The Standard Model EFT (SMEFT)

The SMEFT is a bottom up approach to physics beyond the SM

Recall from the Fermi example we want to take the IR (SM) field content:

$$B_{\mu\nu},\ W^{I}_{\mu\nu},\ G^{A}_{\mu\nu},\ H,\ L,\ Q,\ e,\ d,\ u$$

And form all ops of dim >4

In the Fermi example, Fermi used Paulis ν to form an EFT:

$$e,\ p,\ n,\ F^{\mu\nu} \to p,\ n,\ \nu,\ e,\ F^{\mu\nu}$$

We can do the same w/ the SMEFT

+ RH ν → νSMEFT

+ Dark matter → SMEFT w/ Dark Matter

1) Forming the SMEFT @ D5

In the Fermi thry the leading op is D6, however in the SMEFT we can form a D5 op. This isn't entirely obvious from our tools so far.

Consider $\tilde{H}^\dagger L = (i\sigma_2 H^*)^\dagger L$

This is invar under SU(2), just as the up-quark Yukawa

Under $U(1)_Y$ we have $Y_H = 1/2 \rightarrow Y_{\tilde{H}^\dagger} = 1/2$

$Y_L = -1/2$

So invar under $U(1)_Y$ is also easily established

This is not Lorentz invar as it has an open Dirac index

We could try to form an invariant by multiplying by its dagger:

$$\left([i\sigma_2 H^*]^\dagger L\right)^\dagger = -L^\dagger i\sigma_2 H^* = -L^\dagger \tilde{H}$$

But, recall $L^\dagger L$ was not Lorentz invar:

recall a Dirac Fermion was defined as: $\psi \equiv \begin{pmatrix} \chi_a \\ \xi^{\dagger \dot{a}} \end{pmatrix}$ $\psi^\dagger = (\chi^\dagger_{\dot{a}}, \xi^a)$

$$\psi^\dagger \psi = \chi^\dagger_{\dot{a}} \chi_a + \xi^a \xi^{\dagger \dot{a}}$$

the spinor indices aren't contracted!

Defining the Charge conjugation matrix C in terms of ϵ^{ac}, $\epsilon_{\dot{a}\dot{c}}$, which were "invariant symbols"

$$C \equiv \begin{pmatrix} -\epsilon^{ac} & \\ & -\epsilon_{\dot{a}\dot{c}} \end{pmatrix}$$

We can then consider a new LI made of Dirac Fields:

$$\psi^T C \psi = (\chi_a^T, \xi^{*\dot{a}}) \begin{pmatrix} -\epsilon^{ac} & \\ & -\epsilon_{\dot{a}\dot{c}} \end{pmatrix} \begin{pmatrix} \chi_c \\ \xi^{\dagger\dot{c}} \end{pmatrix}$$

$$= -\chi_a^T \epsilon^{ac} \chi_c - \xi^{*\dot{a}} \epsilon_{\dot{a}\dot{c}} \xi^{\dagger\dot{c}}$$

$$= -\chi_a^T \chi^a - \xi^{*\dot{a}} \xi^\dagger_{\dot{a}}$$

Which is a Lorentz invar quantity.

Contrasting this w/

$$\bar{\psi}\psi = \xi^a \chi_a + \chi_a^\dagger \xi^{\dagger\dot{a}}$$

We see it is essentially a mass term which doesn't mix components of the Dirac field.

So instead of $L^\dagger L$ we can use this invar:

$$c^{(5)} (\tilde{H}^\dagger L)^T C (\tilde{H}^\dagger L)$$

Expanding H about its vev in unitary gauge:

$$\frac{c^{(5)}}{\Lambda} (L^T \tilde{H}^*) C (\tilde{H}^\dagger L) \to \underbrace{\frac{-c^{(5)}}{2\Lambda} v^2}_{m_\nu = \frac{c^{(5)} v^2}{2\Lambda}} \nu_L^T C \nu_L + \text{higgs interactions}$$

2) Tools needed to form the D6 SMEFT

Physical observables in QFTs are invariant under redefinitions of the fields

→ this is sometimes referred to as the equivalence thrm

We wont prove this, but will consider some examples as justification

ex: free scalar theory

$$\mathcal{L} = +\tfrac{1}{2}(\partial^\mu \varphi)^2 - \tfrac{1}{2}m^2\varphi^2$$

take $\varphi \to \varphi + \lambda\varphi^3$

$$\mathcal{L} \to \mathcal{L}' = +\tfrac{1}{2}(\partial^\mu\varphi)^2 - \tfrac{1}{2}m^2\varphi^2 \underbrace{- \lambda(\partial^\mu\varphi)(\partial_\mu\varphi^3)}_{-3\varphi^2(\partial^\mu\varphi)^2} - \frac{\lambda^2}{2}(\partial_\mu\varphi^3)^2 - \lambda m^2\varphi^4 - \frac{\lambda^2 m^2}{2}\varphi^6$$

considering $2\to 2$ scattering:

 $= \dfrac{i}{p^2 - m^2}$

 $= -24i\lambda m^2 - 12i\lambda(\underset{\substack{\uparrow \\ \text{w/a all incoming}}}{p_1\cdot p_2} + p_1\cdot p_3 + p_1\cdot p_4 + p_2\cdot p_3 + p_2\cdot p_4 + p_3\cdot p_4)$

So $i\mathcal{M}_{12\to 34} = -24i\lambda m^2 - 12i\lambda(p_1\cdot p_2 - p_1\cdot p_3 - p_1\cdot p_4 - p_2\cdot p_3 - p_2\cdot p_4 + p_3\cdot p_4)$

− comes from p_3 outgoing

$$s \equiv (p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1\cdot p_2 = 2m^2 + 2p_1\cdot p_2 \quad \to p_1\cdot p_2 = \tfrac{1}{2}(s-2m^2)$$
$$= (p_3+p_4)^2 \quad \to p_3\cdot p_4 = \tfrac{1}{2}(s-2m^2)$$
$$t = (p_1-p_3)^2 = 2m^2 - 2p_1\cdot p_3 \quad \to p_1\cdot p_3 = -\tfrac{1}{2}(t-2m^2)$$
$$= (p_2-p_4)^2 \quad \to p_2\cdot p_4 = -\tfrac{1}{2}(t-2m^2)$$
$$u = (p_1-p_4)^2 = 2m^2 - 2p_1\cdot p_4 \quad \to p_1\cdot p_4 = -\tfrac{1}{2}(u-2m^2)$$
$$= (p_3-p_2)^2 \quad \to p_3\cdot p_2 = -\tfrac{1}{2}(u-2m^2)$$

Subbing in gives:

$$i\mathcal{M}_{12\to34} = -24i\lambda m^2 - 12i\lambda\left[(s-2m^2)+(t-2m^2)+(u-2m^2)\right]$$

recall $s+t+u=\sum_i m_i^2$ w/ i the external particles

$$= -24i\lambda m^2 - 12i\lambda\left[4m^2-6m^2\right]$$

$$=0$$

which is the scattering amplitude in free thry by definition

Now consider the EFT

$$\mathcal{L} = (D_\mu H)^\dagger(D_\mu H) + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2 + C_H(H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}(H^\dagger D_\mu H)(D_\mu H)^\dagger H + C_{HD2}(D_\mu H)^\dagger(D_\mu H)(H^\dagger H)$$

If we make the substitution:

$$H \to H + \frac{a}{\Lambda^2}(H^\dagger H)H$$

note RHS transforms under SM symmetries the same as the LHS

$$\mu^2 H^\dagger H \to \mu^2\left(H^\dagger + \frac{a}{\Lambda^2}(H^\dagger H)H^\dagger\right)\left(H + \frac{a}{\Lambda^2}(H^\dagger H)H\right)$$

$$= \mu^2\left[H^\dagger H + \frac{a}{\Lambda^2}(H^\dagger H)^2 + \frac{a}{\Lambda^2}H^\dagger\underbrace{(H^\dagger H)}_{\text{singlet commutes}}H + \frac{a^2}{\Lambda^4}(H^\dagger H)^3\right]$$

$$-\lambda(H^\dagger H)^2 \to -\lambda H^\dagger H - \frac{4\lambda a}{\Lambda^2}(H^\dagger H)^3 + O\left(\frac{1}{\Lambda^4}\right)$$

$$|DH|^2 \to \left[(D_\mu H)^\dagger + \frac{a}{\Lambda^2}\underbrace{D_\mu(H^\dagger H)}_{D_\mu(\text{singlet})\to\partial_\mu(\text{singlet})}H^\dagger + \frac{a}{\Lambda^2}(H^\dagger H)(D_\mu H)^\dagger\right]\left[D_\mu H + \frac{a}{\Lambda^2}\partial_\mu(H^\dagger H)H + \frac{a}{\Lambda^2}(H^\dagger H)D_\mu H\right]$$

$$= (D_\mu H)^\dagger(D_\mu H) + \frac{a}{\Lambda^2}\left[\partial_\mu(H^\dagger H)H^\dagger(D_\mu H) + \partial_\mu(H^\dagger H)(D_\mu H)^\dagger H + 2(H^\dagger H)(D_\mu H)^\dagger(D_\mu H)\right] + O\left(\frac{1}{\Lambda^4}\right)$$

All $D6 \to D6 + O\left(\frac{1}{\Lambda^4}\right)$

We can simplify the derivatives part:

$$\partial_\mu (H^\dagger H) H^\dagger (D_\mu H) + \partial_\mu (H^\dagger H)(D_\mu H)^\dagger H$$

$$= -(H^\dagger H)\Box(H^\dagger H) - \partial_\mu (H^\dagger H)(D_\mu H)^\dagger H + \partial_\mu (H^\dagger H)(D_\mu H)^\dagger H$$

Then we have:

$$\mathcal{L} \to (D_\mu H)^\dagger (D_\mu H) + \mu^2 (H^\dagger H) - \left(\lambda - \frac{2a\mu^2}{\Lambda^2}\right)(H^\dagger H)^2 + \left(\frac{C_H}{\Lambda^2} + \frac{a^2\mu^2}{\Lambda^4} - \frac{4\lambda a}{\Lambda^2}\right)(H^\dagger H)^3$$

$$+ \left(\frac{C_{H\Box}}{\Lambda^2} - \frac{a}{\Lambda^2}\right)(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{\Lambda^2}(H^\dagger DH)(DH)^\dagger H + \left(\frac{C_{HD2}}{\Lambda^2} + \frac{2a}{\Lambda^2}\right)(H^\dagger H)(D_\mu H)^\dagger (D_\mu H)$$

$$+ \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

So we can choose a such that Q_{HD2} or $Q_{H\Box}$ are removed from our $\mathcal{L}$, and the error in this approach is the next order in our power counting.

Removing $(H^\dagger H)^3$ doesn't simplify calculations much since we rarely study processes w/ more than one Higgs

Consider the H EOM you derived in wk 1: (we neglected Yukawas above so we do here)

$$D^2 H - \mu^2 H + 2\lambda H (H^\dagger H) = 0$$

Multiply by $(H^\dagger H) H^\dagger$:

$$(H^\dagger H) H^\dagger D^2 H - \mu^2 (H^\dagger H)^2 + 2\lambda (H^\dagger H)^3 = 0$$

Add the h.c:

$$(H^\dagger H)(D^2 H)^\dagger H + (H^\dagger H) H^\dagger D^2 H - 2\mu^2 (H^\dagger H)^2 + 4\lambda (H^\dagger H)^3 = 0$$

Integrating by parts:

$$-\partial_\mu (H^\dagger H)(D_\mu H)^\dagger H - \partial_\mu (H^\dagger H) H^\dagger D_\mu H - 2(H^\dagger H)(D_\mu H)^\dagger (D_\mu H) - 2\mu^2 (H^\dagger H)^2 + 4\lambda (H^\dagger H)^3 = 0$$

Using our previous result:

$$+(H^\dagger H)\Box(H^\dagger H) - 2(H^\dagger H)(D_\mu H)^\dagger (D_\mu H) - 2\mu^2 (H^\dagger H)^2 + 4\lambda (H^\dagger H)^3 = 0$$

This is the same result as above, since we only considered the EOM of the D4 Lagrangian we infer the error in this eqn is $O(\frac{1}{\Lambda^4})$

3) The D6 SMEFT

let $X = B_{\mu\nu}, W_{\mu\nu},$ or $G_{\mu\nu}$ we drop $\epsilon^{\mu\nu\rho\sigma} X_{\rho\sigma}$ for simplicity

$\psi = L, Q, e_R, u_R, d_R$

$H = H$

$D = D_\mu$ (any of the above)

recall $[X] = [\partial_\mu V_\mu] = 2$

$[\psi] = 3/2$

$[H] = 1$

$[D_\mu] = 1$

<u>Starting w/ purely bosonic D6 ops</u> we can infer the following op forms

X^3, $\underset{(e)}{\cancel{X^2D^2}}$, $\underset{(a)}{\cancel{XH^4}}$, $\underset{(b)}{\cancel{XD^4}}$ $\underset{(d)}{\cancel{XH^2D^2}}$, H^6, H^4D^2, $\underset{(c)}{\cancel{H^2D^4}}$,

X^2H^2

(a) XH^4 vanishes, the only way to contract Lorentz indices is $X_{\mu\mu} = 0$ bc X is antisymmetric

(b) $XD^4 \to X^2D^2$ bc X is antisymmetric $\to [D_\mu, D_\nu] \sim X_{\mu\nu}$

this is an exercise

(c) HD^4H are the only ops bc others are related by total derivatives

any ordering of D's that gives $[D_\mu, D_\nu] \implies XH^2D^2$ or X^2H^2

this only leaves terms w/ $D_\mu D_\mu H$ which by EOM $\Rightarrow H^4D^2$, $\psi^2 HD^2$

$$D^2H = \mu^2 H - \lambda(H^\dagger H)H - \text{Yukawas}$$

(d) $H^2XD^2 \propto X_{\mu\nu} D_\mu D_\nu H^2$

integrate by parts for $(D_\mu H)^\dagger (D_\nu H)$ removes those ops

if both D's are on X or an H $\rightarrow [D_\mu, D_\nu] \rightarrow X^2 H^2$

for $(DX)H(DH)$ we can remove by X's EOM: $\rightarrow H^4 D^2,\ \psi^2 HDH$

$$(D^\rho G_{\rho\mu})^A = \frac{g_s}{2}\left(\bar{Q}\gamma_\mu \lambda^A Q + \bar{u}_R \gamma_\mu \lambda^A u_R + \bar{d}_R \gamma_\mu \lambda^A d_R\right)$$

$$(D^\rho W_{\rho\mu})^I = \frac{g_2}{2}\left(H^\dagger i \overleftrightarrow{D}^I_\mu H + \bar{L}\gamma_\mu \sigma^I L + \bar{Q}\gamma_\mu \sigma^I Q\right)$$

$$(\partial^\rho B_{\rho\mu}) = g_1 Y_H H^\dagger i \overleftrightarrow{D}_\mu H + g_1 Y_\psi \bar{\psi}\gamma_\mu \psi$$

(e) X^2D^2 we can use integration by parts to place both D on one X

$$D_\mu X_{\mu\nu} = - D_\mu X_{\nu\mu} \rightarrow \text{EOM} \quad X\psi^2 D \text{ or } XD^2H$$

$$X_{\mu\nu} D_\rho D_\mu X_{\rho\nu} = X_{\mu\nu} D_\mu \underbrace{D_\rho X_{\rho\nu}}_{\text{EOM}} - \underbrace{X_{\mu\nu}[D_\mu, D_\rho] X_{\rho\nu}}_{X^3}$$

$$\rightarrow X\psi^2 D \text{ or } XD^2H$$

$$X_{\mu\nu} D_\rho D_\rho X_{\mu\nu} = -X_{\mu\nu}\left(D_\rho D_\mu X_{\nu\rho} + D_\rho D_\nu X_{\rho\mu}\right) \quad \text{by Bianchi} \quad D_{[\rho} X_{\mu\nu]} = 0$$

$$D_\rho D_\mu = \underbrace{[D_\rho, D_\mu]}_{X^3} + \underbrace{D_\mu D_\rho}_{\text{EOM} \rightarrow X\psi^2 D \text{ or } XD^2H}$$

So we only have X^3, X^2H^2, H^6, H^4D^2 left

X^3: $B_{\mu\nu} B_{\nu\rho} B_{\rho\mu} \to 0$ (antisymmetry of B)

$\epsilon^{IJK} W^I_{\mu\nu} W^J_{\nu\rho} W^K_{\rho\mu}$

$f^{ABC} G^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\sigma}$

I've entirely ignored: $\tilde{V}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} V_{\rho\sigma}$

X^2H^2: $(H^\dagger H) B_{\mu\nu} B_{\mu\nu}$

$(H^\dagger H) G^A_{\mu\nu} G^A_{\mu\nu}$

$(H^\dagger H) W^I_{\mu\nu} W^I_{\mu\nu}$

$(H^\dagger \frac{\sigma^I}{2} H) B_{\mu\nu} W^I_{\mu\nu}$

H^6: $(H^\dagger H)^3$

$(H^\dagger \sigma^A H)$ terms will be redundant

H^4D^2: $(H^\dagger H) \square (H^\dagger H)$

$(H^\dagger DH)(DH)^\dagger H$

others redundant by $\partial(\quad)=0$

Next we consider Ops w/ 2 fermions:

$[\bar\psi\psi] = 3$ + 3 combos of γ matrices: $\{\mathbb{1}, \gamma_\mu, \sigma_{\mu\nu}\}$

$\cancel{\psi^2 D^3}$ (a) $\quad \cancel{\psi^2 H D^2}$ (b) $\quad \cancel{\psi^2 X D}$ (c) $\quad \psi^2 H^3 \quad \psi^2 X H \quad \psi^2 H^2 D$

(a) 3 Lorentz indices $\rightarrow \bar\psi \gamma_\mu \psi D_\mu D^2$

Use integration by parts to remove $D\bar\psi$

Since $\not{D}\psi \propto$ Yukawas by EOM we can remove these for $\psi^2 H D^2$

b) 2 Lorentz indices $\rightarrow \bar\psi_L \psi_R$ & $\bar\psi_L \sigma_{\mu\nu} \psi_R$

Integrate by parts so no $D\bar\psi$

$\bar\psi \sigma^{\mu\nu} \psi D_\mu D_\nu H$ & $\bar\psi \sigma_{\mu\nu} D_\mu D_\nu \psi H \quad \propto [D_\mu, D_\nu] \rightarrow \psi^2 X D$

$\bar\psi \psi D^2 H \rightarrow \bar\psi\psi(\mu^2 H + H^3 + \bar\psi\psi)$

$\bar\psi D^2 \psi H = \bar\psi D_\mu D_\nu g_{\mu\nu} \psi H = \bar\psi D_\mu D_\nu (\gamma_\mu \gamma_\nu - i\sigma_{\mu\nu}) \psi H$

$\bar\psi \not{D}\not{D} \psi H \rightarrow \bar\psi D\psi H^2$

$\bar\psi \sigma_{\mu\nu} D_\mu D_\nu \psi H \rightarrow \bar\psi\psi X H$

$(DH)\bar\psi \sigma^{\mu\nu} D_\nu \psi = \frac{i}{2}(D_\mu H)\bar\psi(\gamma^\mu \not{D} - \not{D}\gamma^\mu)\psi = i(D_\mu H)\bar\psi \gamma^\mu \not{D}\psi - \frac{i}{2}(D_\mu H)\bar\psi \underbrace{\{\gamma^\mu, \gamma_\nu\}}_{+2g_{\mu\nu}} D_\nu \psi$

$= i(D_\mu H)\bar\psi \gamma_\mu \not{D}\psi + i(D_\mu H)\bar\psi D_\mu \psi$

$\rightarrow \psi^2 H D + (D_\mu H)\bar\psi D_\mu \psi$

$(D_\mu H)\bar{\psi} D_\mu \psi \propto (D_\mu H)\bar{\psi}(\gamma_\mu \not{D} + \not{D}\gamma_\mu)\psi$

$= (D_\mu H)\bar{\psi}\gamma_\mu \not{D}\psi + (D_\nu \bar{\psi})\gamma_\nu \gamma_\mu \psi D_\mu H + \bar{\psi}\gamma_\mu \gamma_\nu \psi D_\mu D_\nu H$

$\not{D}\psi$ & $D_\nu\bar{\psi}\gamma_\nu$ are EOM $\Rightarrow DH^2\psi^2$

$\bar{\psi}\gamma^\nu\gamma^\mu \psi D_\nu D_\mu H = \bar{\psi}\psi D^2 H - i\bar{\psi}\sigma^{\mu\nu}\psi D_\nu D_\mu H$

$\rightarrow \bar{\psi}\psi(mH + H^3 + \bar{\psi}\psi)$

c) $\psi^2 XD \rightarrow 3$ Lorentz indices, only $\bar{\psi}\gamma_\mu\psi$

the D must be contracted w/ X

For $D_\mu X_{\mu\nu}$, $\psi^2 DX \rightarrow \psi^2(\bar{\psi}\psi + HDH)$

Integrating by parts (DF) puts remaining Ds on ψ

$X_{\mu\nu}\bar{\psi}\gamma_\mu D_\nu \psi = X_{\mu\nu}\bar{\psi}\gamma_\mu g_{\nu\rho} D_\rho \psi$

$= \frac{1}{2} X_{\mu\nu}\bar{\psi}\gamma_\mu \{\gamma_\nu, \gamma_\rho\} D_\rho \psi$ $\qquad g_{\mu\nu} = \frac{1}{2}\{\gamma_\mu, \gamma_\nu\}$

$= \frac{1}{2} X_{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu \not{D} + \gamma_\mu \not{D}\gamma_\nu)\psi$

$= \frac{1}{2} X_{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu \not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi + X_{\mu\nu}\bar{\psi} D_\mu \gamma_\nu \psi$

original op form w/ $\mu \leftrightarrow \nu$

$= \frac{1}{4} X_{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi$

integrate by parts

$= \frac{1}{4} X_{\mu\nu}\bar{\psi}\gamma_\mu\gamma_\nu\not{D}\psi + \frac{1}{4}X_{\mu\nu}\bar{\psi}\overleftarrow{\not{D}}\gamma_\mu\gamma_\nu\psi + \frac{1}{4}\underbrace{\bar{\psi}\gamma_\rho\gamma_\mu\gamma_\nu\psi}_{g_{\rho\mu}\gamma_\nu + g_{\mu\nu}\gamma_\rho - g_{\rho\nu}\gamma_\mu - i\epsilon_{\rho\mu\nu\sigma}\gamma^\sigma\gamma_5} D^\rho X^{\mu\nu}$

$\rightarrow X_{\mu\nu}\bar{\psi}(\psi H) + X_{\mu\nu}(\bar{\psi}H)\psi + \bar{\psi}\psi(HDH + \bar{\psi}\psi)$ + bianchi

$D_{[\rho} X_{\mu\nu]} = 0$

d) $\psi^2 H^3$ - in this case we won't reduce the basis, but identify the op forms

no Lorentz indices $\rightarrow \bar{\psi}\psi \rightarrow$ LR or RL

so one doublet one singlet

H doesn't have color $\rightarrow \bar{\psi}\psi$ is color singlet

For SU(2) closure we need H^3 is a doublet

note: $HH \rightarrow \epsilon_{ij} H_i H_j = 0$

$H^\dagger \tilde{H} \rightarrow \epsilon_{ij} H_i^* H_j^* = 0$

This leaves only $H^\dagger H \times$ Yukawa, eg $H^\dagger H \bar{Q} H d_R$ + h.c.

e) $\psi^2 X H \rightarrow$ 2 Lorentz indices

$\bar{\psi}\sigma_{\mu\nu}\psi X_{\mu\nu} H \rightarrow$ LR or RL

Hypercharge invariance requires these be Yukawa like

$(\bar{Q}\sigma_{\mu\nu} u_R) B_{\mu\nu} H$

$(\bar{Q}\sigma_{\mu\nu}\sigma^I u_R) W^I_{\mu\nu} H$

$(\bar{Q}\sigma_{\mu\nu}\lambda^A u_R) G^A_{\mu\nu} H$

f) $\psi^2 H^2 D$

1 lorentz index → $\bar{\psi}\gamma_\mu\psi$

For $D\bar{\psi}$ & $D\psi$ EOM → $\psi^2 H^3$ case

$H^\dagger$ & H → singlets & triplets of SU(2)

→ $\bar{\psi}\psi$ must mimic this for invariance

We need to check if h.c is unique/indep:

$$H^\dagger(D_\mu + \underbrace{\overleftarrow{D}_\mu}_{\text{h.c}})H\bar{\psi}\gamma_\mu\psi = \partial_\mu(H^\dagger H)\bar{\psi}\gamma_\mu\psi$$

$$= (H^\dagger H)\bar{\psi}(\not{D} + \overleftarrow{\not{D}})\psi$$

So the h.c. is related to $\psi^2 H^3$ by EOM & can be ignored

But we want to write these ops in a manifestly hermitian way:

$$i H^\dagger \overleftrightarrow{D}_\mu H = i[(H^\dagger D_\mu H) - (D_\mu H)^\dagger H]$$

$$i H^\dagger \overleftrightarrow{D}^I_\mu H = i\left[(H^\dagger D_\mu \tfrac{\sigma^I}{2} H) - (D_\mu H)^\dagger \tfrac{\sigma^I}{2} H\right]$$

So we have:

$\psi^2 H^3$: $(H^\dagger H)\,\bar{L} e_R H$

$(H^\dagger H)\,\bar{Q} u_R \tilde{H}$

$(H^\dagger H)\,\bar{Q} d_R H$

$\psi^2 X H$: $(\bar{\psi}_L \sigma_{\mu\nu} \psi_R) B_{\mu\nu} H$

$(\bar{\psi}_L \sigma_{\mu\nu} \frac{\sigma^I}{2} \psi_R) W^I_{\mu\nu} H$

$(\bar{\psi}_L \sigma_{\mu\nu} \frac{\lambda^A}{2} \psi_R) G^A_{\mu\nu} H$

$\psi^2 H^2 D$: $(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\psi} \gamma_\mu \psi)$

$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{\psi} \gamma_\mu \frac{\sigma^I}{2} \psi)$

$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_R \gamma_\mu d_R)$ + h.c.

Four Fermion operators:

recall $\psi^c = i\gamma_2 \psi^*$
choice, can be diff

choosing a basis of u, v onshell spinors on can show

$(u_\uparrow)^c = v_\downarrow \qquad (v_\uparrow)^c = u_\downarrow$

$(u_\downarrow)^c = v_\uparrow \qquad (v_\downarrow)^c = u_\uparrow$

so c-conj takes particles $\leftrightarrow$ antiparticles & spin $\leftrightarrow$ -spin

We'll take all RH fermions & cconj them so we only have LH fermions
Then the set of all 4-ferm ops obeying Lorentz & Hypercharge invar:
(ie ignoring SU(2))

$$(A)\quad \begin{cases} \bar\psi^c\psi^c\bar\chi^c\chi^c & \text{for } \psi,\chi\in\psi_R,\chi_R \\ \bar\psi\psi\bar\chi\chi & \text{for } \psi,\chi\in\psi_L,\chi_L \\ \bar\psi^c\psi^c\bar\chi\chi & \text{for } \psi\in\psi_R,\ \chi\in\chi_R \end{cases}$$

(B) these come from simply trying all combos

$$\begin{cases} L\bar e^c_R d^c_R Q,\quad Q u^c_R Q d^c_R,\quad L e^c_R Q u^c_R,\quad QQQL,\quad d^c_R u^c_R u^c_R e^c_R \\ QQ\bar u^c_R \bar e^c_R \quad QL\bar u^c_R d^c_R \end{cases}$$

(A) all are vector currents: $(\bar\psi^c_L \psi^c_L) =$

Fierz Identities: relate products of fermionic fields, e.g:

$$(\bar{\psi}_\pm \gamma_\mu \psi_\pm)(\bar{\chi}_\pm \gamma^\mu \chi_\pm) = (\bar{\psi}_\pm \gamma_\mu \chi_\pm)(\bar{\chi}_\pm \gamma_\mu \psi_\pm)$$

Note in (A) we don't have to consider $(\bar{\psi}^c \chi^c)(\bar{\chi}^c \psi^c)$ because of this

Four Fermion Ops

Recall: $\bar{\Psi}\chi_{\pm} \to \bar{\Psi}_{\mp}\chi_{\pm}$ w/ $\chi_{+} = \chi_R$, $\chi_{-} = \chi_L$

$\bar{\Psi}\gamma_\mu\chi_{\pm} \to \bar{\Psi}_{\pm}\gamma_\mu\chi_{\pm}$

$\bar{\Psi}\sigma_{\mu\nu}\chi_{\pm} \to \bar{\Psi}_{\mp}\sigma_{\mu\nu}\chi_{\pm}$

We can form 4-fermion ops of these 3 "bilinears"

Note bc of Lorentz invar. we will only have

$(\bar{\Psi}\chi)(\bar{\Psi}'\chi')$

$(\bar{\Psi}\gamma_\mu\chi)(\bar{\Psi}'\gamma_\mu\chi')$

$(\bar{\Psi}\sigma_{\mu\nu}\chi)(\bar{\Psi}'\sigma_{\mu\nu}\chi')$

Further the vector currents dont mix chirality so they will give ops

w/ $(\bar{L}L)(\bar{L}L)$, $(\bar{L}L)(\bar{R}R)$ and $(\bar{R}R)(\bar{R}R)$ forms

$SU(2)_L$ invariance will be harder for the others.

Starting w/ the easy ops:

$(\bar{L}L)(\bar{L}L)$	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$
$(\bar{L}\gamma_\mu L)(\bar{L}\gamma_\mu L)$	$(\bar{e}\gamma_\mu e)^2$	$(\bar{L}\gamma_\mu L)(\bar{e}\gamma_\mu e)$
$(\bar{Q}\gamma_\mu Q)(\bar{Q}\gamma_\mu Q)$	$(\bar{u}\gamma_\mu u)^2$	$(\bar{L}\gamma_\mu L)(\bar{u}\gamma_\mu u)$
$(\bar{Q}\gamma_\mu\sigma^I Q)(\bar{Q}\gamma_\mu\sigma^I Q)$	$(\bar{d}\gamma_\mu d)^2$	$(\bar{L}\gamma_\mu L)(\bar{d}\gamma_\mu d)$
$(\bar{L}\gamma_\mu L)(\bar{Q}\gamma_\mu Q)$	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$	$(\bar{Q}\gamma_\mu Q)(\bar{e}\gamma_\mu e)$
$(\bar{L}\gamma_\mu\sigma^I L)(\bar{Q}\gamma_\mu\sigma^I Q)$	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$	$(\bar{Q}\gamma_\mu Q)(\bar{u}\gamma_\mu u)$
	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$	$(\bar{Q}\gamma_\mu Q)(\bar{d}\gamma_\mu d)$
	$(\bar{u}\gamma_\mu\lambda^A u)(\bar{d}\gamma_\mu\lambda^A d)$	$(\bar{Q}\gamma_\mu\lambda^A Q)(\bar{u}\gamma_\mu\lambda^A u)$
		$(\bar{Q}\gamma_\mu\lambda^A Q)(\bar{d}\gamma_\mu\lambda^A d)$

Note for $(\bar{L}\gamma_\mu L)^2$ we don't have $(\bar{L}\gamma_\mu \sigma^I L)^2$ this is because

$$\sigma^I_{ij}\sigma^I_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$$

So they are equivalent.

This isn't the case for $(\bar{Q}\gamma_\mu \sigma^I Q)^2$ bc Q has color

Similarly we dont have: $(\bar{u}\gamma_\mu \lambda^A u)^2$, $(\bar{d}\gamma_\mu \lambda^A d)^2$, $(\bar{Q}\gamma_\mu \lambda^A Q)^2$ because:

$$\lambda^A_{\alpha\beta}\lambda^A_{\rho\sigma} = \frac{1}{2}\delta_{\alpha\sigma}\delta_{\beta\rho} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\rho\sigma}$$

And we dont have $(\bar{Q}\gamma_\mu \lambda^A \sigma^I Q)$ because of the above two identities

(When taking flavor into account we also need the Fierz identity:

$$(\bar{\psi}_L \gamma_\mu \psi'_L)(\bar{\chi}_L \gamma_\mu \chi'_L) = (\bar{\psi}_L \gamma_\mu \chi'_L)(\bar{\chi}_L \gamma_\mu \psi'_L)$$

The remaining operators aren't as easy to obtain. We can start by identifying all $U(1)_Y$ invariant combinations (excluding those above)

(note: Lorentz invar requires $\bar{\psi}\chi$ form or $(\psi^T C \chi)$

$$\left.\begin{matrix} \bar{Q}\bar{Q}du & \bar{Q}\bar{L}eu & \bar{Q}\bar{e}dL \\ \bar{d}\bar{u}QQ & \bar{e}\bar{u}QL & \bar{d}\bar{L}eQ \end{matrix}\right\} \text{h.c.s of one another}$$

We can write these 4-fermion ops by requiring SU(2) & SU(3) closure:

$$\bar{Q}\bar{Q}du \rightarrow (\bar{Q}u)(\bar{Q}d)$$

$\bar{Q} \rightarrow e^{-i\theta\cdot\frac{\sigma}{2}}$ so we can use $i\sigma_2 \rightarrow \epsilon_{ij}$ as in yukawas

$$\rightarrow (\bar{Q}_i u)\epsilon_{ij}(\bar{Q}_j d)$$

This is already invar under SU(3), w/ $(\bar{Q}_i^\alpha u^\alpha)\epsilon_{ij}(\bar{Q}_j^\beta d^\beta)$ + h.c.

But we can also have SU(3) invar w/ $(\bar{Q}_i \lambda^A u)\epsilon_{ij}(\bar{Q}_j \lambda^A d)$ + h.c.

Remember we need both scalar,

$$(\bar{\Psi}\chi)(\bar{\Psi}'\chi')$$

and tensor combinations:

$$(\bar{\Psi}\sigma^{\mu\nu}\chi)(\bar{\Psi}'\sigma^{\mu\nu}\chi')$$

But there is a Fierz identity that allows us to consider only the scalar combos:

$$(\bar{\Psi}_1 \sigma_{\mu\nu} P_L \psi_2)(\bar{\Psi}_3 \sigma_{\mu\nu} P_L \psi_4) = -4(\bar{\Psi}_1 P_L \psi_4)(\bar{\Psi}_3 P_L \psi_2) - 8(\bar{\Psi}_1 P_L \psi_2)(\bar{\Psi}_3 P_L \psi_4)$$

Notice: $(\bar{Q}_1 u_2)(\bar{Q}_3 d_4) \sim (\bar{Q}_1 d_4)(\bar{Q}_3 u_2)$

so the two ops on the Right are included by considering just the one combo of $(\bar{Q}u)(\bar{Q}d)$

For $(\bar{Q}d)(\bar{e}L)$ we cando the same, in this case $\bar{Q}L$ forms an SU(2) singlet so we have:

$$(\bar{Q}_i d)(\bar{e}L_i)$$

Again we can use the Fierz ID to remove tensor operators by noting $(\bar{Q}L)(\bar{e}d) = 0$ as $\bar{\Psi}_+ \psi_+ = \bar{\Psi}_+ P_- P_+ \psi = 0$

For $(\bar{Q}u)(\bar{L}e)$ again SU(2) closure comes from:

$$(\bar{Q}_i u)(\bar{L}_i e)$$

In this case we do not remove the tensor operator so that the color indices are closed w/in the bilinear.

So we have 2 ops:

$$(\bar{Q}u)(\bar{L}e) \;\; \& \;\; (\bar{Q}\sigma_{\mu\nu} u)(\bar{L}\sigma_{\mu\nu} e)$$

Repeating w/ $\psi_i^T C$ we have the following Hypercharge invariants:

$$(Q^c Q)(Q^c L) \quad (Q^c Q)(u^c e)$$

$$(d^c u)(Q^c L) \quad (d^c u)(u^c e)$$

Notice all of these have 2 barred quarks and 1 quark field so the violate Baryon number conservation,

$$\psi_q \to e^{i\frac{\theta_B}{3}}$$

They can be put in the following forms that are $SU(2)\times SU(3)$ invar:

$$\epsilon^{\alpha\beta\gamma}\epsilon_{ik}\epsilon_{\ell m}[(Q^{\alpha i})^T C Q^{\beta\ell}][Q^{\gamma m} C L^k]$$

$$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}[(Q^{\alpha j})^T C Q^{\beta k}][(u^{\gamma})^T C e]$$

$$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}[(d^{\alpha})^T C u^{\beta}][(Q^{\gamma})^T C L^k]$$

$$\epsilon^{\alpha\beta\gamma}[(d^{\alpha})^T C u^{\beta}][(u^{\gamma})^T C e]$$

Again the Fierz Identity allows us to remove tensor currents, as in the case of $(\bar{Q}L)(\bar{e}d)$