The Standard Model

1) Field content

So far we've seen

electron: e

up-like quark: u

down-like quark: d

We'll add neutrinos: ν

We can split these into their chiralities

$$P_L e = e_L \qquad\qquad e \to e_L$$
$$P_R e = e_R \quad \Rightarrow \quad e \to e_R$$

$$u \to u_L$$
$$u \to u_R$$

$$d \to d_L$$
$$d \to d_R$$

$\nu \to \nu_L$ ← no RH neutrinos in the SM!

The SM is a "chiral" thry → L & RH fields are charged under different gauge groups

We combine the LH fields into "doublets"

$$L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

For simplicity, we drop the R label on RH fermions

$$e_R \to e$$
$$u_R \to u$$
$$d_R \to d$$

These are the fundamental matter fields (fermions) of the SM/nature

$L,\ Q,\ e,\ u,\ d$ $\leftarrow$ 5 in total

Nature liked them so much it gave us 3 "generations" of each

$$L_1 = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \qquad Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$ $\leftarrow$ 6 quarks $\rightarrow$ "flavors"

$e_R,\ \mu_R,\ \tau_R$

$u_R,\ c_R,\ t_R$

$d_R,\ s_R,\ b_R$

The gauge group of the SM is:

$$U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$$

hyperchage · weak isospin · color, QCD

The following charges are given to the 5 fermions:

$$(\text{Color}, \text{Isospin})_{\text{hypercharge}}$$

Color: rep for $SU(3)_C$

Isospin: rep for $SU(2)_L$

hypercharge: charge, Y, under $U(1)_Y$

$L_i: \quad (1, 2)_{-1/2} \leftarrow Y = -1/2$

doublet of $SU(2)_L$ (ie in the fundamental)

1: singlet of $SU(3)_C$, ie not charged under $SU(3)_C$

L_i: all generations the same

$Q_i: \quad (3, 2)_{1/6}$

3: in the fundamental of $SU(3)_C$

$e: \quad (1, 1)_{-1}$

$u: \quad (3, 1)_{2/3}$

$d: \quad (3, 1)_{-1/3}$

Notice RH are singlets under $SU(2)_L$

LH are doublets under $SU(2)_L$

L: Left

The $SU(2)_L$ gauge fields only see LH fermions!

$U(1)_Y$ " " see both, but interacts differently w/ L- & R-H fields

The gauge fields are denoted:

$U(1)_Y$: B_μ

$SU(2)_L$: W_μ^I ← $SU(2)_L$ adjoint → $3 = (2^2-1)$ fields

$SU(3)_c$: G_μ^A ← $SU(3)_c$ ajoint → 8 fields

So the covariant derivative of a fermion ψ is:

$$D_\mu \psi = (\partial_\mu + i g_1 \underbrace{Y_\psi}_{\text{hypercharge}} B_\mu + i g_2 \frac{\sigma^I}{2} W_\mu^I + i g_3 \frac{\lambda^A}{2} G_\mu^A)\psi$$

← notice coupling to B dictated by Y_ψ, coupling to W/G is dictated by representation

g_i is the gauge coupling for $(S)U(i)$

$\frac{\sigma^I}{2}$ are the generators of $SU(2)_L$, σ^I = pauli matrices

$\frac{\lambda^A}{2}$ are the generators of $SU(3)_c$, λ^A = Gell-Mann matrices

If a fermion isn't charged under one of the gauge groups (is a singlet of a gauge group) we drop that term from D_μ

Ex:

$$D_\mu L = (\partial_\mu + i g_1 Y_L B_\mu + i g_2 \frac{\sigma^I}{2} W_\mu^I) L$$

$$D_\mu Q = (\partial_\mu + i g_1 Y_Q B_\mu + i g_2 \frac{\sigma^I}{2} W_\mu^I + i g_3 \frac{\lambda^A}{2} G_\mu^A) Q$$

$$D_\mu e = (\partial_\mu + i g_1 Y_e B_\mu) e$$

However some of the gauge fields are massive, and the chiral charge assignments forbid fermion masses:

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

L & RH fermions charged differently so this isn't gauge invariant!

To resolve this we add a scalar field, the Higgs boson:

$$H: \quad (1,2)_{1/2}$$

charged, so H is complex

Degrees of freedom:

$B_\mu \to$ 2 spin

$W^I_\mu \to$ (2 spin)x(3 isospin)

$G^A_\mu \to$ (2 spin)x(8 color)

$L \to$ (2 spin)x(2 isospin)x(2 $\mathbb{C}$ field)

$Q \to$ (2 spin)x(2 isospin)x(3 color)x(2 $\mathbb{C}$ field)

$e \to$ (2x spin)x(2 $\mathbb{C}$ field)

$u \to$ (2x spin)x(3 color)x(2 $\mathbb{C}$ field)

$d \to$ (2x spin)x(3 color)x(2 $\mathbb{C}$ field)

All x 3 generations

$H \to \underbrace{(1 \times \text{spin}) \times (2\ \text{isospin}) \times (2\ \mathbb{C}\ \text{field})}_{4\ \text{degrees of freedom}}$

3 of these will become longitudinal modes of the W's via the Higgs Mechanism resulting in massive gauge bosons

2) Gauge xforms of the fields & SM $\mathcal{L}$:

For fermions & the Higgs the general gauge xform is:

$$F \to \underbrace{e^{iY_F g_1 \Theta_Y} e^{ig_2 \Theta_i \cdot \frac{\sigma}{2}} e^{ig_3 \Theta_c \cdot \frac{\lambda}{2}}}_{\text{again drop terms under which F isnt charged}} F$$

Ex: $Q \to e^{i\frac{1}{6}g_1\Theta_Y} e^{ig_2\Theta_i\cdot\frac{\sigma}{2}} e^{ig_3\Theta_c\cdot\lambda/2} Q$

$d \to e^{-i\frac{1}{3}g_1\Theta_Y} e^{ig_3\Theta_c\cdot\frac{\lambda}{2}} d$

$H \to e^{i\frac{1}{2}g_1\Theta_Y} e^{ig_2\Theta_i\cdot\frac{\sigma}{2}}$

The gauge bosons xform per our discussion of YM:

$$B_\mu \to U_Y B_\mu U_Y^{-1} - \frac{i}{g_1} U_Y(\partial_\mu U_Y^{-1}) \to B_\mu - \partial_\mu \Theta_Y \qquad \text{w/ } U_Y = e^{ig_1\Theta_Y}$$

$$W_\mu \equiv W_\mu^i \frac{\sigma^i}{2} \to U_L W_\mu U_L - \frac{i}{g_2} U_L(\partial_\mu U_L^{-1}) \qquad U_L = e^{ig_2\Theta_i\cdot\frac{\sigma}{2}}$$

$$G_\mu \equiv G_\mu^A \frac{\lambda^A}{2} \to U_c G_\mu U_c - \frac{i}{g_3} U_c(\partial_\mu U_c^{-1}) \qquad U_c = e^{ig_3\Theta_c\cdot\frac{\lambda}{2}}$$

From this we can write all the familiar terms in $\mathcal{L}_{SM}$ from YM:

$$\mathcal{L}_{Gauge} = -\frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

$$\mathcal{L}_{Fermion} = i\bar{L}\not{D}L + i\bar{Q}\not{D}Q + i\bar{e}\not{D}e + i\bar{u}\not{D}u + i\bar{d}\not{D}d \qquad \leftarrow \text{5 massless chiral fermions}$$

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger(D^\mu H) + \underbrace{|\mu|^2(H^\dagger H) - \lambda(H^\dagger H)^2}_{-V(H) \text{ has wrong sign mass, you'll explore this in the HW}}$$

If fermions were observed to be massless this would be enough

Fermion masses require:

$$\mathcal{L}_{Yukawa} = Y_e\bar{L}He + Y_d\bar{Q}Hd + Y_u\bar{Q}i\sigma_2H^*u \quad \text{+h.c.}$$

In general we can have:

eg $c_{ij} \bar{L}_i \not{D} L_j + c'_{ij} \bar{e}_i \not{D} e_j + Y_{ij} \bar{L}_i H e_j$

But we can make field redefinitions $e_{Li} \to e^{i\theta_{ij}} e_{Lj}$

$\nu_{Li} \to e^{i\theta_{ij}} \nu_{Lj}$

$e_{Ri} \to e^{i\theta_{ij}} e_{Rj}$

To arrive at a diagonal form:

$$\bar{L}_i \not{D} L_i + \bar{e}_i \not{D} e_i + Y_{ii} L_i H e_i$$

For quarks there is not enough freedom $\to$ CKM matrix

See Schwartz 29.3.2

This is important for "flavor" physics, but we will neglect it taking

$$\bar{Q}_i \not{D} Q_i + \bar{u}_i \not{D} u_i + \bar{d}_i \not{D} d_i + Y_{dii} \bar{Q}_i H d_i + Y_{uii} \bar{Q}_i (i\sigma_2) u_i + hc$$

So the SM $\mathcal{L}$ depends on

3 gauge coupling: g_1, g_2, g_3

3x3 Yukawa couplings: Y_{eii}, Y_{dii}, Y_{uii}

2 parameters in $\mathcal{L}_H$: μ, λ

$\Rightarrow$ 14 parameters

$+$ 4 CKM

$+$ 1 "strong CP" (again we wont discuss, See Schwartz 29.5.3)

$\Rightarrow$ 19 parameters

3) Gauge fixing (Srednicki 71)

Our previous method of gauge fixing let us see exactly how we were manipulating the path Integral, now we will derive a more general method that allows us to skip intermediate steps

This will allow us to quickly gauge fix models w/ spontaneous symmetry breaking

We had,

$$Z[J] \propto \int DA_\mu \, e^{iS_{YM}(A,J)}$$

Consider the normal integral:

$$Z = \int dx \, e^{iS(x)}$$

$$= \int dx\,dy \, \underbrace{\delta(y)}_{\text{can shift } \delta \text{ by a } \xi(x) \text{ w/ changing the integral}} e^{iS(x)}$$

$$= \int dx\,dy \, \delta(y-\xi(x)) e^{iS(x)}$$

Recall a general $\delta(G(x,y))$ can be written interms of y as:

$$\delta(G(x,y)) = \frac{\delta(y-\xi(x))}{|\partial G/\partial y|} \rightarrow \left|\frac{\partial G}{\partial y}\right| \delta(G) = \delta(y-\xi(x))$$

So we have:

$$Z = \int dx\,dy \, \frac{\partial G}{\partial y} \delta(G) e^{iS}$$

Generalizing to n integrations over x & y:

$$Z = \int d^n x \, d^n y \, \det\left(\frac{\partial G_i}{\partial y_i}\right) \Pi_i \, \delta(G_i) e^{iS}$$

$\uparrow$ $G_i(x,y)$ are n functions fixing all n components of y

For our Path integral this gives:

$$Z[J] \propto \int DA_\mu \det\left(\frac{\delta G}{\delta \theta}\right) \Pi_{x,a} \, \delta(G) e^{iS_{YM}}$$

$\uparrow$ y becomes the gauge redundancy ie set of all gauge xforms

Now we can choose any convenient G and quickly gauge fix

For YM we take:

$$G^a(x) \equiv \partial^\mu A^a_\mu(x) - \alpha^a(x)$$

We had:

$$\delta A^a_\mu = -\partial_\mu \alpha^a + g f^{abc} \alpha^b A^c_\mu$$
$$= -(\partial_\mu \delta^{ac} - g f^{abc} A^b_\mu)\alpha^c$$
$$\equiv - D^{ab}_\mu \alpha^b$$

So $\delta G^a = -\partial^\mu D^{ab}_\mu \alpha^b$

$$\frac{\delta G^a(x)}{\delta \theta^b(y)} = -\partial^\mu D^{ab}_\mu \delta^4(x-y)$$

But we know

$$\det(-\partial^\mu D_\mu) \propto \int Dc\, D\bar{c}\, e^{+i\int d^4x \underbrace{\bar{c}^a(-\partial^\mu D^{ab}_\mu)c^b}_{=(\partial_\mu \bar{c}^a)(\partial_\mu \delta^{ac} - g f^{abc} A^b_\mu)c^c}}$$

in QED $f \to 0$, so in QED we "have" ghosts, but they don't interact and we instead can fully integrate the PI for an overall constant

Note: $G^a(x)$ depends on the arbitrary α^a, however the PI is indep of α^a so we can multiply by an arbitrary functional of α^a, and this only changes the normalization of $Z[J]$, choosing:

$$\exp\left[-\frac{i}{2\xi}\int d^4x\, \alpha^a \alpha^a\right]$$

We then apply $\delta(G^a)$ w/ $G^a = \partial^\mu A^a_\mu - \alpha^a$, replacing α^a w/ $\partial^\mu A^a_\mu$:

$$\exp[\,] \to \exp\left[\frac{-i}{2\xi}\int d^4x\, G^a G^a\right]$$
$$= \exp\left[\frac{-i}{2\xi}\int d^4x\, (\partial_\mu A^a_\mu)^2\right] = \exp\left[i\int d^4x\, \mathcal{L}_{GF}\right]$$

And we recover our previous result

$$\mathcal{L}_{YM} \to \mathcal{L}_{YM} + \mathcal{L}_{GF} + \mathcal{L}_{ghost}$$

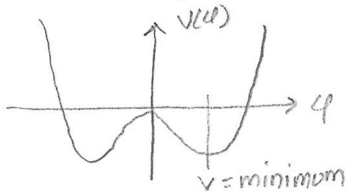
Note: we will generally define $G^a = \partial^\mu A^a_\mu + \cdots$ w/o the α^a dep. as we know the δ sets $\alpha \to G$

4) Spontaneous Symmetry Breaking (SSB): Abelian Higgs Model

$U(1)$ scalar gauge theory:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu \varphi^* - ieA_\mu \varphi^*)(\partial_\mu \varphi + ieA_\mu \varphi) + \underset{\substack{\uparrow \\ \text{wrong sign!}}}{m^2|\varphi|^2} - \frac{\lambda}{4}|\varphi|^4$$

$$V(\varphi) = -m^2|\varphi|^2 + \frac{\lambda}{4}|\varphi|^4$$



rotate around origin → potential bc φ is $\mathbb{C}$

→ ∞ many ground states

we rewrite $\varphi(x) = \dfrac{v + \sigma(x) + i\pi(x)}{\sqrt{2}}$

σ & π are $2\,\mathbb{R}$ dof

φ is $1\,\mathbb{C}$ dof → $2\,\mathbb{R}$ dof

v is the vacuum expectation value, expanding about v expands about one of the minima

$$\mathcal{L} \to -\frac{1}{4}F_{\mu\nu}^2 + \partial_\mu\left(\frac{\sigma - i\pi}{\sqrt{2}}\right)\partial_\mu\left(\frac{\sigma + i\pi}{\sqrt{2}}\right) + \partial_\mu\left(\frac{\sigma - i\pi}{\sqrt{2}}\right) ieA_\mu\left(\frac{v+\sigma+i\pi}{\sqrt{2}}\right) - ieA_\mu\left(\frac{v+\sigma-i\pi}{\sqrt{2}}\right)\partial_\mu\left(\frac{\sigma+i\pi}{\sqrt{2}}\right)$$

$$+ e^2A^2\left|\frac{v+\sigma+i\pi}{\sqrt{2}}\right|^2 + \underbrace{m^2\left|\frac{v+\sigma+i\pi}{\sqrt{2}}\right|^2 - \frac{\lambda}{4}\left|\frac{v+\sigma+i\pi}{\sqrt{2}}\right|^4}_{-V(\sigma,\pi)}$$

$$V(\sigma,\pi) = \frac{v^2}{16}\underbrace{(8m^2 - \lambda v^2)}_{\text{cosm const}} + \underbrace{\left(m^2 - \frac{\lambda v^2}{4}\right)}_{\text{require}=0} v\sigma + \underset{\substack{\uparrow \\ \text{massless}}}{\phi\pi} - \underbrace{\left(\frac{3\lambda v^2}{4} - m^2\right)}_{=\frac{5m^2}{4}}\sigma^2 - \underbrace{\left(\frac{\lambda v^2}{4} - m^2\right)}_{0}\pi^2 + \phi\sigma\pi + \dots$$

$\to \dfrac{\lambda v^2}{4} = m^2$

$\to \lambda = \dfrac{4m^2}{v^2}$

follows from this

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2 + \underbrace{\frac{e^2v^2}{2}A^2}_{\text{A is massive!}} + \underbrace{\frac{1}{2}ev(\partial_\mu\pi)A_\mu + \frac{1}{2}ev(\partial_\mu\pi)A_\mu}_{\text{mixing between } \pi \text{ \& } A} + \text{interactions} + V(\pi,\sigma)$$

Notice: $m_\pi = 0$, see goldstone thrm

since the "goldstone boson" mixes w/ A_μ it isn't actually a goldstone, sometimes we call it a goldstone boson anyway, sometimes "pseudogoldstone"

When gauge fixing QED we were able to impose:

$$\partial^\mu A_\mu = 0$$

Through the procedure we followed.

If we instead impose

$$\partial^\mu A_\mu - \xi e v \pi = 0$$

We can remove the mixing at the cost of introducing ghost.

You will show:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial^\mu A_\mu)(\partial^\nu A_\nu) \underbrace{-evA_\mu(\partial^\mu \pi)}_{\text{removes the mixing}} \underbrace{-\frac{1}{2}\xi e^2 v^2 \pi^2}_{\text{gives } \pi \text{ a gauge } (\xi) \text{ dependent mass}}$$

$$\mathcal{L}_{gh} = +(\partial^\mu \bar{c})(\partial_\mu c) \underbrace{-\xi e^2 v^2 \bar{c}c}_{\text{ghosts are massive!}} \underbrace{-\xi e^2 v \sigma \bar{c}c}_{\text{interaction between ghosts \& } \sigma}$$

Since the (pseudo) goldstone bosons & ghosts have masses $\propto \xi$ we can choose the gauge $\xi \to \infty$ ("Unitary Gauge") to remove them from the spectrum:

$$\mathcal{L} + \mathcal{L}_{GF} + \mathcal{L}_{gh} \to -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{e^2 v^2}{2}A^2 - \frac{5m^2}{4}\sigma^2 + \text{interactions}$$

↑ massive gauge boson ↑ massive σ

Counting the dof:	before SSB:
Massive $A_\mu \to 3$	Massless $A_\mu \to 2$
$\mathbb{R}$ scalar $\sigma \to 1$	$\mathbb{C}$ scalar $\to 2$

So the # of dof didn't change $\to$ we say the gauge boson "ate" the goldstone boson

5) The SM in the "broken phase"

The Higgs potential,

$$V(H) = -|\mu|^2 (H^\dagger H) + \lambda (H^\dagger H)$$

Has degenerate minima, we can choose to expand about a specific vacuum by taking:

$$H \to \begin{pmatrix} w^+ \\ \frac{v+h+iw^0}{\sqrt{2}} \end{pmatrix}$$ w/ v the vacuum expectation value

$w^{\pm,0}$ are the (pseudo) goldstones

Notice: $$(D_\mu H)^\dagger (D_\mu H) \to \frac{g_1 v}{2} B_\mu (\partial_\mu w^0) + \frac{g_2 v}{2\sqrt{2}} W^1_\mu \partial_\mu (w^+ + w^-)$$
$$+ \frac{i g_2 v}{2\sqrt{2}} W^2_\mu \partial_\mu (w^+ - w^-) - \frac{g_2 v}{2} W^3_\mu (\partial_\mu w^0)$$
$$+ \dots$$

Just as in the Abelian case we have mixing between the vectors & scalars

A suitable choice of gauge fixing term removes the mixing and gives the goldstones & ghosts mass prop. to $\sqrt{\xi}\, v$

In the Unitary gauge $\xi \to \infty$ and these particles decouple allowing us to take:

$$H \to \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

We will assume Unitary gauge from now on, unless specified otherwise

Returning to the potential we have:

$$V = \underbrace{\left(\frac{\lambda v^2}{2} - |\mu|^2\right)\frac{v^2}{2}}_{\text{some const}} + \underbrace{(\lambda v^2 - |\mu|^2)}_{\substack{=0 \\ \to |\mu|^2 = \lambda v^2}} v h + \frac{1}{2}\underbrace{(3\lambda v^2 - |\mu|^2)}_{m_H^2 = 2\lambda v^2} h^2 + \underbrace{\lambda v h^3 + \frac{\lambda}{4} h^4}_{\text{interactions}}$$

Our Fermion masses come from $\mathcal{L}_{Yuk}$:

$$(Y_e)_i \bar{L}_i H e_i \to (Y_e)_i \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}_i^T \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} e_{Ri} = \underbrace{\frac{(Y_e)_i v}{\sqrt{2}}}_{\equiv m_{ei}} (\bar{e}_L)_i (e_R)_i + \frac{(Y_e)_i}{\sqrt{2}} h (\bar{e}_L)_i (e_R)_i$$

$$= m_{ei} \bar{e}_{Li} e_{Ri} + \underbrace{\frac{m_{ei}}{v} h \bar{e}_{Li} e_{Ri}}_{\substack{\text{Interaction } h\bar{\psi}\psi \text{ has} \\ \text{coupling prop to } m_\psi}}$$

Masses of vector bosons come from $|D_\mu H|^2$:

$$D_\mu H = \left(\partial_\mu + i g_1 Y_H B_\mu + i g_2 \frac{\sigma^I}{2} W_\mu^I\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

define: $W_\mu^1 = \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}}$ $\quad W_\mu^2 = \frac{W_\mu^- - W_\mu^+}{i\sqrt{2}}$

$$\left.\begin{array}{l} \sigma^1 = T^+ + T^- \\ \sigma^2 = iT^- - iT^+ \end{array}\right\} \text{w/} \quad \begin{array}{l} T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{array}$$

$$D_\mu H \to \begin{pmatrix} \frac{i g_2 v}{2\sqrt{2}}\left[\frac{W_\mu^+ + W_\mu^-}{\sqrt{2}} - \frac{W_\mu^- - W_\mu^+}{\sqrt{2}}\right] \\ \frac{i g_1 v}{2\sqrt{2}} B_\mu - \frac{i g_2 v}{2\sqrt{2}} W^3 \end{pmatrix} = \begin{pmatrix} \frac{i g_2 v}{2} W_\mu^+ \\ \frac{i g_1 v}{2\sqrt{2}} B_\mu - \frac{i g_2 v}{2\sqrt{2}} W_\mu^3 \end{pmatrix}$$

$$(D_\mu H)^\dagger (D_\mu H) \to \frac{g_2^2 v^2}{4} W_\mu^+ W_\mu^- + \left(\frac{g_1 v}{2\sqrt{2}} B_\mu - \frac{g_2 v}{2\sqrt{2}} W_\mu^3\right)^2$$

$$= \underbrace{\frac{g_2^2 v^2}{4}}_{m_W^2 = \frac{g_2^2 v^2}{4}} W_\mu^+ W_\mu^- + \frac{1}{2}\underbrace{\left(\frac{g_1^2 v^2}{4} B_\mu^2 + \frac{g_2^2 v^2}{4}(W^3)^2 - \frac{2 g_1 g_2 v^2}{4} B_\mu W_\mu^3\right)}_{\text{not mass eigenstates}}$$

To obtain mass eigenstates for the other 2 bosons we take:

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}^T \begin{pmatrix} \frac{g_1^2 v^2}{4} & -\frac{g_1 g_2 v^2}{4} \\ -\frac{g_1 g_2 v^2}{4} & \frac{g_1^2 v^2}{4} \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}^T \begin{pmatrix} c_W & +s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} \frac{g_1^2 v^2}{4} & -\frac{g_1 g_2 v^2}{4} \\ -\frac{g_1 g_2 v^2}{4} & \frac{g_1^2 v^2}{4} \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$= \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}^T \begin{pmatrix} \frac{v^2}{4}\underline{(g_1 c_W - g_2 s_W)}^2 & -\frac{v^2}{4}(c_W g_2 + g_1 s_W)\underline{(g_1 c_W - g_2 s_W)} \\ -\frac{v^2}{4}(g_2 c_W + g_1 s_W)\underline{(g_1 c_W - g_2 s_W)} & \frac{v^2}{4}(g_1 s_W + g_2 c_W)^2 \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

requiring $g_1 c_W = g_2 s_W$ → removes mixing

→ renders A_μ massless (photon!)

$$= \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & \frac{v^2}{4}\left(\frac{g_2 s_W^2}{c_W} + \frac{g_2 c_W^2}{c_W}\right)^2 \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$= \frac{g_2^2 v^2}{4 c_W^2}(Z_\mu)^2$$

So: $(D_\mu H)^\dagger (D_\mu H) \rightarrow \underbrace{\frac{g_2^2 v^2}{4}}_{m_W^2} W^+_\mu W^-_\mu + \frac{1}{2}\underbrace{\frac{g_2^2 v^2}{4 c_W^2}}_{m_Z^2} Z_\mu Z^\mu$

w/ $g_1 c_W = g_2 s_W \rightarrow s_W^2 + c_W^2 = 1 = c_W^2(1 + \tan^2\theta_W)$

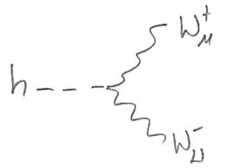
$$\downarrow \qquad\qquad = c_W^2\left(1 + \frac{g_1^2}{g_2^2}\right)$$

$\frac{g_1}{g_2} = \tan^2\theta_W$ $\qquad \rightarrow c_W^2 = \frac{g_2^2}{g_1^2 + g_2^2}$ $\qquad$ similarly: $s_W^2 = \frac{g_1^2}{g_1^2 + g_2^2}$

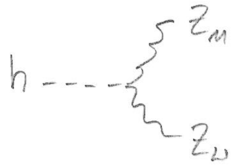
Also: $\rho \equiv \frac{m_W^2 c_W^2}{m_Z^2} = 1$ (at tree level) is an important test of the SM

Expanding the rest of $\mathcal{L}_{Higgs}$:

$$\mathcal{L}_{Higgs} = \frac{1}{2}(\partial_\mu h)^2 + m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$
$$+ \frac{2m_W^2}{v} h W_\mu^+ W_\mu^- + \frac{1}{2}\frac{2m_Z^2}{v} h Z_\mu Z^\mu + \frac{m_W^2}{v^2} h^2 W_\mu^+ W_\mu^- + \frac{1}{2}\frac{m_Z^2}{v^2} h^2 Z_\mu Z^\mu$$

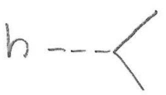
This gives 

$= \frac{2im_W^2}{v}\eta_{\mu\nu}$

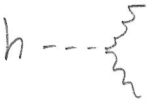


$= \frac{2im_Z^2}{v}\eta_{\mu\nu}$ ← extra $\frac{1}{2}$ not here bc Z's aren't distinguishable

So we've found:



$\propto \frac{m_f}{v}$

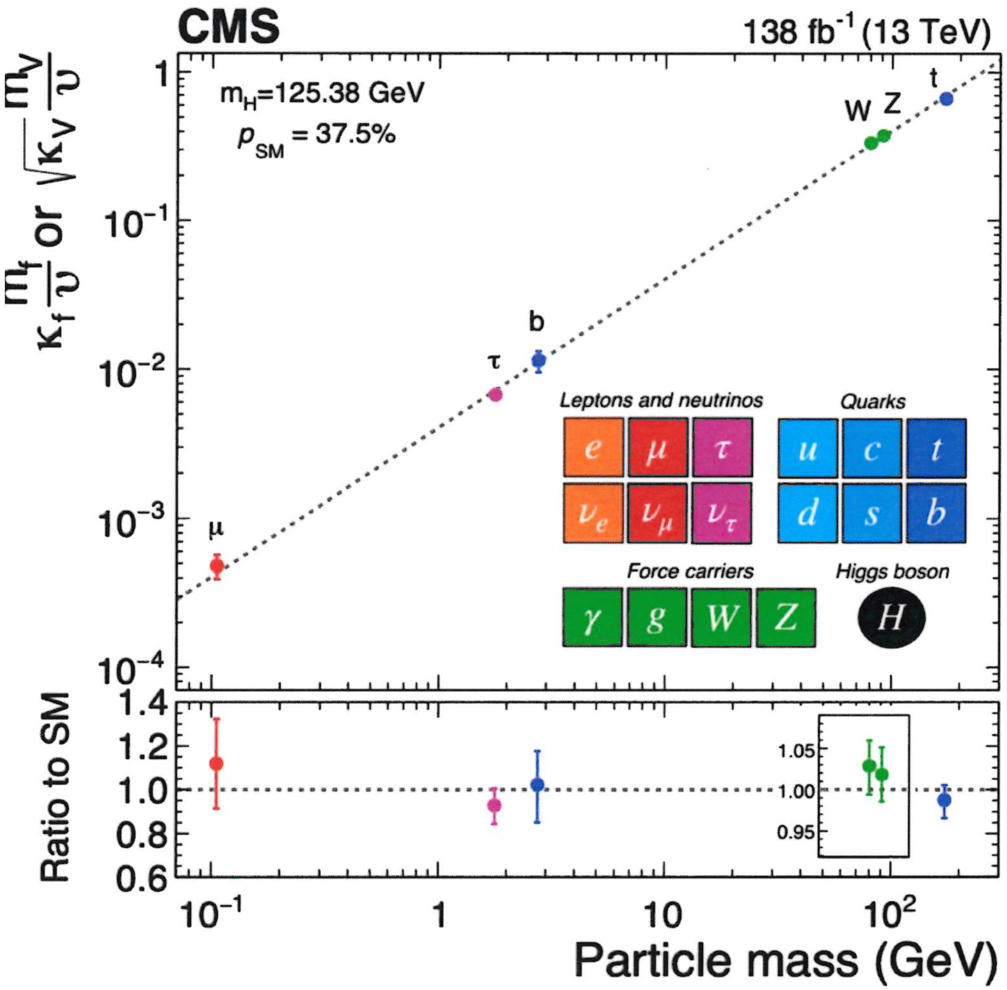


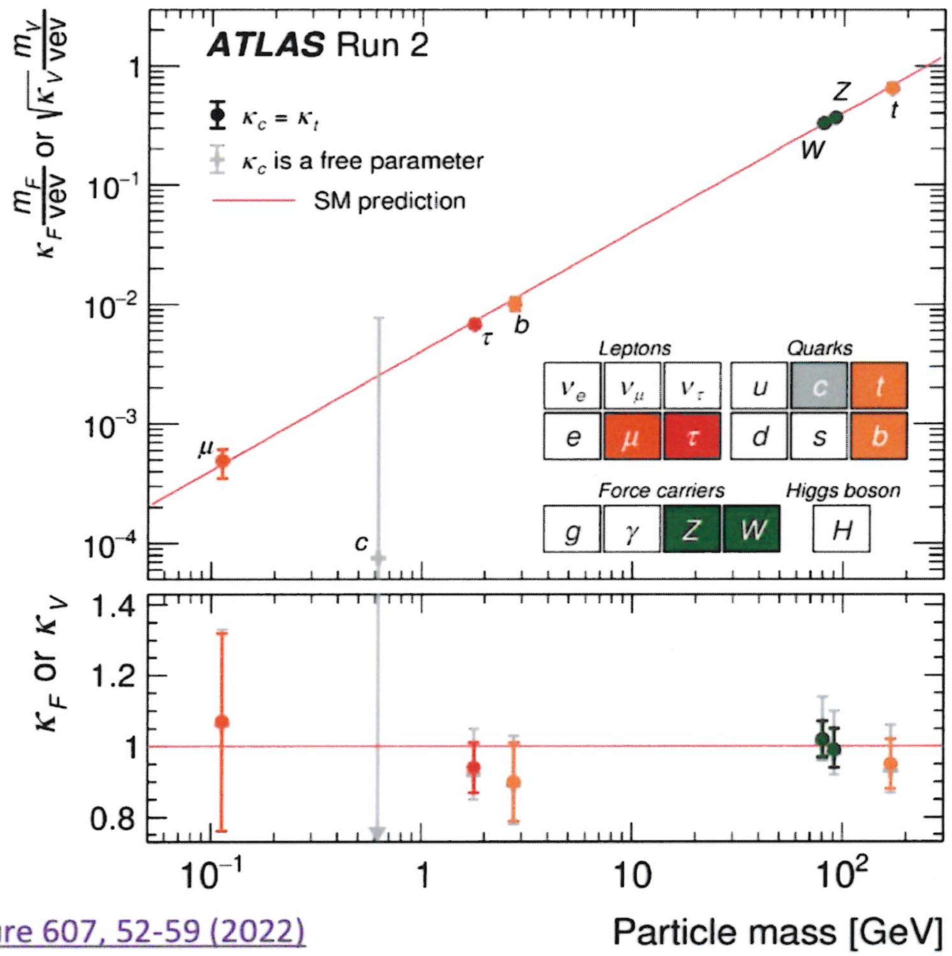
$\propto \frac{m_V^2}{v}$

The attached plot shows LHC $\overbrace{\text{(ATLAS \& CMS)}}^{\text{detectors @ LHC}}$ measurements of these couplings ($\kappa_i = 1$ in SM)

Note they've rescaled the W/Z couplings so everything is on the same line

$m_H = 125.22 \pm 0.14$ GeV





Nature 607, 52-59 (2022)

Consider an ex from $\mathcal{L}_{ferm}$:

$$D_\mu e = (\partial_\mu + i g_1 Y_e B_\mu) e \rightarrow i g_1 (c_W A_\mu - s_W Z_\mu) Y_e e$$

$$= \underbrace{i Y_e g_1 c_W A_\mu e}_{Q_e e A_\mu} - i g_1 s_W Y_e Z_\mu e$$

$$Q_e e A_\mu \rightarrow g_1 c_W = e$$
$$Q_e = Y_e$$

$Y_e = Q_e$ because e doesn't couple to $SU(2)_R$

$g_1 c_W = e$, but $g_1 c_W = g_2 s_W \rightarrow g_2 s_W = e$

$$\rightarrow e^2 = \frac{g_1 g_2}{g_1^2 + g_2^2}$$

Consider:

$$D_\mu L = (\partial_\mu + i g_1 Y_L B_\mu + i g_2 \frac{\sigma_2 \cdot W_\mu}{2}) L$$

$$\rightarrow \left[i g_1 Y_L (c_W A_\mu - s_W Z_\mu) \mathbb{1} + \frac{i g_2}{2} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} (s_W A_\mu + c_W Z_\mu) + W^\pm \right] \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\left.\begin{aligned} (i g_1 Y_L c_W + \tfrac{1}{2} i g_2 s_W) A_\mu \nu_L &= i e (Y_L + \tfrac{1}{2}) A_\mu \nu_L \\ (i g_1 Y_L c_W - \tfrac{1}{2} i g_2 s_W) A_\mu e_L &= i e (Y_L - \tfrac{1}{2}) A_\mu e_L \end{aligned}\right\} \text{ for } Y_L = -\tfrac{1}{2}$$

ν_L is uncharged

e_L has (-1) charge

In general $\hat{Q} = \frac{1}{2} \hat{T}_3 + \hat{Y}$

↑ 3rd comp isospin

ex: $\hat{Q} u_L = \frac{1}{2}(+1) + (1/6) = 4/6 = 2/3$

↑ recall $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ so $\hat{T}_3 = +1$

For a general fermion ψ: (Homework):

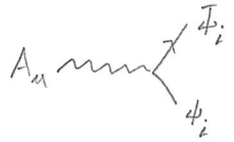
$$D_\mu \psi = \left[\partial_\mu + \frac{ig_2}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) + ig_Z\left(\frac{T_3}{2} - s_W^2 Q_\psi\right)Z_\mu + iQ_\psi e A_\mu\right]\psi$$

$g_Z \equiv \frac{g_2}{c_W}$

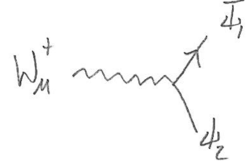
If ψ is an $SU(2)$ singlet, $T^\pm, T^3 \to 0$

Note: H isn't charged under QCD so it changes nothing, I've neglected QCD here for simplicity

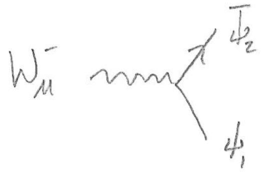
$\mathcal{L}_{Ferm} = i\bar{\psi}\not{D}\psi$ gives the FRs $\left(\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right)$



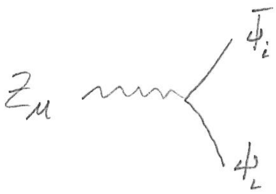
$= ieQ_\psi \gamma^\mu P_\pm$ ← same for both L & RH, sum gives $ieQ_\psi\gamma^\mu$ as in QED



$= \frac{ig_2}{\sqrt{2}}\gamma^\mu P_L$



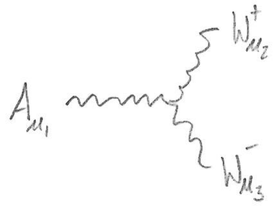
$= \frac{ig_2}{\sqrt{2}}\gamma^\mu P_L$



$= ig_Z\left(\frac{T_3}{2} - s_W^2 Q_\psi\right)P_\pm$ ← + if $T_3 \to 0$, ie uncharged under $SU(2)$

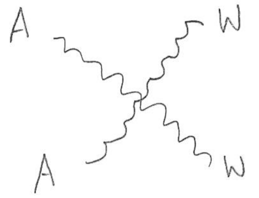
− if $T_3 \neq 0$

The only missing piece is $\mathcal{L}_{Gauge}$, this gives triple & quartic gauge couplings:

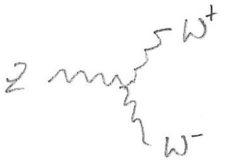


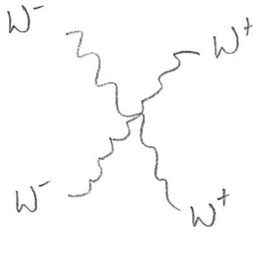
$$= ie\left[(p_1-p_2)^{\mu_3}\eta_{\mu_1\mu_2} + (p_3-p_1)^{\mu_2}\eta_{\mu_1\mu_3} + (p_2-p_3)^{\mu_1}\eta_{\mu_2\mu_3}\right]$$

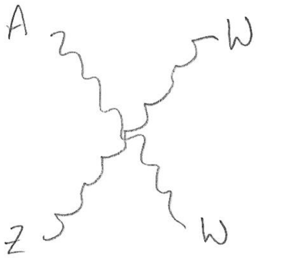
prop to e! so we conclude $\pm$ in $W^\pm$ label W's charge under QED

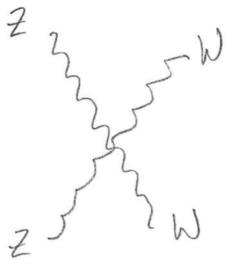


$\propto ie^2$ ← also consistent w/ $Q_W = \pm 1$









We started w/ $SU(2)_L \otimes U(1)_Y$, 3 (pseudo)goldstone bosons ↔ 3 massive gauge bosons

→ indicates 3 generators broken

→ that we don't have a B_μ field & we have a photon that couples as in QED

$\Rightarrow SU(2)_L \otimes U(1)_Y \to U(1)_{QED}$