<u>Problem Set #5</u>

PSE Masters in Economics

Quantitative Macro, FALL 2023

Due Date: 17 November 2023 before the tutorial

Please hand in your answers, the matlab programme and the figures with results using file names that contain all group members' last names (e.g. BROER_ELINA_PS_3.m).

This problem set studies the standard Real-Business-Cycle Model, adding endogenous labor supply and shocks to the neoclassical growth model you studied in the previous problem set. You can build on the solutions you found there.

Consider the following problem solved by a social planner for $t = 0, 1, ...,$, identical to that of the previous problem set

$$\begin{aligned}
\max_{\{C_t, N_t, K_{t+1}\}} \quad & E_0 && \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{\theta}{1+\varphi} N_t^{1+\varphi}\right] && (1)\\
\text{s.t. } C_t + K_{t+1} \quad & = && A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t && (2)\\
K_{t+1}, C_t \quad & \geq && 0, N_t \in [0,1] && (3)\\
\log(A_{t+1}) \quad & = && \rho \log(A_t) + \varepsilon_{t+1}, \varepsilon_{t+1} \sim i.i.d.N(0, \sigma^2) && (4)\\
& K_0 \text{ and } A_0 \text{ given.}
\end{aligned}$$

where C_t denotes consumption, N_t hours worked, K_t capital, and A_t is aggregate productivity. Consider the following parameter values:

ρ	σ	γ	φ
0.95	0.007	2	1

Problem 1 Value function iteration

Solve the above problem by value function iteration. Choose an equally-spaced grid $\mathbb{K} = \{K_1 < K_2 < ...K_N\}$ with $K_1 = 0.9k^*$ and $K_N = 1.1k^*$, for k^* the deterministic steady-state value of k. Choose N=50. For this discretize the process for $log(A_t)$ using Rouwenhorst's method, with $M = 5$ support points. Interpolate the continuation value function at any choice of k'. In particular,

1. Choose a criterion ε for convergence of the value function (choose e.g. 10^{-6}).

2. Choose an initial value function $v_0(K_i, z_j)$, an $N \times M$ matrix.

3. Calculate for all $i = 1, .., N$, $j = 1, ..., M$

$$v_{s+1}(K_i, z_j) = \max_{k', L}\{F(K_i, z_j, k', L) + \beta E[v_s(k', z')|z_j]\} \tag{5}$$

for $s = 0, 1, 2, ...$

4. If $\max_{i,j} |v_{s,i,j} - v_{s-1,i,j}| \geq \varepsilon$ go back to 2). Otherwise stop.

Note that you need to solve for k' and L jointly using the first-order condition for labor supply in (5).

Evaluate the Euler equation errors.

Problem 2 Endogenous-grid point method

Solve the above problem by using the method of endogenous grid points. Choose an equally-spaced grid for the choice of capital today $k' \in \mathbb{K} = \{K_1 < K_2 < ...K_N\}$ with $K_1 = 0.9k^*$ and $K_N = 1.1k^*$, for k^* the deterministic steady-state value of k. Choose N=50. Discretize the process for $log(A_t)$ using Rouwenhorst's method, with $M = 5$ support points, as in the previous problem set. Denote the support of $log(A_t)$ $\mathbb{A}$. Apply the following algorithm:

0. Grids and initial guesses

   i. Choose finite grid $\mathbb{K}$ for k_{t+1} [!] with $K_1 = 0.9k^*$ and $K_N = 1.1k^*$, for k^* the deterministic steady-state value of k. Choose N=50.

   ii. Guess a policy $c_0(k', A')$.

1. For all $k' \in \mathbb{K}$, $A \in \mathbb{A}$, given $k'_i(k', A'), l_i(k', A'), c_i(k', A')$, calculate

$$\hat{c}(k', A) = u'_{inv}\{\beta E[u'(c(k', A'))[\alpha A' k'^{\alpha-1} l(k', A')^{1-\alpha} + (1-\delta)]|A]\} \tag{6}$$

   for inverse marg utility u'_{inv}.

2. For every k', A, and given $\hat{c}(k', A)$, calculate $\hat{k}$ and $\hat{l}(\hat{k}, A)$ that solve the budget constraint and first order condition for labor supply.

3. Update $k'_{i+1}(k', A')$, $l_{i+1}(k', A')$ and $c_{i+1}(k', A')$ by interpolation.

4. If c_{i+1} is close to c_i, stop. O/w go back to 1.

Evaluate the Euler equation errors.

Problem 3 Simulation

Simulate the stochastic model. For this, draw first 100 independent realizations of the discrete Markov chain over 1000 periods $\{Z_t\}_{t=0}^{100}$ with K_0 equal to the steady-state value K^*.

Figure 1: Draw the first simulation of capital (upper panel) and consumption (lower panel) for both solution methods.

Calculate the standard deviations of output, labor supply, consumption, and investment for each draw (discarding the first 100 periods), and calculate the average across 100 simulations. Also calculate the correlation of the three remaining variables with output. Comment and compare to the stylised data facts seen in class.