2)

$$\psi = (P_L^2 + P_R^2)\psi$$

$$= P_L\psi_L + P_R\psi_R$$

$$\bar{\psi} = (\psi_L^\dagger P_L + \psi_R^\dagger P_R)\gamma_0$$

$$= \bar{\psi}_L P_R + \bar{\psi}_R P_L$$

a)

I forgot $\bar{\psi}\chi$, but take unbarred $\psi \to \chi$

$$\bar{\psi}\psi = (\bar{\psi}_L P_R + \bar{\psi}_R P_L)(P_L\psi_L + P_R\psi_R)$$

$$= \bar{\psi}_L \underbrace{P_R P_L}_{0} \psi_L + \bar{\psi}_L \underbrace{P_R P_R}_{P_R} \psi_R + \bar{\psi}_R \underbrace{P_L P_L}_{P_L} \psi_L + \bar{\psi}_R \underbrace{P_L P_R}_{0} \psi_R$$

$$= \bar{\psi}_L \underbrace{P_R \psi_R}_{\psi_R} + \bar{\psi}_R \underbrace{P_L \psi_L}_{\psi_L}$$

$$= \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$$

$$\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L P_R + \bar{\psi}_R P_L)\gamma^\mu(P_L\psi_L + P_R\psi_R)$$

$$= (\bar{\psi}_L \gamma^\mu P_L + \bar{\psi}_R \gamma^\mu P_R)(P_L\psi_L + P_R\psi_R)$$

$$= \bar{\psi}_L \gamma^\mu \psi_L + \bar{\psi}_R \gamma^\mu \psi_R$$

$$\bar{\psi}\sigma^{\mu\nu}\psi = (\bar{\psi}_L P_R + \bar{\psi}_R P_L)\sigma^{\mu\nu}(P_L\psi_L + P_R\psi_R)$$

$$= (\bar{\psi}_L \sigma^{\mu\nu} P_R + \bar{\psi}_R \sigma^{\mu\nu} P_L)(P_L\psi_L + P_R\psi_R)$$

$$= \bar{\psi}_L \sigma^{\mu\nu} \psi_R + \bar{\psi}_R \sigma^{\mu\nu} \psi_L$$

$$\gamma_5 P_\pm = \gamma_5 \tfrac{1}{2}(\mathbb{1} \pm \gamma_5)$$
$$= \tfrac{1}{2}(\gamma_5 \pm \mathbb{1}) \qquad \gamma_5^2 = \mathbb{1}$$
$$= \pm \tfrac{1}{2}(\mathbb{1} \pm \gamma_5)$$

$$\bar\psi \gamma_5 \psi = (\bar\psi_L P_R + \bar\psi_R P_L)\gamma_5 (P_L \psi_L + P_R \psi_R)$$
$$= (+\bar\psi_L \gamma_5 P_R - \bar\psi_R \gamma_5 P_L)(P_L \psi_L + P_R \psi_R)$$
$$= \bar\psi_L \gamma_5 \psi_R - \bar\psi_R \gamma_5 \psi_L$$

$$\bar\psi \gamma_5 \gamma^\mu \psi = (\bar\psi_L P_R + \bar\psi_R P_L)\gamma_5 \gamma^\mu (P_L \psi_L + P_R \psi_R)$$
$$= (+\bar\psi_L \gamma_5 \gamma^\mu P_L - \bar\psi_R \gamma_5 \gamma^\mu P_R)(P_L \psi_L + P_R \psi_R)$$
$$= \bar\psi_L \gamma_5 \gamma^\mu \psi_L - \bar\psi_R \gamma_5 \gamma^\mu \psi_R$$

b) $$\bar\psi P_R \chi = (\bar\psi_L P_R + \bar\psi_R P_L)(P_R \chi_R)$$
$$= \bar\psi_L \chi_R$$

$$\bar\psi P_L \chi = \bar\psi_R \chi_L$$

$$\bar\psi \gamma^\mu P_R \chi = (\bar\psi_L \gamma^\mu P_L + \bar\psi_R \gamma^\mu P_R) P_R \chi_R$$
$$= \bar\psi_R \gamma^\mu \chi_R$$

$$\bar\psi \gamma^\mu P_L \chi = \bar\psi_L \gamma^\mu \chi_L$$

$$\bar\psi \sigma^{\mu\nu} \chi = \bar\psi_L \sigma^{\mu\nu} \chi_R + \bar\psi_R \sigma^{\mu\nu} \chi_L \quad \text{(from a)}$$

3)

$$\left[i\bar{\psi}\gamma_\mu\partial^\mu\psi\right]^\dagger = -i(\partial^\mu\psi^\dagger)\gamma_\mu^\dagger\underset{\substack{\uparrow \\ \gamma_0^\dagger=\gamma_0}}{\gamma_0^\dagger}\psi$$

$$= -i(\partial^\mu\psi^\dagger)\underbrace{\gamma_0\gamma_0}_{\mathbb{1}}\underbrace{\gamma_\mu^\dagger\gamma_0}_{\gamma_0\gamma_\mu^\dagger\gamma_0=\gamma_\mu}\psi$$

$$= -i(\partial^\mu\bar{\psi})\gamma_\mu\psi$$

$$\partial_\mu(i\bar{\psi}\gamma^\mu\psi) = i\bar{\psi}\gamma^\mu\partial_\mu\psi + i(\partial_\mu\bar{\psi})\gamma^\mu\psi \underset{\uparrow}{=} 0$$

we mean the total divergence vanishes after integrating $\int d^4x$ in the action

So $-i(\partial_\mu\bar{\psi})\gamma^\mu\psi = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \cancel{\partial_\mu(i\bar{\psi}\gamma^\mu\psi)}$

We conclude the free $\mathcal{L}$ is hermitian up to a total divergence

4)

$$Z[\bar\eta,\eta] = \int D\bar\Phi\, D\Phi\, \exp\left[i\int d^4x\, c\,\bar\Phi_a(x)\Phi_a(x)\bar\Phi_b(x)\Phi_b(x)\right]\exp\left[-i\int d^4x\, d^4y\, \bar\eta_c(y)G_{cd}(x-y)\eta_d(x)\right]$$

$$\langle 0|T\{\Phi_\xi(x_1)\bar\Phi_{\xi'}(x_2)\bar\Phi_g(z)\Phi_g(z)\bar\Phi_h(z)\Phi_h(z)\}|0\rangle$$

$$\sim \partial_{\bar\eta\xi 1}\,\partial_{\eta\xi' 2}\,\partial_{\eta g z}\,\partial_{\bar\eta g z}\,\partial_{\eta h z}\,\partial_{\bar\eta h z}\,\underbrace{e^{\bar\eta_{cy}G_{cdyx}\eta_{dx}}\Big|_{\stackrel{(-)}{\eta}\to 0}}_{\equiv e}$$

$$= \partial^5\left[G_{hdzx}\eta_{dx}\right]e$$

$$= \partial^4\left[\cancel{(-1)^{2}}{}^{1}\,G_{hdzx}\eta_{dx}\bar\eta_{cy}G_{chyz} + G_{hhzz}\right]e$$

anticommute!

$$= \partial^3\left[\cancel{(-1)^{2}}{}^{1}\,G_{hdzx}\eta_{dx}\bar\eta_{cy}G_{chyz}G_{gd'zx'}\eta_{d'x'} + (-1)G_{hdzx}\eta_{dx}G_{ghzz} + G_{hhzz}G_{gdzx}\eta_{dx}\right]e$$

$$= \partial^2\Big[G_{hgzz}\bar\eta_{cy}G_{chyz}G_{gd'zx'}\eta_{d'x'} + \cancel{(-1)^{2}}{}^{1}\,G_{hdzx}\eta_{dx}\bar\eta_{cy}G_{chyz}G_{ggzz}$$

$$- \cancel{(-1)^{2}}{}^{1}\,G_{hdzx}\eta_{dx}G_{ghzz}\bar\eta_{cy}G_{cgyz} - G_{hgzz}G_{ghzz}$$

$$+ \cancel{(-1)^{2}}{}^{1}\,G_{hhzz}G_{gdzx}\eta_{dx}\bar\eta_{cy}G_{cgyz} + G_{hhzz}G_{ggzz} + \text{too many sources}\Big]e$$

$$= \partial\Big[-G_{hgzz}\bar\eta_{cy}G_{chyz}G_{g\xi' z x_2} + G_{h\xi' z x_2}\bar\eta_{cy}G_{chyz}G_{ggzz}$$

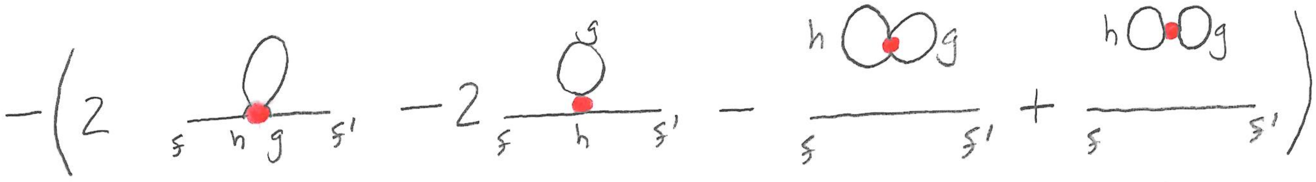
$$- G_{h\xi' z x_2}G_{ghzz}\bar\eta_{cy}G_{cgyz} - (-1)G_{hgzz}G_{ghzz}\bar\eta_{cy}G_{c\xi' y x_2}$$

$$+ G_{hhzz}G_{g\xi' z x_2}\bar\eta_{cy}G_{cgyz} + (-1)G_{hhzz}G_{ggzz}\bar\eta_{cy}G_{c\xi' y x_2} + \text{too many sources}\Big]e$$

$$= \Big[-G_{hgzz}G_{\xi h x_1 z}G_{g\xi' z x_2} + G_{h\xi' z x_2}G_{\xi h x_1 z}G_{ggzz}$$

$$- G_{h\xi' z x_2}G_{ghzz}G_{\xi g x_1 z} + G_{hgzz}G_{ghzz}G_{\xi\xi' x_1 x_2}$$

$$+ G_{hhzz}G_{g\xi z x_2}G_{\xi g x_1 z} - G_{hhzz}G_{ggzz}G_{\xi\xi' y x_2}\Big]$$



no closed loop $(-1)^0$

1 closed loop $(-1)^1$

1 closed loop $(-1)^1$

2 closed loops $(-1)^2$

We can restore the prefactors

$(-i)^3$ for 3x props

ic for vertex

$\Rightarrow -c$