Quantitative Macro Problem Set 4

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This problem set studies the standard Real-Business-Cycle Model, adding endogenous labor supply and shocks to the neoclassical growth model you studied in the previous problem set. You can build on the solutions you found there.
Consider the following problem solved by a social planner for $t = 0, 1, \ldots$.

$$\begin{aligned} \max_{\{C_t, N_t, K_{t+1}\}} \quad & E_0 \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{\theta}{1+\varphi} N_t^{1+\varphi} \right] \\ \text{s.t. } C_t + K_{t+1} \quad & = \quad A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t \\ K_{t+1}, C_t \quad & \geq \quad 0, N_t \in [0,1] \\ \log(A_{t+1}) \quad & = \quad \rho \log(A_t) + \varepsilon_{t+1}, \varepsilon_{t+1} \sim \text{ i.i.d.N } \left(0, \sigma^2\right) (4) \end{aligned}$$

K_0 and A_0 given.
where C_t denotes consumption, N_t hours worked, K_t capital, and A_t is aggregate productivity. Consider the following parameter values:

ρ	σ	γ	φ
0.95	0.007	2	1

Problem 1 Calibration

Write down the four conditions that (together with the initial levels of capital K_0 and productivity A_0, and the transversality condition that you can omit) characterize the optimal allocation. From this, derive the deterministic steady state of the model, where average productivity is $\bar{A} = 1$, with quantities $\bar{K}, \bar{C}, \bar{Y}, \bar{N}$. Use this for a quarterly calibration of $\alpha, \theta, \beta, \delta$ in line with an investment-to-capital ratio of 5 percent, an average net real interest rate (equal to the average return on capital minus depreciation) of 1 percent and a labor supply of one third. (Use the answers you derived in a previous problem set.)

First, we set $\alpha = 1/3$ as in the previous problem set.
Social planner solves the Lagrangian of this problem as below,

$$\mathcal{L} = E_0 \left[\sum_{t=0}^{\infty} \beta^t \Big(\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{\theta}{1+\varphi} N_t^{1+\varphi} + \lambda_t (A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t - K_{t+1} - C_t \Big) \right]$$

Then, take the first-order conditions as,

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow C_t^{-\gamma} = \lambda_t \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = 0 \Leftrightarrow \theta N_t^{\varphi} = \lambda_t (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta E_t \left[\lambda_{t+1} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta \right) \right] \tag{3}$$

By 1 and 2, we obtain intratemporal optimality condition,

$$\theta N_t^{\varphi} = C_t^{-\gamma}(1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} \tag{4}$$

By 1 and 3, we obtain intertemporal optimality condition (Euler equation),

$$1 = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta \right) \right] \tag{5}$$

Together with 4, 5, feasibility constraint $C_t + K_{t+1} = A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t$ and technological process $\log(A_{t+1}) = \rho \log(A_t) + \varepsilon_{t+1}$, (also with initial conditions on K and Z and transversality condition), we can characterize the optimal allocations.

Then, we can write

- the average rate of return for capital as $r_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}$
- the average rate of return for labor as $W_t = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha}$
- Production function as $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$
- Law of motion as $K_{t+1} = (1-\delta)K_t + I_t$

Now, let's characterize the deterministic steady-state of the model. From 5 and average return on capital,

$$1 = \beta E_t[\bar{r} + 1 - \delta] \Leftrightarrow \bar{r} = \frac{1}{\beta} - (1-\delta) \tag{6}$$

Then, to get the capital-labor ratio,

$$\bar{r} = \alpha \bar{A} \bar{K}^{\alpha-1} \bar{N}^{1-\alpha} \Leftrightarrow \frac{\bar{K}}{\bar{N}} = \left(\frac{\bar{r}}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{7}$$

and thus, we can get the wage at the steady state as,

$$\bar{W} = (1-\alpha)\bar{A} \left(\frac{\bar{K}}{\bar{N}} \right)^{\alpha} \Longleftrightarrow \bar{W} = (1-\alpha) \left(\frac{\bar{r}}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} \tag{8}$$

For the investment-to-capital ratio,

$$\bar{K} = (1-\delta)\bar{K} + \bar{I} \Longleftrightarrow \frac{\bar{I}}{\bar{K}} = \delta \tag{9}$$

Then, we can get the steady-state value for the production function as,

$$\bar{Y} = \frac{\bar{r}\bar{K}}{\alpha} = \frac{\bar{W}\bar{N}}{1-\alpha} \tag{10}$$

To get $\bar{C}$, we use resource constraint, 9 and 10,

$$\bar{Y} = \bar{C} + \bar{I} \Longleftrightarrow \frac{\bar{r}\bar{K}}{\alpha} = \bar{C} + \delta \bar{K} \Longleftrightarrow \bar{C} = \left(\frac{\bar{r}}{\alpha} - \delta \right) \bar{K} \tag{11}$$

Let's try to get $\bar{N}$ now. Recall from 4, we can write it at the steady state as $\theta \bar{N}^{\varphi} = \bar{C}^{-\gamma}(1-\alpha)\bar{A}\bar{K}^{\alpha}\bar{N}^{-\alpha}$. By using 7 and 11,

$$\theta \bar{N}^{\varphi} = \left[\left(\frac{\bar{r}}{\alpha} - \delta\right)\bar{K}\right]^{-\gamma}(1-\alpha)\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

By using 8,

$$\bar{N}^{\varphi} = \left[\left(\frac{\bar{r}}{\alpha} - \delta\right)\bar{K}\right]^{-\gamma}\frac{\bar{W}}{\theta}$$

By using 7 again,

$$\bar{N}^{\varphi} = \left[\left(\frac{\bar{r}}{\alpha} - \delta\right)\left(\frac{\bar{r}}{\alpha}\right)^{\frac{1}{\alpha-1}}\bar{N}\right]^{-\gamma}\frac{\bar{W}}{\theta}$$

which results,

$$\bar{N} = \left[\frac{\bar{W}}{\theta\left(\frac{\bar{r}}{\alpha} - \delta\right)^{\gamma}\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\gamma}{\alpha-1}}}\right]^{\frac{1}{\gamma+\varphi}} \tag{12}$$

Now, let's focus on the calibration of our parameters. By 9 in line with an investment-to-capital ratio of 5 percent, we get the value of δ as,

$$\delta = \frac{\bar{I}}{\bar{K}} = 0.05$$

By 6, in line with an average net real investment rate of 1 percent,

$$\beta = \frac{1}{\bar{r} + 1 - \delta} = \frac{1}{1.01} = 0.99$$

Now, by using $\alpha = 1/3$ for the return of capital, we will calibrate the θ as,

$$\theta = \frac{\bar{W}\bar{N}^{-(\gamma+\varphi)}}{\left(\frac{\bar{r}}{\alpha} - \delta\right)^{\gamma}\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\gamma}{\alpha-1}}} = \frac{\bar{W}\bar{N}^{-(\gamma+\varphi)}}{\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha} - \delta\right)^{\gamma}\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha}\right)^{\frac{\gamma}{\alpha-1}}} = 14.64 \tag{13}$$

Problem 2.a Numerical solution I: Log-linearisation around the steady state

For solving the system of linearised conditions, you can use the function solab.m by Paul Klein (http://paulklein.ca/newsite/codes/codes.php).

1. **Log-linearise the Euler equation for capital investment, the first-order condition for labor supply, and the feasibility constraint around the deterministic steady state, and write the resulting system as**

$$Ax_{t+1} = Bx_t$$

where $x_t = [K_t, z_t, C_t, N_t]'$ is a column vector whose entries are the log deviations of, respectively, capital, productivity, consumption, and labor supply from their log-steady state values.

Euler Equation:

Recall from the 5 that we found $1 = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}\left(\alpha A_{t+1}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}+1-\delta\right)\right]$. Take the exponential of logarithm,

$$1 = E_t\Big[exp(ln\beta + \gamma(lnC_t - lnC_{t+1}) + ln(\alpha A_{t+1}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha} + 1 - \delta))\Big]$$

For x close enough to 0, the exponential function can be approximated as $e^x \approx 1 + x$. Since the expected value of the exponential is 1, the exponent must be close enough to 0. Rewrite,

$$1 = E_t\Big[1 + ln\beta + \gamma(lnC_t - lnC_{t+1}) + ln(\alpha A_{t+1}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha} + 1 - \delta)\Big] \tag{14}$$

Since this is the expectation of a linear function, we can separate the linear terms,

$$\gamma E_t\left[lnC_{t+1}\right] - E_t\left[ln\left(\alpha A_{t+1}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha} + 1 - \delta\right)\right] = ln\beta + \gamma lnC_t$$

and apply the first-order approximation,

$$\gamma ln\bar{C} + \gamma\frac{E_t(C_{t+1}) - \bar{C}}{\bar{C}} - ln(\bar{r} + 1 - \delta) - \frac{\bar{r}}{\bar{r} + 1 - \delta}\Big[\frac{E_t(A_{t+1}) - \bar{A}}{\bar{A}} - (1-\alpha)\Big(\frac{K_{t+1} - \bar{K}}{\bar{K}} - \frac{E_t(N_{t+1}) - \bar{N}}{\bar{N}}\Big)\Big]$$
$$= ln\beta + \gamma ln\bar{C} + \gamma\frac{C_t - \bar{C}}{\bar{C}}$$

Note that since we decide K_{t+1} at period t, we do not need to express it inside the expectation operator. Terms with ln cancel out since we already know that 14 will hold for every period, including the steady state as well. Then,

$$\gamma E_t\hat{c}_{t+1} - \frac{\bar{r}}{\bar{r} + 1 - \delta}\left[E_t\hat{a}_{t+1} - (1-\alpha)\left(\hat{k}_{t+1} - E_t\hat{n}_{t+1}\right)\right] = \gamma\hat{c}_t$$

by using 6, rewrite as

$$\gamma E_t\hat{c}_{t+1} - \beta\bar{r}E_t\hat{a}_{t+1} + \beta(1-\alpha)\bar{r}\hat{k}_{t+1} - \beta(1-\alpha)\bar{r}E_t\hat{n}_{t+1} = \gamma\hat{c}_t \tag{15}$$

Labor Supply:

Recall from 4, we found $\theta N_t^{\varphi} = C_t^{-\gamma}(1-\alpha)A_tK_t^{\alpha}N_t^{-\alpha}$. Let's take logarithm,

$$\ln\theta + \varphi\ln N_t = -\gamma\ln C_t + \ln(1-\alpha) + \ln A_t + \alpha\ln K_t - \alpha\ln N_t$$

Then, apply first-order Taylor approximation,

$$\begin{aligned} ln\theta + \varphi ln\bar{N} + \varphi\frac{N_t - \bar{N}}{\bar{N}} =& -\gamma ln\bar{C} - \gamma\frac{C_t - \bar{C}}{\bar{C}} + ln(1-\alpha) + ln\bar{A} + \frac{A_t - \bar{A}}{\bar{A}} \\ &+ \alpha ln\bar{K} + \alpha\frac{K_t - \bar{K}}{\bar{K}} - \alpha ln\bar{N} - \alpha\frac{N_t - \bar{N}}{\bar{N}} \end{aligned} \tag{16}$$

which results,

$$-\gamma \hat{c}_t + \hat{a}_t + \alpha \hat{k}_t - (\alpha + \varphi)\hat{n}_t = 0 \tag{17}$$

Feasibility constraint:

Recall that feasibility constraint is $C_t + K_{t+1} = A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t$. Now, take log,

$$\ln(C_t + K_{t+1}) = \ln(A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t)$$

Take the first-order Taylor approximation,

$$\ln(\bar{C} + \bar{K}) + \frac{\left(C_t - \bar{C}\right) + \left(K_{t+1} - \bar{K}\right)}{\bar{C} + \bar{K}} = \ln\left[\bar{A}\bar{K}^{\alpha}\bar{N}^{1-\alpha} + (1-\delta)\bar{K}\right] + \frac{\bar{Y}\frac{A_t - \bar{A}}{\bar{A}} + (\bar{r} + 1 - \delta)\left(K_t - \bar{K}\right) + \bar{W}\left(N_t - \bar{N}\right)}{\bar{A}\bar{K}^{\alpha}\bar{N}^{1-\alpha} + (1-\delta)\bar{K}}$$

By using steady-state definition and 6,

$$\left(C_t - \bar{C}\right) + \left(K_{t+1} - \bar{K}\right) = \bar{Y}\frac{A_t - \bar{A}}{\bar{A}} + \frac{1}{\beta}\left(K_t - \bar{K}\right) + \bar{W}\left(N_t - \bar{N}\right)$$

and by some modification,

$$\begin{aligned} \bar{C}\frac{C_t - \bar{C}}{\bar{C}} + \bar{K}\frac{K_{t+1} - \bar{K}}{\bar{K}} &= \bar{Y}\hat{A}_t + \frac{1}{\beta}\frac{K_t - \bar{K}}{\bar{K}}\bar{K} + \bar{W}\frac{N_t - \bar{N}}{\bar{N}}\bar{N} \\ \bar{C}\hat{C}_t + \bar{K}\hat{K}_{t+1} &= \bar{Y}\hat{A}_t + \frac{1}{\beta}\bar{K}\hat{K}_t + \bar{W}\bar{N}\hat{N}_t \end{aligned}$$

and by 10,

$$\frac{\bar{K}}{\bar{Y}}\hat{k}_{t+1} = \hat{a}_t + \frac{1}{\beta}\frac{\bar{K}}{\bar{Y}}\hat{k}_t + (1-\alpha)\hat{n}_t - \frac{\bar{C}}{\bar{Y}}\hat{c}_t \tag{18}$$

Technological progress:

Recall that $log(A_{t+1}) = \rho log(A_t) + \epsilon_{t+1}$. Then, take log and eliminate the terms in In since $log(A_{t+1}) = \rho log(A_t) + \epsilon_{t+1}$ holds for $\forall t$. We obtain,

$$\hat{a}_{t+1} = \rho\hat{a}_t + \varepsilon_{t+1} \tag{19}$$

Matrix form:

By using 15,17,18 and 19, we can construct the following matrix representation of a log-linearized RBC model,

$$\underbrace{\begin{pmatrix} \beta(1-\alpha)\bar{r} & -\beta\bar{r} & \gamma & -\beta(1-\alpha)\bar{r} \\ 0 & 0 & 0 & 0 \\ \frac{\bar{K}}{\bar{Y}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}_{A} \times \underbrace{\begin{pmatrix} \hat{k}_{t+1} \\ \hat{a}_{t+1} \\ \hat{c}_{t+1} \\ \hat{n}_{t+1} \end{pmatrix}}_{x_{t+1}} = \underbrace{\begin{pmatrix} 0 & 0 & \gamma & 0 \\ \alpha & 1 & -\gamma & -(\alpha+\varphi) \\ \frac{1}{\beta}\frac{\bar{K}}{\bar{Y}} & 1 & -\frac{\bar{C}}{\bar{Y}} & 1-\alpha \\ 0 & \rho & 0 & 0 \end{pmatrix}}_{B} \times \underbrace{\begin{pmatrix} \hat{k}_t \\ \hat{a}_t \\ \hat{c}_t \\ \hat{n}_t \end{pmatrix}}_{x_t} + C \underbrace{\begin{pmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ \varepsilon_{t+1} \end{pmatrix}}_{\epsilon_{t+1}},$$

Remember that: $Ax_{t+1} = Bx_t + C\epsilon_{t+1}$. Now, we need to write $AE_t x_{t+1} = Bx_t + CE_t\epsilon_{t+1}$.

However, we know that $E_t\varepsilon_{t+1} = 0$ since $\varepsilon_t \sim \text{iidN}\left(0, \sigma^2\right)$. Consequently, $E_t\epsilon_{t+1} = \overrightarrow{0}$, where $\overrightarrow{0}$ is a 4-element column of zeros. Thus, we get,

$$AE_t x_{t+1} = Bx_t.$$

2. **Transform this equation to a lower-triangular one and solve for the policy functions and the law of motion (solab.m does this for you).**

We use the generalized Shur decomposition to do this. The formula is shown below:

$$\begin{aligned} \mathbb{E}_t\left[x_{t+1}\right] &= Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}x_t \\ x_{t+1} &= \underbrace{Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}}_{P}x_t + \varepsilon_{t+1} \end{aligned} \tag{20}$$

where $Z_{i,j}$, $S_{i,j}$ and $T_{i,j}$ are corresponding partitioned matrices matrix in the upper triangular matrices generated by the Shur decomposition. x_t is the vector of state variables. We apply `solab.m` to find the policy function. The result is shown below:

$$\begin{pmatrix} \hat{c}_t \\ \hat{n}_t \\ \hat{k}_{t+1} \end{pmatrix} = \begin{pmatrix} 0.3471 & 0.4241 \\ -0.2706 & 0.1138 \\ 0.9324 & 0.1385 \end{pmatrix} \cdot \begin{pmatrix} \hat{k}_t \\ \hat{a}_t \end{pmatrix} \tag{21}$$

Problem 2.b Numerical solution II: Deterministic impulse response function

Consider a deterministic path of the economy starting at $K_0 = K^*$ and $\varepsilon_0 = \log(z_0) = \sigma$, where $z_t, t = 1, 2, \ldots$ follows (4). Assume the economy converges to steady state in at most T periods. Write a program that takes paths of $\{z_t\}_{t=0}^{T}$, capital $\{K_{t+1}\}_{t=0}^{T}$, labor supply $\{N_{t+1}\}_{t=0}^{T}$ and consumption $\{C_t\}_{t=0}^{T}$ as inputs and outputs a $3T \times 1$ vector with errors of the Euler equation for capital investment, the intratemporal condition for labor supply, and the feasibility constraint. Write a program that solves for the optimal path starting at K_0.

We use the `fsolve` function to find the root for the function that output the errors for the Euler equation, intratemporal condition for labor supply, and the feasibility constraint. The result is shown in figure 1.

The impulse responses show classical patterns of an RBC model. After a positive technology shock at $t = 0$ with $\epsilon_0 > 0$, output, consumption, and investment increase at the same time. This co-movement can be explained by the consumption-smoothing: as productivity increases, the output increases, and thus consumers have more income at hand, leading to a higher consumption; on the other hand, due to consumption smoothing, forward-looking consumers will invest more in order to enjoy for a longer period the income growth resulting from the technology advancement that will extend to future periods. Therefore, the representative agent will not consume all of the extra outputs in the first period since, under rational expectation, household understands that $\triangle A_t$ will die out overtime.

Due to the investment motivated by consumption smoothing, the capital k_t increases as well. However, as the technology growth begins to diminish, output will decrease. As k_t, capital stock level, is now higher and A_t is lower, investment will decline due to the decreasing marginal return to capital. The interest rate could even fall below the steady-state level after some periods, same for the investment level. This could be explained by the fact that, in later stages, labour supply N_t is lower than steady state level (explained in next paragraph), yet capital stock K_t remains much higher than steady-state level, leading to a very low level MPK_t, therefore low incentives for investment and low interest rate compared with steady state. Simultaneously, consumption declines as more percentage of additional outputs are consumed rather than saved. Capital lags in response and it will decline after a peak, as the decreasing investment can no longer compensate for the depreciation.

As we endogenize labor supply, a positive technology shock propagate to the labor supply through the channel of intra-temporal labor-consumption trade-off. First of all, from equation 4, we can tell that income effects of higher wage decrease labour supply and substitution effects increase

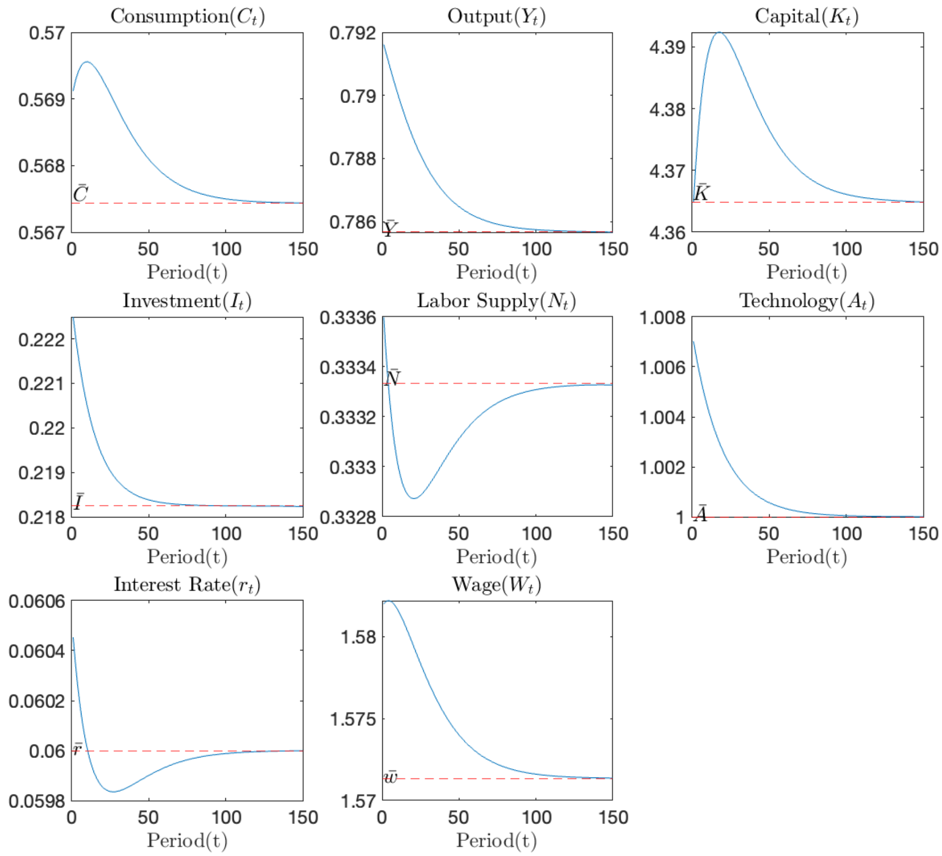

Figure 1: Impulse Response, Unit Sd. Technology Shock

labour supply. Initially, the labor supply increases, as the substitution effect between labor and consumption dominates. By working more, consumers can save more, consume more and enjoy a higher future consumption flow. As technology shock begins to decay, the income effect becomes dominant, and the labor supply drops below the steady state level. As A_t goes back to the steady state level, household now has close to steady-state level of marginal productivity, yet they can eat up some capital instead of working, and wealth effects dominate. One can play with CRRA and Frisch elasticity coefficient to see how income and substitution effects on labour supply alter and different degree of propagation towards labor supply dynamics.

With the introduction of endogenous labor supply, the consumption response is attenuated, since agents can now choose to slack off. Consumers work hard to consume today and also save for the future so that they can have higher future utility return when productivity is lower again. Therefore, positive productivity shocks propagate through higher labor supply. And consumption is smoothed through rational expectation and risk aversion. To conclude, the volatility of consumption is supposed to be smaller than that of investment and output due to endogenous labor supply and consumption smoothing. We can see this in the computation of moments in problem 3.

In addition, the calibration of ρ, persistency of productivity shock, matters for the impulse responses. When ρ goes to 1, there is a permanent upper drift for productivity level and future income, therefore no incentives for working more today. The propagation channel of endogenous labor supply disappear in such a scenario.

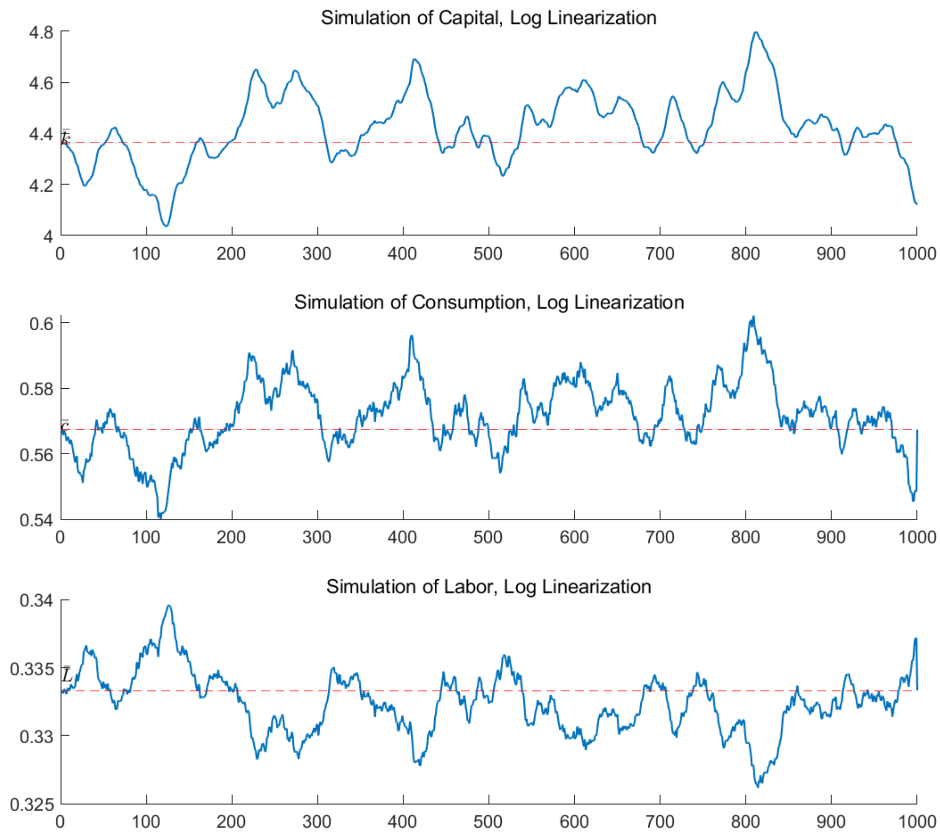

Figure 2: RBC Simulation, Log Linearization, Continuous Markov Process

Problem 3 Simulation

Simulate the stochastic model. For this, draw first 100 independent realizations of the continuous or discrete markov chain over 1000 periods $\{Z_t\}_{t=0}^{100}$ with K_0 equal to the steady-state value K^*.
Figure 1: Draw the first simulation of capital (upper panel) and consumption (lower panel) for both solution methods.
Calculate the standard deviations of output, labor supply, consumption, and investment for each draw (discarding the first 100 periods), and calculate the average across 100 simulations. Also calculate the correlation of the three remaining variables with output. Comment and compare to the stylised data facts seen in class.

We use the policy function computed by the log-linearized model to simulate the economy. We draw the simulation for the first realization of the continuous and discrete Markov chain over 1000 periods in figure 2 and figure 3 respectively. Firstly, we note that consumption and output comove, which corresponds to the analysis in the section of impulse response. On the other hand, we note that labor supply is counter-cyclical, which is kind of not straightforward. We argue that this is due to the choice of γ in calibration, which is the inverse of the inter-temporal elasticity. We will come back to this later when we talk about the calculated moments.
Note that when we use the discrete Markov process to simulate the TFP shocks, the figure seems to be extremely discontinuous. This is expected as we only choose to have 3 states for the discrete Markov chain.
We calculate the standard deviations for the simulated technology shocks, the standard deviations for real variables and the correlation of variables with output in table 1, 2 and 3 respectively.

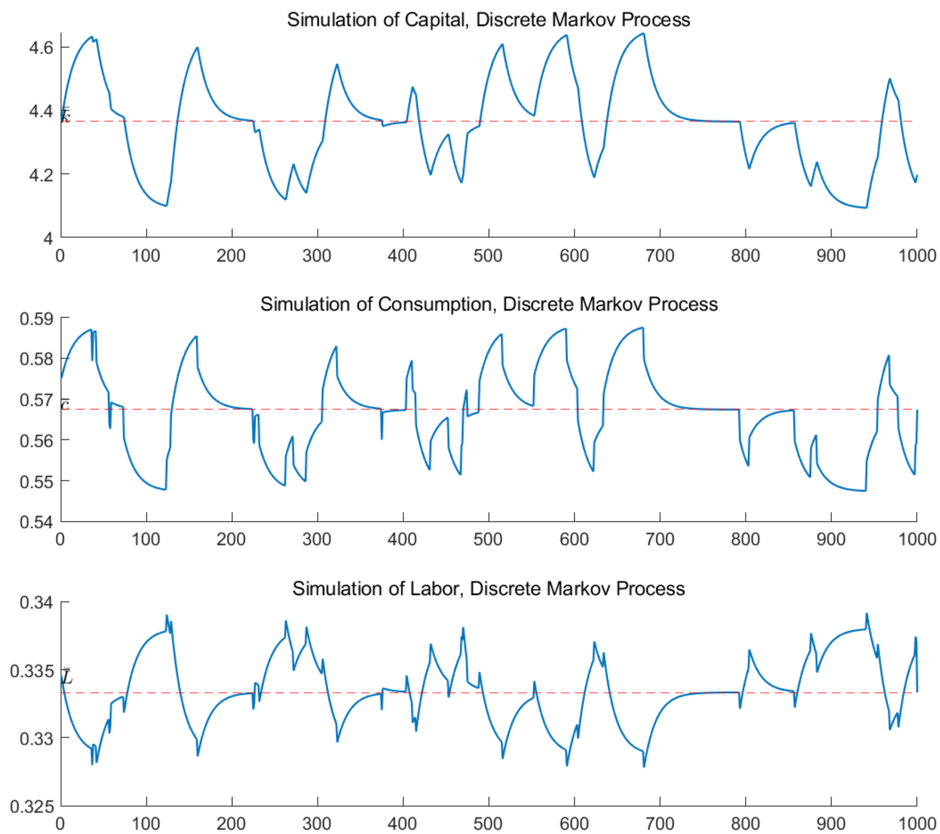

Figure 3: RBC Simulation, Log Linearization, Discrete Markov Process

From table 1, we can see that the simulated technology shocks can emulate the real process well, both discrete and continuous Markov process. We calculate the mean of the autocorrelation of the technology shocks across simulations as well. The discrete shocks give an autocorrelation of 0.9454, while the continuous shocks give an autocorrelation of 0.9472, both are close to the true value 0.95.

Discrete Z Shock Sd.	Continuous Z Shock Sd.	Real Z Schock Sd.
0.0215	0.0220	0.0224

Table 1: Standard Deviations of Simulated and Real Technology Shock

Table 2 gives the mean of the standard deviation of the real variables across simulations. Also, we calculate the coefficient of variation for consumption and investment. From the standard deviation, we cannot actually see that investment is more volatile than output, as predicted by theory. However, when we look at the coefficient of variation, we can learn that investment is more volatile than output and consumption. This is due to consumption smoothing as discussed in the last section. Investment adjusts rapidly to smooth out consumption over time. We also include the case where $\gamma = 1.001$, to compare the result with the standard case of $\gamma = 2$. When γ becomes lower, we can see that output becomes more volatile both from the standard deviation and the coefficient of variation. Consumption becomes less volatile in the case of $\gamma = 1.001$ than that in the case of $\gamma = 2$: as γ decreases, the intertemporal substitution effect becomes stronger, thus stronger consumption smoothing.

	Output Sd.	Labor Sd.	Consumption Sd.	Investment Sd.
Continuous Shock	0.0219	0.0025	0.0112	0.0116
Discrete Shock	0.0239	0.0024	0.0107	0.0138
$\gamma = 1.001$	0.0271	0.0019	0.0160	0.0128
	Output CV.	Labor CV.	Consumption CV.	Investment CV.
Continuous Shock	0.0477	0.0197	0.0431	0.0614
Discrete Shock	0.0609	0.0194	0.0426	0.1137
$\gamma = 1.001$	0.0345	0.0057	0.0281	0.0585

Table 2: Monte Carlo Simulation, Standard Deviation and Coefficient of Variation

Now we show in table 3 the correlation of consumption, investment and labor with output respectively. Output strongly and positively correlates with consumption and investment, as seen in the class. We see that the output-labor correlation is negative, which doesn't correspond to what we see in class. We guess that this is due to the fact that γ is too high, which means a high income effect compared with the substitution effect, thus leading to a counter-cyclical labor. when we change γ to 1.001, as expected, the output-labor correlation becomes positive. Note that labor here is not very volatile compared with the data (see figure 4). This is result of the assumption that there's no labor friction.

	Output-Consumption Corr.	Output-Investment Corr.	Output-Labor Corr.
Continuous Shock	0.9561	0.9605	-0.6429
Discrete Shock	0.9697	0.9820	-0.7080
$\gamma = 1.001$	0.9670	0.9288	0.5983

Table 3: Monte Carlo Simulation, Correlation

	SD		Rel. SD		Corr Y_t		Autocorr	
	Data	Model	Data	Model	Data	Model	Data	Model
Y_t	0.017	0.015	1.00	1.00	1.00	1.00	0.85	0.72
C_t	0.009	0.006	0.53	0.40	0.76	0.95	0.79	0.78
I_t	0.047	0.041	2.76	2.73	0.79	0.99	0.87	0.72
N_t	0.019	0.005	1.12	0.33	0.88	0.98	0.90	0.72
W_t	0.009	0.010	0.53	0.66	0.10	0.996	0.73	0.74
R_t	0.004	0.015	0.24	1.0	0.00	0.97	0.42	0.71
A_t	0.012	0.012	0.71	0.80	0.76	0.999	0.75	0.72

Figure 4: Stylized Facts in Class