Effective Field Theories:

1) Fermi thry from matching

consider μ decay in the SM

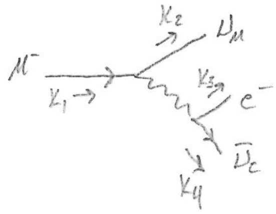
$$\mathcal{L} = i\bar{L}_i \not{D} L_i \qquad L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

expanding in terms of $SU(2)_L$ components gives interactions:

$$\mathcal{L}_I = \frac{g_2}{\sqrt{2}} \left(\bar{\nu}_i \not{W}^+ P_L e_i + h.c. \right) + \ldots$$

In the last lecture we had, for $\mu \to e \bar{\nu} \nu$, the amplitude:

$$i\mathcal{M} \sim \frac{ig_2^2}{2} \frac{[\bar{u}_2 \gamma^\mu P_L u_1][\bar{u}_3 \gamma^\mu P_L v_4]}{(k_3+k_4)^2 - m_W^2} \quad \leftarrow \text{in } m_e \to 0 \text{ limit}$$



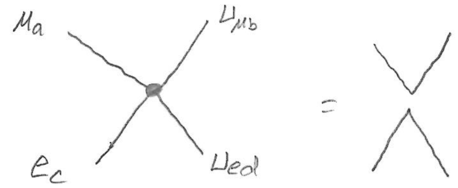
We have the kinematic constraint $(k_3+k_4)^2 \leq m_\mu^2$ so:

$$i\mathcal{M} \sim \underbrace{-\frac{ig_2^2}{2m_W^2} [\bar{u}_2 \gamma^\mu P_L u_1][\bar{u}_3 \gamma^\mu P_L v_4]}_{\substack{\text{this looks like a 4-fermion} \\ \text{interaction instead of a} \\ W\text{-exchange}}} + \underbrace{\frac{ig_2^2}{2m_W^4} (k_3+k_4)^2 [\bar{u}_2 \gamma^\mu P_L u_1][\bar{u}_3 \gamma^\mu P_L v_4]}_{\text{4-fermion int w/ 2 } \partial_\mu} + \ldots$$

Imagine a world where we don't know of the W boson, such as that of Fermi in 1933, we could guess an interaction:

$$\mathcal{L}_{IR} = \underbrace{\frac{-4G_F}{\sqrt{2}}}_{\text{convention}} \left[\bar{e} \gamma^\mu P_L \nu_e + \bar{\mu} \gamma^\mu P_L \nu_\mu \right] \left[\bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu \right]$$

This IR (Infrared) Lagrangian or effective Lagrangian gives the Feynman Rule:



$$= -i\frac{4G_F}{\sqrt{2}}[\gamma^\mu P_L]_{ab}\,[\gamma^\mu P_L]_{cd}$$

↑ Dirac indices

The corresponding amplitude is:

$$i\mathcal{M}_{IR} = \frac{-4i}{\sqrt{2}}G_F[\bar{u}_2\gamma^\mu P_L u_1][\bar{u}_3\gamma^\mu P_L v_4]$$

Comparing this w/ our result from the SM in the large m_W limit:

$$i\mathcal{M} = \frac{-ig_2^2}{2m_W^2}[\bar{u}_2\gamma^\mu P_L u_1][\bar{u}_3\gamma^\mu P_L v_4]$$

We have that the two things agree if:

$$\frac{4G_F}{\sqrt{2}} = \frac{g_2^2}{2m_W^2} \rightarrow G_F = \frac{1}{2\sqrt{2}}\frac{g_2^2}{2m_W^2} = \frac{1}{\sqrt{2}v^2} \qquad \left(\text{recall } m_W^2 = \frac{g_2^2v^2}{4}\right)$$

This works given $1/m_W^2$ is sufficiently small, if not we need an operator for the $1/m_W^4$ or higher order terms.

negligible, in practice, means $\ll$ experimental error

$\mathcal{L}_{IR}$ is the "infrared" Lagrangian describing the low E effects of heavy particles in the "ultraviolet" theory.

The method of calculating in both theories and requiring they agree at a given order in $1/m$ is called "matching"

1) know the UV
2) write all possible IR ops
3) calculate observables in both theories
4) determine low E constants (like G_F) by requiring amplitudes agree

→ it is not always obvious which ops to include

2) Integrating out heavy fields (Donaghue, "Dynamics of the SM")

Consider a heavy scalar coupled to some light fields

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)^2 - \frac{M^2}{2}H^2 + JH$$

$\uparrow$ e.g. $J = \bar{\psi}\psi$ or φ^3

We can obtain the effective field thry describing the interactions of the light fields, l_i, by:

$$e^{iZ_{eff}[l_i]} \equiv \int [dH] e^{i\int d^4x \, \mathcal{L}(H,l)} \Big/ \int [dH] e^{i\int d^4x \, \mathcal{L}(H,0)}$$

That this quotient removes the dep on H should be familiar from our discussion of path integrals and generating functionals which allowed us to remove disconnected diagrams

letting $D \equiv \Box_x + M^2$

$$\int d^4x \, \mathcal{L}(H,J) = \int d^4x \left[-\frac{1}{2}HDH + JH\right]$$

$$= -\frac{1}{2}\int d^4x \left[HDH - JH - JH + \underbrace{JD^{-1}J - JD^{-1}J}_{=0}\right]$$

$$= -\frac{1}{2}\int d^4x \left[HDH - HDD^{-1}J + \underbrace{D^{-1}JDH}_{} + D^{-1}JDD^{-1}J - JD^{-1}J\right]$$

$$JH = (DD^{-1}J)H = (\partial_\mu \partial^\mu D^{-1}J)H$$

$$= -(\partial^\mu D^{-1}J)\partial_\mu H + \partial_\mu(\quad)$$

$$= (D^{-1}J)(DH) + \partial_\mu(\quad)$$ integration by parts

$$= -\frac{1}{2}\int d^4x \left[(H - D^{-1}J)D(H - D^{-1}J) - JD^{-1}J\right]$$

$$= -\frac{1}{2}\int d^4x \left[H'DH' - JD^{-1}J\right] \quad \text{w/ } H' \equiv H - D^{-1}J$$

So we find:

$$e^{iZ_{eff}[l_i]} = \int [dH'] \exp[-\tfrac{i}{2}\int d^4x (H'DH' - JD^{-1}J)] \Big/ \int [dH] \exp(-\tfrac{i}{2}\int d^4x\, HDH)$$

since we're integrating over all values of $H^{(\prime)}$ we can take $H=H'$ leaving us with:

$$e^{iZ_{eff}[l_i]} = \exp(\tfrac{i}{2}\int d^4x\, JD^{-1}J)$$

Now consider, $D = \Box + M^2 \rightarrow D^{-1} \sim$ propagator

$$Z_{eff}[l_i] = -\frac{1}{2}\int d^4x\, d^4y\, J(x) G_F(x-y) J(y)$$

Notice the propagator is peaked at small distances (large mta $\sim M^2$) so we can expand J at small distances:

$$J(y) = J(x) + (y-x)^\mu (\partial_\mu J(y)]_{y=x} + \cdots$$

Recalling our propagator in configuration space:

$$i\,G_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-M^2} e^{ip(x-y)}$$

$$\int d^4y\, G_F(x-y) = \int d^4y \frac{d^4p}{(2\pi)^4} \frac{1}{p^2-M^2} e^{ip\cdot x} e^{-ip\cdot y}$$

$$= \int \frac{d^4p}{(2\pi)^4} (2\pi)^4 \delta^4(p) \frac{1}{p^2-M^2} e^{ip\cdot x}$$

$$= -\frac{1}{M^2}$$

This gives:

$$Z_{eff}[l_i] = -\frac{1}{2}\int d^4x\, J(x) J(x) \frac{-1}{M^2} + \cdots \rightarrow \mathcal{L}_{eff} = \frac{1}{2M^2} J^2 + \cdots$$

A more general approach can be found in Henning et al. arXiv: 1412.1837

The effective action is defined as the integral over the heavy fields:

$$e^{i S_{eff}[l_i](\mu)} = \int DH \, e^{iS[l_i, H](\mu)}$$

The (μ) dep indicates the need to specify a matching scale at one-loop, it is generally useful to use $\mu = M$, the heavy scale, as this removes large logs like $\ln \frac{\mu^2}{M^2}$ that would otherwise appear at the low E scale of the EFT

S_{eff} is computed using the "saddle pt" approx, expanding $H = H_c + \eta$:

$$\frac{\delta S[l, H]}{\delta H} = 0 \quad \rightarrow \text{ minimum}$$

This defines $H_c[l]$, notice $\frac{\delta S}{\delta H} = 0$ is the Equation of motion (EOM) of H!

$$\delta S = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial H} \delta H + \frac{\partial \mathcal{L}}{\partial(\partial_\mu H)} \delta(\partial_\mu H)\right] + (\delta l \text{ terms})$$

$$= \int d^4x \left(\left[\frac{\partial \mathcal{L}}{\partial H} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu H)}\right] \delta H + \underbrace{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu H)} \delta H\right]}_{\text{surface term} \to 0}\right)$$

$$\frac{\delta S}{\delta H} = 0 \Rightarrow \frac{\partial \mathcal{L}}{\partial H} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu H)} = 0 \quad \text{the EOM defines } H_c[l]$$

Expanding S about this minimum:

$$S[l, H] = S[H_c] + \frac{1}{2} \left.\frac{\delta^2 S}{\delta H^2}\right|_{H_c} \eta^2 + O(\eta^3)$$

Notice the $\frac{\delta S}{\delta H}$ term is by definition zero

Our expression for the effective action becomes:

$DH \to D\eta$ as H_c depends only on the light fields

$$e^{iS_{eff}[\ell]} = \int D\eta \, e^{iS[H_c]} \, e^{\frac{i}{2} \frac{\delta^2 S}{\delta H^2}\big|_{H_c} \eta^2 + \dots}$$

$$= e^{iS[H_c]} \underbrace{\left[\det \left(-\frac{i}{2} \frac{\delta^2 S}{\delta H^2}\Big|_{H_c} \right) \right]^{-1/2}}_{\text{this is just our gaussian integral from at the beginning of the semester!}}$$

Recalling: $\det(e^A) = e^{\mathrm{tr} A} \to \det(e^{\ln A}) = e^{\mathrm{tr} \ln A}$

We have:

$$e^{iS_{eff}[\ell]} = e^{iS[H_c] + \frac{i}{2} \mathrm{Tr} \ln \left(-\frac{\delta^2 S}{\delta H^2}\big|_{H_c} \right)}$$

$S[H_c]$ is the tree level term, while the latter is one-loop

we will only focus on the tree level part

3) The Fermi thry from the EOM

To obtain the EOM of the $W^\pm$ we need to look at the SM $\mathcal{L}$ parts which dep on the W's, for simplicity we consider only the coupling to L:

$$\mathcal{L}^W_{SM} = -\frac{1}{4} W^I_{\mu\nu} W^I_{\mu\nu} + i\bar{L}\not{D}L + m_W^2 W^+_\mu W^-_\mu$$

$$= -\frac{1}{2}(\partial_\nu W^+_\mu - \partial_\mu W^+_\nu)(\partial_\nu W^-_\mu - \partial_\mu W^-_\nu) + g_2 \bar{L}_i \gamma^\mu \left[\frac{T^+}{\sqrt{2}} W^+_\mu + \frac{T^-}{\sqrt{2}} W^-_\mu\right] L_i + m_W^2 W^+_\mu W^-_\mu$$

↑ recall $2T^\pm = \sigma_1 \pm i\sigma_2$

$$\frac{\delta \mathcal{L}}{\delta W^+_\beta} = g_2 (\bar{L}\gamma_\mu \frac{T^+}{\sqrt{2}} L)\eta^{\mu\beta} + m_W^2 W^-_\mu \eta^{\mu\beta}$$

$$\frac{\delta \mathcal{L}}{\delta(\partial_\alpha W^+_\beta)} = -\frac{1}{2}(\eta^{\nu\alpha}\eta^{\mu\beta} - \eta^{\mu\alpha}\eta^{\nu\beta})(\partial_\nu W^-_\mu - \partial_\mu W^-_\nu)$$

$$= \partial^\beta W^{-\alpha} - \partial^\alpha W^{-\beta}$$

$$\partial_\alpha \frac{\delta \mathcal{L}}{\delta(\partial_\alpha W^+_\beta)} = \partial_\alpha \partial^\beta W^{-\alpha} - \Box W^{-\beta} = (\partial_\alpha \partial^\beta \eta^{\rho\alpha} - \Box \eta^{\beta\rho}) W_\rho$$

$$= \frac{\delta \mathcal{L}}{\delta W^+_\beta} = m_W^2 W^{-\beta} + g_2 (\bar{L}_i \gamma^\beta \frac{T^+}{\sqrt{2}} L_i)$$

$$\Rightarrow \quad (\partial_\alpha \partial^\beta \eta^{\rho\alpha} - \Box \eta^{\beta\rho} - m_W^2 \eta^{\beta\rho}) W^-_\rho = g_2 (\bar{L}_i \gamma^\beta \frac{T^+}{\sqrt{2}} L_i)$$

To solve this for W^- we make a perturbative expansion:

consider: $(\Delta - M^2)\varphi = A$

↑ ∂'s, but can also contain fields

$$\varphi_c = \underbrace{(\Delta - M^2)^{-1}}_{\text{weird, but we've done this before}} A$$

$$(\Delta - M^2)^{-1} = -\frac{1}{M^2}\left(1 - \frac{\Delta}{M^2}\right)^{-1}$$

↑ small if $M^2 \gg$ mta & "field configurations"

$$\sim -\frac{1}{M^2}\left(1 - \frac{\Delta}{M^2} + \frac{\Delta^2}{M^2} + \dots\right)$$

So to leading order the soln to the EOM, ie $W_c^\pm$, is:

$$W^-_{c\rho} \sim \frac{-g_2}{m_W^2}\left(\bar{L}_i \gamma_\rho \frac{T^+}{\sqrt{2}} L_i\right) + \underbrace{\cdots}$$

this term looks like $\frac{\partial_\alpha \partial^\beta \eta^{\rho\alpha} - \Box \eta^{\beta\rho}}{m_W^2}\left(\bar{L}\gamma_\rho \frac{T^+}{\sqrt{2}} L\right)$

which is exactly the mtm dep terms we talked about in part (1)

$$W^+_{c\rho} = (W^-_{c\rho})^\dagger \sim \frac{-g_2}{m_W^2}\left(\bar{L}_i \gamma_\rho \frac{T^-}{\sqrt{2}} L_i\right)$$

We plug these solutions back in to $\mathcal{L}^W$:

$$\mathcal{L}^W_{SM} = \underbrace{-\frac{1}{2}(\partial_\nu W^+_\mu - \partial_\mu W^+_\nu)(\partial_\nu W^-_\mu - \partial_\mu W^-_\nu)}_{\propto W_c^2 \propto \frac{1}{m_W^4}} + \underbrace{\underbrace{g_2 \bar{L}_i \gamma^\mu\left[\frac{T^+}{\sqrt{2}} W^+_\mu + \frac{T^-}{\sqrt{2}} W^-_\mu\right] L_i}_{\propto W_c \propto \frac{1}{m_W^2}} + \underbrace{m_W^2 W^+_\mu W^-_\mu}_{\propto m_W^2 W_c^2 \propto \frac{1}{m_W^2}}}_{\text{these give the leading terms}}$$

$$= -\frac{g_2^2}{m_W^2} \bar{L}_i \gamma^\mu \frac{T^+}{\sqrt{2}} \underbrace{\left[\bar{L}_j \gamma_\mu \frac{T^-}{\sqrt{2}} L_j\right]}_{\substack{\text{This is an } SU(2) \\ \text{singlet and can be} \\ \text{pulled out}}} L_i - \frac{g_2^2}{m_W^2}\left(\bar{L}_i \gamma^\mu \frac{T^-}{\sqrt{2}} L_i\right)\left(\bar{L}_j \gamma_\mu \frac{T^+}{\sqrt{2}} L_j\right) + \frac{g_2^2}{m_W^2}\left(\bar{L}_i \gamma^\mu \frac{T^+}{\sqrt{2}} L_i\right)\left(\bar{L}_j \gamma_\mu \frac{T^-}{\sqrt{2}} L_j\right)$$

$$= \left(-\frac{1}{2} - \frac{1}{2} + \frac{1}{2}\right)\frac{g_2^2}{m_W^2}\left(\bar{L}_i \gamma^\mu T^+ L_i\right)\left(\bar{L}_j \gamma^\mu T^- L_j\right)$$

$$T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow \bar{L}_i \gamma^\mu T^+ L_i = \bar{\nu}_i \gamma^\mu P_L e_i$$

$$T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \rightarrow \bar{L}_i \gamma^\mu T^- L_i = \bar{e}_i \gamma^\mu P_L \nu_i$$

$$= \underbrace{-\frac{g_2^2}{2m_W^2}}_{\frac{-4G_F}{\sqrt{2}}}\left(\bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu\right)\left(\bar{e}\gamma^\mu P_L \nu_e + \bar{\mu}\gamma^\mu P_L e\right) + \text{Tau stuff}$$

This agrees w/ our result from matching

4) Top down, bottom up, and Fermi theory as a bottom up EFT

Top down EFTs:

1) have preferred UV model in mind

2) "match" onto the IR model

3) make predictions of low E behavior

4) low E constants (AKA Wilson coefficients) are given by matching/integrating out & correlations between ops are predicted:

eg in Fermi thry we have:

$$\underbrace{\frac{c_6}{m_W^2}(\bar{L}\gamma_\mu L)^2}_{\mathcal{O}(\frac{1}{m_W^2})} + \underbrace{\frac{c_8}{m_W^4}\partial^2(\bar{L}\gamma_\mu L)^2}_{\mathcal{O}(\frac{1}{m_W^4})}$$

c_6 & c_8 are given by matching & are functions of g_2

5) Symmetries of the UV are embedded in the IR

→ In the SM W's couple only to LH fermions

→ In the EFT the operators only couple LH fermions

Bottom up EFTs

1) We don't know the UV, only the IR (eg QED in Fermi's case)
2) We know the low E degrees of freedom

   (eg $e, \gamma,$ + Pauli's proposed ν)

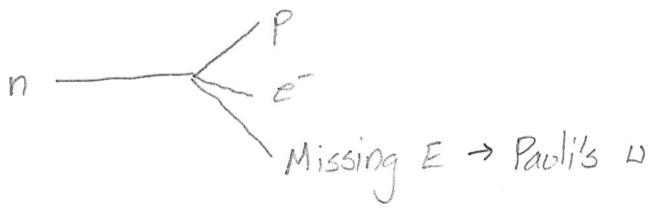
3) We form all possible ops and use experiments to constrain the UV w/o knowing the UV thry

   → there are ∞ ops so we need a power counting to organize our IR thry
   for us this is $\frac{1}{M}$

4) Symmetries in the UV will manifest as experiments constrain the low E constants

Fermi, Bottom up:

Fermi had QED, p, n, e, γ and wanted to describe:



$\mathcal{L}_I \sim G_F (\bar{p} n)(\bar{e} \nu) + h.c.$

Dealing w/ Hadrons (p, n) is difficult, so we'll stick w/ μ decay example

μ decay, bottom up:

1) Fields: e, μ, ν_e, ν_μ

2) Symmetries of IR (QED): $e \to e^{i\alpha(x)} e$

$$\mu \to e^{i\alpha(x)} \mu$$

$$\nu \to \nu$$

3) Since QED is vector-like (ie couples to L & RH fields the same) $e^{i\alpha(x)}$ commutes w/ our basis of Dirac Matrices

eg the chiral basis $\Gamma = \{ \underbrace{P_L, P_R}_{\substack{\text{Scalar}\\ S}}, \underbrace{\gamma_\mu P_L, \gamma_\mu P_R}_{\substack{\text{Vector}\\ V}}, \underbrace{\sigma^{\mu\nu} P_L, \sigma^{\mu\nu} P_R}_{\substack{\text{Tensor}\\ T}} \}$

4) So we form our ops of fields $\{e, \mu, \nu_e, \nu_\mu\}$ and the chiral basis:

$$\mathcal{L}_I = \sum_{\substack{\mu,\epsilon = L,R\\ \gamma = S,V,T}} g^{\gamma}_{\epsilon\mu} (\bar{e}_\epsilon \Gamma^\gamma \nu_e)(\bar{\nu}_\mu \Gamma^\gamma \mu_\mu)$$

notice the labels ϵ, μ also dictate the chirality of the ν's:

$$\bar{e}_\pm \Gamma^S \nu_e = e_\pm^\dagger P_\pm \gamma_0 \mathbb{1} \nu_e$$

$$= \bar{e}_\pm P_\mp \nu_e$$

$$= \bar{e}_\pm \nu_{e\mp}$$

The ops $(\bar{e} \Gamma \nu_e)(\bar{\nu}_\mu \Gamma \mu)$ are dimension-six

recall $[\psi] = 3/2$

This means $g^\gamma_{\epsilon\mu} \sim \frac{1}{M^2}$, for bottom up we often use Λ instead of M

$\mu^- \to e \bar{\nu}_e \nu_\mu$ is insufficient to fully constrain the set of ops

So we (Fetscher et al. 1986) include inverse "μ decay": $\nu_\mu + e^- \to \mu^- + \nu_e$

cross section S of *inverse* muon decay, normalized to the V-A value, yields [2]

$$|g^S_{LL}|^2 \leq 4(1 - S) \tag{58.13a}$$

and

$$|g^V_{LL}|^2 = S\,. \tag{58.13b}$$

Thus the Standard Model assumption of a pure V-A leptonic charged weak interaction of e and μ is derived (within errors) from experiments at energies far below the mass of the $W^{\pm}$: Eq. (58.13 b) gives a lower limit for V-A, and Eqs. (58.12 a, b, c) and (58.13 a) give upper limits for the other four-fermion interactions. The existence of such upper limits may also be seen from $Q_{RR}+Q_{RL} = (1-\xi')/2$ (e^+ longitudinal polarization) and $Q_{RR}+Q_{LR} = \frac{1}{2}(1+\xi/3-16\,\xi\delta/9)$ (decay asymmetry). Table 58.1 gives the current experimental limits on the magnitudes of the $g^{\gamma}_{\varepsilon\mu}$'s. More stringent limits on the six coupling constants g^S_{LR}, g^V_{LR}, g^T_{LR}, g^S_{RL}, g^V_{RL}, and g^T_{RL} have been derived from upper limits on the neutrino mass [15]. Limits on the "charge retention" coordinates, as used in the older literature (*e.g.*, Ref. [16]), are given by Burkard *et al.* [17].

Table 58.1: Coupling constants $g^{\gamma}_{\varepsilon\mu}$ and some combinations of them. Ninety-percent confidence level experimental limits. The limits on $|g^S_{LL}|$ and $|g^V_{LL}|$ are from [18–20], and the others from a general analysis of muon decay measurements. Top three rows: [21], fourth row: [22], next three rows: [23], last row: [24]. The experimental uncertainty on the muon polarization in pion decay is included. Note that, by definition, $|g^S_{\varepsilon\mu}| \leq 2$, $|g^V_{\varepsilon\mu}| \leq 1$ and $|g^T_{\varepsilon\mu}| \leq 1/\sqrt{3}$.

$\lvert g^S_{RR}\rvert < 0.035$	$\lvert g^V_{RR}\rvert < 0.017$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.050$	$\lvert g^V_{LR}\rvert < 0.023$	$\lvert g^T_{LR}\rvert < 0.015$
$\lvert g^S_{RL}\rvert < 0.420$	$\lvert g^V_{RL}\rvert < 0.105$	$\lvert g^T_{RL}\rvert < 0.105$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6\,g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6\,g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2\,g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2\,g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2\,g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2\,g^T_{RL}\rvert < 0.418$	
$Q_{RR} + Q_{LR} < 8.2 \times 10^{-4}$		

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5) Beyond the SM top down EFTs

Next week we will discuss the bottom up approach to BSM physics

we finish this wk w/ an example of top down:

Consider a heavy scalar S that is a singlet of the SM gauge group

ie $S \to S$ under the SM gauge xforms

We want to write the UV (renormalizable) $\mathcal{L}$:

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{1}{2}M^2 S^2 - V_S - V_{SH}$$

$$V_S = \frac{g}{3!}S^3 + \frac{\lambda_S}{4!}S^4$$

$$V_{SH} = g_{SH} S (H^\dagger H) + \frac{\lambda_{SH}}{2!} S^2 (H^\dagger H)$$

The only gauge invar op of $d \leq 4$ is $(H^\dagger H)$ so it is the only term we combine w/ S to write terms in $\mathcal{L}_S$ $\to$ this is the full set of ops

$$\mathcal{L}_{full} = \mathcal{L}_{SM} + \mathcal{L}_S$$

To integrate out S at tree level we need its EOM:

$$\frac{\partial \mathcal{L}}{\partial S} = -M^2 S - \frac{g}{2}S^2 - \frac{\lambda_S}{3!}S^3 - g_{SH}(H^\dagger H) - \lambda_{SH} S (H^\dagger H) = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu S)} = \Box S$$

$$\Rightarrow \underbrace{\left[\Box + M^2 + \lambda_{SH}(H^\dagger H)\right]}_{\equiv \Delta + M^2} S = \frac{-\lambda_S}{3!}S^3 - g_{SH}(H^\dagger H) - \frac{g}{2}S^2$$

So we have:

$$M^2\underbrace{\left(1+\frac{\Delta}{M^2}\right)}S = -\frac{g}{2}S^2 - \frac{\lambda_s}{3!}S^3 - g_{SH}(H^\dagger H)$$

we approx the inverse of this as:

$$\left(1-\frac{\Delta}{M^2}\right)^{-1} \sim 1 - \frac{\Delta}{M^2} + \frac{\Delta^2}{M^4} + \cdots$$

We have:

$$S = \frac{1}{M^2}\left(1-\frac{\Delta}{M^2}+\frac{\Delta^2}{M^4}+\cdots\right)\left(-\frac{g^2}{2}S^2 - \frac{\lambda_s}{3!}S^3 - g_{SH}(H^\dagger H)\right)$$

We will approximate S_c perturbatively: (note $[g]=1$, so we need to retain terms of $O(\frac{g^2}{M^4})$)

$$S_c = \frac{1}{M^2}S_0 + \frac{1}{M^4}S_1 + \cdots$$

$O(\frac{1}{M^2})$: $$\frac{S_0}{M^2} = \frac{1}{M^2}\Big(1-\underbrace{\cancel{\frac{\Delta}{M^2}}}_{O(\frac{1}{M^4})}+\cdots\Big)\left(-\frac{g^2}{2}\cancel{\frac{S_0^2}{M^4}} - \frac{\lambda_s}{3!}\cancel{\frac{S_0^3}{M^6}} - g_{SH}(H^\dagger H)\right)$$

$$= \frac{-1}{M^2}g_{SH}(H^\dagger H)$$

$$\rightarrow S_0 = -g_{SH}(H^\dagger H)$$

$O(\frac{1}{M^4})$: $$\left(\frac{S_0}{M^2}+\frac{S_1}{M^4}+\cdots\right) = \frac{1}{M^2}\left(1-\frac{\Delta}{M^2}+\cdots\right)\Big(-\frac{g}{2}\underbrace{\cancel{\frac{S_0^2}{M^4}}}_{O(\frac{1}{M^6})} - g_{SH}(H^\dagger H) + \cdots\Big)$$

$$\cancel{\frac{-g_{SH}(H^\dagger H)}{M^2}} + \frac{S_1}{M^4} = \cancel{-\frac{g_{SH}(H^\dagger H)}{M^2}} - \frac{g_{SH}}{M^4}\Delta(H^\dagger H) + O\left(\frac{1}{M^6}\right)$$

$$\rightarrow S_1 = -g_{SH}\Delta(H^\dagger H)$$

We conclude: $S_c = \frac{-g_{SH}}{M^2}(H^\dagger H) - \frac{g_{SH}}{M^4}\Delta(H^\dagger H) + O(\frac{1}{M^6}) = \frac{-g_{SH}}{M^2}(H^\dagger H) - \frac{g_{SH}}{M^4}\Box(H^\dagger H) - \frac{g_{SH}}{M^4}(H^\dagger H)^2$

We then plug S_c into our $\mathcal{L}$:

$$\mathcal{L}_S = -\frac{1}{2} S\Box S - \frac{1}{2}M^2S^2 - \frac{g}{3!}S^3 - \frac{\lambda_S}{4!}S^4 - g_{SH}S(H^\dagger H) - \frac{\lambda_{SH}}{2!}S^2(H^\dagger H)$$

notice: $S_c \sim \frac{g}{M^2} + \cdots$

$[1/M]$

so $S^4 \sim 1/M^4$ is too high order

$gS^3 \sim \left(\frac{g}{M^2}\right)^3 g \rightarrow \left[\frac{g^4}{M^6}\right] = [1/M^2]$ so we have to keep this

Term by term:

$$S\Box S \rightarrow \frac{g_{SH}^2}{M^4}\underbrace{(H^\dagger H)\Box(H^\dagger H)}_{\text{A D6 op}} + O\left(\frac{g^2}{M^6}\right)$$

remember $[g]=1$, so this is like $\frac{1}{M^4}$

$$M^2S^2 \rightarrow \frac{g_{SH}^2}{M^2}\left[(H^\dagger H) + \frac{1}{M^2}\Box(H^\dagger H) + \frac{1}{M^2}(H^\dagger H)^2\right]^2$$

$$= \frac{g_{SH}^2}{M^2}(H^\dagger H)^2 + \frac{2g_{SH}^2}{M^4}(H^\dagger H)\Box(H^\dagger H) + \frac{2g_{SH}^2}{M^4}(H^\dagger H)^3 + O\left(\frac{g^2}{M^6}\right)$$

$$-g_{SH}S(H^\dagger H) \rightarrow \frac{g_{SH}^2}{M^2}\underbrace{(H^\dagger H)^2}_{\text{a D4 correction!}} + \frac{g_{SH}^2}{M^4}(H^\dagger H)\Box(H^\dagger H) + \frac{g_{SH}^2}{M^4}\underbrace{(H^\dagger H)^3}_{\text{another D6 op}} + O\left(\frac{g^2}{M^6}\right)$$

$$S^2(H^\dagger H) \rightarrow \frac{g_{SH}^2}{M^4}(H^\dagger H)^3$$

$$S^3 \rightarrow \frac{-g_{SH}^3}{M^6}(H^\dagger H)^3$$

Plugging all of this into $\mathcal{L}_S$:

$$\mathcal{L}_S \to -\frac{g_{SH}^2}{2M^4}(H^\dagger H)\Box(H^\dagger H)$$

$$-\frac{g_{SH}^2}{2M^2}(H^\dagger H)^2 - \frac{g_{SH}^2}{M^4}(H^\dagger H)\Box(H^\dagger H) - \frac{g_{SH}^2}{M^4}(H^\dagger H)^3$$

$$+\frac{g_{SH}^2}{M^2}(H^\dagger H) + \frac{g_{SH}^2}{M^4}(H^\dagger H)\Box(H^\dagger H) + \frac{g_{SH}^2}{M^4}(H^\dagger H)^3$$

$$-\frac{\lambda_{SH}\, g_{SH}^2}{2M^4}(H^\dagger H)^3$$

$$+\frac{g}{3!}\frac{g_{SH}^3}{M^6}(H^\dagger H)^3$$

$$= \frac{g_{SH}^2}{2M^2}\underbrace{(H^\dagger H)}_{\equiv Q_H^{(4)}} - \frac{g_{SH}^2}{2M^4}\underbrace{(H^\dagger H)\Box(H^\dagger H)}_{\equiv Q_{H\Box}^{(6)}} + \frac{g_{SH}^2}{M^4}\left(\frac{g\, g_{SH}}{3!\, M^2} - \frac{\lambda_{SH}}{2}\right)\underbrace{(H^\dagger H)^3}_{\equiv Q_H^{(6)}}$$

$$\equiv C_H^{(4)} Q_H^{(4)} + C_{H\Box}^{(6)} Q_{H\Box} + C_H^{(6)} Q_H^{(6)}$$

$Q_i^{(n)}$ are ops of dim n, w/ label i

$C_i^{(n)}$ are the low E constants or Wilson coefficients

$Q_{H\Box}^{(6)}$ and $Q_H^{(6)}$ are two of many dim 6 operators in the "SMEFT"

SM → Standard Model
EFT → Effective Field Thry

We also saw $(\bar{L}\gamma_\mu L)(\bar{L}\gamma_\mu L)$, this could also be generated by BSM (Beyond the SM) physics, e.g. a heavy new vector coupling to LH leptons (aka a W')

<u>Decoupling Theorem</u>: if the low-energy thry is renormalizable, then all effects of the heavy particle appear as 1) a renormalization of the couplings in the thry (e.g. $C_H^{(4)}(H^\dagger H)^2$)

or 2) are suppressed by powers of the heavy particle mass

(e.g. all our D6 ops)