Problem Set 9

We have seen that radiative corrections to the process $e^+e^- \to Z \to \bar{\psi}\psi$ are necessary in order to find agreement between the SM and experimental data. In [1, 2], the authors use a convenient parameterization of various Z-pole processes up to and including two-loop diagrams. These include parameterizations of $\Gamma_{Z\to \text{leptons}}$, $\sin\theta_{\text{eff}}^{\text{leptons}}$, and m_W:

$$\begin{aligned}\Gamma_{Z\to ee} &= X_0 + a_1 L_H + a_2 L_H^2 + a_3 L_H^4 + a_4\Delta_H + a_5\Delta_t + a_6\Delta_t^2 + a_7\Delta_t L_H + a_8\Delta_t L_H^2 + a_9\Delta_{\alpha_s} \\ &\quad + a_{10}\Delta_{\alpha_s}^2 + a_{11}\Delta_{\alpha_s}\Delta_H + a_{12}\Delta_{\alpha_s}\Delta_t + a_{13}\Delta_\alpha + a_{14}\Delta_\alpha\Delta_H + a_{15}\Delta_\alpha\Delta_t + a_{16}\Delta_Z\end{aligned} \tag{1}$$

Here we have:

$$\begin{gathered} L_H = \log\tfrac{m_H}{125.7\text{ GeV}}, \qquad \Delta_H = \tfrac{m_H}{125.7\text{ GeV}} - 1, \qquad \Delta_t = \left(\tfrac{m_t}{173.2}\right)^2 - 1, \\ \Delta_{\alpha_s} = \tfrac{\alpha_s}{0.1184} - 1, \qquad \Delta_\alpha = \tfrac{\Delta\alpha}{0.059} - 1, \qquad \Delta_Z = \tfrac{m_Z}{91.1876\text{ GeV}} - 1 \\ X_0 = 83.983, \qquad a_1 = -0.1202, \qquad a_2 = -0.06919, \qquad a_3 = 0.00383, \\ a_4 = 0.0597, \qquad a_5 = 0.8037, \qquad a_6 = -0.015, \qquad a_7 = -0.0195, \qquad a_8 = 0.0032, \\ a_9 = -0.0956, \qquad a_{10} = -0.0078, \qquad a_{11} = -0.0095, \qquad a_{12} = 0.25, \qquad a_{13} = -1.08 \\ a_{14} = 0.056, \qquad a_{15} = -0.37, \qquad a_{16} = 286 \end{gathered} \tag{2}$$

As well as:

$$\begin{aligned} 10^4 \times \sin^2\theta_{\text{eff}}^{\text{leptons}} &= s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4\Delta_\alpha + d_5\Delta_t + d_6\Delta_t^2 \\ &\quad + d_7\Delta_t L_H + d_8\Delta_{\alpha_s} + d_9\Delta_{\alpha_s}\Delta_t + d_{10}\Delta_Z \end{aligned} \tag{3}$$

with:

$$\begin{gathered} s_0 = 2314.64, \qquad d_1 = 4.616, \qquad d_2 = 0.539, \qquad d_3 = -0.0737, \qquad d_4 = 206, \\ d_5 = -25.71, \qquad d_6 = 4.00, \qquad d_7 = 0.288, \qquad d_8 = 3.88, \qquad d_9 = -6.49, \qquad d_{10} = -6560 \end{gathered} \tag{4}$$

The parameterization for m_W is a little different:

$$\begin{aligned} m_W &= m_W^0 - c_1 dH - c_2 dH^2 + c_3 dH^4 + c_4(dh - 1) - c_5 d\alpha + c_6 dt - c_7 dt^2 - c_8 dH dt \\ &\quad + c_9 dh dt - c_{10} d\alpha_s + c_{11} dZ \end{aligned} \tag{5}$$

with:

$$\begin{gathered} dH = \ln\left(\tfrac{m_H}{100\text{ GeV}}\right), \qquad dh = \left(\tfrac{m_h}{100 GeV}\right)^2, \qquad dt = \left(\tfrac{m_t}{174.3\text{ GeV}}\right)^2 - 1, \qquad dZ = \tfrac{m_Z}{91.1875\text{ GeV}} - 1, \\ d\alpha = \tfrac{\Delta\alpha}{0.05907} - 1, \qquad d\alpha_s \tfrac{\alpha_s}{0.119} - 1, \qquad M_W^0 = 80.3779, \qquad c_1 = 0.05427, \\ c_2 = 0.008931\text{ GeV}, \qquad c_3 = 0.0000883\text{ GeV}, \qquad c_4 = 0.000161\text{ GeV}, \qquad c_5 = 1.070\text{ GeV}, \\ c_6 = 0.5237\text{ GeV}, \qquad c_7 = 0.0679\text{ GeV}, \qquad c_8 = 0.00179\text{ GeV}, \qquad c_9 = 0.0000664\text{ GeV}, \\ c_{10} = 0.0795\text{ GeV}, \qquad c_{11} = 114.9\text{ GeV} \end{gathered} \tag{6}$$

The top quark was discovered at Tevatron in 1995 and the Higgs boson was discovered at the LHC in 2012. Taking the above parameterizations as well as the experimental measured values of $\Gamma_{Z\to\text{leptons}}$, $\sin\theta_{\text{eff}}^{\text{leptons}}$, and m_W we can predict the top quark and Higgs boson masses using known values of the other parameters.

We'll use the following experimental measurements as inputs:

$$\begin{aligned}\alpha_s &= 0.1184\\ \Delta_\alpha &= 0.059\\ m_Z &= 91.1876\text{ GeV}\end{aligned}\tag{7}$$

And the following experimental measurements [3] for comparison with the theory predictions:

$$\begin{aligned}\Gamma_{Z\to\text{leptons}} &= 83.92\pm0.12\text{ MeV}\\ \sin^2\theta_{\text{eff}}^{\text{leptons}} &= 0.23153\pm0.00016\\ m_W &= 80.379\pm0.012\text{ GeV}\end{aligned}\tag{8}$$

Define,

$$\chi^2=\sum_i\left(\frac{\mu_{\text{exp,i}}-\mu_{\text{th,i}}}{\delta\mu_{\text{exp,i}}}\right)^2,\tag{9}$$

where $\mu_{\text{exp,i}}$ is the experimental measurement of $i\in\{\Gamma_{Z\to\text{leptons}},\sin^2\theta_{\text{eff}}^{\text{leptons}},m_W\}$, and $\delta\mu_{\text{exp,i}}$ is the associated experimental error, while $\mu_{\text{th,i}}$ are the theoretical predictions made by the above parameterizations. Bear in mind we are neglecting theory errors and correlations between the measured values for simplicity. The below problems should be done numerically using a computer algebra system, the solutions will be presented using Mathematica.

1) Minimize χ^2 with respect to m_H and m_t, the values of m_H and m_t at the minimum are the best fit points/predictions for these two parameters of the SM. This is considered an indirect determination of the mass as we are only seeing the quantum effects of the particles.

2) Hold m_H constant at its best fit value and vary m_t about its minimum until you find $\chi^2-\chi^2_{\text{min}}=1$ for values of m_t both above and below its best fit value, this is the 1σ error in our predicted value of m_t. Switch the roles of m_t and m_H to obtain the error in the prediction of m_H.

3) The top quark mass is now measured to be 173 GeV. Using this value what is the predicted mass of the Higgs boson and the error in the prediction?

4) The Higgs boson mass is now measured to be 125 GeV. Using this $m_t=173$ what is the value of $\chi^2-\chi^2_{\text{min}}(m_t=173)$, where χ^2_{min} is the value of χ^2 at its minimum as found in 3) above. What does this number mean?

The above methodology was used to help the Tevatron search for and find the top quark. This methodology was used to support the design of the LHC as the machine that would find the Higgs boson. These fits were performed with many more experimental measurements, such as the other partial widths of the Z and other asymmetries.

References

[1] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, and J. Usovitsch, *Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy*, *JHEP* **08** (2019) 113, [1906.08815].

[2] M. Awramik, M. Czakon, A. Freitas, and G. Weiglein, *Precise prediction for the W boson mass in the standard model*, *Phys. Rev. D* **69** (2004) 053006, [hep-ph/0311148].

[3] **ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group** Collaboration, S. Schael *et al.*, *Precision electroweak measurements on the Z resonance*, *Phys. Rept.* **427** (2006) 257–454, [hep-ex/0509008].