PSE Masters in Economics
Quantitative Macroeconomics
TANK models

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HANK

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Since Great Recession: strong interest in NK models with heterogeneous consumers. Why?

1. Allows to ask new questions that standard model cannot answer
2. Gives different answers to 'old' questions - may solve puzzles of standard NK model

HANK: New questions

1. Which welfare effects of business cycles?: Average welfare strongly depends on consumption-income distribution, and its evolution over time / cycle.
2. Which determinants of inequality? Might include aggregate fluctuations and macro-policy.
3. Does inequality change the propagation of shocks?

The puzzles of the textbook model

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- log-linear intertemp budget constraint, impose *EE* (see Bilbiie's appendix)

$$c_t = (1-\beta)\sum_{k=0}^{\infty}\beta^k E_t y_{t+k} - \sigma^{-1}\beta\sum_{k=0}^{\infty}\beta^k E_t r_{t+k} \quad (1)$$

- (PE) MPC out of income change with persistence p: $\omega = \frac{1-\beta}{1-\beta p}$
  ***PI* Contradicts evidence of high MPC out of transitory incomes**
- (PE) MPC out of future change with persistence p: $\omega = \frac{\beta(1-\beta)}{1-\beta p}$
  ***PII* Contradicts evidence of low MPC out of future incomes**
- $c_t = y_t$, $c_{t+1} = y_{t+1}$ gives GE Euler equation

$$r_t = \sigma E_t[y_{t+1} - y_t] = -\sigma(1-p)y_t \quad (2)$$

  ***PIII* Contradicts evidence of low effect of changes in RIR r_t**
- Fiscal multiplier: with lump-sum taxes
  ($G_t > 0, G_s = 0, s = t+1, ..., \{\tau_t\} >= 0$ s.t. intertemporal BC holds)
  - nothing changed but $y_t = \hat{y}_t - \tau_t$ for $\hat{y}_t$ pre-tax income
  - At unchanged $\{r_t\}$, output-multiplier is approx. 1
  - With e.g. Taylor rule, r_t rises, so output multiplier is < 1
- Forward guidance puzzle: consider change in r_{t+s} given $r_k, k \neq s$
  - *EE*: increases $c_t, c_{t+1}, ..., c_{t+s-1}$ by same amount
  - At zero lower bound on i_t, inflation rises by PDV of future marginal cost increase, reduces real interest rate: even stronger effect

  ***PIV* Seems too strong.**

Wealth inequality may help...

- Data: many households have low (liquid) asset levels
- With borrowing limits, asset-poor have...
  - high MPCs out of current y
  - low MPCs out of future y
  - low interest-rate elasticities
- But:
  - HANK models usually analytically intractable
  - Quantitative solution complex
  - Is it worth the effort, given "approximate aggregation" result (KS98)?
  - $\Rightarrow$ Yes, but postpone "full HANK" to Axelle's part.
  - Two alternatives
    1. "Simple" HANK models
    2. Two-agent NK (TANK) models

Simple HANK models

- Alternative I: CARA w/o borrowing contraints: linear consumption rule with exogenous labor supply (Acharya and Dogra 2019)
- Alternative II: conditions such that individual allocation independent of aggregate quantities (Werning 2015)
  1. log-preferences
  2. borrowing-limit proportional to agg output
  3. Acyclical income risk (individuals earn constant proportion of agg income
  4. Cyclical income risk: can amplify or dampen stabilisation policy measures
- Alternative III: no-liquidity limit ($B, b = 0$), no capital: wealth distribution degenerate; can write EE for 'marginal saver''s c i.t.o. C (Broer et al (2019), Ravn and Sterck (2018))
- Alternative IV (today): simplify heterogeneity - "TANK" models (Bilbiie (various), Broer et al (2019), Gali and Debortoli (2018))

Two-agent NK (TANK) models

- Start from RANK model
- Split rep household in two types of agents (where types may change over time) that differ in any of:
  1. Asset market participation (hand-to-mouth vs smoothers, Bilbiie)
  2. Ownership of firms (workers vs. capitalists, Broer et al)
  3. Labor market status (employed vs unemployed, Ravn and Sterck)
- May capture: inequality; heterogeneity in MPCs; cyclicality of risk
- Key question: how are firm profits distributed?

Two-agent NK (TANK) models: Outline

- Point of departure: Galí textbook model
- Campbell and Mankiw (1989)
- Bilbiie (2008,17): Limited participation - some HHs don't trade assets
- Broer et al (19): Workers vs capitalists

Point of departure: Galí textbook model

Galí textbook model: Summary of log-linearized equilibrium

$$
\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: &\quad \hat{\omega}_t = \varphi \hat{n}_t + \frac{1}{\sigma}\hat{c}_t \\
\text{Market clearing}: &\quad \hat{c}_t = \hat{y}_t \\
\text{HH BC}: &\quad \hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t \\
\text{Production} &\quad \hat{y}_t = \hat{n}_t
\end{aligned}
$$

where $\bar{S} = \frac{W_t N_t}{Y_t P_t} = \frac{\epsilon_p - 1}{\epsilon_p}$ is the steady-state labor share

An early TANK model (Campbell and Mankiw 89)

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- Fraction λ of "hand-to-mouth" agents consumes their share of agg inc $c_t^H = y_t$
- Rest: smoothers (S);
- Resource constraint: $\lambda c_t^H + (1-\lambda)c_t^S = c_t = y_t$
- S's log-linear intertemp BC (w. CRRA σ, see Bilbiie's apdx)

$$c_t^S = (1-\beta)\sum_{k=0}^{\infty}\beta^k E_t y_{t+k} - \sigma^{-1}\beta\sum_{k=0}^{\infty}\beta^k E_t r_{t+k} \tag{3}$$

- use $c_t = \lambda c_t^H + (1-\lambda)c_t^S = y_t$, $c_t^H = y_t$, imply

$$c_t = (1-\beta(1-\lambda))y_t + (1-\beta)(1-\lambda)\sum_{k=1}^{\infty}\beta^k E_t y_{t+k} - (1-\lambda)\sigma^{-1}\beta\sum_{k=0}^{\infty}\beta^k E_t r_{t+k}$$

- "PE" aggregate MPC out of one-off change in current income: $\omega = 1-\beta(1-\lambda) > 1-\beta$
- ... in future income $(1-\lambda)(1-\beta) < 1-\beta$
- $c_t = y_t$ gives GE Euler equation as in RANK

$$c_t = E_t c_{t+1} - \sigma^{-1} r_t \tag{5}$$

- implies
  1. $MPC^{TANK} > MPC^{RANK}$
  2. Ricardian equivalence does not hold
  3. Interest elasticity $\Omega^{TANK} = \Omega^{RANK}$, but indirect effect (through dy) larger, and direct effect (through dr) smaller, than in RANK

TANK I: Limited participation (Bilbiie 08,17,...)

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- Departure point: Galí textbook model (but tax steady state profits and subsidise labor to have undistorted steady state)
- Two agents
  - $u(C, L) = \frac{C^{1-\sigma}-1}{1-\sigma} - \frac{(1-L)^{1+\psi}}{1+\psi}$
  - Asset market participants: fraction $1-\lambda$, with BC

$$C_t^S + B_{t+1}^S = W_t N_t^S + (1+i)B_t^S + \frac{1}{1-\lambda} D_t + T_t^S \tag{6}$$

  - 0 net supply of bonds: priced but not traded in equilibrium

$$C_t^S = W_t N_t^S + \frac{1}{1-\lambda} D_t + T_t^S = Y_t^S \tag{7}$$

  - Non-participants / Hand-to-mouth : fraction λ, consume income

$$C_t^H = W_t N_t^H + T_t^H = Y_t^H \tag{8}$$

- What to do with profits D_t? Give to participants, but redistribute through taxes

$$\lambda T_t^H = \tau^D D_t = (1-\lambda) T_t^S \tag{9}$$

TANK I: equilibrium conditions

Equilibrium condition	Log-linearized
$U_C\left(C_t^S\right) = \beta R_t E_t\left[U_C\left(C_{t+1}^S\right)\right]$	$c_t^S = E_t c_{t+1}^S - \sigma^{-1} r_t$
$\frac{W_t}{P_t} U_C\left(C_t^S\right) = -U_N\left(N_t^S\right)$	$\varphi n_t^S = w_t - \sigma c_t^S$
$\frac{W_t}{P_t} U_C\left(C_t^H\right) = -U_N\left(N_t^H\right)$	$\varphi n_t^H = w_t - \sigma c_t^H$
$C_t^H = \frac{W_t}{P_t} N_t^H + \frac{\tau^D}{\lambda} D_t$	$c_t^H = w_t + n_t^H + \frac{\tau^D}{\lambda} d_t$
$D_t = \left(1+\tau^S\right) Y_t - \frac{W_t}{P_t} N_t - T_t^F$	$d_t = -w_t$
$Y_t = C_t \equiv \lambda C_t^H + (1-\lambda) C_t^S$	$y_t = c_t \equiv \lambda c_t^H + (1-\lambda) c_t^S$
$N_t = \lambda N_t^H + (1-\lambda) N_t^S$	$n_t = \lambda n_t^H + (1-\lambda) n_t^S$
$Y_t = N_t$	$y_t = n_t$

(10)

Hand-to-mouth consumption: proportional to output

- Aggregate labor supply plus $y_t = c_t = n_t$ gives

$$w_t = (\psi + \sigma) y_t \tag{11}$$

- H labor supply solved for n_t^H

$$n_t^H = \frac{1}{\psi}((\psi + \sigma) y_t - \sigma c_t^H) \tag{12}$$

- In BC for H, with $d_t = -w_t$

$$c_t^H = w_t + n_t^H + \frac{\tau^D}{\lambda} d_t \tag{13}$$

$$c_t^H (1 + \frac{\sigma}{\psi}) = (\psi + \sigma)(\frac{1}{\psi} + (1 - \frac{\tau^D}{\lambda})) y_t \tag{14}$$

$$c_t^H = \left[1 + \psi(1 - \frac{\tau^D}{\lambda})\right] y_t \equiv \chi y_t \tag{15}$$

- $\chi = \left[1 + \psi(1 - \frac{\tau^D}{\lambda})\right]$ elasticity of H's income to aggregate income
- $c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t$
- $\chi > 1$ depending on redistribution: if H's share of profits lower than S's ($\tau^D < \lambda$), $\chi > 1$ and vice versa
- "cyclical income inequality"

MPC and interest elasticity

- log-linear intertemp budget constraint

$$c_t^S = (1-\beta)\sum_{k=0}^{\infty}\beta^k E_t y_{t+k}^s - \sigma^{-1}\beta\sum_{k=0}^{\infty}\beta^k E_t r_{t+k} \tag{16}$$

$$\tag{17}$$

- $c_t = \lambda c_t^H + (1-\lambda)c_t^S = y_t$ implies

$$c_t = (1-\beta(1-\lambda\chi))y_t + (1-\lambda\chi)(1-\beta)\sum_{k=1}^{\infty}\beta^k E_t y_{t+k} - (1-\lambda)\sigma^{-1}\beta\sum_{k=0}^{\infty}\beta^k E_t r_{t+k} \tag{18}$$

- "Partial equilibrium" aggregate MPC out of income change with persistence p: $\omega = \frac{1-\beta(1-\lambda\chi)}{1-p\beta(1-\lambda\chi)}$
- $c_t = y_t$ gives GE Euler equation as in RANK but with different interest elasticity

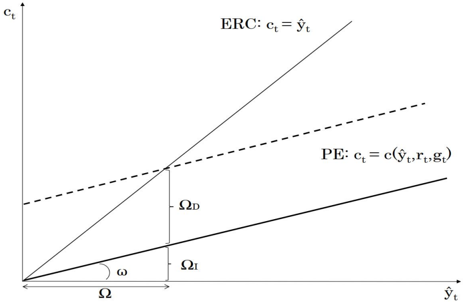
$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi}\sigma^{-1} r_t \tag{19}$$

- implies
  1. $MPC^{TANK} > MPC^{RANK}$
  2. $\chi > 1$ implies interest elasticity $\Omega^{TANK} > \Omega^{RANK}$

The new Keynesian Cross

The New Kenesian Cross

$$c_t = \omega \hat{y}_t - (1-\omega)\,\Omega r_t + (1-\omega)\,(M-1)\,g_t$$



Introduce income risk

- Probability to stay S and H is, resp. s and h, implying $\lambda = \frac{1-s}{2-s-h}$
- S's EE becomes $c_t^S = sE_t c_{t+1}^S + (1-s)E_t c_{t+1}^H - \sigma^{-1} r_t$. Yields aggregate EE

$$c_t = \delta E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi}\sigma^{-1} r_t \tag{20}$$

$$\delta = 1 + (\chi - 1)\frac{1-s}{1-\lambda\chi} \tag{21}$$

  implying $\delta > (<) 1$ iff $\chi > (<) 1$
- Cyclicality of **income inequality** determines compounding / discounting of future consumption changes
- $\chi > 1$: news about aggregate future income y_s is amplified as c_s^H overreacts to y_s
- $\chi < 1$: news about aggregate future income y_s is discounted as c_s^H underreacts to y_s - may help solve FG puzzle!
- NB: income risk is acyclical here

TANK II: Workers vs Capitalists

Broer, Harbo Hansen, Krusell and Öberg, The New Keynesian Transmission Mechanism: A Heterogenous-Agent Perspective" (BKHÖ)

BKHÖ

- Takes standard ‘textbook’ NK model, adds two simple dimensions of heterogeneity
  1. Workers vs capitalists (crucial)
  2. Idiosyncratic income risk (not crucial)
- ‘Tricks’ make solution as simple as textbook model.
- But: flex-wage version has NO output response to monetary policy!
- With rigid wages, model behaves similar to textbook.
- Why?

Intuition: Transmission in simple NK model

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- Standard intuition: Demand-based
  1. With sticky prices, a rise in i raises real interest rate R
  2. Rise in R makes HH want to postpone consumption via EE

$$U_C(C_t, N_t) = \beta E_t R_t U_C(C_{t+1}, N_{t+1})$$

  3. $Y_t = C_t$ implies drop in output, followed by gradual recovery

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  3. $Y_t = C_t$ implies drop in output, followed by gradual recovery
- This paper: Simple extension with heterogeneous agents highlights importance of supply side of the NK economy, and of the source of nominal friction.

Intuition: Labor supply and transmission in the simple NK model

$$W_t U_C(C_t, N_t) \quad = \quad -U_N(C_t, N_t)$$

- Fall in consumption demand implies fall in labour demand and W_t
  1. $\Rightarrow$ Fall in labor supply (substitution effect)

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  2. $\Rightarrow$ Also: Fall in leisure demand (income effect)
- Net effect with standard (BGP) preferences ($f(N_t) = \frac{W_t}{C_t}$)
  - RANK: $C_t^{RA} = W_t N_t + D_t$
    - $C_t > W_t N_t$, countercyclical D_t: profits dampen income effect
  - SHANK: $C_t^w = W_t N_t$, $C_t^{cap} = D_t$

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    - Income & substitution effects cancel: N_t and $Y_t = AF(N_t)$ constant!

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    - Income & substitution effects cancel: N_t and $Y_t = AF(N_t)$ constant!
- RANK transmission - supply side view:
  Workers work less because they own firms and get higher (!) profits in a recession. This wealth shock makes them take more leisure.

Overview: Baseline 'Worker-capitalist' model with flexible wages

- Departure point: Galí (2009), Ch. 3
  - Representative household
  - No capital
  - Monopolistic competition, Calvo price-setting
- SHANK
  - Two HH types: Workers and Capitalists
  - Capitalists collect firm dividends, workers do not
  - Both face idiosyncratic productivity risk and a participation cost of working
  - Both choose how much to work, consume and save in a risk-free bond
  - (but capitalists choose not to work)
- Rest: identical
- Motivation: Tractable form of type-heterogeneity that matches
  1. A small share of the households own almost all financial wealth (Piketty-Zucman, 2015)
  2. At the top of the wealth distribution, labor income is a small share of total income (Gornemann-Kuester-Nakajima, 2016)
- Assumptions that make SHANK tractable:
  1. Fixed cost of working, few capitalists: only workers supply labor
  2. Bonds in 0 net supply: degenerate asset distribution
  3. Together: income-rich workers have highest incentive to save, price the bond

SHANK model: Households

- Worker $j \in [0, 1]$ solves:

$$
\begin{aligned}
\max_{C_{jt}, N_{jt}, B_{jt}} \quad & E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right) \\
\text{s.t.} \quad & P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} \cancel{+ P_t D_{jt}} \\
& B_{jt} \geq 0
\end{aligned}
$$

- Capitalist $j \in [1, m_c]$ solves:

$$
\begin{aligned}
\max_{C_{jt}, N_{jt}, B_{jt}} \quad & E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right) \\
\text{s.t.} \quad & P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} + P_t D_{jt} \\
& B_{jt} \geq 0
\end{aligned}
$$

- Assumption 1: m_c small $\rightarrow$ capitalists choose not to work
- Assumption 2: in $t = 0$, capitalists are endowed with equal shares of the well-diversified portfolio of firm claims, $\rightarrow D_{jt} = \frac{D_t}{m_c}$
- Implies no heterogeneity between capitalists

SHANK model: Production side

- Final goods sector as in Galí textbook model $\rightarrow$ CES demand for intermediate goods :

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = \int_{j=0}^{1} A_{jt} N_{jit} dj$$

- where A_{jt} is the productivity of household j, A_j is drawn from an i.i.d. distribution with finite support and $E(A_j) = 1$
- Calvo friction and intermediate firm maximization problem otherwise identical

Equilibrium conditions

- Textbook model:

$$\begin{aligned} C_t &= Y_t \\ B_t &= 0 \end{aligned}$$

- SHANK:

$$\begin{aligned} \int_{j=0}^{1+m_c} C_{jt}\,dj &= Y_t \\ \int_{j=0}^{1+m_c} B_{jt}\,dj &= 0 \end{aligned}$$

SHANK: solution

- Textbook RANK model is easy to solve
  - Up to the first order, the state space consists only of aggregate variables
  - (Fluctuations in price dispersion are second order)
- HANK models are in general harder to solve
  - The wealth distribution is an endogenous state variable
- Our SHANK model is easy to solve
  - Zero borrowing constraint $\rightarrow$ Degenerate equilibrium bond wealth distribution (Krusell-Mukoyama-Smith, 2011)
    - "Autarky solution" $\rightarrow$ workers (capitalists) consume labor (profit) income hand-to-mouth
  - Labour supply of workers is homogeneous
  - Individual consumption is linear in aggregate consumption

Equilibrium in SHANK: aggregation I

- Autarky consumption
  No borrowing + zero net supply of assets = no lending in equilibrium:
  $B_{jt} = 0 \forall j \in [0, 1 + m_c]$

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} \forall j \in [0, m_c]$$

$$C_{jt} = \frac{D_t}{m_c} \forall j \in (1, m_c]$$

- Homogeneous labour supply
  BGP preferences, autarky: Income and substitution effects cancel
  $\frac{W_{jt}}{P_t} = MRS_{jt} = N_{jt}^{\varphi} C_{jt}$

$$N_{jt} = 1 \forall j \in [0, 1] \tag{22}$$

Equilibrium in SHANK: aggregation II

- Worker Euler Equation
  - Worker consumption $C_{jt} = \frac{W_{jt}}{P_t} N_{jt} = A_{jt} \frac{W_t}{P_t} N_t = A_{jt} C_t$ where $C_t \equiv \frac{W_t}{P_t} N_t$

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  - Implies

$$Q_t = \beta E_t \left\{ \left(\frac{A_{jt+1}}{A_{jt}} \frac{C_{t+1}}{C_t} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

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- Capitalist Euler Equation:

$$Q_t = \beta E_t \left\{ \left(\frac{D_{t+1}}{D_t} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

Equilibrium in SHANK: aggregation II

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- Assume small aggregate shocks: "marginal saver" is worker with highest productivity:

$$Q_t = \beta^{eff} E_t \left\{ \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}} \right\}$$

$$\beta^{eff} = \beta \max\{E_s \left(\frac{A_{js+1}}{A_{js}} \right)^{-1} \} > \beta$$

Equilibrium in SHANK: aggregation II

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$$\beta^{eff} = \beta \max\{E_s \left(\frac{A_{js+1}}{A_{js}} \right)^{-1} \} > \beta$$

- Log-linear Euler equation for aggregate worker consumption:

$$\hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1})$$

$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

SHANK: other conditions

- On firm side, log-linearization of first order condition implies the standard Phillips curve:

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{mc}_t$$

  - where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$
  - with CRS production technology, $\hat{mc}_t = \hat{\omega}_t$

- Intratemporal optimality condition:

$$\frac{W_t}{P_t} = -\frac{U_n}{U_c} = A_{jt} C_t N_t^{\varphi} \Rightarrow \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$

Summary of log-linearized equilibrium

- Our Worker-capitalist model:

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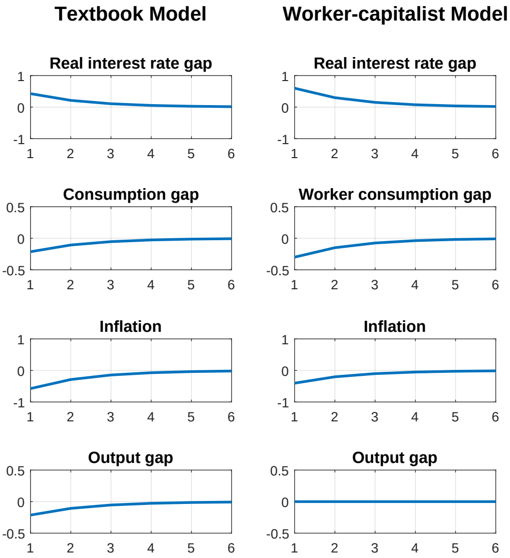
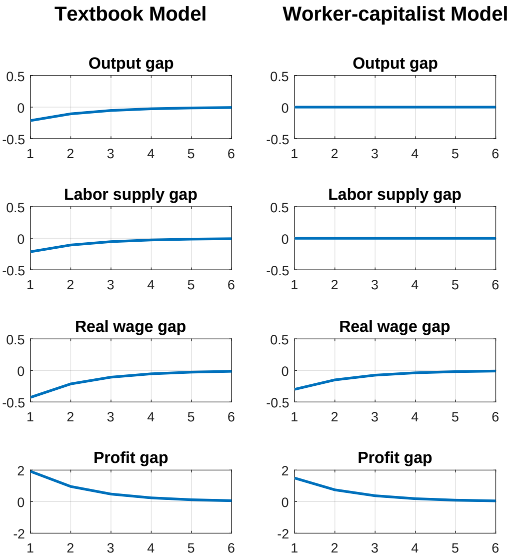
A monetary experiment

- Assume AR(1): $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_{\nu t}$
- Feed in a 25 basis point shock with $\rho_\nu = 0.5$
- How do the two models respond?
- Standard parameterization, follows Galí (2008)

Monetary Shock: Consumption, Output and Inflation



Monetary Shock: Labor supply, wages and profits



Intuition: The determination of labor supply

- Intratemporal first order condition:

$$\frac{W_t}{P_t} = C_t N_t^{\varphi} \Rightarrow \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$

- Elasticity of labor supply:

$$\frac{\partial \hat{n}_t}{\partial \hat{\omega}_t} = \frac{1}{\varphi}\left(1 - \frac{\partial \hat{c}_t}{\partial \hat{\omega}_t}\right)$$

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$$\frac{\partial \hat{c}_t}{\partial \hat{\omega}_t} = 1 + \frac{\partial \hat{n}_t}{\partial \hat{\omega}_t}$$

- The zero result in the WC model is due to BGP preferences, but the mechanism extends to all standard preference specifications.

Introducing rigid wages

- Summary so far
  - Worker-capitalist model undid the influence of monopolistic profits on the IRFs
  - The difference between the worker-capitalist model and the textbook model arose in the labor supply choice
- But
  - In many cases, we think of employment fluctuations as a result of involuntary unemployment fluctuations
  - Leading hypothesis behind such fluctuations: rigid wages
- Now: Introduce rigid wages to our setting, again comparing our model to its corresponding textbook version

Overview rigid wage model

- Households supply differentiated labor-type and face CES-demand curve; interact via monopolistic competition
- Households reset their wages s.t. a quadratic adjustment cost (Rotemberg, 1987)
- Timing within period:
  1. Aggregate shock is realized
  2. Households choose whether to participate and set their wages
  3. Idiosyncratic shocks are realized
  4. Goods and bonds are traded
- RANK: Adds wage Phillips curve, quantities respond similarly
- SHANK: all workers can adjust wages s.t. same cost, iid idiosyncratic shocks and $B_{jt} = 0 \forall j, t$:
  - Households are homogeneous at time of wage choice, set the same W_{jt}
  - $\rightarrow$ we can once more aggregate the model analytically
- Vs. Calvo friction as in Erceg-Henderson-Levin (2000)
  - produces observationally equivalent wage Phillips curve
  - but the wage transitions depend on history of aggregate shocks $\rightarrow$ aggregation of the Euler equation fails

Production technology

- Households supply differentiated labor inputs; efficiency units aggregated by intermediate goods firms using CES aggregator
- $\rightarrow$ Downward-sloping demand curve:

$$N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t$$

- and wage index:

$$W_t = \left[\int_{j=0}^{1} \left(\frac{W_{jt}}{A_{jt}} \right)^{1-\epsilon_w} dj \right]^{\frac{1}{1-\epsilon_w}}$$

SHANK: Worker problem

- Conditional on participating, worker j chooses $C_{jt+k}, N_{jt+k}, W_{jt+k}$ to maximize :

$$
\begin{aligned}
& E_t \sum_{k=0}^{\infty} \beta^k \left(\log C_{jt+k} - \frac{N_{jt+k}^{1+\varphi}}{1+\varphi} - \vartheta \right) \\
\text{s.t.} \quad & P_{t+k} C_{jt+k} + Q_{t+k} B_{jt+k} = \\
& W_{jt+k} N_{jt+k} - \frac{\xi}{2} \left(\frac{W_{jt+k}}{W_{jt+k-1}} - 1 \right)^2 W_{jt+k} N_{jt+k} + B_{jt+k-1} \\
& B_{jt+k} \geq 0 \\
& N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t
\end{aligned}
$$

- As before, we set parametric conditions so that the capitalists choose not to participate.

Equilibrium implications I - aggregation of consumption

- Because idiosyncratic shocks are iid and realized after wages are set, all households set the same wage $\rightarrow$ individual worker income is proportional to average worker income:

$$W_{jt+k} N_{jt+k} = \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} W_{t+k} N_{t+k}$$

- Since all households set the same wage, the adjustment cost is identical to all workers

- $\rightarrow$ individual worker consumption is still proportional to average worker consumption:

$$\begin{aligned} C_{jt+k} &= \left(1 - \frac{\phi}{2} \left(\Pi_{t+k}^{w} - 1\right)^2\right) \frac{W_{jt+k}}{P_{t+k}} N_{jt+k} \\ &= \left(1 - \frac{\phi}{2} \left(\Pi_{t+k}^{w} - 1\right)^2\right) \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} \frac{W_{t+k}}{P_{t+k}} N_{t+k} \\ &= \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} C_{t+k} \end{aligned}$$

Equilibrium implications II - aggregate Phillips curve and aggregate worker EE

- Since all households set the same wage, and worker consumption is proportional to aggregate consumption, we retrieve the standard wage Phillips curve:

$$\begin{aligned} \pi_t^w &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - \hat{mrs}_t) \\ &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \end{aligned}$$

where $\lambda_w = \frac{\epsilon_w - 1}{\xi}$ (same as in textbook model, due to log preferences)

- Because idiosyncratic shocks are iid and worker consumption is proportional to aggregate consumption, the euler equation aggregates as before:

$$\hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1})$$

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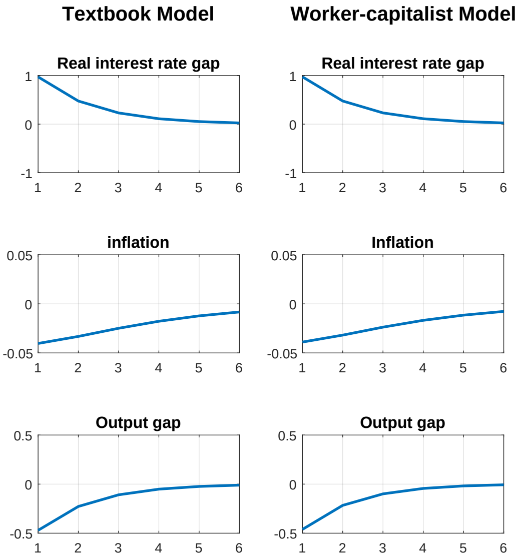
A monetary experiment

- Assume AR(1): $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_{\nu t}$
- Feed in a 25 basis point shock with $\rho_\nu = 0.5$
- How do the two models respond?
- Parameterization: standard, we set ξ so that the textbook wage Phillips curve has the same slope as in Galí (2008)

Monetary Shock: Consumption, Output and Inflation



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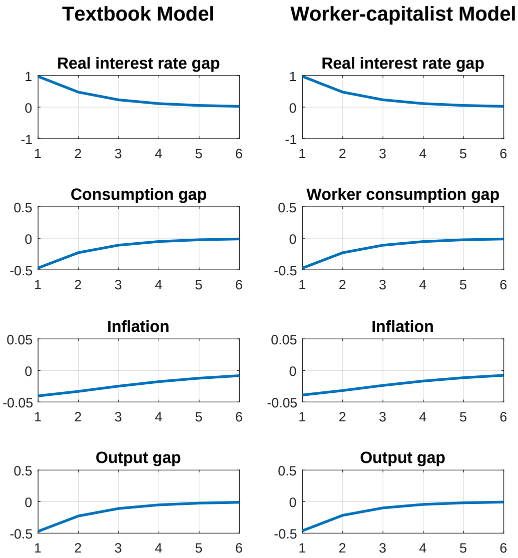

Rigid wages: Interpretation

- Rigid prices: As before, real interest rate increases and worker consumption demand falls
- Rigid wages: No role for income and substitution effects - Labor usage becomes "demand-determined"
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive in both models
- RANK: output response similar to rigid price model
- SHANK:
  - Calibration implies ($\approx$) constant real wage and ($\approx$) constant MPL: profits (= capitalist cons.) and labour income (=worker cons.) are constant shares of output
  - Total output moves proportional to labour income, which follows Euler equation - so output follows standard EE
  - Less wage rigidity would imply smaller output response, larger redistribution from MP shocks

Summary

- Simple HANK model with two dimensions of heterogeneity: workers vs capialist; idiosyncratic labour productivity shocks
- Zero bond holdings, fixed cost of working imply near-analytical solution

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- Simple HANK model with two dimensions of heterogeneity: workers vs capialist; idiosyncratic labour productivity shocks
- Zero bond holdings, fixed cost of working imply near-analytical solution
- Results highlight the unreasonable transmission mechanism in the textbook RANK model
    1. MP affects output because 1) profits are distributed to working households and 2) profits respond countercyclically.
    2. Wage setting mechanism does not matter much for transmission to quantities.
- A simple HANK model with wage rigidities seems a better alternative:
    - Captures first order characteristics of ineqality in data.
    - Captures consensus view of monetary transmission in simple 5-equation framework, without counterfactual redistribution.
    - Wage setting institutions are key for transmission, as in data (Olivei-Tenreyro, 2007, 2010; Bjorklund-Carlsson-Skans, 2016).
    - Rich interaction between inequality in factor incomes, but not wage inequality, and monetary business cycle.

Discussion

- Semi-analytical solution to HANK model
- What's the price?
  - $c_{it} = y_{it}$ counterfactual
  - No other assets / capital
- How do the implications compare to a more realistic HANK model?
  - Kaplan and Violante (2008) find an elasticity of output to MP shock that is 1/30 of their benchmark when profits are not redistributed to workers

Take-aways

- Can analyze HANK models with rep-agent tools. But cost is high.
- The textbook RANK model has an unreasonable transmission mechanism:
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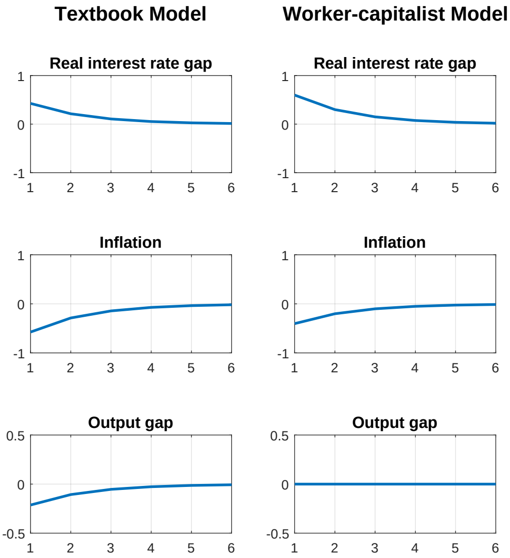
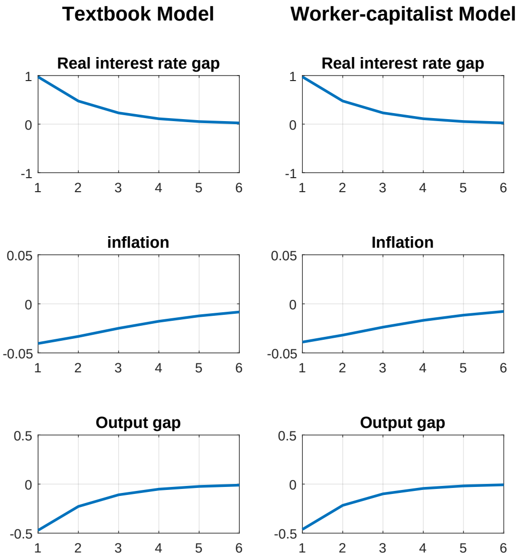
Baseline model: IRF to Monetary Shock



Model with rigid wages: IRF to Monetary Shock



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