CMP_SC: Homework 2
Due: 9:15 am, February 12, 2015

Answers to the following problems must be handed in class. Unless otherwise stated, the running time of an algorithm is its running time in the worst case. There are a total of 20 points and will contribute towards 2% of the final grade. You must explain why your algorithm runs in the stated time bounds in order to get full credit.

1. Consider the following computational problem.

   Input: A sequence $a_1, \ldots, a_n$ of n integers stored in an array A.
   Output: A doubly-indexed array B of size $n \times n$ such that

   $$B[i,j] = \begin{cases} 0 & \text{if } j < i \\ \max\{a_k | i \leq k \leq j\} & \text{otherwise.} \end{cases}$$

   For example, if $n = 3$, $a_1 = -3, a_2 = 2$ then A is the array

-3	2	1

   and B is the 2-dimensional array:

-3	2	2
0	2	2
0	0	1

   Give an algorithm that solves the above computational problem in $O(n^2)$ running time. (4 points).

2. Consider the following computational problem.

   Input: A *sorted* sequence $a_1, \ldots, a_n$ of n integers stored in an array A and a number v.
   Output: The number of times v occurs in A.
   For example, if A is the array

-3	1	1	5	6

   and v is 1, then v occurs 2 times in A.

   Give an algorithm that solves the above problem in $O(\log n)$ running time. (4 points)

3. We want to solve the following computational problem.

   Input: A sequence $a_1, \ldots, a_n$ of n integers stored in an array A and a number $k \leq n$.
   Output: A one-dimensional array B such that

$$B[i] = \begin{cases} 0 & \text{if } i < k \\ k\text{-th smallest number of } A[1 \ldots i] & \text{otherwise.} \end{cases}$$

   For example, if A is the array

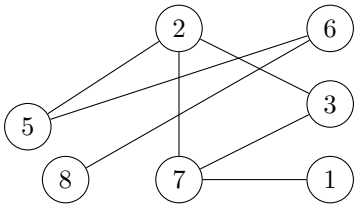
2	−3	5	1	6

   and k is 3, then B is

0	0	5	2	2

   Give an algorithm that solves the above problem in $O(n \log k)$ running time. (5 points).

4. Consider the following undirected graph G:



   (a) Give the adjacency matrix representation of G. (2 points)

   (b) Give the adjacency list representation of G. (2 points)

   (c) Give the distances and the BFS tree generated by running the Breadth First Search algorithm on G with 3 as the source vertex. (3 points)