Problem Set 8

1. Gauge fixing the Abelian Higgs

   (a) Starting from the gauge transformations of A_μ and ϕ,

   $$\begin{aligned} \delta A_\mu &= -\partial_\mu \alpha\,, \\ \delta\phi &= ie\alpha\phi\,, \\ \delta\phi^* &= -ie\alpha\phi^*\,, \end{aligned} \tag{1}$$

   derive the transformation of π:

   $$\delta\pi = \alpha e(v+\sigma) \tag{2}$$

   (b) using this show:

   $$\delta G = \left[-\Box - \xi e^2 v(v+\sigma)\right]\alpha \tag{3}$$

   (c) Conclude that the ghost Lagrangian is:

   $$\mathcal{L}_{\text{gh}} = (\partial_\mu \bar{c}^a)(\partial^\mu c^a) - \xi e^2 v^2 \bar{c}^a c^a - \xi e^2 v\sigma \bar{c}^a c^a \tag{4}$$

   (d) Show:

   $$\mathcal{L}_{\text{gf}} = \frac{-1}{2\xi}(\partial_\mu A_\mu)^2 - \frac{\xi e^2 v^2}{2}\pi^2 - evA_\mu(\partial_\mu \pi)\,, \tag{5}$$

   The last term exactly cancels the $A-\pi$ mixing from the "classical Lagrangian" (i.e. the Lagrangian before gauge fixing)

2. Ward Identities for Abelian Higgs

   (a) using the transformations in Eq. 1, derive the "Master Ward Identity:"

   $$\partial_\mu \frac{\delta\Gamma}{\delta A_\mu} + ie\alpha\left(\frac{\delta\Gamma}{\delta\phi}\phi - \frac{\delta\Gamma}{\delta\phi^*}\phi^*\right) = 0 \tag{6}$$

   (b) Starting with the Master Ward ID, set all fields to their "background values"/vacuum expectation values, use that their cannot be a photon tadpole (it violates Lorentz invar.), and use the chain rule to rewrite the variation w/r to ϕ as a sum of variations w/r to σ and π recalling:

   $$\phi = \frac{v+\sigma+i\pi}{\sqrt{2}} \tag{7}$$

   Show that the π particle does not develop a Tadpole, i.e.:

   $$\frac{\delta\Gamma}{\delta\pi} = 0 \tag{8}$$

   (c) Derive the following Ward ID from Eq. 6:

   $$\partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta A_\nu} = ev\frac{\delta^2\Gamma}{\delta A_\nu \delta\pi} \tag{9}$$

   We conclude that the photon has a Longitudinal component, and it is proportional to the mixing between the photon and the (pseudo-) Goldstone boson.

(d) Further derive the following identity:

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta A_\mu \delta \pi} = ev \frac{\delta^2 \Gamma}{\delta \pi \delta \pi} + v \frac{\delta \Gamma}{\delta \sigma} \tag{10}$$

We conclude there is a further relation between the $A - \pi$ mixing, the π two-point function, and the tadpole diagram (for σ). These Ward identities have direct analogues in the SM.