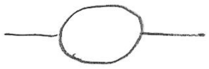
1)

$G_B \sim \frac{1}{\ell^2}$

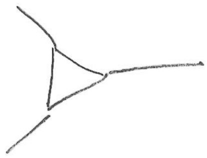


$G_F \sim \frac{1}{\ell}$

$BB \sim \int \frac{d^4\ell}{\ell^4} \sim \ln\Lambda$

$BF \sim \int \frac{d^4\ell}{\ell^3} \sim \Lambda$

$FF \sim \int \frac{d^4\ell}{\ell^2} \sim \Lambda^2$

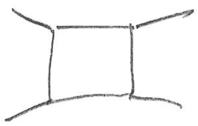


$BBB \sim \int \frac{d^4\ell}{\ell^6} \sim \frac{1}{\Lambda^2}$

$BBF \sim \int \frac{d^4\ell}{\ell^5} \sim \frac{1}{\Lambda}$

$BFF \sim \int \frac{d^4\ell}{\ell^4} \sim \ln\Lambda$

$FFF \sim \int \frac{d^4\ell}{\ell^3} \sim \Lambda$



$BBBB \sim \int \frac{d^4\ell}{\ell^8} \sim \frac{1}{\Lambda^4}$

$BBBF \sim \int \frac{d^4\ell}{\ell^7} \sim \frac{1}{\Lambda^3}$

$BBFF \sim \int \frac{d^4\ell}{\ell^6} \sim \frac{1}{\Lambda^2}$

$BFFF \sim \int \frac{d^4\ell}{\ell^5} \sim \frac{1}{\Lambda}$

$FFFF \sim \int \frac{d^4\ell}{\ell^4} \sim \ln\Lambda$

We conclude pentagons & higher are all UV-finite

2) $$\frac{1}{AB} = \frac{(A-B)^2}{AB(A-B)^2}$$

$$= \frac{A^2 - AB - BA + B^2}{AB(A-B)^2}$$

$$= \frac{1}{A-B}\left(\frac{A(A-B)}{AB(A-B)} - \frac{B(A-B)}{AB(A-B)}\right)$$

$$= \frac{1}{A-B}\left(\frac{1}{B} - \frac{1}{A}\right)$$

$$= \frac{1}{A-B}\int_B^A \frac{dz}{z^2}$$

let $x \equiv (z-B)/(A-B)$, $dx = \frac{dz}{A-B}$, $z = (A-B)x + B = xA + (1-x)B$

$$\frac{1}{AB} = \int_0^1 \frac{dx}{[xA + (1-x)B]^2}$$

$$= \int_0^1 dx\,dy \, \frac{\delta(1-x-y)}{(xA+yB)^2}$$

3)

$$\int_0^\infty ds_i\, e^{-s_i A_i} = \frac{1}{A_i}$$

$$\frac{1}{A_1 \cdots A_n} = \int_0^\infty ds_1 \cdots ds_n\, e^{-s_1 A_1 - \cdots - s_n A_n}$$

change variables to $\alpha = s_1 + \cdots s_n$
$\alpha_{i \neq n} = s_i/\alpha$

$$J = \begin{vmatrix} \alpha & 0 & \cdots & -\alpha \\ 0 & \alpha & & -\alpha \\ \vdots & & & \vdots \\ \alpha_1 & \alpha_2 & \cdots & 1-\Sigma_i \alpha_i \end{vmatrix} = \alpha^{n-1}$$

$$\frac{1}{A_1 \cdots A_n} = \int_0^1 d\alpha_1 \cdots \int^{1-\alpha_1-\cdots-\alpha_{n-1}} d\alpha_{n-1} \int_0^\infty d\alpha\, \alpha^{n-1} \exp[-\alpha(\alpha_1 A_1 + \cdots \alpha_{n-1} A_{n-1} + (1-\alpha_1 - \cdots \alpha_{n-1}) A_n$$

Notice:

$$\frac{\partial^{n-1}}{\partial(-x)^{n-1}} \int_0^\infty d\alpha\, e^{-\alpha x} = \int_0^\infty d\alpha\, \alpha^{n-1} e^{-\alpha x} = \frac{(n-1)!}{x^n}$$

Giving:

$$\frac{1}{A_1 \cdots A_n} = (n-1)! \int_0^1 d\alpha_1 \cdots \int_0^{1-\alpha_1-\cdots-\alpha_{n-1}} \frac{1}{[\alpha_1 A_1 + \cdots + \alpha_{n-1} A_{n-1} + (1-\alpha_1 - \cdots - \alpha_{n-1}) A_n]^n}$$

$$= \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \cdots \int^{1-\alpha_1-\cdots-\alpha_{n-2}} d\alpha_{n-1} \int d\alpha_n \frac{\Gamma(n)\,\delta(1-\alpha_1 - \cdots - \alpha_n)}{[\alpha_1 A_1 + \cdots + \alpha_{n-1} A_{n-1} + \alpha_n A_n]^n}$$

4)

Notice/Recall Gaussians:

$$\int d^d r\, e^{-r^2} = \left(\int_{-\infty}^{\infty} dx\, e^{-x^2}\right)^d = (\sqrt{\pi})^d$$

$$\int_0^{\infty} dr\, r^{d-1} e^{-r^2} = \frac{\Gamma(d/2)}{2}$$

Combining w/ $\int d^d K = \int d\Omega_d \int K^{d-1} dK$

$$\Rightarrow \int d^d r\, e^{-r^2} = (\sqrt{\pi})^d = \int d\Omega_d \int_0^{\infty} dr\, r^{d-1} e^{-r^2} = \Omega_d \frac{\Gamma(d/2)}{2}$$

$$\rightarrow \Omega_d = \frac{2(\sqrt{\pi})^d}{\Gamma(d/2)}$$

5)

Instead for the onshell condition we had:

$$Z_m = 1 + \frac{\lambda_R}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right)$$

$$\mu\frac{dZ_m}{d\mu} = \frac{1}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right)\mu\frac{d\lambda_R}{d\mu}$$
$$+ \frac{\lambda_R}{32\pi^2}2 - \frac{\lambda_R}{32\pi^2}\mu\frac{dm_R^2}{d\mu}$$

$$0 = \left(\mu\frac{dm_R^2}{d\mu}\right)Z_m + m_R^2\frac{1}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right)\mu\frac{d\lambda_R}{d\mu}$$
$$+ \frac{\lambda_R}{16\pi^2} - \frac{\lambda_R}{32\pi^2}\mu\frac{dm_R^2}{d\mu}$$

Dividing by $m_R^2 Z_m$:

$$0 = \mu\frac{d\ln m_R^2}{d\mu} + \frac{1}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right)/Z_m \; \mu\frac{d\lambda_R}{d\mu}$$
$$+ \frac{\lambda_R}{16\pi^2} - \frac{\lambda_R}{32\pi^2}\mu\frac{d\ln m_R^2}{d\mu}/Z_m$$

2 loop $\to Z_m \to 1$

$$\left(1 - \frac{\lambda_R}{32\pi^2}\right)\mu\frac{d\ln m_R^2}{d\mu} = -\frac{\lambda_R}{32\pi^2}\left(\frac{1}{\epsilon} + 1 + \ln\frac{4\pi\mu^2}{e^{\gamma} m_R^2}\right)(-2\epsilon)\Big(1 - \underbrace{\frac{3\lambda_R}{32\pi^2\epsilon}}_{2\text{ loop}} + \cdots\Big)$$
$$- \frac{\lambda_R}{16\pi^2}$$
$$= \frac{\lambda_R}{16\pi^2}(1-1) + O(\epsilon)$$
$$= 0$$

So the anomalous dim of m_R vanishes, ie m_R doesn't change w/ μ, which it shouldn't since our renorm condition sets m_R to the physical pole mass.