For QED we had $G = \partial_\mu A_\mu$

Now we choose $G = \partial_\mu A_\mu - \xi e v \pi$

$$\delta A_\mu = -\partial_\mu \alpha$$

$$\left.\begin{aligned} \delta\varphi &= +ie\alpha\varphi \\ \delta\varphi^* &= -ie\alpha\varphi^* \end{aligned}\right\} \quad \pi = -i\sqrt{2}\,\mathrm{Im}(\varphi) \rightarrow \delta\pi = -i\sqrt{2}\,\frac{\delta\varphi - \delta\varphi^*}{2}$$

$$= \frac{-i\sqrt{2}}{2}\left(+ie\alpha\varphi + ie\alpha\varphi^*\right)$$

$$= +\frac{e\sqrt{2}}{2}\,\alpha\,\frac{2(v+\sigma)}{\sqrt{2}} = +e(v+\sigma)\alpha$$

$$\delta G = -\partial_\mu \partial^\mu \alpha + \xi e v(-e(v+\sigma))\alpha$$

$$= \left[-\Box - \xi e^2 v(v+\sigma)\right]\alpha$$

$$\frac{\delta G}{\delta \alpha} = -\Box - \xi e^2 v(v+\sigma)$$

So $\mathcal{L}_{gh} = +\bar{c}^a\left(-\partial_\mu\partial^\mu - \xi e^2 v(v+\sigma)\right)c^a$

$$= (\partial_\mu \bar{c}^a)(\partial^\mu c^a) \underbrace{- \xi e^2 v^2 \bar{c}^a c^a}_{\substack{\text{ghost mass} \\ m_{gh}^2 = \xi e^2 v^2}} \underbrace{- \xi e^2 v \sigma \bar{c}^a c^a}_{\text{ghost-}\sigma\text{ interaction}}$$

And $\mathcal{L}_{GF} = \frac{-1}{2\xi} G^2 = \frac{-1}{2\xi}\left(\partial_\mu A_\mu - \xi e v \pi\right)^2$

$$= \frac{-1}{2\xi}\left[(\partial_\mu A_\mu)^2 - 2\xi e v \pi\, \partial_\mu A_\mu + \xi^2 e^2 v^2 \pi^2\right]$$

$$= \underbrace{\frac{-1}{2\xi}(\partial_\mu A_\mu)^2}_{\substack{\text{normal GF} \\ \text{term}}} + \underbrace{e v \pi (\partial_\mu A_\mu)}_{\substack{\text{IBP} \rightarrow = -e v A_\mu (\partial_\mu \pi) \\ \text{exactly cancels} \\ A\text{-}\pi \text{ mixing}}} - \frac{\xi e^2 v^2}{2}\pi^2$$

$m_\pi^2 = \xi e^2 v^2$ (π is $\mathbb{R}$ so $\frac{1}{2}$ isnt part of m_π^2)

We have:

$$\delta A_\mu = -\partial_\mu \alpha$$

$$\delta \varphi = ie\alpha\varphi$$

$$\delta \varphi^* = -ie\alpha\varphi^*$$

So taking

$$\delta\Gamma = 0 = \frac{\delta\Gamma}{\delta A_\mu}(-\partial_\mu\alpha) + \frac{\delta\Gamma}{\delta\varphi} ie\alpha\varphi + \frac{\delta\Gamma}{\delta\varphi^*}(-ie\alpha\varphi^*)$$

$$= \underbrace{\left[\partial_\mu \frac{\delta\Gamma}{\delta A_\mu} + ie\left(\frac{\delta\Gamma}{\delta\varphi}\varphi - \frac{\delta\Gamma}{\delta\varphi^*}\varphi^*\right)\right]}_{=0 \text{ bc } \alpha \text{ arbitrary}}\alpha$$

Taking a variation w/r to A_ν, then setting all fields to their vevs:

$$\partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta A_\nu} + ie\left(\frac{\delta^2\Gamma}{\delta A_\nu \delta\varphi}\langle\varphi\rangle - \frac{\delta^2\Gamma}{\delta A_\nu \delta\varphi^*}\langle\varphi^*\rangle\right) = 0$$

both $\frac{v}{\sqrt{2}}$

$$\varphi = \frac{v+\sigma+i\pi}{\sqrt{2}} \rightarrow \frac{\delta}{\delta\varphi} = \underbrace{\frac{\delta\varphi}{\delta\sigma}}_{1/\sqrt{2}}\frac{\delta}{\delta\sigma} + \underbrace{\frac{\delta\varphi}{\delta\pi}}_{i/\sqrt{2}}\frac{\delta}{\delta\pi}, \quad \frac{\delta}{\delta\varphi^*} = \frac{1}{\sqrt{2}}\frac{\delta}{\delta\sigma} - \frac{i}{\sqrt{2}}\frac{\delta}{\delta\pi}$$

$$\partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta A_\nu} + \frac{iev}{2}\left(\cancel{\frac{\delta^2\Gamma}{\delta A_\nu \delta\sigma}} + i\frac{\delta^2\Gamma}{\delta A_\nu \delta\pi} - \cancel{\frac{\delta^2\Gamma}{\delta A_\nu \delta\sigma}} + i\frac{\delta^2\Gamma}{\delta A_\nu \delta\pi}\right) = 0$$

$$\rightarrow \partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta A_\nu} = ev\frac{\delta^2\Gamma}{\delta A_\nu \delta\pi}$$

↑

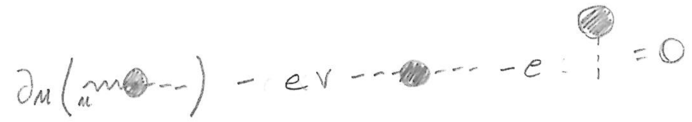
Longitudinal 2pt is prop to A-π mixing! $\partial_\mu\Big($  $\Big) = e\alpha v\Big($  $\Big)$

Returning to the master Ward ID & varying w/r to π:

$$\partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta\pi} + ie\left(\frac{\delta^2\Gamma}{\delta\varphi\delta\pi}\langle\varphi\rangle + \frac{\delta\Gamma}{\delta\varphi}\frac{\delta\varphi}{\delta\pi} - \frac{\delta^2\Gamma}{\delta\varphi^*\delta\pi}\langle\varphi^*\rangle - \frac{\delta\Gamma}{\delta\varphi^*}\frac{\delta\varphi^*}{\delta\pi}\right) = 0$$

$$= \partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta\pi} + ie\left(\cancel{\frac{1}{\sqrt2}\frac{\delta^2\Gamma}{\delta\sigma\delta\pi}\frac{v}{\sqrt2}} + \frac{i}{\sqrt2}\frac{\delta^2\Gamma}{\delta\pi\delta\pi}\frac{v}{\sqrt2} + \frac{1}{\sqrt2}\frac{\delta\Gamma}{\delta\sigma}\frac{i}{\sqrt2} + \cancel{\frac{i}{\sqrt2}\frac{\delta\Gamma}{\delta\pi}\frac{i}{\sqrt2}} \right.$$
$$\left. - \cancel{\frac{1}{\sqrt2}\frac{\delta^2\Gamma}{\delta\sigma\delta\pi}\frac{v}{\sqrt2}} + \frac{i}{\sqrt2}\frac{\delta^2\Gamma}{\delta\pi\delta\pi}\frac{v}{\sqrt2} - \frac{1}{\sqrt2}\frac{\delta\Gamma}{\delta\sigma}\left(-\frac{i}{\sqrt2}\right) + \cancel{\frac{i}{\sqrt2}\frac{\delta\Gamma}{\delta\pi}\left(-\frac{i}{\sqrt2}\right)}\right) = 0$$

$$= \partial_\mu \frac{\delta^2\Gamma}{\delta A_\mu \delta\pi} - e\left(\frac{\delta^2\Gamma}{\delta\pi\delta\pi} v + \frac{\delta\Gamma}{\delta\sigma}\right) = 0$$



Setting all fields in the master Ward ID to zero we see:

$$\underbrace{\partial_\mu \frac{\delta\Gamma}{\delta A_\mu}}_{=0,\ \text{no } A_\mu \text{ tadpole}} + ie\,\frac{v}{\sqrt2}\left(\frac{1}{\sqrt2}\frac{\delta\Gamma}{\delta\sigma} + \frac{i}{\sqrt2}\frac{\delta\Gamma}{\delta\pi} - \cancel{\frac{1}{\sqrt2}\frac{\delta\Gamma}{\delta\sigma}} + \frac{i}{\sqrt2}\frac{\delta\Gamma}{\delta\pi}\right) = 0$$

$\Rightarrow \frac{\delta\Gamma}{\delta\pi} v = 0$, since $v \neq 0$ we conclude $\frac{\delta\Gamma}{\delta\pi} = 0$

$\bar{Q}\tilde{H}u_R$ invar

$\tilde{H} = i\sigma_2 H^*$

$H^* \to e^{-ig_1/2\,\theta_Y + \frac{-ig_2}{2}\theta_L\cdot\sigma^*}$ (recall $H \to e^{ig_1/2\,\theta_Y + \frac{ig_2}{2}\theta_L\cdot\sigma}$)

$\bar{Q} \to e^{-ig_1/6\,\theta_Y - \frac{i}{2}g_2\theta_L\cdot\sigma - \frac{i}{2}g_3\theta_c\cdot\lambda}$

$u_R \to e^{ig_1\theta_Y 2/3 + \frac{i}{2}g_3\theta_c\cdot\lambda}$

So $U(1)_Y$ is trivial: $e^{-ig_1/2\,\theta_Y} \cdot e^{-ig_1/6\,\theta_Y} e^{ig_1\theta_Y 2/3} = e^{ig_1(\frac{1}{2}-\frac{1}{6}+2/3)\theta_Y} = \mathbb{1}$

$U(3)_c$ is too: $e^{-\frac{i}{2}g_3\theta_3\cdot\lambda} e^{\frac{i}{2}g_3\theta_3\cdot\lambda} = \mathbb{1}$

For $SU(2)_L$ recall: $(\sigma^1)^* = \sigma^1$, $(\sigma^2)^* = -\sigma^2$, $(\sigma^3)^* = \sigma^3$

$$\sigma^i\sigma^2 + \sigma^2\sigma^{i*} = \begin{cases} i=1: & \sigma^1\sigma^2 + \sigma^2\sigma^{1*} = \{\sigma^1,\sigma^2\} = 0 \\ 2: & \sigma^2\sigma^2 + \sigma^2\sigma^{2*} = 0 \\ 3: & \sigma^3\sigma^2 + \sigma^2\sigma^{3*} = \{\sigma^3,\sigma^2\} = 0 \end{cases}$$

So $e^{-\frac{i}{2}g_2\theta_L\cdot\sigma}\, i\sigma_2\, e^{-\frac{i}{2}g_2\theta_L\cdot\sigma^*} \to (\mathbb{1} - \frac{ig_2}{2}\delta\theta_L\cdot\sigma)\, i\sigma_2 (\mathbb{1} - \frac{i}{2}g_2\delta\theta_L\cdot\sigma^*)$

$$= i\sigma_2 + \frac{g_2}{2}\delta\theta_L^i\sigma^i\sigma_2 + \frac{g_2}{2}\sigma_2\delta\theta_L^i(\sigma^i)^* + O(\delta\theta_L^2)$$

$$= i\sigma_2 + \frac{g_2}{2}\delta\theta_L \underbrace{(\sigma^i\sigma_2 + \sigma_2\sigma^{i*})}_{=0}$$

$$= i\sigma_2$$

So $\bar{Q}\tilde{H}u_R$ is $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ invar.