Problem Set #2

PSE Masters in Economics
Quantitative Macro, Fall 2023

Due Date: Thursday 12 October, 12 noon

Please hand in your answers, the matlab programme and the figures with results using file names that contain all group members' last names (e.g. BROER_ELINA_PS_1.m).

Consider the following problem solved by a social planner for $t = 0, 1, ..., T$.

$$\max_{\{c_t, k_{t+1}, i_t\}_{t=1}^{T}} E_0 \sum_{t=0}^{T} \beta^t u(c_t)$$
$$\text{s.t.}$$
$$c_t + i_t \leq k_t^\alpha$$
$$k_{t+1} = (1-\delta)k_t + i_t$$
$$k_0 \text{ given}$$
$$\text{for} \quad u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

1. Calculate analytically the constant "steady-state" levels of capital $\bar{k}$, consumption $\bar{c}$, the net interest rate defined as the marginal product of capital minus depreciation, and the capital output ratio. "Calibrate" the parameters β, α and δ at quarterly frequency, such that the steady state of the model replicates long-run features of developed economies, in particular an annualised net interest rate equal to 4 percent, an income share of capital (defined as the marginal product of capital times the stock of capital, divided by total output) equal to 1/3, and a quarterly capital-output ratio of 10. Set γ to 1.0001.

   If you cannot achieve this, consider the following parameter values:

β	α	σ	δ
0.99	0.3	1.0001	0.025

2. Now solve the model using log-linear techniques seen in class:

   (a) Log-linearise the Euler equation for capital investment and the feasibility constraint around the deterministic steady state, and write the resulting system as

   $$Ax_{t+1} = Bx_t \tag{1}$$

   where $x_t = [c_t, k_t]'$ is a column vector.

(b) Transform this equation to

$$x_{t+1} = Dx_t \tag{2}$$

(c) Diagonalise the system and solve for the policy rule for consumption $\hat{c}_t = \alpha_1 \hat{k}_t$, where $\hat{x}_t = log(x_t) - log(\bar{x})$ denotes log deviations of x from its steady-state value $\bar{x}$.

(d) Draw the time series of c_t and k_t when the initial level of capital equals $k_0 = 0.9\bar{k}$. Comment. (Feel free to compare to that you solved for in the previous problem set).

3. Now consider an alternative version of the model, where labor supply l_t is time varying. In particular, consider a period utility function that also takes labor as an argument

$$u(c_t, l_t) = u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \theta \frac{l_t^{1+\mu}}{1+\mu} \tag{3}$$

(a) Derive the first-order condition for labor supply. Calculate the steady-state labor supply $\bar{l}$.

(b) Calibrate θ to have $\bar{l} = 1/3$. Use $\mu = 1$.

(c) Log-linearise the first-order condition for labor supply in period t. Write the resulting system as

$$Ax_{t+1} = Bx_t \tag{4}$$

Either do d) or e).

(d) Use the Schur decomposition of A and B to solve the system. For this, you can use pre-programmed routines, like Paul Klein's solab.m (which you can download from his website).

(e) Alternatively, solve the first-order condition for labor for l_t, and substitute for it in the Euler-equation, and the feasibility condition. Then proceed as in 2.

(f) Draw the time series of c_t, k_t and l_t when the initial level of capital equals $k_0 = 0.9\bar{k}$. Compare to that in 2.