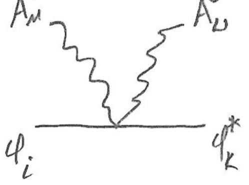
$$(D_\mu\varphi)_i^\dagger = (\partial_\mu\delta_{ij} - igA_\mu^a T_{ij}^a)\varphi_j^\dagger$$

Note: for $SU(N)$ $(T^a)^\dagger = T^a$

$$|D_\mu\varphi|^2 = |(\partial_\mu\varphi)_i|^2 - igA_\mu^a\varphi_i^* T_{ij}^a\partial_\mu\varphi_j + igA_\mu^a\partial_\mu\varphi_i^* T_{ij}^a\varphi_j + g^2A_\mu^a A_\mu^b\varphi_i^* T_{ij}^a T_{jk}^b\varphi_k$$



$\sim g p^\mu T_{ij}^a$



$\sim g^2 \eta_{\mu\nu} T_{ij}^a T_{jk}^b$

2)

$$0 = \delta\Gamma = \frac{\delta\Gamma}{\delta A^a_\mu}\delta A^a_\mu + \frac{\delta\Gamma}{\delta\psi_i}\delta\psi_i + \frac{\delta\Gamma}{\delta\bar\psi_i}\delta\bar\psi_i$$

$$= \frac{\delta\Gamma}{\delta A^a_\mu}\left(-\partial_\mu\alpha^a + g f^{abc}\alpha^b A^c_\mu\right) + \frac{\delta\Gamma}{\delta\psi_i}\left(-ig T^a_{ij}\alpha^a\psi_j\right) + \frac{\delta\Gamma}{\delta\bar\psi_i}\left(ig T^a_{ij}\alpha^a\bar\psi_j\right)$$

$$= \left[\partial_\mu\frac{\delta\Gamma}{\delta A^a_\mu}\delta^{ab} + g f^{abc} A^c_\mu \frac{\delta\Gamma}{\delta A^a_\mu} - ig\frac{\delta\Gamma}{\delta\psi_i}T^b_{ij}\psi_j + ig\frac{\delta\Gamma}{\delta\bar\psi_i}T^b_{ij}\bar\psi_j\right]\alpha^b$$

So we conclude:

$$\partial_\mu \frac{\delta\Gamma}{\delta A^a_\mu} - g f^{abc}\frac{\delta\Gamma}{\delta A^b_\mu}A^c_\mu - ig\frac{\delta\Gamma}{\delta\psi_i}T^b_{ij}\psi_j + ig \frac{\delta\Gamma}{\delta\bar\psi_i}T^b_{ij}\bar\psi_j = 0$$

Taking a variation w/r to A^d_ν and setting all fields →0:

$$\partial_\mu\frac{\delta^2\Gamma}{\delta A^d_\nu\,\delta A^a_\mu} - g f^{abc}\cancel{\frac{\delta\Gamma}{\delta A^b_\mu}}\,\delta^{cd} = 0$$

↑ The gauge field tadpole|1-point function =0 in a Lorentz invar. thry

So we conclude:

$$\partial_\mu \frac{\delta^2\Gamma}{\delta A^a_\nu\,\delta A^b_\mu}=0 \quad\to\quad p_\mu \frac{\delta^2\Gamma}{\delta A^a_\nu\,\delta A^b_\mu}=0$$

$\Rightarrow$  $\propto \Pi_T^{\mu\nu}$