DSP Formelsammlung Sedlmeier, Toni

1 Zahlenformate

		Numeric		Dynamic
Bits	Format	Range	Precision	Range
8	Unsigned integer	$0 \rightarrow +255$	1	$\approx 48~\mathrm{dB}$
8	Signed integer	$-128 \rightarrow +127$	1	$\approx 48~\mathrm{dB}$
16	Unsigned integer	$0 \to +65,536$	1	$\approx 96~\mathrm{dB}$
16	Signed integer	$-32,768 \to +32,767$	1	$\approx 96~\mathrm{dB}$
16	Fixed-point (Q12)	$-8.0 \rightarrow \approx +7.999756$	≈ 0.000244	$\approx 96~\mathrm{dB}$
16	Fixed-point (Q15)	$-1.0 \rightarrow \approx +0.9999695$	≈ 0.0000305	$\approx 96~\mathrm{dB}$
32	Unsigned integer	$0 \to +4,294,967,296$	1	$\approx 193~\mathrm{dB}$
32	Signed integer	$-2, 147, 483, 648 \rightarrow +2, 147, 483, 647$	1	$\approx 193~\mathrm{dB}$
32	Single-precision	$\approx \pm 3.402823 \times 10^{38}$	$\approx 1.19 \times 10^{-7}$	$\approx 138~\mathrm{dB}$
64	Double-precision	$\approx \pm 1.797693 \times 10^{308}$	$\approx 2.22 \times 10^{-16}$	$\approx 314~\mathrm{dB}$

1.1 Zweierkomplement

Umwandlung: Bsp. 8-Bit $(-4)_{10}$ (Funktioniert in beide Richtungen)

- Vorzeichen Ignorieren $(4)_{10} = (00000100)_2$
- Bits Invertieren $(0000\ 0100)_2 \rightarrow (1111\ 1011)_2$
- Eins Addieren $(1111\ 1011)_2 + (0000\ 0001)_2 = (1111\ 1100)_2$

1.2 Fixed Point (unsigned)

Qk.l mit k = Vorkomma und l = Nachkomma

$$x_{(10)} = \sum_{i=0}^{k-1} b_i \cdot 2^i + \sum_{j=-l}^{-1} b_j \cdot 2^j$$
 (1)

Bsp Q4.5
$$a = (01010110)_2$$

 $0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} = 5.375$

1.3 Fixed Point (signed)

Bsp. $Q3.3 (100.001)_2$

- Vorzeichen Merken (100.001) \rightarrow -1
- Bits Invertieren $(100\ 001)_2 \rightarrow (011\ 110)_2$
- $1 \cdot 2^{-k}$ Addieren $(011\ 110)_2 + (000\ 001)_2 = (011\ 111)_2 = -3.875$

2 Filter in c

2.1 FIR

$$A(0)y(k) = \sum_{i=0}^{N} B(i)x(k-i)$$
 (2)

2.2 IIR

$$A(0)y(k) = \sum_{i=0}^{N} B(i)x(k-i) - \sum_{i=0}^{N} A(i)y(k-i)$$
 (3)

2.3 Notch-IIR-Filter

$$H(z) = k \frac{(z - \beta_1)(z - \beta_2)}{(z - \alpha_1)(z - \alpha_2)} = k \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$$
(4)

 $f_0 = \text{Kerbfrequenz}, f_s = \text{Abtastfrequenz} \ B_{3dB} = \text{Kerbbreite}$ te

$$k = \frac{1 - 2r\cos(\omega_0) + r^2}{1 - 2\cos(\omega_0) + 1} \tag{5}$$

$$\omega_0 = 2\pi \frac{f_0}{f} \tag{6}$$

$$r = 1 - \left(\frac{B_{3dB}}{f_s}\right)\pi\tag{7}$$

2.4 IIR-Oszillator

$$sin(\omega_0 k) \leftrightarrow \frac{sin(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$$

$$(8)$$

$$y(n) = sin(\omega_0)x(n-1) + 2cos(\omega_0)y(n-1) - y(n-2)$$

$$(9)$$

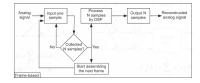
2.5 DDS-Oszillator

$$\varphi_{inc} = 2\pi \frac{f_0}{f_s} \tag{10}$$

$$\varphi = \varphi + \varphi_{inc} \tag{11}$$

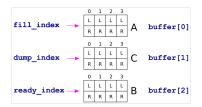
$$x(n) = Asin(n\varphi) \tag{12}$$

3 Blocksignalverarbeitung



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ISR schreibt N samples nach buffer[fill_index] und setzt ready index = fill index (buffer[fill index] ist jetzt dran mit ProcessBuffer). Jeder Sample generiert einen Interrupt



- fill index wird von ADC gefüllt
- dump_index wird an DAC geschrieben
- ready index Buffer für Blocksignalverarbeitung

```
// buffer length in samples
#define NUM BUFFERS
volatile float buffer[NUM_BUFFERS][2][BUFFER_LENGTH];
void ProcessBuffer()
// Processes the data in buffer[ready_index]
    volatile float *pL = buffer[ready_index][LEFT];
    volatile float *pR = buffer[ready_index][RIGHT];
    // Do the Process
    buffer_ready = 0;
                         // means were done here
}
interrupt void Codec_ISR()
    static Uint8 fill_index = INITIAL_FILL_INDEX; // index of buffer to fill
static Uint8 dump_index = INITIAL_DUMP_INDEX; // index of buffer to dump
    static Uint32 sample_count = 0; // current sample count in buffer
    // get input data samples
          CodecDataIn.UINT = ReadCodecData();
           // IN
    buffer[fill_index][ LEFT][sample_count] = LEFT + RIGHT; // cropped
    buffer[fill_index][RIGHT][sample_count] = RIGHT + LEFT; // cropped
    CodecDataOut.channel[ LEFT] = buffer[dump_index][LEFT][sample_count];
    CodecDataOut.channel[RIGHT] = buffer[dump_index][RIGHT][sample_count];
       update sample count and swap buffers when filled
    if(++sample_count >= BUFFER_LENGTH) {
        sample_count = 0;
        ready_index = fill_index;
        if(++fill_index >= NUM_BUFFERS)
fill_index = 0;
        if(++dump_index >= NUM_BUFFERS)
             dump_index = 0;
        if(buffer_ready == 1) // set a flag if buffer isn't processed in time
             over_run = 1;
        buffer_ready = 1;
    }
        WriteCodecData(CodecDataOut.UINT); // send output data to port
}
```

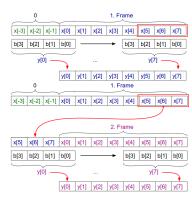
3.1 Blocksignalverarbeitung mit DMA

- DMA kopiert Sample von ADC nach Eingangsbuffer
- DMA kopiert Processed von Ausgangsbuffer nach DAC
- DMA generiert Interrupt, wenn N Samples transfert wurden \rightarrow Buffer-Swap

```
interrupt void EDMA_ISR()
    if(++ready_index >= NUM_BUFFERS)
        ready_index = 0;
    if(buffer_ready ==1) //buffer isnt processed in time
        over_run = 1;
    buffer_ready = 1; //buffer is now ready for processing
}
```

3.2FIR mit Blocksignalverarbeitung

Bsp. Ordnung Filter N = 4, Framesize = 8



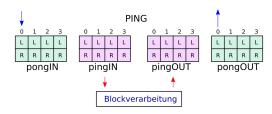
Problem: Bei Frame-Übergängen müssen die N-1 letzten Werte des letzten Frames berücksichtigt werden.

Lösung: Framesize += N

- Left[N|FRAMESIZE], buffer[FRAMESIZE]
- Left[N:FRAMESIZE+N] = buffer[0:FRAMESIZE]
- buffer[0:FRAMESIZE] = Left * B (B wird)drüber FRAMESIZE-mal drüber geschoben s.o)
- Left[0:N] = Left[FRAMESIZE:FRAMESIZE+N]

```
void ProcessBuffer()
    short *pBuf = buffer[ready_index];
    // extra buffer room for convolution "edge effects"
    // N is filter order from coeff.h
    static float Left[BUFFER_COUNT+N]={0}, Right[BUFFER_COUNT+N]={0};
    float *pL = Left, *pR = Right;
    float yLeft, yRight;
    int i, j, k;
    // offset pointers to start filling after N elements
    pL += N;
    // extract data to float buffers
    for(i = 0; i < BUFFER_COUNT; i++)</pre>
        *pR++ = *pBuf++;
        *pL++ = *pBuf++;
    // reinitialize pointer before FOR loop
    pBuf = buffer[ready_index];
    // Implement FIR filter
    for(i=0; i < BUFFER_COUNT; i++)</pre>
        yLeft = 0; // initialize the LEFT output value
        yRight = 0; // initialize the RIGHT output value
        for(j=0,k=i+N; j <= N; j++,k--)
            \label{eq:left} \mbox{yLeft += Left[k] * B[j]; // perform the LEFT dot-product}
            yRight += Right[k] * B[j]; // perform the RIGHT dot-product
        // pack into buffer after bounding (must be right then left)
        *pBuf++ = _spint(yRight * 65536) >> 16;
        *pBuf++ = _spint(yLeft * 65536) >> 16;
    // save end values at end of buffer array for next pass
    // by placing at beginning of buffer array
    for(i=BUFFER_COUNT,j=0; i < BUFFER_COUNT+N; i++,j++)</pre>
        Left[j] = Left[i];
        Right[j] = Right[i];
    buffer_ready = 0; // signal we are done
```

Ping-Pong 3.3



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- gleiche Latenz (=Durchlaufzeit 2 Buffer)
- Ping-Pong einfacher zu verwalten mit DMA

4 FFT

$$w_N^{nk} = e^{-j\frac{2\pi nk}{N}} \tag{13}$$

$$w_N^{2nk} = e^{-j\frac{4\pi nk}{N}} = w_{\frac{N}{N}}^{nk} \tag{14}$$

1. Aufteilung in $Y(n) = Y_{even}(n) + Y_{odd}(n)$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_N^{2nk} + \sum_{k=0}^{N/2-1} y(2k+1) w_N^{(2k+1)n}$$

$$(15)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_N^{2nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1) w_N^{2kn}$$

$$(16)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1) w_{\frac{N}{2}}^{nk}$$

$$(17)$$

$$Y(n + \frac{N}{2}) = \sum_{k=0}^{N/2-1} y(2k) w_{\frac{N}{2}}^{(n+\frac{N}{2})k} + w_N^{n+\frac{N}{2}} \sum_{k=0}^{N/2-1} y(2k+1) w_{\frac{N}{2}}^{(n+\frac{N}{2})k}$$

$$(18)$$

2. Aufteilung in $Y(n) = Y_{even}(n) + Y_{odd}(n)$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_N^{2nk} + \sum_{k=0}^{N/2-1} y(2k+1) w_N^{(2k+1)n}$$

$$(19)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_N^{2nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1) w_N^{2kn}$$

$$(20)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k) w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1) w_{\frac{N}{2}}^{nk}$$

$$(21)$$

5 NCO

NCO = Counter, der Jeden Takt um ein Phaseninkrement μ (hier 32) erhöht wird. Der Ausgang des Counters wird mit LUT in Signalform (sin,cos,sägezahn) umgewandelt. Die LUT ist mit $N=2^n$ 12-bit breiten Werten gefüllt

$$\mu = N \frac{f_d}{f_s} \tag{22}$$