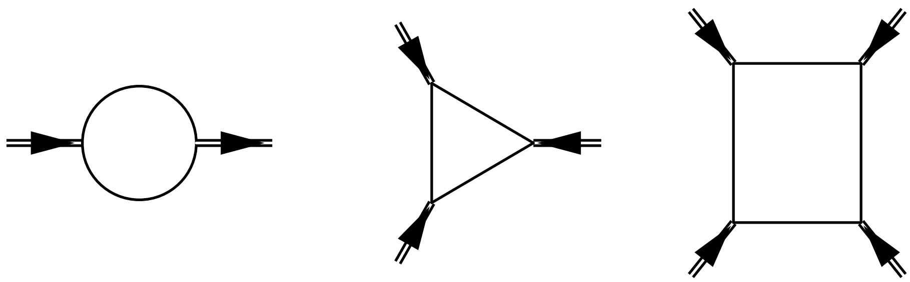
Problem Set 5

1. Superficial degree of divergence:
   For $p^2 \to \infty$ the bosonic and fermionic propagators scale as:

$$G_B = \frac{i}{p^2} \tag{1}$$

$$G_F = \frac{i}{p} \tag{2}$$

   (a) From this derive the superficial degree of divergence for the bubble, triangle, and box diagrams:



   Consider all possible combinations of internal propagators. For example for the triangle diagram we could have two bosonic and one fermionic propagator leading to:

$$BBF \sim \int d^4l \left(\frac{1}{l^2}\right)^2 \frac{1}{l} = \int d\Omega_3 \, l^3 dl \left(\frac{1}{l^2}\right)^2 \frac{1}{l} \to \frac{1}{\Lambda} \tag{3}$$

   (b) From this (limited) analysis, what can we conclude about pentagon and higher one-loop diagrams?
   (for this discussion we are ignoring that vertices can have momentum dependence)

2. Feynman Parameterization I:
   Derive Equation B.1 of Schwartz,

$$\frac{1}{AB} = \int_0^1 \frac{1}{[A + (B-A)x]^2} = \int_0^1 dx dy \delta(x + y - 1) \frac{1}{[xA + yB]^2} \tag{4}$$

3. Feynman Parameterization II:
   Derive Equation 14.9 of Srednicki, See also Peskin Chapter 6.3,

$$\frac{1}{A_1 \cdots A_n} = \int_0^1 d\alpha_1 \cdots \int_0^{1-\alpha_1-\cdots-\alpha_{n-2}} d\alpha_{n-1} \int_0^1 d\alpha_n \frac{\Gamma(n)\delta(1 - \alpha_1 - \cdots - \alpha_n)}{[\alpha_1 A_1 + \cdots + \alpha_{n-1} A_{n-1} + \alpha_n A_n]^n} \tag{5}$$

   Hints,
   a) begin with $1/A = \int_0^\infty ds_i \exp(-s_i A_i)$,
   b) take the product of n of these integrals,
   c) change variables to $\alpha = s_1 + \cdots + s_n$ and $\alpha_{i \neq n} = s_i/\alpha$,
   d) take the α integral,
   e) finally insert a δ function.

4. d-dimensional solid angle:
   From the (Equation B.22 of Schwartz) d-dimensional volume element,

$$\int d^d k = \int d\Omega_d \int k^{d-1} dk \tag{6}$$

   derive the (Equation B.28 of Schwartz) d-dimensional solid angle:

$$\Omega_d = \int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}, \tag{7}$$

   Hint, ♡ Gaussian integrals.

5. Anomalous dimension in the on-shell scheme:
   Derive the anomalous dimension of the m_R for $\lambda\phi^4$ theory for the on-shell renomalization scheme. Recall for this scheme we found:

$$Z_m = 1 + \frac{\lambda_R}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right) \tag{8}$$

   hint: Z_m now also has explicit μ^2 dependence, don't forget it!
   also: You will need to drop terms that are formally two-loop, as we did in the notes!