# Problem Set 7

1. Charged scalars: we so far have only charged fermions under $U(1)_{\text{QED}}$ and $SU(N)$, here we consider scalars charged under these groups.

   (a) For a complex scalar field the free Lagrangian is:

   $$\mathcal{L} = a(\partial_\mu \phi^*)(\partial^\mu \phi) - am^2 \phi^* \phi \tag{1}$$

   For the real scalar we had something like $a = 1/2$. In the case of the complex scalar $a = 1$, why?
   hint: If you don't know derive the propagator without setting $a = 1$ and think about our on-shell renormalization conditions.

   (b) Assume the $\phi$ have electric charge $Q_\phi = 1$. Gauge the theory and identify the vertices, their dependence on the coupling $e$, momentum, etc. (Don't derive the Feynman rules, just indicate the general dependence as we did for Yang Mills coupled to fermions.

   (c) Repeat the last point for $SU(N)$, make sure to identify the generator dependence of the vertices. Keep in mind, the generators of $SU(N)$ have the property $(T^a)^\dagger = T^a$.

2. Ward Identity

   Assuming the effective action, $\Gamma$, is gauge invariant show the gluon is transverse. Keep in mind that the gauge transformation of the Yang Mills field is:

   $$\delta A^a_\mu = -\partial_\mu \alpha^a + g f^{abc} \alpha^b A^c_\mu \tag{2}$$

   The second term *was not present* for QED.

3. Read Schwartz 25.3 about conserved currents in Yang Mills.