Problem Set 4

1. Gauge Invariance:
   Demonstrate that the gauge-kinetic term for the photon,
   $$\mathcal{L}_{\text{free}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1}$$
   Is invariant under the gauge transformation:
   $$A_\mu \to A_\mu + \partial_\mu \alpha(x) \tag{2}$$

2. Interaction vertex for QED:
   Derivate the Feynman rule coupling the photon A_μ and the electron field ψ from the interaction Lagrangian:
   $$\mathcal{L}_{\text{I}} = -e\bar{\psi}_a(x)\gamma^\mu_{ab}\psi_b(x)A_\mu(x) \tag{3}$$
   Where a, b are Dirac indices.

3. Traces of Dirac Matrices:
   Derive the following Dirac trace identities:
   (a) $\text{Tr}[\text{odd } \#\gamma] = 0$
   (b) $\text{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$
   (c) $\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$

4. Feynman Diagrams for QED at tree level:
   Draw the tree-level Feynman diagram(s) corresponding to the following processes:
   (a) Bhabha scattering: $e^+e^- \to e^+e^-$
   (b) Møller scattering: $e^-e^- \to e^-e^-$
   (c) $e^+e^- \to \mu^+\mu^-$
   (d) Compton scattering: $e^-\gamma \to e^-\gamma$

5. Feynman Diagrams for QED at one loop:
   Draw the one-loop Feynman diagram(s) corresponding to:
   (a) the photon propagator correction (aka self-energy, vacuum polarization)
   (b) the electron propagator correction
   (c) the vertex coupling a photon, an electron, and a positron
   (d) the vertex coupling four photons

6. Calculate the cross section for $e^+e^- \to \mu^+\mu^-$, spin average (sum) the incoming (outgoing) spinors.
   hint: Apply the Dirac equation relations to simplify your life,
   $$(\not{p} - m)u_s(p) = \bar{u}_s(p)(\not{p} - m) = 0 \tag{4}$$
   $$(\not{p} + m)v_s(p) = \bar{v}_s(p)(\not{p} + m) = 0 \tag{5}$$
   The Mandelstam variables also need to be reevaluated for the different masses in this problem from our example in class.
   Refer to the Phase space integration note on Moodle to get the phase space correct.
   Refer to Schwartz 13.3 for the full calculation.