1)

We had:

$$\frac{\partial^3 \Gamma}{\partial \varphi^3} = \left(\frac{\partial^3 W}{\partial J^3}\right) \Big/ \left(\frac{\partial^2 W}{\partial J^2}\right)^3 \qquad i\left(\frac{\partial^2 W}{\partial J^2}\right)^{-1} = -i\,G^{-1}$$

$$\left(\frac{\partial^2 W}{\partial J^2}\right)^{-1} = -i\,G^{-1}$$

$$\frac{\partial}{\partial \varphi} = \frac{1}{i} G^{-1} \frac{\partial}{\partial J}$$

So

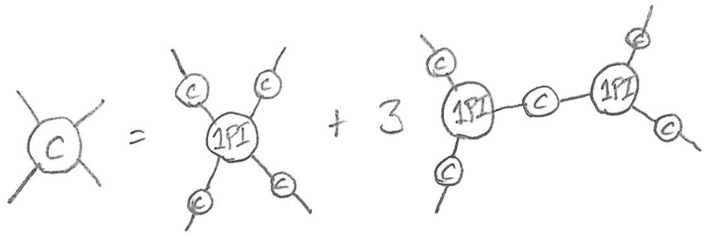
$$\frac{\partial^4 \Gamma}{\partial \varphi^4} = \frac{1}{i} G^{-1} \frac{\partial}{\partial J}\left[\frac{\partial^3 W}{\partial J^3} \Big/ \left(\frac{\partial^2 W}{\partial J^2}\right)^3\right]$$

$$= \frac{1}{i} G^{-1}\left[\left(\frac{\partial^4 W}{\partial J^4}\right) \Big/ \left(\frac{\partial^2 W}{\partial J^2}\right)^3 - 3\left(\frac{\partial^3 W}{\partial J^3}\right) \Big/ \left(\frac{\partial^2 W}{\partial J^2}\right)^4\right]$$

$$= G^{-4} \frac{\partial^4 W}{\partial J^4} + 3i\,G^{-5} \underbrace{\left(\frac{\partial^3 W}{\partial J^3}\right)^2}_{\left(i^3 G^3 \frac{\partial^3 \Gamma}{\partial \varphi^3}\right)^2}$$

$$\frac{\partial^4 W}{\partial J^4} = G^4 \frac{\partial^4 \Gamma}{\partial \varphi^4} + 3i\,G^5 \left(\frac{\partial^3 \Gamma}{\partial \varphi^3}\right)^2$$

$$i\,\frac{\partial^4 W}{\partial J^4} = i\,G^4 \frac{\partial^4 \Gamma}{\partial \varphi^4} + 3\left(i\,\frac{\partial^3 \Gamma}{\partial \varphi^3}\right) G \left(i\,\frac{\partial^3 \Gamma}{\partial \varphi^3}\right) G^4$$



2)

$$\mathcal{L}_I = c\varphi^2 \Box \varphi \qquad S_I = \int d^4x\, \mathcal{L}_I$$

We have:

$$\left.\frac{\partial^3 \Gamma}{\partial \varphi^3}\right|_{\varphi=0} = \frac{1}{(2\pi)^{2\cdot 4}} \delta^4(k_1+k_2+k_3)\, i\Gamma^{(3)}(k_1,k_2,k_3)$$

$$iS_I = i\int d^4x\, d^4y\, d^4z\, \delta^4(x-y)\delta^4(x-z)\, c\varphi(x)\varphi(y)\Box\varphi(z)$$

$$= i\int d^4x\, d^4y\, d^4z\, \delta^4(x-y)\delta^4(x-z)\int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{3\cdot 4}} e^{-ip_1\cdot x}\varphi(p_1) e^{-ip_2\cdot x}\varphi(p_2) \underbrace{\Box e^{-ip_3\cdot x}}_{(-ip_3)^2}\varphi(p_3)$$

$$= i\int d^4x \int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{3\cdot 4}} e^{-i(p_1+p_2+p_3)\cdot x}(-p_3^2)\varphi(p_1)\varphi(p_2)\varphi(p_3)$$

$$= i\int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{3\cdot 4}} (2\pi)^4\delta^4(p_1+p_2+p_3)(-p_3^2)\varphi(p_1)\varphi(p_2)\varphi(p_3)$$

$$\frac{\partial^3 iS_I}{\partial\varphi_1\partial\varphi_2\partial\varphi_3} = i\int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{3\cdot 4}}(2\pi)^4\delta^4(p_1+p_2+p_3)(-p_3)^2 c$$

$$\times\left[\delta_{11}\delta_{22}\delta_{33} + \delta_{11}\delta_{23}\delta_{32} + \delta_{12}\delta_{21}\delta_{33} + \delta_{12}\delta_{23}\delta_{31} + \delta_{13}\delta_{21}\delta_{32} + \delta_{13}\delta_{22}\delta_{31}\right] \quad \text{w/ } \delta_{ij} = \delta^4(k_i - p_j)$$

$\varphi_i = \varphi(k_i)$

$$= \frac{ic}{(2\pi)^{2\cdot 4}}\delta^4(k_1+k_2+k_3)\left[-k_3^2 - k_2^2 - k_3^2 - k_2^2 - k_1^2 - k_1^2\right]$$

$$= \frac{1}{(2\pi)^{2\cdot 4}}\delta^4(k_1+k_2+k_3)\underbrace{(-2ic)(k_1^2+k_2^2+k_3^2)}_{i\Gamma^{(3)}(k_1,k_2,k_3)}$$