Pheno in the SMEFT

1) Recall the "canonical" kinetic terms for our fields:

$$\frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m^2 S^2$$

$$i\bar{\psi}(\partial\!\!\!/ - m)\psi$$

$$-\frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + m^2 V_\mu V_\mu$$

In the SMEFT many of these are shifted by the new operators;

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H) = \frac{1}{4}(v^2 + 2vh + h^2)\Box(v^2 + 2vh + h^2) \qquad \text{(unitary gauge)}$$

$$= \frac{4}{4}v^2 h\Box h + \frac{v^2}{4}\Box h^2 + \text{(interactions)}$$

$$= v^2\left(1 + \frac{1}{4} + \frac{1}{4}\right)h\Box h + \frac{2v^2}{4}(\partial_\mu h)^2 + \cdots$$

$$= \frac{-3v^2}{2}(\partial_\mu h)^2 + \frac{v^2}{2}(\partial_\mu h)^2$$

$$= -v^2(\partial_\mu h)^2$$

Recall in the SM:

$$(D_\mu H)^\dagger(D_\mu H) \to \underbrace{\frac{1}{2}}_{\text{canonical}}(\partial_\mu h)^2 + \cdots$$

Adding these terms we have:

$$(D_\mu H)^\dagger(D_\mu H) + C_{H\Box}Q_{H\Box} = \frac{1}{2}\underbrace{(1 - 2C_{H\Box}v^2)}_{\text{not canonical}}(\partial_\mu h)^2 + \cdots$$

To have a correctly defined propagator we want a pole at m^2 and residue 1, this is why we want canonical normalizations.

To achieve this, let $h \to k_H h'$

$$(D_\mu H)^\dagger (D_\mu H) + C_{H\Box} Q_{H\Box} \to \frac{1}{2}(1 - 2C_{H\Box} v^2) k_H^2 (\partial_\mu h)^2 + \cdots$$

choose $k_H^2 = (1 - 2C_{H\Box} v^2)^{-1}$

$$k_H = (1 - 2C_{H\Box} v^2)^{-1/2} \sim 1 + C_{H\Box} v^2 + O\left(\frac{1}{\Lambda^4}\right)$$

This results in a canonical h' kinetic term, but shifts all h couplings

e.g. $\quad Y_\ell \bar{L} H e_R \to Y_\ell \bar{e}_L e_R h \to Y_\ell k_H \bar{e}_L e_R h$

$$= Y_\ell (1 + C_{H\Box} v^2) \bar{e}_L e_R h'$$

2) Consider the H potential:

$$V_H = -\mu^2 (H^\dagger H) + \lambda (H^\dagger H)^2 - C_H (H^\dagger H)^3$$

requiring we expand about the true minimum changes the definition of v:

$$V_H \to \left(\frac{\lambda v^4}{4} - \frac{\mu^2 v^2}{2} - \frac{C_H v^6}{8}\right) + \left(\lambda v^3 - \mu^2 v - \frac{3C_H v^5}{4}\right) h + \left(\frac{3v^2\lambda}{2} - \frac{\mu^2}{2} - \frac{15 C_H v^4}{8}\right) h^2 + \cdots$$

$$= (\text{constant}) + \underbrace{\left(\lambda v^3 - \mu^2 v - \frac{3C_H v^5}{4}\right)}_{\equiv 0} k_H h' + \underbrace{\left(\frac{3v^2\lambda}{2} - \frac{\mu^2}{2} - \frac{15 C_H v^4}{8}\right)}_{\frac{1}{2}\bar{m}_H^2} k_H^2 (h')^2 + \cdots$$

↑ bar notation means we've include SMEFT corrections

$$\left(\lambda v^3 - \mu^2 v - \frac{3C_H v^5}{4}\right) k_H = \left(\lambda v^3 - \mu^2 v - \frac{3C_H v^5}{4}\right)(1 + C_{H\Box} v^2) + O\left(\frac{1}{\Lambda^4}\right)$$

$$= \lambda v^3 - \mu^2 v + \lambda v^5 C_{H\Box} - \mu^2 v^3 C_{H\Box} - \frac{3C_H v^5}{4} + O\left(\frac{1}{\Lambda^4}\right) = 0$$

$$\mu^2 = \frac{4\lambda v^2 + 4C_{H\Box}\lambda v^4 - 3C_H v^4}{4(1 + C_{H\Box} v^2)}$$

$$= \underset{\text{SM}}{\underset{\uparrow}{\lambda v^2}} - \underset{\substack{\text{SMEFT}\\ \text{correction}}}{\underset{\uparrow}{\frac{3C_H v^4}{4}}} + O\left(\frac{1}{\Lambda^4}\right)$$

$$\left(\frac{3v^2\lambda}{2} - \frac{\mu^2}{2} - \frac{15C_H v^4}{8}\right) k_H^2 = \left(\frac{3v^2\lambda}{2} - \frac{\lambda v^2}{2} - \frac{3C_H v^4}{8} - \frac{15C_H v^4}{8}\right)(1 + 2C_{H\Box} v^2)$$

$$= \frac{1}{2}\underbrace{\left(\underset{\text{SM}}{\underset{\uparrow}{2\lambda v^2}} \underbrace{- 3C_H v^4 + 4\lambda v^4 C_{H\Box}}_{\text{SMEFT correction}}\right)}_{\overline{m}_H^2}$$

Notice we've forgotten $Q_{HD} = |H^\dagger D_\mu H|^2$

So $k_H \to \left(1 + C_{H\Box} v^2 - \frac{C_{HD} v^2}{4}\right)$

$$\overline{m}_H^2 \to 2\lambda v^2 - 3C_H v^4 + 4\lambda v^4 C_{H\Box} - \lambda v^4 C_{HD}$$

3) For fermions we've removed all the $D\phi$ terms, so we don't need to worry about the kinetic term, but the masses do change:

e.g. $\quad C_{eH} Q_{eH} = C_{eH} (H^\dagger H) \bar{L} e_R H$

$$\rightarrow C_{eH} \tfrac{1}{2} (v^2 + 2vh + h^2) \bar{e}_L e_R \frac{(v+h)}{\sqrt{2}} + \ldots \quad \text{(interactions)}$$

$$= \frac{C_{eH} v^3}{2\sqrt{2}} \bar{e}_L e_R$$

Adding the SM:

$$-Y_e \bar{L} e_R H + C_{eH} Q_{eH} = \underbrace{\frac{-v}{\sqrt{2}} \left(Y_e - \frac{C_{eH} v^2}{2} \right)}_{\equiv -m_e} + \ldots$$

$$\Rightarrow Y_e = \frac{\sqrt{2}\, m_e}{v} + \frac{C_{eH} v^2}{2}$$

What about the interactions?

$$-Y_e \bar{L} e_R H + C_{eH} Q_{eH} = -Y_e \frac{(v+h)}{\sqrt{2}} \bar{e}_L e_R + \frac{C_{eH}}{2} (v^2 + 2vh + h^2) \bar{e}_L e_R \frac{(v+h)}{\sqrt{2}}$$

$$= \left(-\frac{Y_e}{\sqrt{2}} + \frac{C_{eH} v^2}{\sqrt{2}} + \frac{C_{eH} v^2}{2\sqrt{2}} \right) \bar{e}_L e_R h + \text{(mass \& interactions)}$$

$$= -\left(\frac{m_e}{v} - \frac{C_{eH} v^2}{\cancel{2}\sqrt{2}} + \frac{C_{eH} v^2}{\sqrt{2}} + \frac{C_{eH} v^2}{2\sqrt{2}} \right) \bar{e}_L e_R h + \ldots$$

In the SMEFT the mass of a fermion and its coupling to the Higgs are no longer correlated!

4) Gauge Bosons:

Again we have to deal w/ the kinetic terms & masses, but also new mixing

Notice:

$$C_{HB} Q_{HB} = C_{HB} (H^\dagger H) B_{\mu\nu} B_{\mu\nu}$$

$$\rightarrow \frac{C_{HB}}{2} (v^2 + 2vh + h^2) B_{\mu\nu} B_{\mu\nu}$$

$$C_{HW} Q_{HW} = C_{HW} (H^\dagger H) W^I_{\mu\nu} W^I_{\mu\nu}$$

$$\rightarrow \frac{C_{HW}}{2} (v^2 + 2vh + h^2) W_{\mu\nu} W_{\mu\nu}$$

These can be taken care of w/ redefining the field:

$$B_\mu \rightarrow k_B B'_\mu \Rightarrow -\frac{1}{4} B_{\mu\nu} B_{\mu\nu} + \frac{C_{HB}}{2} v^2 B_{\mu\nu} B_{\mu\nu}$$

$$\rightarrow -\frac{1}{4} (1 - 2C_{HB} v^2) k_B^2 B'_{\mu\nu} B'_{\mu\nu}$$

$$k_B^2 = (1 - 2C_{HB} v^2)^{-1}$$

$$k_B \sim 1 + C_{HB} v^2$$

But the covariant derivative is affected:

$$D_\mu F \sim (\partial_\mu + i g_1 Y B_\mu) F \rightarrow (\partial_\mu + i \underbrace{g_1 k_B}_{\equiv \bar{g}_1} Y B_\mu) F$$

this amounts to a shift in the definition of the gauge coupling

For the W^I we can do the same:

$$-\frac{1}{4} W^I_{\mu\nu} W^I_{\mu\nu} + C_{HW}(H^\dagger H) W^I_{\mu\nu} W^I_{\mu\nu}$$

$$\rightarrow -\frac{1}{4}(1-2C_{HW}v^2) k_W^2 W^{I\prime}_{\mu\nu} W^{I\prime}_{\mu\nu}$$

$$k_W^2 = (1-2C_{HW}v^2)^{-1}$$

$$k_W \sim 1 + C_{HW}v^2$$

$$D_\mu F \sim \left(\partial_\mu + i g_2 W^I_\mu \frac{\sigma^I}{2}\right) F$$

$$\rightarrow \left(\partial_\mu + i \underbrace{g_2 k_W}_{\equiv \bar{g}_2} W^{I\prime}_\mu \frac{\sigma^I}{2}\right) F$$

Remember the nonAbelian part of $W^I_{\mu\nu}$:

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - i g_2 \epsilon^{IJK} W^J_\mu W^K_\nu$$

$$\rightarrow k_W\left(\partial_\mu W^{I\prime}_\nu - \partial_\nu W^{I\prime}_\mu - i \underbrace{g_2 k_W}_{=\bar{g}_2} \epsilon^{IJK} W^{J\prime}_\mu W^{K\prime}_\nu\right)$$

so this is a consistent strategy!

But we have one more operator:

$$Q_{HWB} = (H^\dagger \sigma^I H) W^I_{\mu\nu} B_{\mu\nu}$$

consider:

$$H^\dagger \sigma_1 H = \frac{1}{2}\begin{pmatrix}0\\v+h\end{pmatrix}^T \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0\\v+h\end{pmatrix} = 0$$

$$H^\dagger \sigma_2 H = \frac{1}{2}\begin{pmatrix}0\\v+h\end{pmatrix}^T \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0\\v+h\end{pmatrix} = 0$$

$$H^\dagger \sigma_3 H = \frac{1}{2}\begin{pmatrix}0\\v+h\end{pmatrix}^T \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0\\v+h\end{pmatrix} = -(v+h)^2/2$$

so (in Unitary gauge) we have:

$$Q_{HWB} = -\frac{(v+h)^2}{2} W^3_{\mu\nu} B_{\mu\nu}$$

$$\rightarrow -\frac{v^2}{2}\underbrace{(\partial_\mu W^3_\nu - \partial_\nu W^3_\mu)(\partial_\mu B_\nu - \partial_\nu B_\mu)}_{} + \text{interactions}$$

kinetic (not mass) mixing!
we need to diagonalize this
simultaneously w/ the mass mixing

let $W^3_{\mu\nu} \equiv (\partial_\mu W^3_\nu - \partial_\nu W^3_\mu)$

Then we have:

$$-\frac{1}{4}\left(1 - \frac{C_{HB}}{2}v^2\right) B_{\mu\nu}B_{\mu\nu} - \frac{1}{4}\left(1 - \frac{C_{HW}}{2}v^2\right) W^3_{\mu\nu}W^3_{\mu\nu} - \frac{v^2}{2} C_{HWB} W^3_{\mu\nu} B_{\mu\nu}$$

$$= -\frac{1}{4} B'_{\mu\nu}B'_{\mu\nu} - \frac{1}{4} W^{3\prime}_{\mu\nu} W^{3\prime}_{\mu\nu} - \frac{v^2}{2} C_{HWB} W^{3\prime}_{\mu\nu} B'_{\mu\nu} + O\left(\frac{1}{\Lambda^4}\right)$$

since C_{HWB} is already $\propto(\frac{1}{\Lambda^4})$
we can just prime these fields

$$= \begin{pmatrix} B'_{\mu\nu} \\ W^{3\prime}_{\mu\nu} \end{pmatrix} \begin{pmatrix} -\frac{1}{4} & -\frac{v^2}{4} C_{HWB} \\ -\frac{v^2}{4} C_{HWB} & -1/4 \end{pmatrix} \begin{pmatrix} B'_{\mu\nu} \\ W^{3\prime}_{\mu\nu} \end{pmatrix}$$

Next consider

$$C_{HD}\,(H^\dagger D_\mu H)(D_\mu H)^\dagger H$$
$$= \underset{\uparrow \atop (\frac{1}{\sqrt{2}})^4\, Y_H^2 \text{ or } \frac{\sigma^3}{2}}{\frac{1}{16}}\, C_{HD}\Big(g_1^2 v^4 B_\mu B_\mu + g_2^2 v^4 W_\mu^3 W_\mu^3 - 2 g_1 g_2 v^4 B_\mu W_\mu^3 \Big) + \text{interactions}\cdots$$

no $W^\pm$ contributions!

So our mass matrix becomes:

$$(D_\mu H)^\dagger (D_\mu H) + C_{HD}(H^\dagger D_\mu H)(D_\mu H)^\dagger H$$
$$= \frac{1}{8}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}^T \underbrace{\begin{pmatrix} g_1^2 v^2 + \frac{g_1^2 v^4 C_{HD}}{2} & -g_1 g_2 v^2 - \frac{g_1 g_2}{2} v^4 C_{HD} \\ -g_1 g_2 v^2 - \frac{g_1 g_2}{2} v^4 C_{HD} & g_2^2 v^2 + \frac{g_2^2 v^4}{2} C_{HD} \end{pmatrix}}_{\text{note } C_{HD} \text{ really just shifts the SM mixing}} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

But we need to write in terms of B'_μ & $(W_\mu^3)'$!

$$\rightarrow \frac{v^2}{8}\begin{pmatrix} B'_\mu \\ W'^3_\mu \end{pmatrix}^T \begin{pmatrix} K_B & 0 \\ 0 & K_W \end{pmatrix}^T \begin{pmatrix} g_1^2 + \frac{g_1^2 v^2}{2} C_{HD} & -g_1 g_2 - \frac{g_1 g_2}{2} v^2 C_{HD} \\ -g_1 g_2 - \frac{g_1 g_2}{2} v^2 C_{HD} & g_2^2 + \frac{g_2^2 v^2}{2} C_{HD} \end{pmatrix} \begin{pmatrix} K_B & 0 \\ 0 & K_W \end{pmatrix} \begin{pmatrix} B'_\mu \\ W'^3_\mu \end{pmatrix}$$

$$= \frac{v^2}{8}\begin{pmatrix} B'_\mu \\ W'^3_\mu \end{pmatrix} \begin{pmatrix} \bar{g}_1^2 + \frac{\bar{g}_1^2 v^2}{2} C_{HD} & -\bar{g}_1 \bar{g}_2 - \frac{\bar{g}_1 \bar{g}_2}{2} v^2 C_{HD} \\ -\bar{g}_1 \bar{g}_2 - \frac{\bar{g}_1 \bar{g}_2}{2} v^2 C_{HD} & \bar{g}_2^2 + \frac{\bar{g}_2^2 v^2}{2} C_{HD} \end{pmatrix} \begin{pmatrix} B'_\mu \\ W'^3_\mu \end{pmatrix}$$

again: $g_i C^{(6)} = \bar{g}_i C^{(6)} + O\left(\frac{1}{\Lambda^4}\right)$

So we arrive at:

$$\mathcal{L}_{\text{gauge-mixing}} = -\frac{1}{4} W^{3'}_{\mu\nu} W^{3'}_{\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'_{\mu\nu} - \frac{1}{2} v^2 C_{HWB} W^{3'}_{\mu\nu} B'_{\mu\nu}$$
$$+ \frac{1}{8} v^2 (\bar{g}_2 W^{3'}_\mu - \bar{g}_1 B'_\mu)^2 + \frac{1}{16} v^4 C_{HD} (\bar{g}_2 W^{3'}_\mu - \bar{g}_1 B'_\mu)^2$$

Taking:

$$\begin{pmatrix} B_\mu' \\ W_\mu^{3\prime} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{v^2}{2}C_{HWB} \\ -\frac{v^2}{2}C_{HWB} & 1 \end{pmatrix}\begin{pmatrix} \bar{c}_w & -\bar{s}_w \\ +\bar{s}_w & \bar{c}_w \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

The mass matrix becomes:

$$\frac{v^2}{8}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}^T \begin{pmatrix} \bar{c}_w & \bar{s}_w \\ -\bar{s}_w & \bar{c}_w \end{pmatrix}\begin{pmatrix} 1 & -\frac{v^2}{2}C_{HWB} \\ -\frac{v^2}{2}C_{HWB} & 1 \end{pmatrix}\begin{pmatrix} \bar{g}_1^2(1+\frac{v^2}{2}C_{HD}) & -\bar{g}_1\bar{g}_2(1+\frac{v^2}{2}C_{HD}) \\ -\bar{g}_1\bar{g}_2(1+\frac{v^2}{2}C_{HD}) & \bar{g}_2^2(1+\frac{v^2}{2}C_{HD}) \end{pmatrix}\begin{pmatrix} 1 & -\frac{v^2}{2}C_{HWB} \\ -\frac{v^2}{2}C_{HWB} & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} \bar{c}_w & -\bar{s}_w \\ \bar{s}_w & \bar{c}_w \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$\equiv \frac{1}{2}\left(1+\frac{v^2}{2}C_{HD}\right)\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}^T \begin{pmatrix} m_A^2 & m_{AZ}^2 \\ m_{AZ}^2 & \tilde{m}_Z^2 \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$\tilde{m}_Z^2 \neq 0$, so m_Z^2 will include $(1+\frac{v^2}{2}C_{HD})$

$$m_A^2 = \frac{1}{4}(\bar{c}_w\bar{g}_1 - \bar{g}_2\bar{s}_w)\underbrace{\left(\bar{c}_w[\bar{g}_1+\bar{g}_2 C_{HWB}v^2] - \bar{s}_w[\bar{g}_2+\bar{g}_1 C_{HWB}v^2]\right)}_{=0,\ \text{the other choice doesn't give } m_{AZ}=0}$$

requiring $m_A^2 = 0 \to \dfrac{\bar{s}_w^2}{\bar{c}_w^2} = \dfrac{(\bar{g}_1+\bar{g}_2 C_{HWB}v^2)}{(\bar{g}_2+\bar{g}_1 C_{HWB}v^2)}$

$$\tan\theta_w = \frac{\bar{g}_1}{\bar{g}_2} + \frac{1}{2}\left(1-\frac{\bar{g}_1^2}{\bar{g}_2^2}\right)C_{HWB}v^2$$

$$1 = \bar{c}_w^2(1+\tan\theta_w^2) \to \bar{c}_w = \frac{\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1 - \frac{\bar{g}_1^2-\bar{g}_2^2}{2(\bar{g}_1^2+\bar{g}_2^2)}\frac{\bar{g}_1}{\bar{g}_2}C_{HWB}v^2\right]$$

$$\to \bar{s}_w = \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1 + \frac{\bar{g}_2^2-\bar{g}_1^2}{2(\bar{g}_1^2+\bar{g}_2^2)}\frac{\bar{g}_2}{\bar{g}_1}C_{HWB}v^2\right]$$

With this $m_{AZ}^2 = 0$

The mass of the Z is:

$$m_Z^2 = \frac{v^2}{4}\left(\bar{c}_w \bar{g}_2 + \bar{g}_1 \bar{s}_w\right)\left(\bar{c}_w\left(\bar{g}_2 + \bar{g}_1 C_{HWB} v^2\right) + \bar{s}_w\left(\bar{g}_1 + \bar{g}_2 C_{HWB} v^2\right)\right)\left(1 + \frac{v^2}{2} C_{HD}\right)$$

cross terms are $O\left(\frac{1}{\Lambda^4}\right)$

$$= \frac{\left(\bar{c}_w \bar{g}_2 + \bar{g}_1 \bar{s}_w\right)^2}{4} v^2 \left[1 + \frac{\bar{g}_1 \bar{c}_w + \bar{g}_2 \bar{s}_w}{\bar{g}_2 \bar{c}_w + \bar{g}_1 \bar{s}_w} C_{HWB} v^2\right]\left(1 + \frac{v^2}{2} C_{HD}\right)$$

$$\equiv \frac{\bar{g}_Z^2 v^2}{4}\left(1 + \frac{v^2}{2} C_{HD}\right)$$

The mass & kinetic terms are diagonalized & canonically normalized

The W-mass is:

$$\bar{m}_W^2 = \frac{\bar{g}_2^2 v^2}{4}$$

@ D8 this receives corrections from

eg: $C_{HD2}^{(8)} (H^\dagger H)(H^\dagger \sigma^a H)(D_\mu H)^\dagger \sigma^a (D_\mu H)$

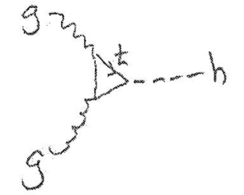
5) With all these definitions we find (dropping primed notation)

$$D_\mu\phi \to \left[\partial_\mu + \frac{i\bar g_2}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) + i\bar g_Z(T_3 - s_W^2 Q_\phi)Z_\mu + iQ_\phi \bar e A_\mu\right]\phi$$

With all the above developments we can do calculations in the SMEFT

Ex 1: The Higgs boson is produced at the LHC via gluon fusion

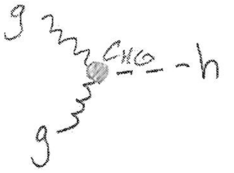
in SM:



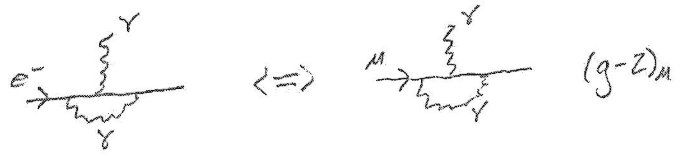
in SMEFT:

$$C_{HG}Q_{HG} = C_{HG}H^\dagger H G^A_{\mu\nu}G^A_{\mu\nu} \to \frac{1}{2}(v^2+2vh+h^2)G^A_{\mu\nu}G^A_{\mu\nu}$$

note we also have to normalize QCD processes



Ex 2: Recall the e^- g-2 (magnetic moment) from APP: ← Advanced Particles or Wk 6



$$\mathcal{M} \propto \frac{ie}{2m}\underbrace{[\bar u\sigma^{\mu\nu}u]}_{\bar\psi\sigma^{\mu\nu}\psi = \bar\psi_L\sigma^{\mu\nu}\psi_R + (L\leftrightarrow R)} p_\nu F_2\left(\frac{p^2}{m^2}\right) \quad \text{w/ } p = p_\gamma$$

in SMEFT: $C_{eB}Q_{eB} = C_{eB}\bar L\sigma^{\mu\nu}e_R H B_{\mu\nu}$ + h.c.

$$= C_{eB}\bar e_L\sigma^{\mu\nu}e_R(v+h)\underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu)}_{\propto p_\gamma} + Z \text{ stuff} + \text{h.c}$$

$\propto C_{eB}v[\bar u\sigma^{\mu\nu}u]p_\mu$

6) Input parameters in the SMEFT:

Again we'll take $\{\alpha, m_Z, G_F\}$

defining:

$$\hat{\alpha} = \frac{\bar{e}^2}{4\pi} = \frac{1}{4\pi}\frac{\bar{g}_1^2\bar{g}_2^2}{\bar{g}_1^2+\bar{g}_2^2}(1+\delta\alpha) \quad w/ \quad \delta\alpha = -\frac{2\bar{g}_1\bar{g}_2}{\bar{g}_1^2+\bar{g}_2^2} C_{HWB} v^2 = -2 s_W c_W C_{HWB} v^2$$

$$\hat{m}_Z^2 = \frac{\bar{g}_Z^2 v^2}{4}\left(1+\frac{v^2}{2}C_{HD}\right) = \frac{\bar{g}_1^2+\bar{g}_2^2}{4} v^2\left(1+\frac{1}{2}\sqrt{\bar{g}_1^2+\bar{g}_2^2}\, C_{HD} v^2 + \frac{2\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}} C_{HWB} v^2\right)$$

$$\equiv m_Z^2(1+\delta m_Z^2)$$

$$\hat{G}_F = \frac{1}{\sqrt{2}v^2}(1+\underbrace{\delta G_F}_{= 2C_{Hl}^{(3)} v^2 + C_{ll} v^2})$$ (we didn't derive this)

We solve for $\{\bar{g}_1, \bar{g}_2, v\}$:

$$v^2 = \underbrace{\frac{1}{\sqrt{2}G_F}}_{\hat{v}^2} + \frac{1}{\sqrt{2}}\frac{\delta G_F}{\hat{G}_F}$$ ← Implicitly we are dropping terms of $O(\frac{1}{\Lambda^4})$

→ ie $v^2 = \hat{v}^2 + \delta v^2$; $\hat{v}$ corresponds to the $\frac{1}{\Lambda^0}$ term

$$\bar{g}_1 = \hat{g}_1\left(1+\frac{1}{2\underset{\uparrow\ \cos 2\theta_W}{\hat{c}_{2W}}}\left[\hat{s}_W^2\left(\sqrt{2}\,\delta G_F + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) - \hat{c}_W^2\,\delta\alpha\right]\right) \equiv \hat{g}_1 + \delta g_1/\hat{g}_1$$

$$\bar{g}_2 = \hat{g}_2\Big(1\underbrace{-\frac{1}{2\hat{c}_{2W}}\left[\hat{c}_W^2\left(\sqrt{2}\,\delta G_F + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) - \hat{s}_W^2\,\delta\alpha\right]}_{\equiv\, \delta g_2/\hat{g}_2}\Big)$$

In the above we have defined:

$$\hat{e} = 4\pi\hat{\alpha}$$

$$\hat{g}_1 = \hat{e}/\hat{c}_W \qquad \hat{g}_2 = \hat{e}/\hat{s}_W$$

$$\hat{s}_W^2 = \frac{1}{2}\left[1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right]$$

We also need:

$$\bar{m}_W^2 = \hat{m}_W^2 + \delta m_W^2$$

$$= \frac{\bar{g}_2^2 v^2}{4} = \frac{\hat{g}_2^2 \hat{v}^2}{4}\left(1 + \frac{2}{\sqrt{2}}\frac{\delta G_F}{\hat{G}_F} + 2\frac{\delta g_2}{\hat{g}_2}\right)$$

$$\bar{s}_W^2 = \hat{s}_W^2 + \delta s_W^2$$

$$= \Bigg[\underbrace{\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}(1+\delta\alpha)\frac{1}{\bar{g}_2}}_{\bar{e}/\bar{g}_2}\Bigg]^2$$

$$= \frac{\hat{g}_1^2}{\hat{g}_1^2+\hat{g}_2^2} + 2\hat{s}_W^2\hat{c}_W^2\left(\frac{\delta g_1}{\hat{g}_1} - \frac{\delta g_2}{\hat{g}_2}\right) - \hat{c}_{2W}\,\delta\alpha$$