Problem Set #4

PSE Masters in Economics

Quantitative Macro, Fall 2023

Due Date: 9 November 2023 before the tutorial

Please hand in your answers, the matlab programme and the figures with results using file names that contain all group members' last names (e.g. BROER_ELINA_PS_3.m).

This problem set studies the standard Real-Business-Cycle Model, adding endogenous labor supply and shocks to the neoclassical growth model you studied in the previous problem set. You can build on the solutions you found there.

Consider the following problem solved by a social planner for $t = 0, 1, ...$

$$
\begin{aligned}
\max_{\{C_t, N_t, K_{t+1}\}} \quad & E_0 \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{\theta}{1+\varphi} N_t^{1+\varphi}\right] && (1) \\
\text{s.t. } C_t + K_{t+1} \quad & = \quad A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t && (2) \\
K_{t+1}, C_t \quad & \geq \quad 0, N_t \in [0,1] && (3) \\
\log(A_{t+1}) \quad & = \quad \rho \log(A_t) + \varepsilon_{t+1}, \varepsilon_{t+1} \sim i.i.d. N(0, \sigma^2) && (4) \\
& K_0 \text{ and } A_0 \text{ given.}
\end{aligned}
$$

where C_t denotes consumption, N_t hours worked, K_t capital, and A_t is aggregate productivity. Consider the following parameter values:

ρ	σ	γ	φ
0.95	0.007	2	1

Problem 1 Calibration

Write down the four conditions that (together with the initial levels of capital K_0 and productivity A_0, and the transversality condition that you can omit) characterise the optimal allocation. From this, derive the deterministic steady state of the model, where average productivity is $\bar{A} = 1$, with quantities $\bar{K}, \bar{C}, \bar{Y}, \bar{N}$. Use this for a quarterly calibration of $\alpha, \theta, \beta, \delta$ in line with an investment-to-capital ratio of 5 percent, an average net real interest rate (equal to the average return on capital minus depreciation) of 1 percent and a labor supply of one third. (Use the answers you derived in a previous problem set.)

Problem 2.a Numerical solution I: Log-linearisation around the steady state

For solving the system of linearised conditions, you can use the function solab.m by Paul Klein (http://paulklein.ca/newsite/codes/codes.php).

1. Log-linearise the Euler equation for capital investment, the first-order condition for labor supply, and the feasibility constraint around the deterministic steady state, and write the resulting system as

$$Ax_{t+1} = Bx_t \tag{5}$$

   where $x_t = [K_t, z_t, C_t, N_t]'$ is a column vector whose entries are the log-deviations of, respectively, capital, productivity, consumption, and labor supply from their log-steady state values.

2. Transform this equation to a lower-triangular one and solve for the policy functions and the law of motion (solab.m does this for you).

Problem 2.b Numerical solution II: Deterministic impulse response function

Consider a deterministic path of the economy starting at $K_0 = K^*$ and $\varepsilon_0 = log(z_0) = \sigma$, where $z_t, t = 1, 2, ...$ follows (4). Assume the economy converges to steady state in at most T periods. Write a program that takes paths of $\{z_t\}_{t=0}^T$, capital $\{K_{t+1}\}_{t=0}^T$, labor supply $\{N_{t+1}\}_{t=0}^T$ and consumption $\{C_t\}_{t=0}^T$ as inputs and outputs a $3T \times 1$ vector with errors of the Euler equation for capital investment, the intratemporal condition for labor supply, and the feasibility constraint. Write a program that solves for the optimal path starting at K_0.

Problem 3 Simulation

Simulate the stochastic model. For this, draw first 100 independent realizations of the continuous or discrete markov chain over 1000 periods $\{Z_t\}_{t=0}^{100}$ with K_0 equal to the steady-state value K^*.

Figure 1: Draw the first simulation of capital (upper panel) and consumption (lower panel) for both solution methods.

Calculate the standard deviations of output, labor supply, consumption, and investment for each draw (discarding the first 100 periods), and calculate the average across 100 simulations. Also calculate the correlation of the three remaining variables with output. Comment and compare to the stylised data facts seen in class.