Quant Macro QM2

January 24

Jan 23 - Class I

ADMIN

$\underbrace{23, 26, 30}_{\text{Jan}}$, $\overbrace{6, 7, 13}^{\text{Feb}}$

3 PS

1/29
2/5
2/16

12 pm (mid-day) Eastern time

Teams of 2

50% grade + 50% exam

SEND ME AN EMAIL

TOMORROW MAX

FRIDAY 4 pm

~~THU 6.15 pm~~

Ch 1_ Introduction Heterogeneity in macroeconomics

Ch 2 _ $\boxed{\text{Steady-states}}$ in Aiyagari models

A] Eq B] Algorithm C] Application

Ch 3 _ Deterministic TRANSITION

Ch 4 _ Aggregate SHOCKS

B] Algorithm

⋮

HANK session

Ch1 _ Heterogeneity

A] History of thoughts (Ben Moll)

Modern Macro _ 70s / 80s

"First generation" Models ... Sargent Lucas

Rational Expectations

└ eq / solve / calibrate

RA { HA + complete markets }

cycle, shocks, ...

$\boxed{\text{• no data,}}$ $\boxed{\text{no computers}}$

"Second generation" ... 2000s ~ / 90s

▷ widening of inequality

▷ computational power

HA + IM

+ welfare .. MACRO ⇒ welfare poor
→ vs rich
⇒ inequality ↑ ?

Bewley
Aiyagari, Huggett, Imrohoroglu
Violante, Storesletten, Heathcote, Rios-Rull, ...

MACRO → HA

rich ~ RA

(linear) ..

Ineq has no effect
on macro dynamics

(3) "Third generation" of HA macro models
Moll, Violante, Auclert, ...
Two-asset models

Second + curvature → HA ⇒ Macro Dynamics

$E[c^i]$

(B) Topics (C) Facts (US)

Ch 2 – Bewley – Huggett – Aiyagari models in steady state

Incomplete

STATIONARY RECURSIVE COMP EQUILIBRIUM

$K_{t+1} = K_t = K$

$N, r, \ldots$

value function

$\{r, w\}$

GE

A] Equilibrium

A1] Income Fluctuation Problem

HH problem with IM

A2] RCE exchange economy

A3] RCE production (RBC + IM)

A1) income fluctuation problem

IM ... a' is not state-contingent

- PIH : IM + quadratic
  - Consumes all of permanent shock

    r of temporary shock

utility

$u' > 0$

$u'' < 0$

Inada condition

PE HH

$$\max_{\{c_t, A_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t. $A_0 = 0$

$$c_t + \frac{A_{t+1}}{1+r} = A_t + y_t$$

$$A_{t+1} \geq \phi$$

$\{y_t\}$: $y_t \in \{\bar{y}_1, \bar{y}_2, \dots, \bar{y}_N\}$

$\bar{y}_N < \infty$

deterministic , stochastic {iid y

$-y$ iid

$$V(A_t + y_t) = \max_{c_t,\ A_{t+1}} u(c_t) + \beta \sum_{y_{t+1}|y_t} \pi(y_{t+1}|\cancel{y_t})\, V(A_{t+1} + y_{t+1})$$

$$A_{t+1} \geq \phi, \quad c_t + \frac{A_{t+1}}{1+r} = A_t + y_t$$

$a \equiv A + y$

$$V(\underline{a}) = \max u(c) + \beta \sum_{s'} \pi(y_s') \, v(\underline{a_s'})$$

$$\frac{A'}{1+r} = \underbrace{(A+y)}_{a} - c$$

$$A' = (1+r)(a-c)$$

$$\boxed{a_s' - y_s' = (1+r)(a-c)} \leftarrow \{ a_s'$$

$$V(a) = \max u(c) + \beta \sum_{s'} \pi(y_s') \cancel{V}\big(\overbrace{(1+r)(a-c) + y_s'}\big)$$

BC

$$A' \geq \phi$$

$$(1+r)(a-c) \geq \phi$$

$$c \leq -\frac{\phi}{1+r} + a$$

$$V(a) = \max_{\substack{0 \leq c \leq a - \frac{\phi}{1+r} \\ \lambda}} u(c) + \beta \sum_{s'} n(y'_{s'}) V\big[(1+r)(a-c) + y'_{s'}\big]$$

$$a = A + y$$

FOC

$$u_c + \beta \sum_{s} n(y'_{s'}) V'\big((1+r)(a-c) + y'_{s'}\big) \cdot \big(-(1+r)\big) - \lambda = 0$$

Env $\quad V'(a) = \beta^{(1+r)} \sum_{s'} n(y'_{s'}) V'(\ldots) + \lambda$

FOC : $u_c = \beta \sum_{s'} n(y'_{s'}) V'(a'_{s'})(1+r) + \lambda$

$= V'(a)$

$$u_c = \cancel{\beta(1+r)} \sum_{s'} \pi(y_{s'}) \, u_{c_{s'}} + \lambda$$

$$\boxed{\beta(1+r) = 1} \qquad u_{c_t} = E_t \, u_{c,t+1} + \lambda$$

$$\lambda \geq 0$$

$$\boxed{U_{c,t} \geq E_t U_{c\,t+1} \geq 0}$$

Path for $\{c_t\}$ in LR ?

3 cases

1. deterministic path for $\{y_t\}$ + ϕ ... natural borrowing limit

$$\lambda = 0 \qquad U_{c_t} = U_{c_{t+1}}$$

$$c_t = c_{t+1} = \bar{c}$$

BC $\boxed{\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j c_j = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_j}$

$$\bar{c} = \frac{r}{1+r}\underbrace{\left\{\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_j\right\}}_{\text{TOTAL "PERMANENT" wealth}}$$

"interest rate"

② determine but $\phi = 0$

$$U_{c_t} = U_{c_{t+1}} + \lambda$$

$\lambda = 0 \Rightarrow \quad c < \frac{\phi}{1+r} \quad \ldots\ldots$

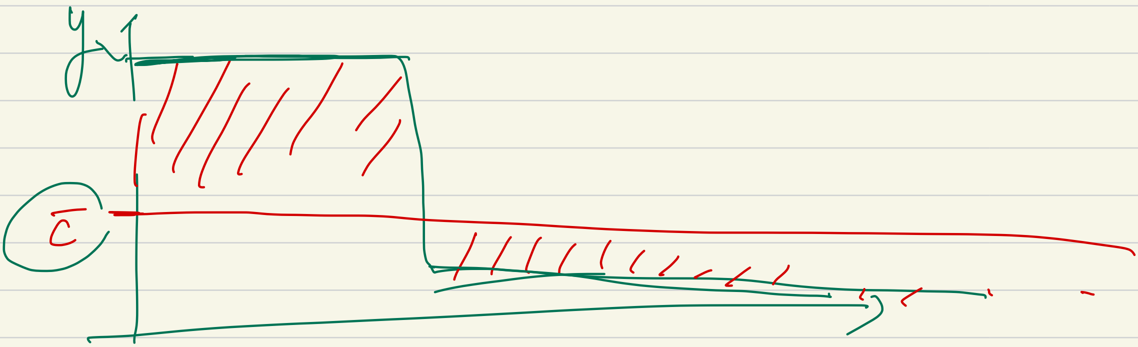
$A' > 0 \longrightarrow \boxed{c_t = c_{t+1}}$

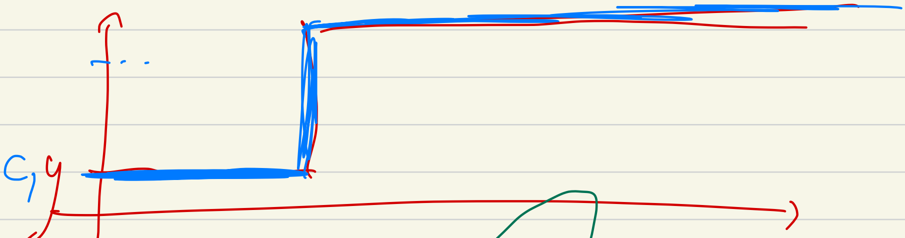
$\lambda > 0 \rightarrow A' = 0 \rightarrow U_{ct} > U_{ct+1}$

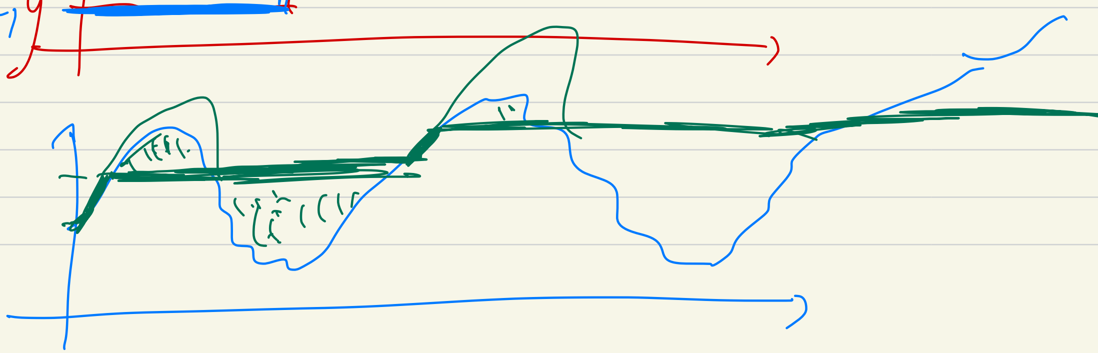
$\boxed{c_t < c_{t+1}}$

$$\lim_{t\to\infty} c_t = \sup_t \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j y_{t+j}$$

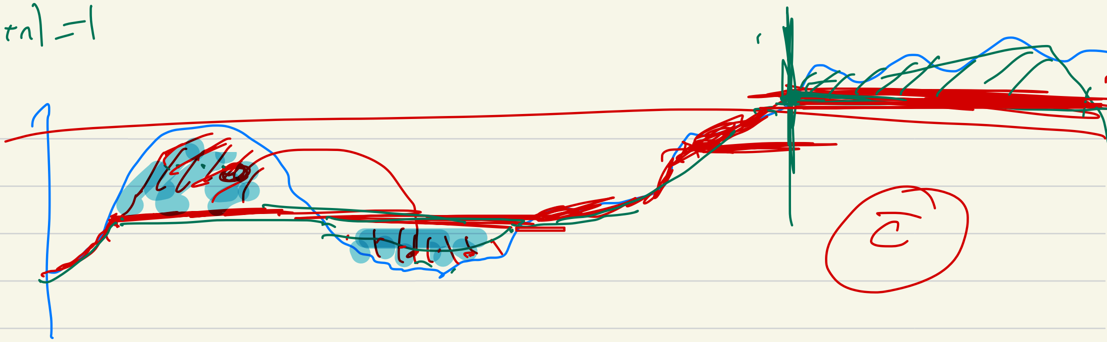
$< \infty$ {y has finite support...}







$\beta(1+r)=1$



③ stochastic case

$$u'(c_t) \geq E_t u'(c_{t+1}) \geq 0$$

Doob (53) — Let M_t supermartingale bounded below

$M_t = E_t M_{t+1}$

by 0

$$\boxed{M_t \geq E_t M_{t+1}} \geq 0$$

$$M_t \to \bar{M} < \infty$$

$u'(c_t)$ RV $\quad u'(c_t) \geq E_t u'(c_{t+1}) \geq 0$

$$u'(c_t) = v'(a_t)$$

Doob $\to$ $u'(c_t) \to \alpha \geq 0$

$\alpha < \infty$

$\alpha \in [0, \alpha^+]$

$V'(a_t) \to V'(a) = \alpha \geq 0$

$< \infty$

<u>if</u> $\alpha > 0 \to$ no

$V'(a) \to \alpha > 0$

$a_t \to V'^{-1}(\alpha)$

$\quad \hookrightarrow \tilde{\alpha}$

$a' = (1+r)(a - c) + y_s'$

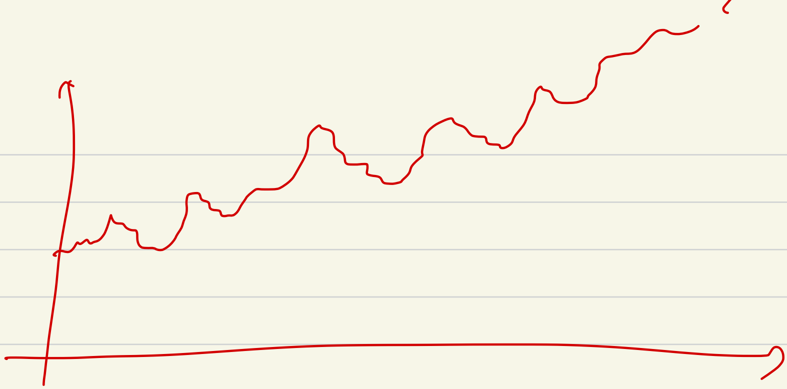
$a' \downarrow \alpha \qquad a \downarrow \alpha \qquad c \downarrow$

$\to a' \to +\infty$

$+\infty \qquad c \qquad +\infty$

$V'(a) \to 0 \Rightarrow u'(c) \to 0$

$c \to +\infty$



Chamberlain Wilson (2000)

IM + $\beta(1+r) = 1$

stochastic $\{y_t\}$ $\longrightarrow$ precautionary

$c_t \to +\infty$

$a_t \to +\infty$

Asset M $\Rightarrow$ $\boxed{1+r < \frac{1}{\beta}}$

$\boxed{r^{IM} < r^{CM}}$

Ch 11.A.2 RCE in an Exchange Economy

$\beta(1+r) < 1$

$r < \frac{1}{\beta} - 1$

HH $\quad \varepsilon_t \in E = \{\varepsilon^1, \varepsilon^2, \dots, \varepsilon^N\}$ endowment

Markov $\quad \pi_{\varepsilon\varepsilon'} = P(\varepsilon_{t+1} = \varepsilon' \mid \varepsilon_t = \varepsilon)$

Unique stationary distribution: $\Pi^* = [\pi_1^* \; \pi_2^* \dots \pi_n^*]$

Mass 1 of hh

Total output / endowment $\quad \boxed{Y = \sum_{k=1}^{N} \pi_k^* \varepsilon^k}$

HH's Problem with idiosyncratic shocks

$$v(a, \varepsilon) = \max_{c, a'} \; u(c) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} v(a', \varepsilon')$$

$$\text{s.t.} \quad c + a' = \varepsilon + (1+r)a, \quad a' \geq b.$$

policy function: $\underline{c}(a, \varepsilon)$, $\underline{a'}(a, \varepsilon)$

distribution $\lambda(a, \varepsilon)$ $\leftarrow \{\varepsilon^1, \varepsilon^2, \dots, \varepsilon^N\}$

continuum $[b, \bar{a}]$

$\{a_1, a_2, \dots, a_M\}$

$$
\lambda_t \quad
\begin{array}{c|cccc|}
 & \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_N \\
a = a_1 & \lambda & \lambda & & \lambda \\
a = a_2 & & 0 & & 0 \\
a = a_3 & 0 & & & 0 \\
\vdots & & 0 & & \\
a = a_M & & & & 0
\end{array}
$$

$$\sum_a \sum_\varepsilon \lambda(a, \varepsilon) = 1$$

$$\int_a \sum_\varepsilon \lambda(a, \varepsilon) = 1$$

Λ_t move ?

transition function

$S = A \times E$ $\qquad$ $\boxed{A = [b, \bar{a}]}$

$(\mathcal{A} \times \mathcal{E})$ subset of σ-algebra defined by $B_A \times P(E)$

$\{\phi, \{\varepsilon^1\}, \{\varepsilon^2\}, \dots$
$\{\varepsilon^1, \varepsilon^2\}, \dots \}$

$Q((a, \varepsilon), \mathcal{A} \times \mathcal{E})$: prob that someone with state (a, ε) ends up next period with $a' \in \mathcal{A}$ and $\varepsilon' \in \mathcal{E}$

$$\lambda(\mathcal{A} \times \mathcal{E}) = \int_{A \times E} Q((a, \varepsilon), \mathcal{A} \times \mathcal{E}) \, d\lambda(a, \varepsilon)$$

$$Q((a, \varepsilon), \mathcal{A} \times \mathcal{E}) = \mathbb{1}_{\{a'(a,\varepsilon) \in \mathcal{A}\}} \sum_{\varepsilon' \in \mathcal{E}} \pi_{\varepsilon \varepsilon'}$$

$Q((a, \varepsilon), A \times E) = 1$

stationarity $\boxed{\lambda' = \lambda}$ (*)

Definition RCE : a VF $v: S \to \mathbb{R}$

PF $c: S \to \mathbb{R}^+$

$a': S \to A$

price r, and a distribution λ^* s.t.:

(1) given r, VF + PF solve HH's problem

(2) Good market must clear

$$\int c(a,\varepsilon)\, d\lambda(a,\varepsilon) = C = Y = \sum_{k=1}^{N} n_k^* \varepsilon^k$$

(3) Assets market have to clear

$$\int a'(a,\varepsilon)\, d\lambda(a,\varepsilon) = 0$$

(4) λ^0 stationary (*)

$$\int a\, d\lambda(a,\varepsilon) = 0$$

class #2

Assets are in zero-net supply

$b = 0 \Rightarrow$ "$a'(a, \varepsilon)$" $= 0$

Ch2A3] RBC model

Jan24

Friday, Jan 26: 16:15 - 17:15

Feb2, 16:45 - 17:45

~~Feb~~ Feb16 _ 16:45 - 18:45

ch 2A3] RBC model

labor, capital - HH, firm

$\boxed{hh}$ continuum of mass 1

consumption, leisure β

ε labor productivity $\pi_{\varepsilon\varepsilon'}$

$$E = \{ \varepsilon^1, \varepsilon^2, \dots, \varepsilon^N \}$$

$$\sum_{k=1}^{N} \pi^*_k = 1 \quad , \quad \pi^* \text{ stationary distribution.}$$

$$c, n, a' \qquad (a, \varepsilon)$$

labor income : $w \varepsilon n(a, \varepsilon)$

Firms : $Y = F(K, N)$ CRS
CE $\rightarrow$ no profits

HH: $$v(a, \varepsilon) = \max_{c, n, a'} u(c, n) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} v(a', \varepsilon')$$

$$c + a' = (1+r)a + w\varepsilon n \quad , \quad a' \geq b$$

FOC

$$u_{c_t} = \beta(1+r) E_t u_{c_{t+1}} + \lambda_t \qquad \text{(Euler)}$$

$$u_{c_t} w_t \varepsilon_t = - u_{n_t} \qquad \text{(intratemp FOC)}$$

Firm:

$$\max_{K_t, N_t} K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t + \delta) K_t$$

$$\begin{cases} w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} \\ r_t + \delta = \alpha \left(\frac{K_t}{N_t}\right)^{\alpha-1} \end{cases}$$

$$w_t = (1-\alpha)\left[\frac{r_t+\delta}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} \quad (*)$$

STATIONARY RCE:

- VF $v : A \times E \to \mathbb{R}$
- PF $a' : A \times E \to A$
  - $n : A \times E \to \mathbb{R}^+$
  - $c : A \times E \to \mathbb{R}^+$

PF for the firm $\{N, K\}$, prices $\{r, w\}$, and a distribution λ^* st: $A \times E$

- Given (r, w), HHs behave optimally
- " " , firm behave optimally

- Asset markets clear:

$$K' = \int a'(a,\varepsilon)\, d\lambda^*(a,\varepsilon)$$
$$= K = \int a \, d\lambda^*(a,\varepsilon)$$

- Labor market clears:

$$N = \int \varepsilon\, n(a,\varepsilon)\, d\lambda^*(a,\varepsilon)$$

- λ^* stationary ... (transition function Q)

ch II A4] Comments on the RBC model

- bounds on r, existence, uniqueness
- key properties of this model

ch II B] ALGORITHM – inelastic labor supply

Structure

Step 0. ε ... AR(1) in logs... $\rho = 0.9$ ... variance

discretize ... $\{\varepsilon^1, \varepsilon^2, \dots, \varepsilon^N\}$

TAUCHEN ... ROUWENHORST

Discretize assets ... $[b, \bar{a}]$

discretize $\{a^1, a^2, \dots, a^M\}$

$a^1 = b, \dots\dots, a^M = \bar{a}$

Step 1: Given r_0 [bounds ... $r_0 \geq -\delta$, $r_0 \leq \frac{1}{\beta} - 1$]

Restart given r_1

Compute w_0 $\left[(1-\alpha) \cdot \left(\frac{r_0+\delta}{\dots} \right) \right]$ (* firm)

Step 2: $\{r_0, w_0\}$ VFI ... v^0, c^0, n^0, a^0

(a, ε)

$\boxed{n^0 = 1}$

$V(a, \varepsilon)$ ... VF, policy function

Step 3: distribution

$a'^{0} \rightarrow \varphi^{0}$

$$\lambda^{0} = \int \varphi^{0} \, d\lambda^{0}$$

Step 4: Check whether markets clear.

$$\boxed{\bar{N} = \sum_{n=1}^{N} \varepsilon^{n} \, n_n^{*}}$$

(inelastic labor supply)

Capital demand firm K^{0} ... given $r_0, w_0, \bar{N}$

$$r_0 + \delta = \alpha \left(\frac{K}{N_0} \right)^{\alpha - 1}$$

$$\left(\frac{r_0 + \delta}{\alpha} \right)^{\frac{1}{\alpha - 1}} \bar{N} = K_0$$

Capital supplied by hh

$K_1 = \int a^0(a, \varepsilon)\, d\lambda^0(a, \varepsilon)$

$I_1 \quad K_0 = K_1 ?$ No...

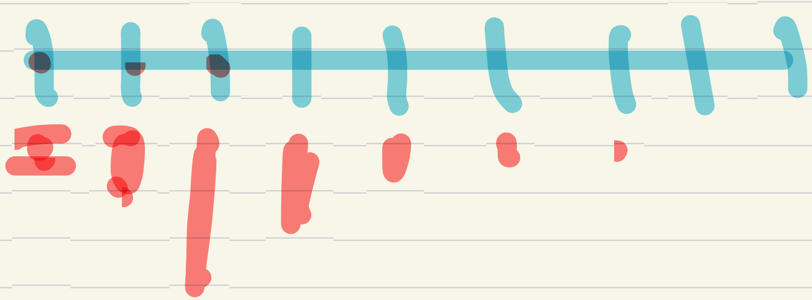
$K_0 > K_1$ ... r_0 was too low

→ increase r

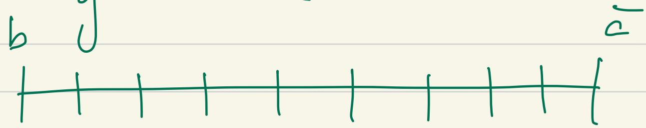
↓

$r_1 > r_0$

$K_0 < K_1$ ... r_0 was too high... decrease r, and call that r_1



- TAUCHEN ... (Rouwenhorst)
- $N=5$, grid on a



equally spaced

- r_0 ... $\longrightarrow r_1$ bissection

- $\boxed{\text{VFI}}$ $v_0 = 0 \quad \forall (a, \varepsilon)$

grid search

$\forall (a, \varepsilon)$, ... compute all a'

$$\hat{V}(a, \varepsilon; a') = u(c, \bar{n}) + \beta \cancel{\sum} \Pi_{\varepsilon\varepsilon'} v_0(a, \varepsilon, a')$$

$$c = w \varepsilon \bar{n} + (1 + r_0) a - a'$$

$\forall a' \rightarrow$ Pick the best a' on the grid

update V ... until converges.

- DISTRIBUTION ... simulate

$\begin{pmatrix} a, \varepsilon \\ \\ \\ \\ \end{pmatrix}$

7500

a' ?

$\varepsilon = \varepsilon^3$

$a = 2$

$$\frac{\sum a}{7500} \ldots \quad K_1 \text{ stable}$$

↳ $K_1 \ldots K_0 \ldots \rightarrow \Omega$? (bisection)

ϕ One guy j

a_1^j, ε_1^j

$a_1^j = 1$ $\varepsilon_1^j = 1$

$\forall j \ldots$ Random draw to comput ε'

Random draw $\pi_{\varepsilon\varepsilon'} \rightarrow \varepsilon_2^j$

$a_2^j = a_0'(a_1^j, \varepsilon_1^j)$

a_2^j, ε_2^j

Rand draw to compt $\varepsilon' \rightarrow \varepsilon_3^j$

$a_3^j = a_0'(a_2^j, \varepsilon_j^2)$

$\varepsilon = \varepsilon^1$

$$\begin{bmatrix} 0.85 & 0.10 & 0.05 & 0\ 0 \\ & & & \\ \end{bmatrix}$$

RD..

$0.4 \longrightarrow \varepsilon^1$

$0.88 \longrightarrow \varepsilon^2$

$0.99 \longrightarrow \varepsilon^3$ ____ - - -

↳ $K^1 = K^0$?

... n —

Initial guess

V_0 increasing in a and in ε

Assume $\boxed{a' = a}$

$$V_0(a, \varepsilon) = u(c) \ldots$$

$$c = (\cancel{1}+r)a + w\varepsilon n - \cancel{a}$$

$$= ra + w\varepsilon n$$

$$v_0(a,\varepsilon) = \frac{[ra + w\varepsilon n]^{1-\mu}}{1-\mu} + \beta \cancel{\sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} v_0(a,\varepsilon')}$$

$v_0(a,\varepsilon)$

$$v_0(a,\varepsilon) = \frac{1}{1-\beta} \frac{[ra + w\varepsilon \bar{n}]^{1-\mu}}{1-\mu}$$

$\cancel{v = 0}$

Howard Step

VFI ① $v^{k}(a,\varepsilon) = \max\limits_{c,\,a'} \; u(c) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} \, v^{k-1}(a',\varepsilon')$

k is for given

$c^{k}(a,\varepsilon),\; a'^{k}(a,\varepsilon)$

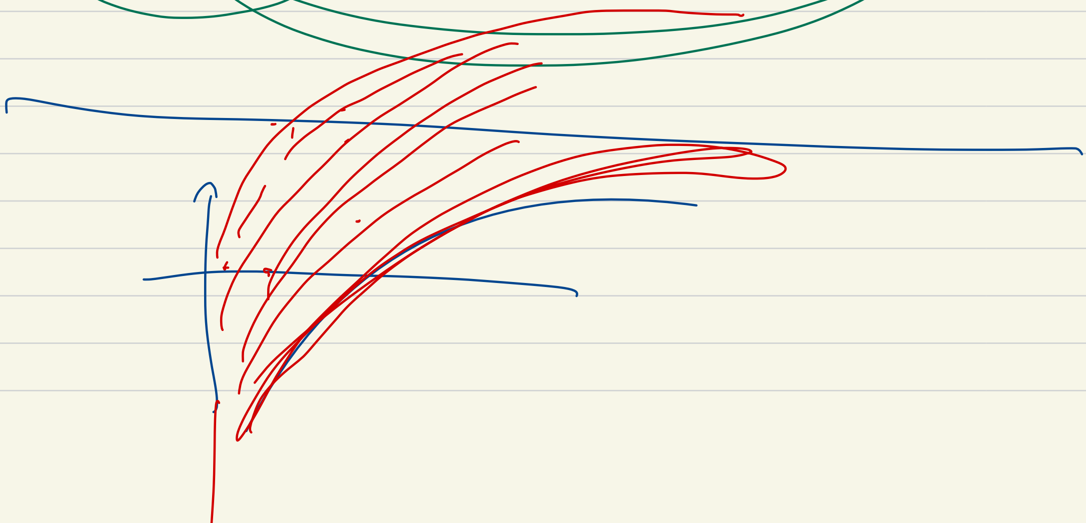
② Howard step

h

$\hat{v}^{k+1}(a,\varepsilon) = u\left(c^{k}(a,\varepsilon)\right) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} \, \hat{v}^{k}\left(a'^{k}(a,\varepsilon),\varepsilon'\right)$

iterate until

new $v^{k}(a,\varepsilon)$



simulation

1) matrix ~~$(\mathcal{E} \times a)$~~ $[N_m \times N_n]$ $(a \times \mathcal{E})$

$$\begin{bmatrix} (a=1, \mathcal{E}=1) & \cdots & (a=1, \mathcal{E}=n) \\ (a=2, \mathcal{E}=1) & & \vdots \\ \vdots & & \vdots \\ (a=m, \mathcal{E}=1) & \cdots & (a=m, \mathcal{E}=n) \end{bmatrix}$$

Vector $[N_m \times N_n, 1]$ = Γ

$$\begin{bmatrix} \lambda(a=1, \mathcal{E}=1) \\ \lambda(a=2, \mathcal{E}=1) \\ \lambda(a=3, \mathcal{E}=1) \\ \vdots \\ \lambda(a=m, \mathcal{E}=1) \\ \lambda(a=1, \mathcal{E}=2) \\ \vdots \\ \lambda(a=m, \mathcal{E}=n) \end{bmatrix}$$

$$\Gamma = Q \cdot \Gamma$$

$$[N_m \times N_n, 1] \quad [N_m \times N_n, \quad N_m \times N_n]$$

$$\begin{bmatrix} \delta^1(a=1, \varepsilon=1) \\ \delta^2(a=2, \varepsilon=1) \\ \vdots \\ \delta^1(a=m, \varepsilon=n) \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & \cdots & q_{1 m\times n} \\ q_{21} & q_{22} & & & \\ & & Q & & \\ & & & & \end{bmatrix} \begin{bmatrix} \delta(a=1, \varepsilon=1) \\ \delta(a=2, \varepsilon=1) \\ \vdots \\ \delta(a=m, \varepsilon=n) \end{bmatrix}$$

$q_{11} = P($hh in state $(a=1, \varepsilon=1)$ goes to $(a'=1, \varepsilon'=1)$ next period$)$

$$Q \qquad q_{11} = \mathbb{1}\left[a'(a, \varepsilon) = a_1\right] \pi\left(\varepsilon' = \varepsilon_1 \,\middle|\, \varepsilon = \varepsilon_1\right)$$

$$q_{12} = \mathbb{1}\left\{a'(a=2, \varepsilon=1) = a_1\right\} \pi\left(\varepsilon' = \varepsilon_1 \mid \varepsilon = \varepsilon_1\right)$$

given Γ ...

$Q\Gamma$ ... $\boxed{\text{until converge}}$

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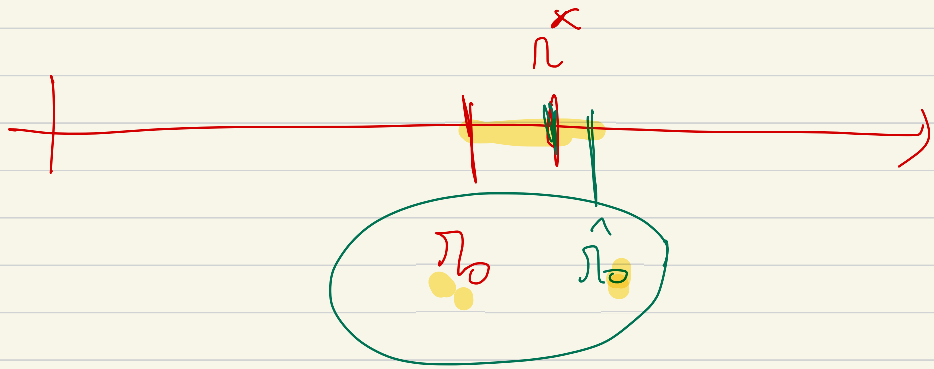
(4) $r_0 \rightarrow K_1$

$\bar{N}$

$$\hat{r}_0 = \alpha \left(\frac{K_1}{\bar{N}}\right)^{\alpha-1} - \delta$$

$$r_1 = 0.9 \times r_0 + 0.1\, \hat{r}_0$$

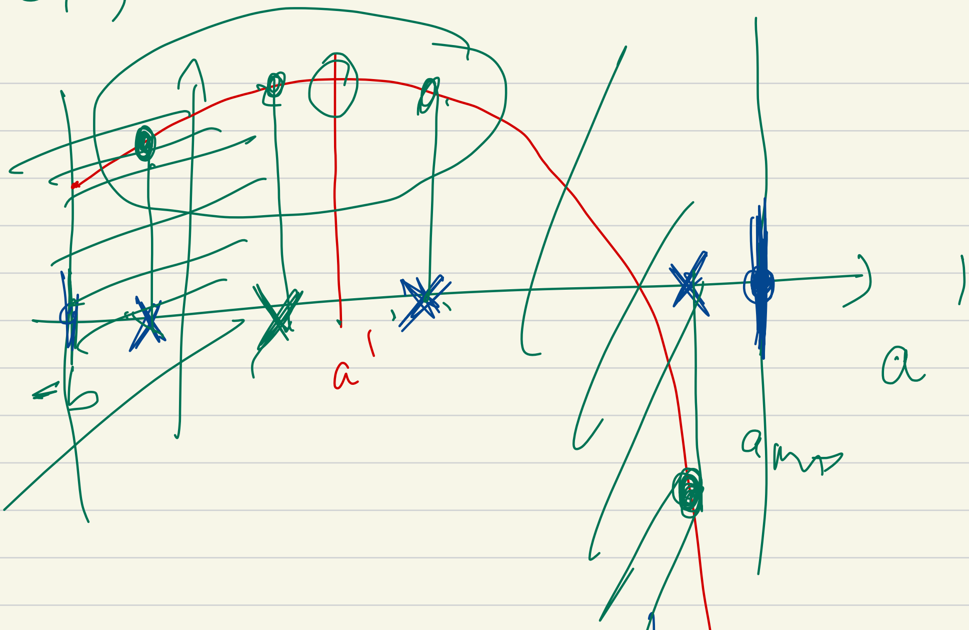


slow weights

Golden search optimal a'

VF concave

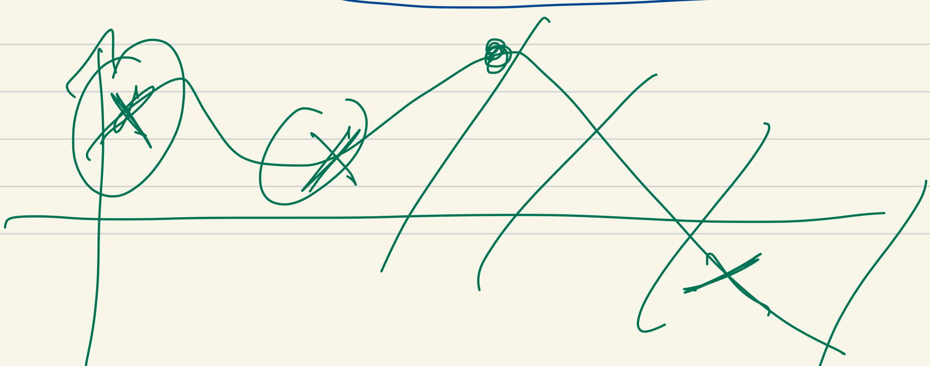
(a, ε)

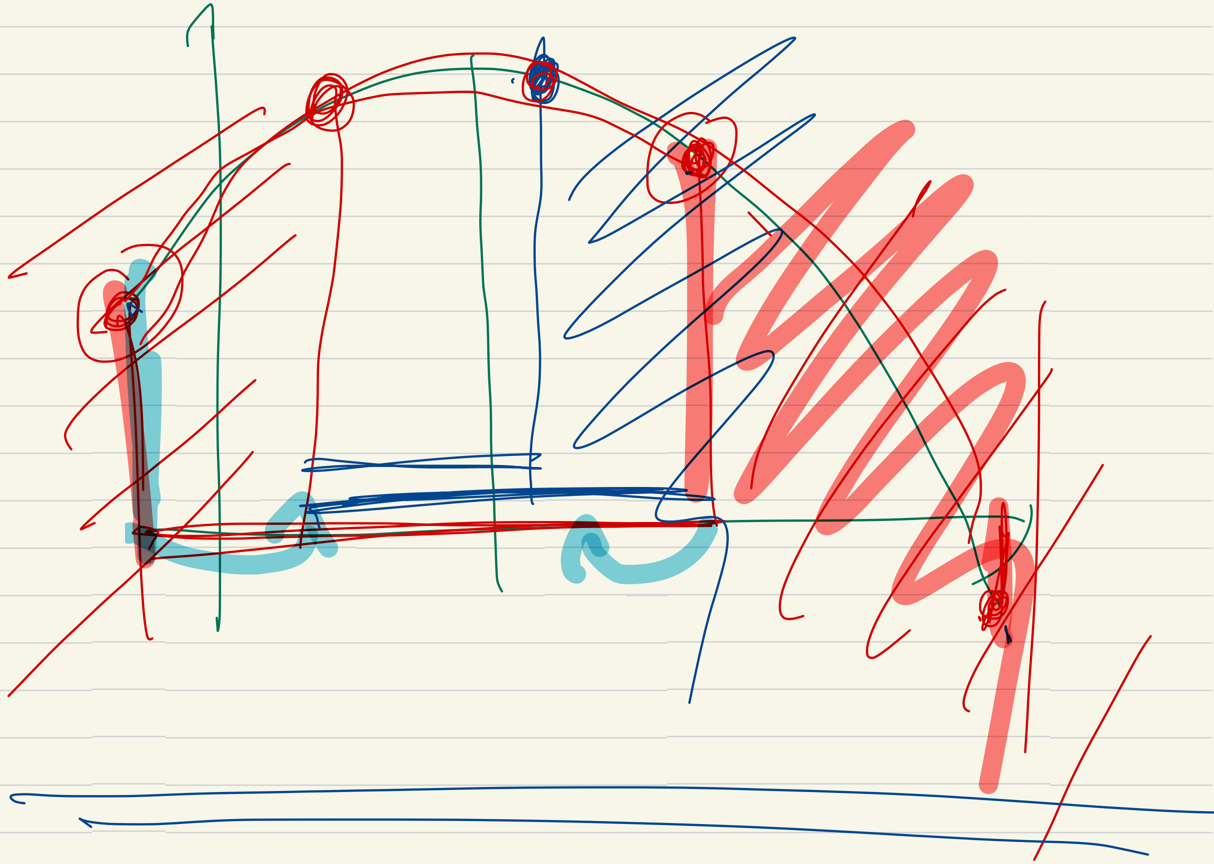


bounds: $[-b, a_{max}$ s.t. $c > 0$

$c \leq (1+\lambda)a + w\varepsilon\lambda - \hat{a} > 0$

$$\boxed{\hat{a} < (1+\lambda)a + w\varepsilon\lambda}$$





VF

$v_0 \to a'$ golden

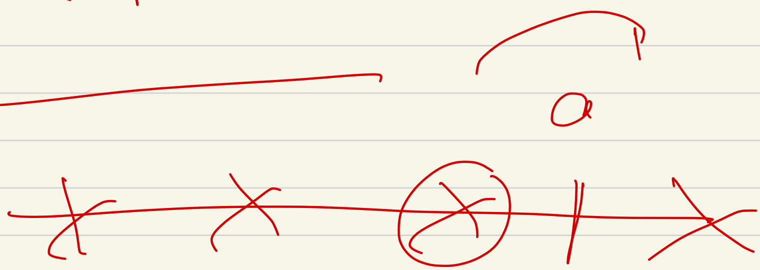
(bounds . . . . golden)

$a'(a, \varepsilon)$

$$u(c(a, \varepsilon, a')) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} v_0(a', \varepsilon')$$

INTERPOLATE

linear interpolation



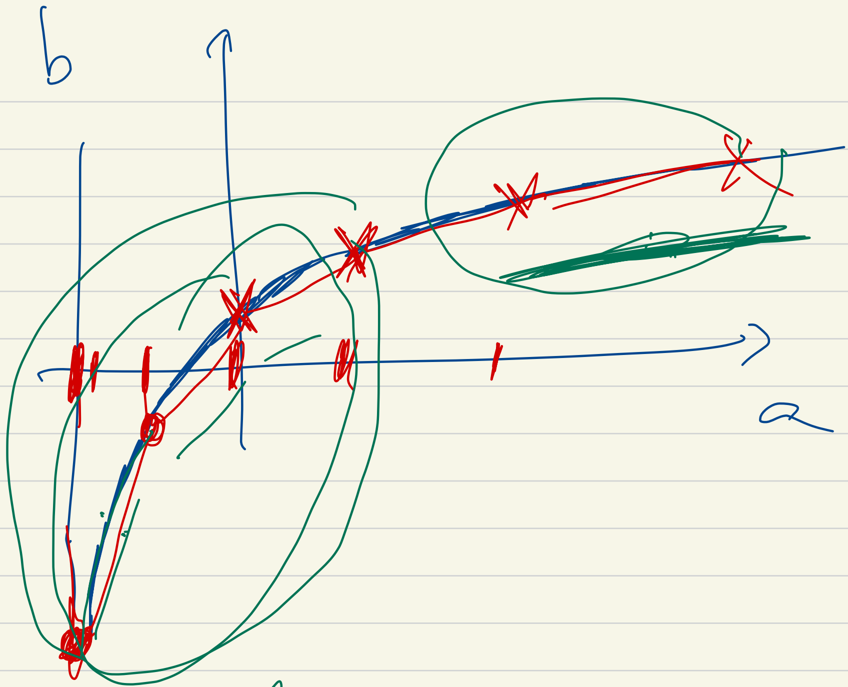
FIND a_i st $a_i < a' \le a_j$

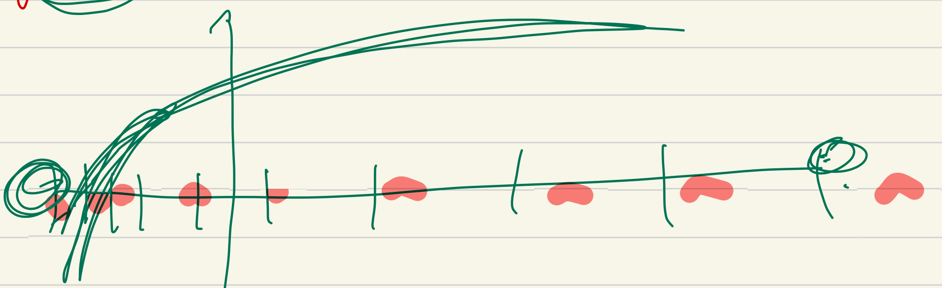
$$v_0(a', \varepsilon') = v_0(a_i, \varepsilon') + (a' - a_i) \dots$$

TRANSITION MATRIX

$$\mathbb{1}\{a'(a=a_m, \varepsilon=\varepsilon_n)=a_k\}\,\pi(\varepsilon'=\varepsilon_s \mid \varepsilon=\varepsilon_n)$$

"stupid"





equally spaced log grid → exp

JAN 30

Ch II A] EQ SS Aiyagari model

+ Existence, Uniqueness

r^* demand for K: firm

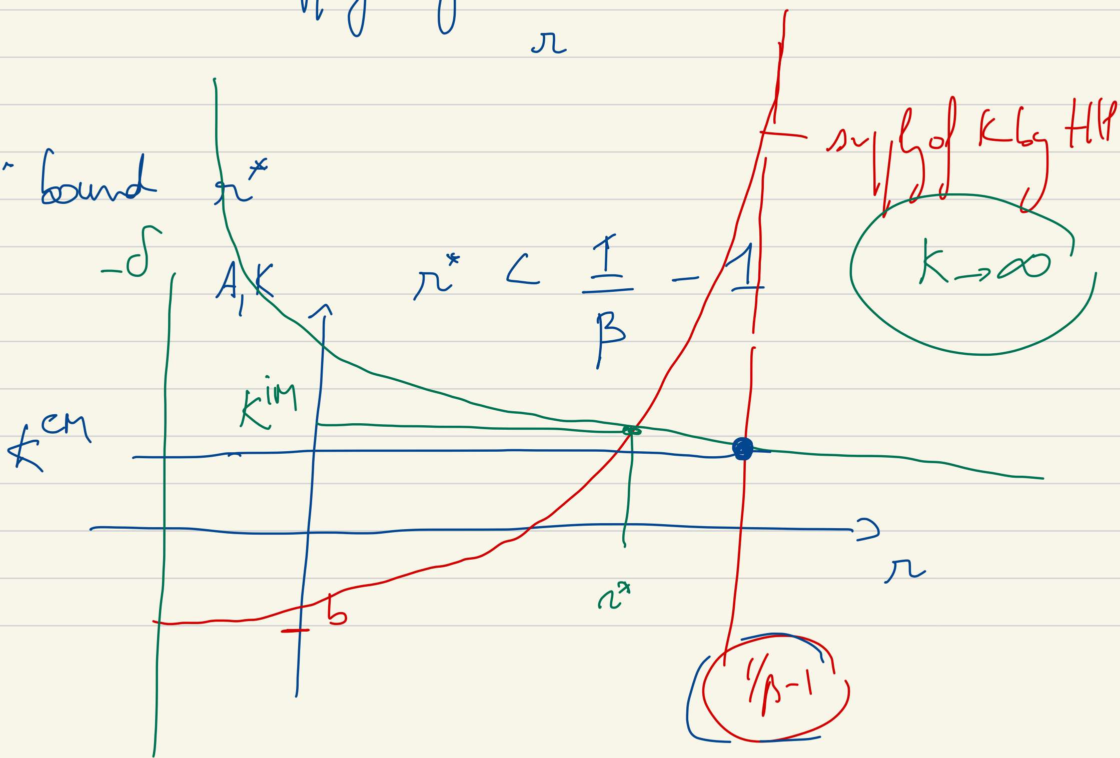
$$r + \delta = F_K(K, \bar{N})$$

supply for K: hh

r

· bound r^*

$$-\delta \qquad A, K \qquad r^* < \frac{1}{\beta} - 1$$



SS

Huggett (93) "Equity premium" puzzle
riskfree rate is so low ...

iM, hh save ... precautionary motive
$\rightarrow r^* \downarrow\downarrow$

Aiyagari (95) $\boxed{K^{iM} > K^{CM}}$
... Quantify ... precautionary savings ?

precautionary savings motive.

CRRA

Case 1: low ...
low RA : log
iid
$\rightarrow \boxed{K^{iM} \simeq K^{CM}}$
$>$

Case 2: RA = 2, AR1 ... $\rho = 0.9$ $\rightarrow$ 25%

Precautionary savings

* Income, assets, consumption.

FOCs,

$$u_c = \beta(1+r)\, E\, u_c'$$

$$u_c\, w\, \varepsilon = -u_n$$

∘ asset / consumption respond after a shock in income

$$\Delta a_{t+1}(a, \varepsilon)$$

mean-reversion

bad ε … $a \downarrow \downarrow$

$\Delta a < 0$

good $\varepsilon \rightarrow$ SAVE

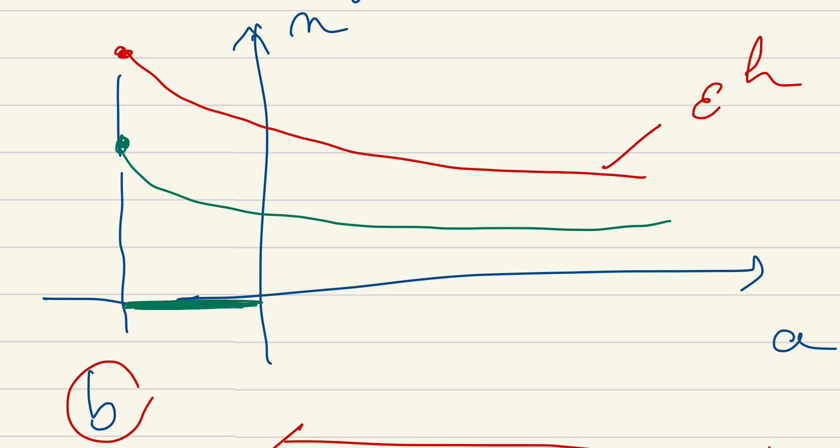
Given $\boxed{\text{process}}$ for $\{\varepsilon\}$ β

↳ distribution of assets

∘ Correlation inc and asset

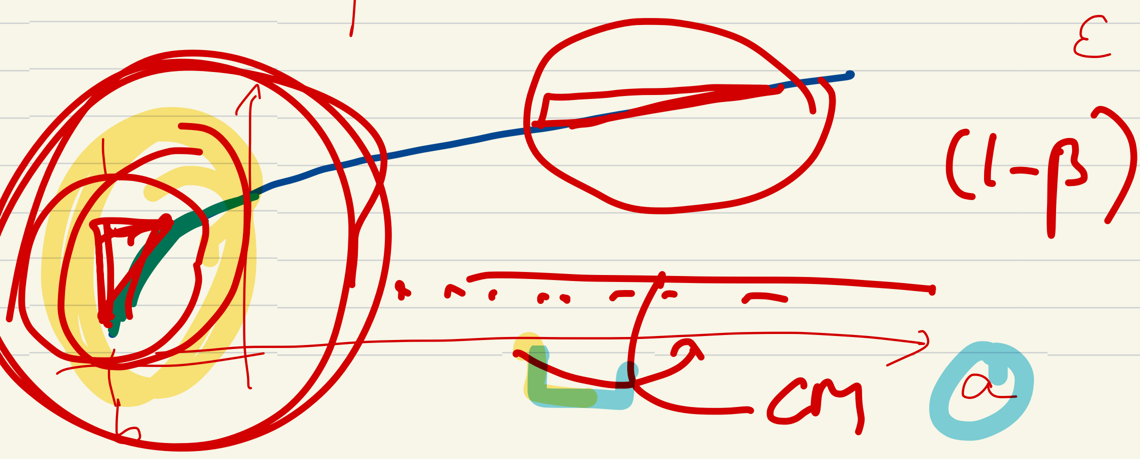
ε^l ε^h

* labor supply



B

* C responds to TRANSFERS



Mass of the BC

10%

Dynamics

Top quintile Asset today
simulate ...

proba ... remains top quintile ...

Ch II B ALGORITHMS

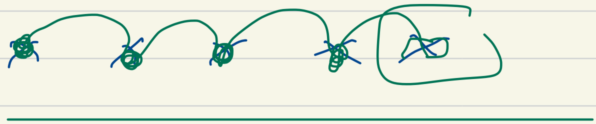
(1) golden

~~interp 2~~

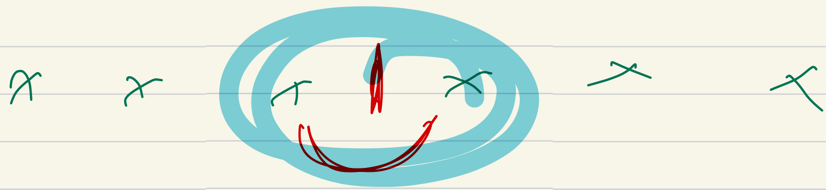
(a, ε)

$$\boxed{\beta \sum_{\varepsilon'} \pi(\varepsilon'|\varepsilon)\, \hat{v}(a', \varepsilon')}$$

- Abar ε'



iterap 1 $(a' \dots)$



$\varepsilon' = \varepsilon_1$

find ... $\{a_i, a_{i+1}\}$ ... $a' \longrightarrow \{\sigma, 6\}$

$\varepsilon' = \varepsilon_2$

find ... ——— ——— $\longrightarrow \{\sigma, 6\}$

Alternative algorithm...

① guess $\hat{v}$

$$\text{(2)}\quad \hat{W}(a,\varepsilon) = \beta \sum \pi(\varepsilon'|\varepsilon)\, \hat{V}(a,\varepsilon')$$

$$\hookrightarrow \quad \max_{c,a'} \; u(c) + \hat{W}(a',\varepsilon)$$

TRICK : Vectorize your code

$$\underbrace{N_k}_{a} \times \underbrace{N_s}_{\varepsilon}$$

$\forall (a,\varepsilon)$

$a_{lb} = a_{min}$

$a_{ub} = (1+r)a + w\varepsilon$

golden

$$S = [N_k \times N_s, 2]$$

$$S = \begin{bmatrix} a_1 & \varepsilon_1 \\ a_2 & \varepsilon_1 \\ a_3 & \varepsilon_1 \\ \vdots & \\ a_k & \varepsilon_1 \\ a_1 & \varepsilon_2 \\ \vdots & \end{bmatrix}$$

$$\begin{bmatrix} \vdots \\ a_k \ \varepsilon_s \end{bmatrix}$$

$$\vec{a}_{lb} = \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} \quad (N_k \times N_s, 1)$$

$$\vec{a}_{us} = \begin{bmatrix} (1+r)\, S(:,1) + w\, S(:,2) \\ \end{bmatrix}$$

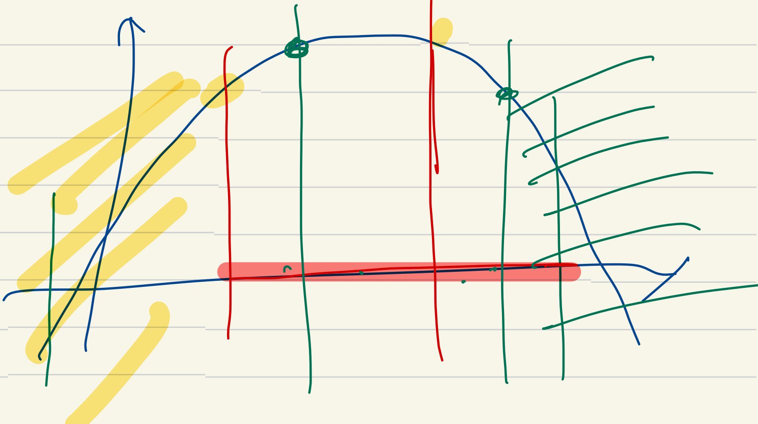
$$\bar{N} = N_s \times N_k$$

$$(N_k \times N_s, 1)$$

Rewrite golden vectors

golden

d'



$\max_{\{a'(a,\varepsilon)\}}$ over obj ...

(a, ε) $\max_{a'} u\left(C(1+r)a - \varepsilon - a'\right) + W(a', \varepsilon)$

$\boxed{\text{EGM}}$

ch II B ...

guess? $c = ra + w\varepsilon$

@] Nk ... N_s ...

1] guess c_pol (a, ε) ... c_0

2] Compute ... using FOC, CRRA

$u_c = c^{-\gamma}$

$$\hat{c}^{-\gamma} = \beta(1+r) \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} [c_0(a', \varepsilon')]^{-\gamma}$$

$\boxed{\forall (a', \varepsilon)}$

$$\hookrightarrow \hat{c}(a', \varepsilon) = \left[\beta(1+r) \sum_{\varepsilon'} \pi_{\varepsilon,\varepsilon'} c_0(a', \varepsilon')^{-\gamma}\right]^{-\frac{1}{\gamma}}$$

$\hat{c}, a', \varepsilon \longrightarrow \dots a$ BC

$(1+r)a + w\varepsilon = c + a'$

$\Rightarrow a = \dfrac{c + a' - w\varepsilon}{1+r}$ — $a(a', \varepsilon)$ interpolat

$(a, \varepsilon) \rightarrow \boxed{\text{new } c(a, \varepsilon)}$

Vectorized + EGM

II C] Fiscal Policy

+ Positive Question..
+ NORMATIVE Question

taxes / transfers... trade-off

efficiency / redistribution

labor supply, dispersion in income

Consumption

ΔU_c

SAVINGS ... dispersion in income / consumption

endogenous

Fiscal instruments

flat tax τ on total income

govt ... G

II C 1. A government ...

hh $\quad v(a,\varepsilon) = \max_{c,a'} u(c) + \beta \sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} v(a',\varepsilon')$

s.t. $a' \geq \underline{a}$, $\quad c + a' = \underline{(1-\tau)}[ra + w\varepsilon] + a$

distortionary taxes: $\quad u_c = \beta(1+(1-\tau)r)\sum_{\varepsilon'} \pi_{\varepsilon\varepsilon'} u_c'$

GOV: BC

$$G = \tau \int (ra + w\varepsilon)\, d\lambda(a,\varepsilon)$$

G fixed exogenous

Algorithm: $\{r^*, \tau^*\}$

Two steps

(1) Outer loop … r_0 … w_0

(2) Inner loop … τ_0

(3) given $\{r_0, w_0, \tau_0\}$
solve HH $\rightarrow$ vf, policy, λ_0

(4) Check Gov BC

$$G = \tau_0 \int (r_0 a + \varepsilon w_0)\, d\lambda_0(a, \varepsilon)\ ?$$

(5) $K = \int a'(a,\varepsilon)\, d\lambda(a,\varepsilon) \rightarrow$ implied r_1
$= \int a\, d\lambda$

Start $r_0 \rightarrow w_0$

$\bar{N} \rightarrow K_0$

$$r_0 + \delta = (1-\alpha)\left(\frac{K_0}{N}\right)^{-\alpha}$$

To st

$$G = T_0 \int (n_0 a + w_0 \varepsilon)\, dA$$

$$= T_0 \left[n_0 \int a + w_0 \int \varepsilon \right]$$

$$= T_0 \left[n_0 K_0 + w_0 \bar{N} \right]$$

$n_t \approx n_0$

labor relative (1..)

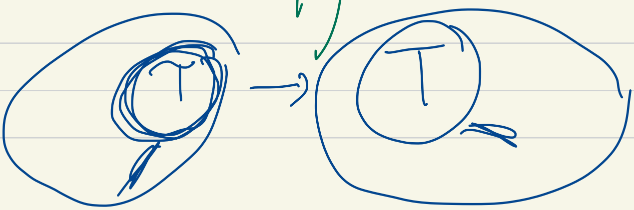
$$N = \int \varepsilon\, n\, dA$$

$$= \int \varepsilon\, \underbrace{n(a,\varepsilon)}_{=1}\, dA(a,\varepsilon)$$

$$\leftarrow \int \varepsilon\, dA$$

Elastic labor supply

Floden Linde (2002)



$$v(a,\varepsilon) = \max_{c,n,a'} u(c,n) + \beta \sum \pi_{\varepsilon\varepsilon'} v(a', \varepsilon')$$

s.t. $c + a' = (1+r)a + (1-\tau) w \varepsilon n + T$

$a' \geq a$

$$\boxed{u_c w \varepsilon = -u_n}$$

$$u(c,n) = \frac{c^{1-\gamma}}{1-\gamma} - B \frac{n^{1+\varphi}}{1+\varphi}$$

$\varphi = 1/\text{frisch}$

B labor disutility

CRRA, separable, isoelastic preferences

FOC

$$B n^{\varphi} - c^{-\gamma} \cdot w\varepsilon = 0$$

$$B n^{\varphi} - w\varepsilon \left[(1+r) a + w \varepsilon n - a' \right]^{-\gamma} = 0 \quad (*)$$

$\hookrightarrow \boxed{n(a, \varepsilon, a')}$ one equation, one unknown

Algorithm: start ...

given $r_0 \to w_0$

given $V \to W$

↳ for each a'

$$m(a,\varepsilon,a')\quad(*)$$

$$u(c,n)+W(a',\varepsilon)$$

$\forall \varepsilon$

$$=(1+r)a+w\varepsilon n-a'$$

$$N=\int \varepsilon\, n(a,\varepsilon)\,d\lambda$$

$$K=\int a'(a,\varepsilon)\,d\lambda$$

$\longrightarrow$ New r ... update

How to compute $n(a,\varepsilon,a')$

$[r,w]$ for each point on

Asset grid $\times$ productivity grid $\times$ Asset grid

(N_k) (N_s) (N_k)

→ bisection

$$Bn^{\varphi}-w\varepsilon\left[(1+r)a+w\varepsilon n-a'\right]^{-\gamma}=0$$

$0 \leq n \leq 1$ $\quad$ nlb s.t. $c > 0$

given $(a, \varepsilon, a') \to$ nlb

$(1+r)a + w\varepsilon n - a' > 0$

$$n > \frac{a' - (1+r)a}{w\varepsilon}$$

$nlb \geq 1$

... your a' too big

if $nlb < 1$ ...

$$nlb = \max\left(0, \frac{a' - (1+r)a}{w\varepsilon}\right)$$

$\boxed{nub = 1}$

$$f(n) = Bn^{\varphi} - w\varepsilon\,[\underbrace{(1+r)a + w\varepsilon n - a'}_{c\ \text{low} \to U_c\ \text{large}}]^{-\gamma} = 0$$

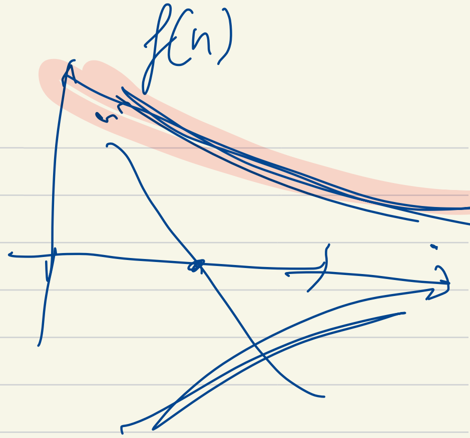
small

(*)

$-nlb \cdot (*) < 0$ ... > 0

$$\boxed{nub\ (*) > 0}$$

nlb, nub



$$n^* = \frac{1}{2}(nlb + nub)$$

(*) ? $> 0 \to$ bound . . .

FOC ~ 0

$$\boxed{m_0(a, \varepsilon, a')} \qquad [\pi_0, w_0)$$

grid Code PSI (1..4)

VF

(a, ε)

a'...

$\hookrightarrow$ CV

C

m

a' . . . .

(a, ε) . . . all a'

$m_0(a, \varepsilon, a')$

① $g_w(r_0, w_0)$

② $n_0(a, \varepsilon, a')$

③ VFI

④ d . . .

---

golden $n(a, \varepsilon, a')$

golden (a, ε)

. . . $a' \to c, n, W$

interpolate W on a'

n on a'

$\text{max}x = n(a, \varepsilon, :) \cdot W(a', \varepsilon)$

Pick a'

$\{a_i \leq a' < a_{i+1}\}$

CV W

$n(q, \varepsilon, s')$

interpolator

golden

~~interp 2 VF~~

~~tinker 3 n~~

I] A] History of thought

B] Applications

heterogeneity + dynamic

recursively . . .

* income ineq { exogenous , endogenous }

→ wealth inequality [endogenous]

* understand wealth inequality

- fiscal policy ...
- income inequality ... + education

wealth ineq

- age dimension (OLG)
  - dynamics ... bequests inheritance tax
- health ... (a, ε, θ)
- family structure / fertility
  - Michele Tertilt
  - Mathias Doepke
- entrepreneurs / workers
- labor market friction + wealth

ϕ spatial macro ... housing

(migration, trade, ...

$$V^h(a, \varepsilon^1, \varepsilon^2, \theta) = \max_{c, a', n^1, n^2}$$

Ch I C] BASIC FACTS on inequality ... US

- survey data (not admin data)

mis-reporting / ..

Accessible

CPS . Current Pop Survey

+ 60k hh

+ 1968 ... → now

+ cross-sectional 72 87

4 months | 8 months ... | 4 months

x x x x | | x x x x

labor market

March supplement: ASEC supplement

annual cross-section

68 → ...

Taxes TAXSIM

~~TRANS~~fers

PSID – Panel Study of Inc Dynamics – –

5k hh

annual 68 – 97 ... biannual ...

68 – 92

PANEL... PSID

income risk ... quivers...

CEX: Consumer Expenditure Survey...

80s... (quarterly)

with hh ... cons-reha...

(FOOD)

(bank data... C° data Spain

CENSUS 10 y 1950, 60 70 ... 10 m°

1800 s... .

IPUMS

SCF Survey of Consumer Finance

bi-annual . –   mid 80s

wealth → { net / gross , liquid , illiquid , housing ... }

Right tail ..

$\underbrace{\text{Aggregate SCF + Forbes 400}}$

~ NIPA Tables

INC

SIPP   TRANSFERS ...

FACTS ...

individual ...

hours
hourly wage

earnings ... individual

hh

... earnings (Σ labor income) CPS

\+ private transfers (pensions, private transfers)

\+ asset income

PRIVATE INCOME CPS

− taxes

\+ transfers

DISPOSABLE income CPS

± SAVINGS ..

Consumption at hh CEX

"Unequal we stand: ....." 2010

Wealth SCF

top 20%
~50-60%

~ now

10% ~ 65%

top 1% ~ 33%

20% ~ 80%

d II B

labor supply

≠ Newton

$x(a, \varepsilon, a')$ ... bisection ...

$$f(x) = 0$$

$$f(x) = Bx^{\varphi} - w\varepsilon\left(w\varepsilon x + (1+r)a - a'\right)^{-\gamma}$$

x^* st $f(x^*) = 0$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$-\ 0 \quad = 0$$

x_1 st $f(x_1) = 0$

given x_0

$f(x_0)$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

* given $x_0 \to f(x_0) = 0$? YES ✓

$x_0 \longrightarrow x_1$

$\curvearrowleft x$

∴ …almost done $\{r, w\}$

$x^*(r_0, w_0) \ldots < x^*(r_1, w_1)$

x_0: labor supply policy function of the previous iteration in ~~path~~ prices.

$$\boxed{Bx^d = w \mathcal{E} C^T}$$

$$Bn^{\varphi} = w\varepsilon[w\varepsilon m + (1+r)a - a']^{-1}$$

$c, \ldots m \ldots$

$w\varepsilon n + (1+r)a - c \ldots$

$\longrightarrow a'$

golden in $c \longrightarrow m$

$\longrightarrow a'$

never interpolate outside of bounds

$V(a')$ if $a' > a_{max}$.

Proble :

G → m

$n = 3.2$

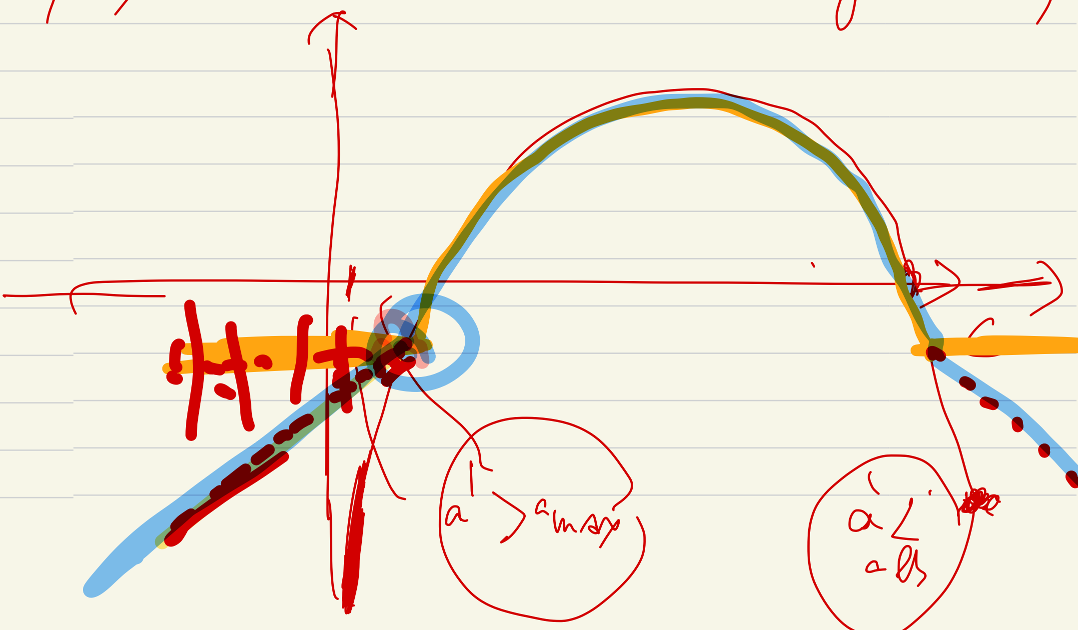
$n = 7$

$Bn^{\varphi} = w \varepsilon c^{-T}$

↳ BC → a'

$V(\hat{a}, \varepsilon; \zeta)$

given (a, ε)



1

$a'(c) < a_{LB}$, $VF = -100\,000 - c$

$a'(c) > a_{max}$, $VF = \cancel{...}\ -100\,000$

$+ c$

2 impose bounds on c or

c_{min} / c_{max} ... bounds for a'

---

a [C] taxes / redistribution

Floden Linde 2001

(τ, T) → to redistribute lump sum to all

distortionary flat labor inc tax

$$v(a,\varepsilon) = \max_{c,n,a'} \frac{c^{1-\gamma}}{1-\gamma} - \frac{B n^{1+\varphi}}{1+\varphi} + \mathbb{E}\left[v(a',\varepsilon') \mid \varepsilon\right]$$

s.t.

$$c + a' = w\varepsilon \boxed{(1-\tau)} n + (1+r)a + T$$

$$c^{-\gamma} w \varepsilon \boxed{(1-\tau)} = B n^{\varphi}$$

$\oplus$ $\uparrow \tau \rightarrow \downarrow n$ ... efficiency cost of taxes

$\therefore \varphi = 1/\text{Frisch}$

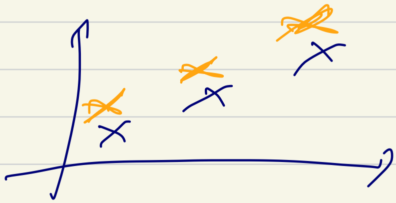
$$\oplus \quad \boxed{T = \tau \int w \varepsilon n(a,\varepsilon)\, d\lambda(a,\varepsilon)}$$

$G = 0$

T good → reduce dispersion in

$\mathbb{V}(\log c)$ ... $\mathbb{V}(c^{-\gamma})$

Welfare losses with dispersion in marginal utilities



$\Rightarrow$ Redistribution GAIN

large if $\{\varepsilon\}$ is unequal

... large if ρ is big

Case 1, $n = 1$ exogenous + inequality of $\{\varepsilon\}$

$(\tau, T) \to \tau = 1$

$\to$ no inequality

$\to$ SAME income after-tax-and-transfer

no more ineq + no more risk

$$\to \quad r^* = \frac{1}{\beta} - 1$$

Case 2: n elastic, $\varepsilon = 1$ $\forall$ hh

↳ no gains from redistribution

$\boxed{T = 0}$ → $\cancel{T} = 0$

Real World... some neg... elastic...

Frisch $\{0.2, 0.4, 0.5\}$

Ineq $\{\varepsilon$ ... Swede ... US $\}$

$\{\tau, T\}$ ... US $\tau = 27\%$

higher 2000s...

$\tau = 45\%$

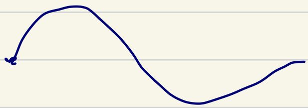
(1) model better fails

(μ, φ)

Feb 7, 2024

Ch3 _ AGG DYNAMICS

- Aggregate Risk / business cycles



(TFP, demand shocks)

RBC/NK

λ

$(K_t, A_t ; \lambda_t) \to$

$N_k \times N_j$

- deterministic transitions

SS ... suddenly / unexpectedly one shock

$t=0$ ... $\to$ no more agg uncertainty

MIT shock

* TAX REFORM

SS ... realworld $\{\tau^n, \tau^k, G\}$ ... λ

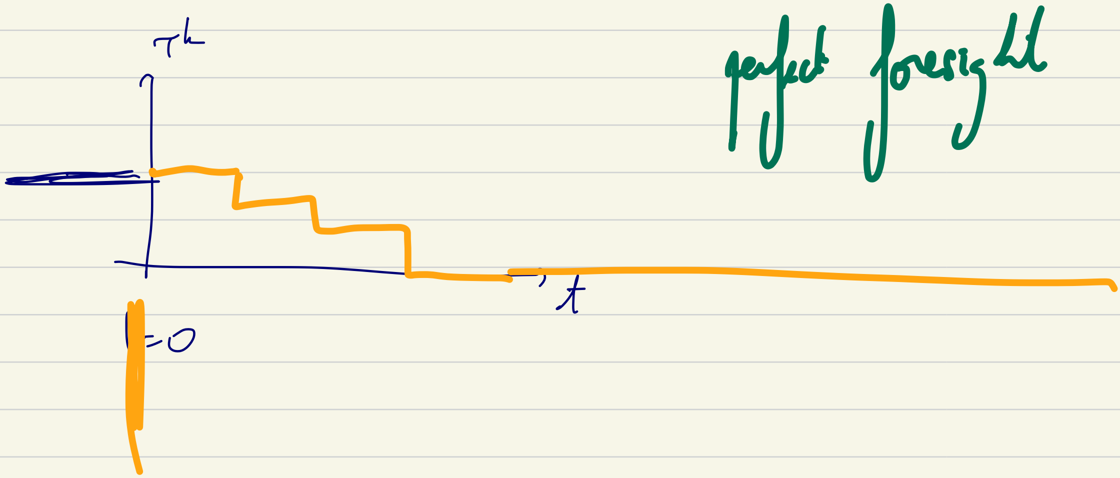
t=0 onwards

new $\hat{\tau}^k$ ... τ_t^n will adjust ...

new SS + transition

welfare

▷ DYNAMIC TAX REFORM



perfect foresight

▷ temporary shock

SS → same SS

* $\uparrow T^n$ by 1% for 1 quarter
  Well calibrated?

* give a transfer T "Grid check"
  2008 ..

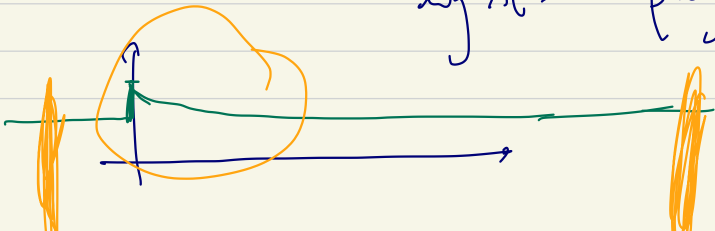
* Contrived
  t=0 Chint → new SS / sustainable
  [implement paths for policies

* "approximate" a business cycle shock

SS, TFP $A = 1$

suddenly... $\log A_0 = \log A + \sigma_A$

$\log A_t = \rho \log A_{t-1} \quad \forall t \geq 0$



$\{A_t\} \to \{Y_t, C_t, N_t\}$

programming of tradition.

$$\log A_t = (1-\rho)\log A + \rho \log A_{t-1}$$

$\rho \in (0,1)$

Domeij Heathcote (2004 IER)

On the distributional effects of reducing capital taxes

Envt

+ Continuum of HH, firm, govt

+ Gov: τ^k, τ^n, G (fixed exogenous wasted)

HH:

$$v_t(a,\varepsilon) = \max_{c_t, a_{t+1}, n_t} u[c_t(a,\varepsilon)] + \beta \sum_{\varepsilon' \in E} v_{t+1}(a_{t+1}(a,\varepsilon), \varepsilon') \, \pi(\varepsilon'/\varepsilon)$$

st $c_t(a,\varepsilon) + a_{t+1}(a,\varepsilon)$

$$= \left[1 + (1-\tau_t^k) r_t\right] a + w_t \cdot \varepsilon \cdot n_t^{(a,c)} (1-\tau_t^n)$$

$$a_{t+1}(a,\varepsilon) \geq \underline{a}$$

SS US 2002 $\tau^k = 30\%$

TAX Reform $\tau^k = 0$, τ_t^n adjusts

$$G = \tau_t^k \int r_t K_t + w_t \tau_t^n \bar{N}$$

$\tau_t^k \to 0$

$$\tau_t^n = \frac{G}{\bar{N} w_t}$$

RCE: Given an initial distribution λ^0, K^0, and sequence of taxes $\{\tau_t^k, \tau_t^n\}_{t=0}^{\infty}$, a RCE is a sequence of VF $\{v_t\}_{t=0}^{\infty}$ and decision rules for the $\{a_{t+1}, c_t\}_{t=0}^{\infty}$, prices

$\{r_t, w_t\}_{t=0}^{\infty}$ and sequence of distribution $\{\lambda_t\}_{t=0}^{\infty}$ s.t.

1. given $\{r_t, w_t\}$ and $\{\tau_t^k, \tau_t^n\}$, VF + DR solve HH problem

2. firm behave optimally

$$r_t + \delta = F_k(K_t, \bar{N}), \quad w_t = F_N(K_t, N)$$

3. labor markets clear

4. asset markets clear

$$K_{t+1} = \int_{A \times E} a_{t+1}(a, \varepsilon) \, d\lambda_t(a, \varepsilon)$$

5. $G = \tau_t^k r_t K_t + \tau_t^n w_t \bar{N}$

6. Measure

$$\lambda_{t+1}(\mathcal{A}, \mathcal{E}) = \int_{A \times E} Q_t((a, \varepsilon), \mathcal{A} \times \mathcal{E}) \, d\lambda_t(a, \varepsilon)$$

$$Q_t((a, \varepsilon), \mathcal{A} \times \mathcal{E}) = \mathbb{1}_{\{a_{t+1}(a, \varepsilon) \in \mathcal{A}\}} \sum_{\varepsilon' \in \mathcal{E}} \Pi(\varepsilon' | \varepsilon)$$

Exn: US, $G \sim 22\% Y$, $\tau^n \sim 27\%$, $\tau^k \sim 40\%$ (Mendoza)

Saddle $\tau^n = 0$

$\tau^m_t \ldots$ Gov BC holds

SS_1 (*) $\qquad\qquad$ SS_2 (**)

Heterogeneity + Transition

ALGORITHM

① SS $\{r^*, \lambda^*, n^*, w^*, K^*, a'^*\}$

② New SS $\{r^{**}, \lambda^{**}, n^{**}, w^{**}, K^{**}, a^{**}, \tau^{n**}\}$

③ given T large enough so the economy has converged to **

$$\lambda_{T-1} \approx \lambda^{**}$$

$T = 100$ $\quad$ $T = 5000$

(1) given $\{\hat{K}_t\}_{t=0}^{T}$

$$\boxed{\begin{aligned}\hat{K}_0 &= K^* \\ \hat{K}_T &= K^{**}\end{aligned}}$$

$(\bar{N}) \to \{\hat{r}_t, \hat{w}_t\}_{t=0}^{T}$

$\Rightarrow$ get $\{\hat{\tau}_t^n\}$ using $\dfrac{G}{\hat{w}_t \bar{N}} = \hat{\tau}_t^n$

(2) HH: $\boxed{\text{backwards}}$

V^{**}

$$v_T(a,\varepsilon) = \max_{c,a'} u(c) + \beta \sum_{\varepsilon'} \pi(\varepsilon'|\varepsilon)\, v^{**}(a',\varepsilon')$$

$$c + a' = [1 + \hat{r}_T]\, a + \hat{w}_T (1 - \hat{\tau}^n_T)\, \varepsilon \bar{n}$$

$$a' \geq \underline{a}$$

$$v_T,\ c_T(a,\varepsilon),\ a_{T+1}(a,\varepsilon)$$

$$\ldots\ v_{T-1}(a,\varepsilon) = \max_{c,a'} u(c) + \beta \sum_{\varepsilon'} \pi(\varepsilon'|\varepsilon)\, v_T(a,\varepsilon)$$

$$c + a' = [1 + \hat{r}_{T-1}]\, a + \hat{w}_{T-1} (1 - \hat{\tau}^n_{T-1})\, \varepsilon \bar{n}$$

$$a' \geq \underline{a}$$

$$\vdots$$

$$\{\hat{v}_t\}_{t=0}^{T} \quad \ldots \quad \{\hat{c}_t, \hat{a}_{t+1}\}_{t=0}^{T}$$

(3) Measure: $\boxed{\text{FRONTWARDS}}$

$$\lambda_0 = \lambda^*$$

$$\hat{\lambda}_1(A\times\mathcal{E}) = \int Q_0\big((a,c), A\times\mathcal{E}\big)\, d\hat{\lambda}_0(a,c)$$

$$Q_0(A, \epsilon) = \mathbb{1}_{\{\hat{a}_1(a,\varepsilon)\in A\}} \sum_{\varepsilon'\in\mathcal{E}} \pi(\varepsilon'|\varepsilon)$$

$\vdots$

$\hat{\lambda}_2 \quad \dots \qquad \hat{a}_2$

$\vdots$

$\hat{\lambda}_T$

---

(4) Check guess for K

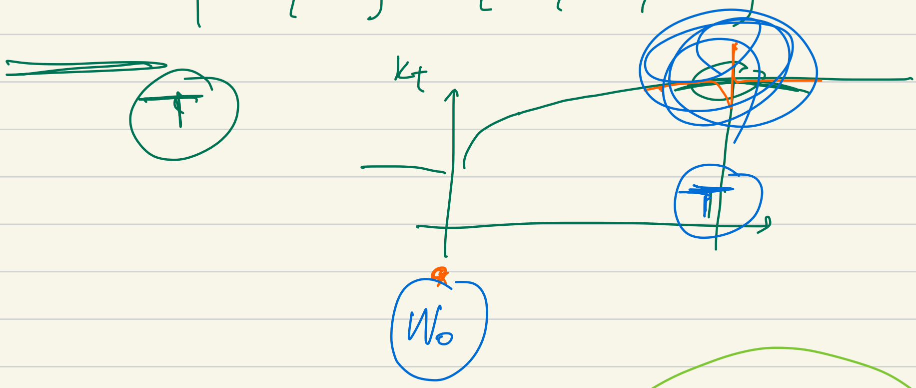
$$\hat{K}_{t+1} = \int_{A\times\mathcal{E}} \hat{a}_{t+1}(a,\varepsilon)\, d\hat{\lambda}_t(a,c)$$

$$\hat{K}_t = \int_{A\times\mathcal{E}} a\, d\hat{\lambda}(a,c)$$

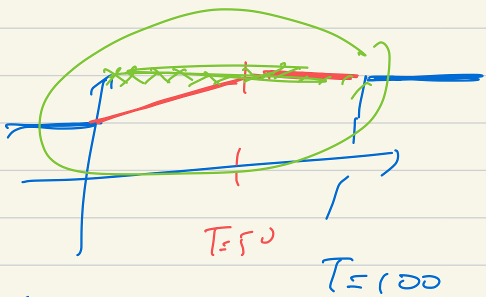
$$\{\hat{K}_{t+1}\}_{t=0}^{T}$$

$\{\hat{\hat{K}}_t\} \tilde{\approx} \{\hat{K}_t\}$?

Eq: $\{K_t\} \to \{r_t, w_t, T_t^h\}$



$K_t = K^{**}$

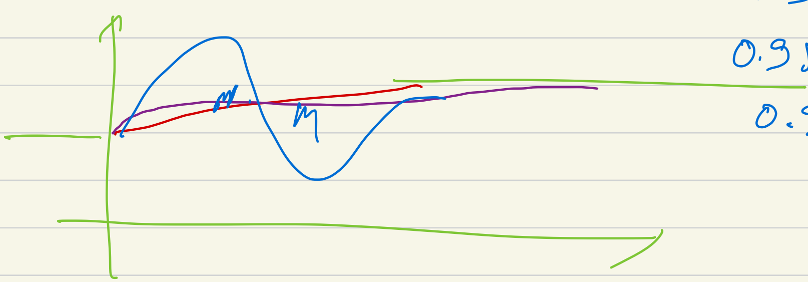


slow weight

0.9995

0.95

0.99



guess Two PATHS

$$\{ r_t , \tau_t^n \}$$

$$\boxed{\{ K_t , N_t \}}$$ (SSJ)

Ch II.C] Fiscal Policy

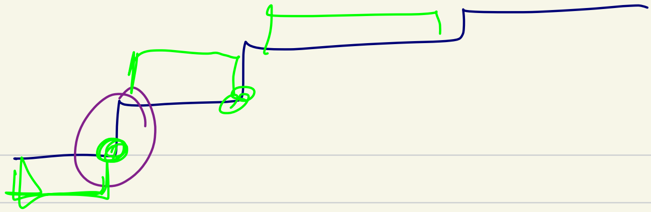
T, T

Real world / fiscal roots

[TAXES] (Fed, Stat, local)

deductible

PROGRESSIVE _ BRACKETS



TAx credit

(EITC, CTC → 0

0 ... −2

ACTC → ny

+ Welfare Program

state level / fed...

UI

Food Stamps

TANF

⋮

Medicaid

Benabou (2002)

Heathcote Storesletten Violante (2017)

(HSV)

log linear tax

inc y

$$y^{AT} = \lambda y^{1-\tau}$$

$$\log y^{AT} = \log \lambda + (1-\tau) \log y$$

λ $(1-\lambda)$ capture level

τ progressivity

$$y^{AT} = \lambda y^{1-\tau}$$

$\tau = 0$ flat $\quad y^{AT} = \lambda y$

flat tax $(1-\lambda)$

$$y^{AT} = y - (1-\lambda)\, y$$

$T = 1$ ... $y^{AT} = \lambda$ Full redistribution

y

$\tau > 0$ may not are increasing in inc (PROGRESSIVE)

$\tau < 0$ ... regressive system

Nice: 2 param (λ, τ)

$$v(a, \varepsilon) = \max \frac{c^{1-\gamma}}{1-\gamma} - \frac{B n^{1+\varphi}}{1+\varphi} + \beta \cdots$$

$$c + a' = \lambda [w \varepsilon n]^{1-\tau} + (1+r)\, a$$

$$a' \geq \underline{a}$$

FOC: $c^{-\rho} \cdot \lambda(1-\tau) n^{-\tau} (w\varepsilon)^{1-\tau} = B n^{\varphi}$

$(a, \varepsilon, e') \dots \; n$

GOV: $G = \int (w\varepsilon n)\, d\hat{\lambda}(a, c)$

$\boxed{y^{AT} = \lambda y^{1-\tau}}$ $\quad - \lambda \int (w\varepsilon n)^{1-\tau}\, d\hat{\lambda}(a, c)$

$y - \lambda y^{1-\tau} = \text{taxes}$

τ DATA $\tau = 0.18$

give τ, (prices, wh n …)

… FIND λ st gov BC holds

---

r, λ

$\hat{r}$. update λ?

$$\underline{\lambda}^{implied} = \frac{\int w \varepsilon n \, d\lambda(a,c) - G}{\int [w \varepsilon n]^{1-\tau} \, d\lambda(a,c)}$$

Is $\lambda^{implied} \approx \lambda^{0}$?

outer loop π

inner loop on λ

(π, λ) .. consistent

FIT DATA ?

$$\log y^{AT} = \log \lambda + (1-\tau) \log y$$

2012 CPS ... y by hh

(TAXSIM + CPS) ~ taxes

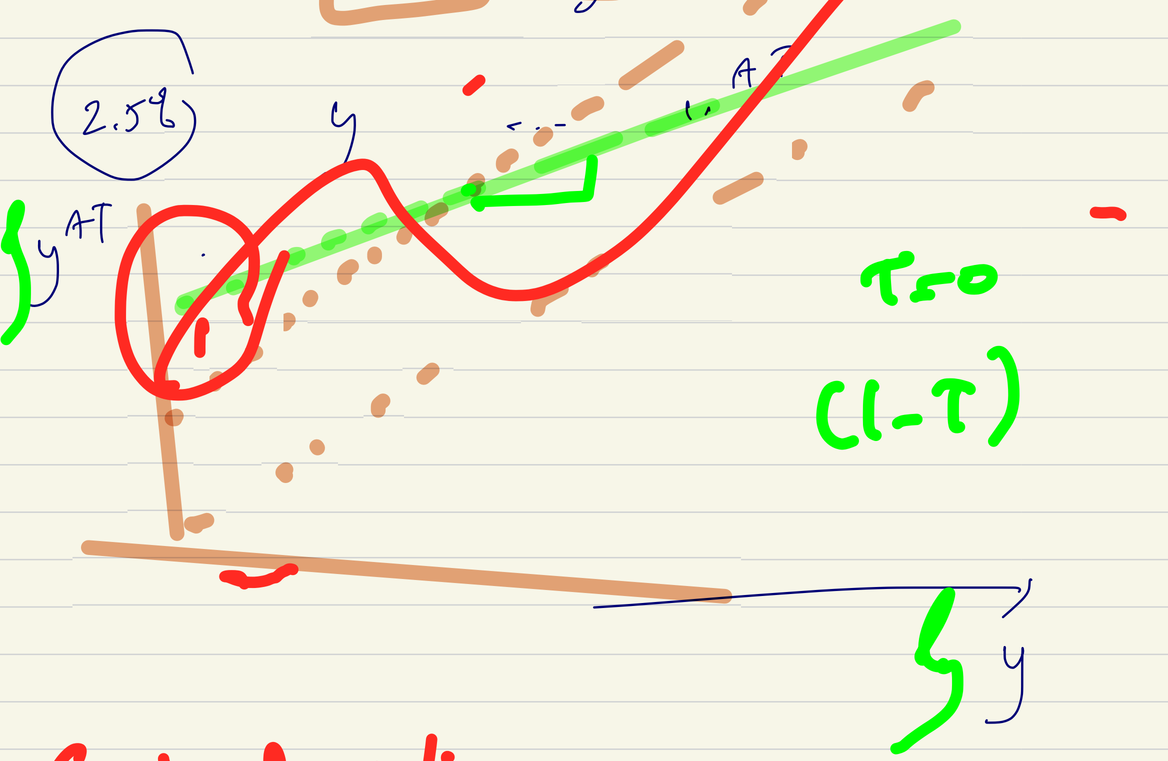
CPS .. transfers

(CFGNV)

LOGS

$$\boxed{y^{AT}} = \boxed{y} - \text{taxes} + \text{transfers}$$

2.5%



Good description ...

T data ...

λ

$G = \int \ldots - \lambda \int \ldots$ T

G/X

ANALYTICAL results

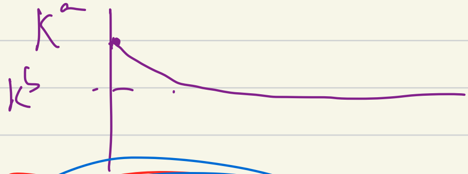
9/PS3

$SS_A \quad \{T_a, \lambda_a\} \quad r_a$

$SS_B \quad \{T_b, \lambda_b\} \quad r_b$

~~Ok~~ permanent / sudden change in τ

SS_A . . . $t=0$ $\boxed{\tau_0 = \tau^b}$



$\{\pi_t, \lambda_t\}_{t=0}^{T}$

G fixed . . . G^A . . .

$\{\hat{\lambda}\}_t$ . . . . N_t . . . t by t

$\{\hat{\pi}\}$ = . . . A_t . . . . . .

$\{\pi_t\}$

. . . more loop $\{\lambda_t\}$

$\{ r_t, \lambda_t \}$ → HH policies

$a_{t+1}(a, \varepsilon)$, $n_t(a, \varepsilon)$

$\{ \hat{r}_t, \hat{\lambda}_t \}$ Gov BC holds

Ch II C] Fiscal Policy

HSV tax function (λ, τ)

+ fit of the US data
+ Qual work
+ tractable / theoretical way

HSV (2017, AER)

. hh hand-to-mouth

$$u: \log c - \frac{B n^{1+\varphi}}{1+\varphi}$$

$$c = \underbrace{\lambda (w \varepsilon n)^{1-\tau}}$$

$$\ldots - \max_{n} \log \lambda + (1-\tau) \log(w\varepsilon) + (1-\tau) \log n - \frac{B n^{1+\varphi}}{1+\varphi}$$

$$n^* = \left[\frac{1-\tau}{B}\right]^{\frac{1}{1+\varphi}}$$

$Y = L$, $w = 1$

ε, λ

$\log$

Gov:

$$G = \int \varepsilon n - \lambda \int (\varepsilon n)^{1-\tau}$$

$$G = n^* \, \mathbb{E}\varepsilon - \lambda \, n^{*\,1-\tau} \cdot \mathbb{E}\varepsilon^{1-\tau}$$

$W = \ldots \ldots (\tau)$

Recent QUANT MACRO

$\varphi^{o} \ldots$

- single framework ... formula
  {mpc} {elasticities}
- big quant

T micro data ... , since 80s ...
2010 ...

$$1 - \tau = \frac{\text{marg} - \text{average tax rate}}{1 - \text{average tax rate}} =$$

AGGREGATE: Average tax rate $\sim$

$$\frac{\text{total fiscal revenues collected by gov}}{\text{TOTAL INC}}$$

$$\text{marg} = \frac{\partial \frac{T(y)}{y}}{\partial y}$$

∂, τ ...

$$av = \frac{y - \lambda y^{1-\tau}}{y} \sim$$

$1 - \tau = \ldots$

Mertens

Romer +

$\{ \tau \} \ldots$ 1913

Ch III — Domeij Heathcote

$\tau^{*k} = 30\%$

$\hookrightarrow \quad \tau^{**k} = 0\% \qquad \tau^n_t \ldots$

Welfare

$$W^* = \int v^*(a, \varepsilon) \, d\lambda^*(a, \varepsilon)$$

$$W^{**} = \int v^{**}(a,\varepsilon)\, d\lambda^{**}(a,\varepsilon)$$

$W^{**} >> W^{*}$ $C^{**} >> C^{*}$

$K^{**} >> K^{*}$

$$W_0 = \int v_0(a,\varepsilon)\, d\boxed{\lambda^{*}(a,\varepsilon)}$$

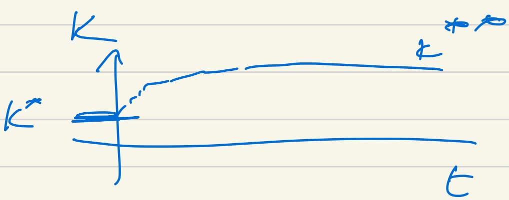
$W_0 < W^{*}$

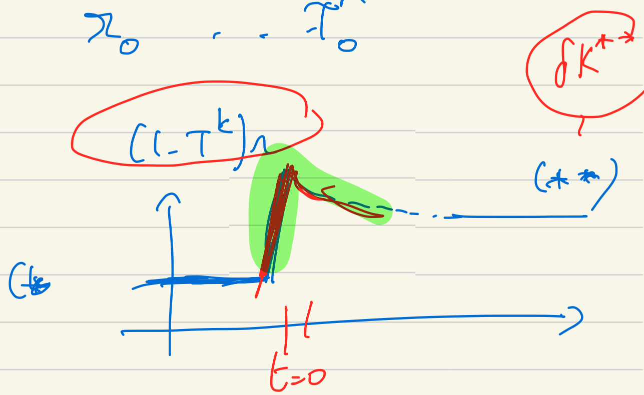
TRANSITION COSTS outweigh LR welfare GAINS.

$V_0(a,\varepsilon)$

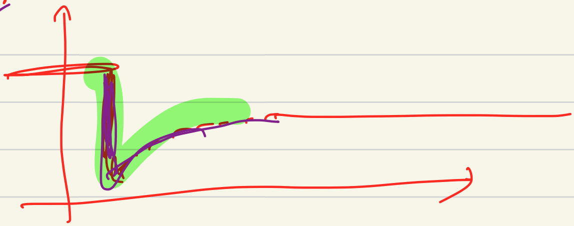
$\tau^k_0 = 0\%$ $\tau_0 \ldots \tau^n_0$

$K^{**} > K^{*}$





$(1-\tau)w$



r overshoot

wage undershoot

RA ... overshooting

$W_0^{RA} > W_*^{RA}$

transitions + heterogeneity

$W_0^{HA} < W^{*HA}$

low a, low c, M_c

labor income shocks

r, Ω ... w

$W_0 < W^*$, $W^{**} > W^*$

Consumption equivalents (in the distribution)

$\Delta(a, \varepsilon)$ .. labor inelastic

$v_0(a, \varepsilon)$ $\qquad v^*(a, \varepsilon)$

$$v^*(a, \varepsilon) = E_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c^*(a_t, \varepsilon_t)\right) \middle| \boxed{a_0 = a, \varepsilon_0 = \varepsilon}\right]$$

$$\hat{v}(a, \varepsilon, \Delta) = E_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+\Delta) c^*(a_t, \varepsilon_t)\right) \middle| a_0 = a, \varepsilon_0 = \varepsilon\right]$$

Find Δ:

$$\boxed{\hat{v}(a, \varepsilon; \Delta) = v_0(a, \varepsilon)}$$

$\Delta(a, \varepsilon)$ $\qquad$ 0.5%

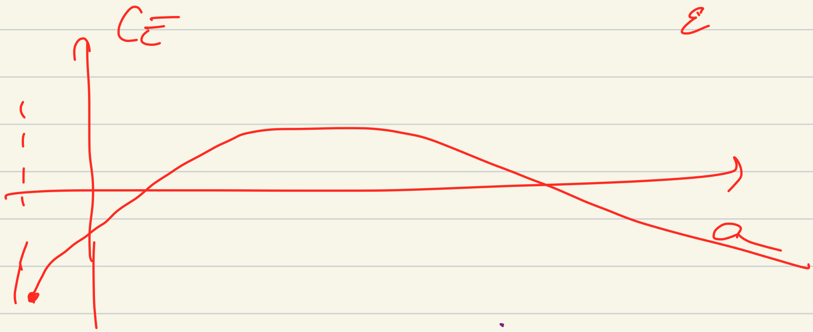
0.1%

$\boxed{v^* < v_0}$ ... $\boxed{\Delta > 0}$ $\qquad \Delta(a, \varepsilon)$

$v_0 < v^*$ $\qquad \boxed{\Delta < 0}$

Δ .. CE .. $\not\sim$ complic

$-128.27 < -120.9$



CRRA .. $u = \dfrac{c^{1-\rho}}{1-\rho}$

$$\hat{v}(a,\varepsilon,\Delta) = E_0\left[\sum_{t=0}^{\infty}\beta^t \left[(1+\Delta)^{1-\rho}\frac{c^*(a_t,\varepsilon_t)^{1-\rho}}{1-\rho}\,\middle|\, a_0 \ldots\right]\right]$$

$$= (1+\Delta)^{1-\rho}\cdot E_0\left[\sum_{t=0}^{\infty}\beta^t \frac{c^{*\,1-\rho}}{1-\rho}\,\middle|\,\ldots\right]$$

$$= (1+\Delta)^{1-\rho}\cdot v^*(a,\varepsilon,\cancel{\Delta})$$

Δ: $\quad (1+\Delta(a,\varepsilon))^{1-\rho}\, v^*(a,\varepsilon) = v_0(a,\varepsilon)$

$$\boxed{\Delta(a,\varepsilon) = \left[\frac{v_0(a,\varepsilon)}{v^*(a,\varepsilon)}\right]^{\frac{1}{1-\gamma}} - 1}$$

$$\boxed{\Delta = \int \Delta(a,\varepsilon)\, d\Lambda^*(a,\varepsilon)}$$

$\bar{\Delta} = -1.42\%$ $\Delta^{RA} = +1.5\%$

... lon-inc.: CE -5%

rd -1%

- wealth equivalent

$$v^*(a+\Delta, \varepsilon) = v^0(a,\varepsilon)$$

$a = \underline{a}$ $v^*(\cancel{a+\Delta}$

$$v^*(a,\varepsilon) = v^0(a + \Delta \quad)$$

Δ wealth $\rightarrow$ \$ numbers . .

$$\bar{y} = \int [w \varepsilon n(a, \varepsilon) + r a] \, d\lambda(a, \varepsilon)$$

US ~ 60/90k

Fr ~ 45k ...

.... losing -5% CE ... 2 000 \$

-1% CE ~ 20 000 \$

CE / labor elastic ...

$$v(a, \varepsilon, \Delta) = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\Delta^{1-\rho} \frac{c^*(a_t, \varepsilon_t)^{1-\rho}}{1-\rho} \middle| a_0 \right] \right.$$

$$\left. - E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{n^*(a_t, \varepsilon_t)^{1+\varphi}}{1+\varphi} \right] \right]$$

Simulation $\quad \Omega_c = E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c^*(a_t, \varepsilon_t)^{1-\rho}}{1-\rho} \middle| a_0, \varepsilon_0 \right]$

$$\ell_N = -E_0\left[\sum_{t=0} \beta^t n^t \ldots\right]$$

$\{a_0, \varepsilon_0\} \to t=1$

$t=2 \ldots$

$t=50, \; t=60 \ldots$

$$\boxed{V_0(a,\varepsilon) = (1+\Delta)^{1-\gamma} \cdot \left[\ell_C(a,\varepsilon)\right] + \ell_N(a,\varepsilon)}$$

Deterministic transition

HANK

NK model – Consumer (save real terms)

~~Product~~: FGP

IGP → nominal rigidity (Rotemberg)

PC, MT,

monetary . . .

$\{\pi_t, w_t, d_t\} \ldots \{Y_t, \Pi_t\}$

HANK

AD

expansion

paychecks

fixed

shocks

...

MPC

Marginal Propensity to Consume

$$MPC(a, \varepsilon) = \frac{c(a+T, \varepsilon) - c(a, \varepsilon)}{T}$$

Model : persistence of T

if permanent $\quad MPC = 1$

transitory $\sim \left(\frac{1}{\beta} - 1\right) \sim \boxed{0.02}$

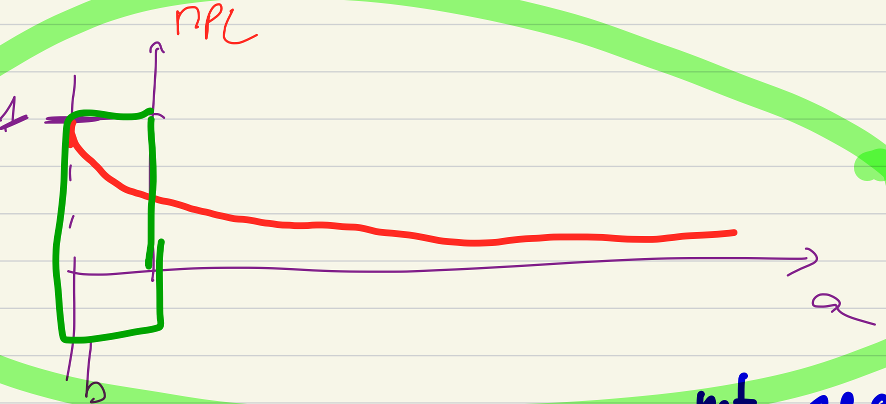
RA

HA ... borrowing constraint

MPC $\downarrow$ a converge ~ 0.02

$\sim \beta\left[\frac{1}{\beta} - 1\right]$

So $a = \bar{a}$ MPC $= 1$



[$\sim 10\%$ HH with net neg wealth $a < 0$

$\overline{MPC} > 0.02 \ldots \quad 0.04$

DATA [+ Blundell Pistaferri Preston (2008)]

+ "QUASI experiments"

Alaska 2001

2008 ...

Card

Kayla Vislatin

Parker

CEX data...

$\boxed{MPC \sim 0.25 - 0.4}$

+ huge mass at 1

• depends on size ~~[illegible]~~ transfer

FAILINGS

+ wealth inequality too small

Top 5% (top 1% wealth

+ MPC

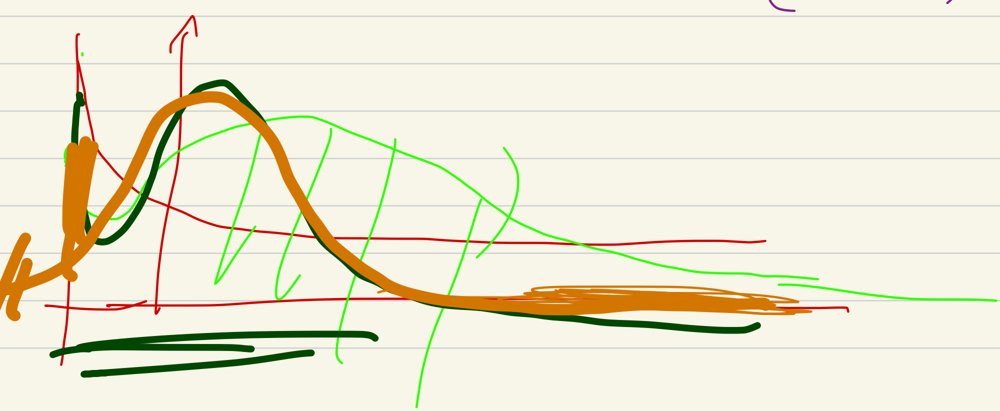
simplest...   OPTION 1

β ... $\frac{\text{Wealth}}{\text{inc}}$ $\frac{K}{\text{inc}}$

$\hat{\beta}$ ... $\int a \, d\,f(a, \varepsilon) =$ $\boxed{\text{TOTAL LIQUID wealth in SCF}}$

TOTAL $\frac{\text{wealth}}{\text{OUTPUT}}$ ~ 3-4 (annual)

MPC ~ 1 (check)



SR effect.. / K adjustments

option 2 : heterogeneous β

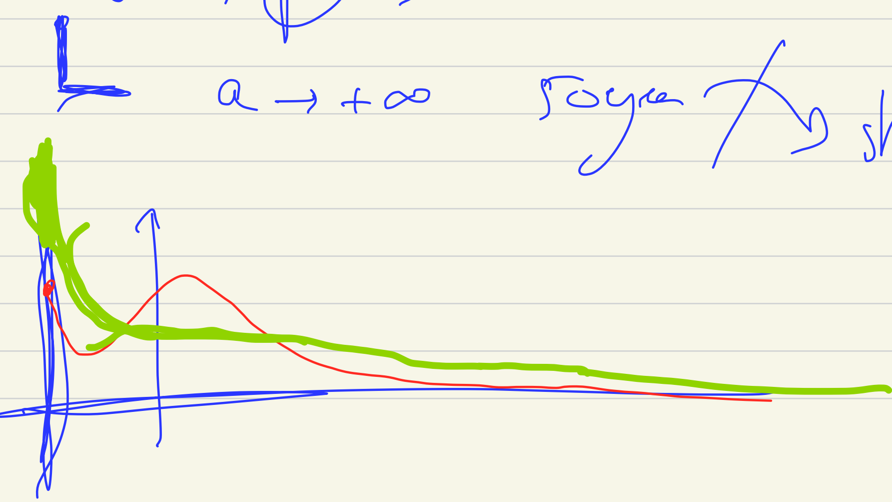
C MPC + wealth inequality

Carroll, ... (JEDC)
Krusell Smith...

β stochastic

$(1+r)\beta^h > 1$

$a \to +\infty$



distribut of MPC is policy invariant

OPTION 3: Two-Asset Model

Kaplan Violat Ecta 2015

- data ... MPC .. 0.25

- $\{m, h, \varepsilon\}$

liquid, R_m ℓ_m, adjust, $m \geq -b$

illiquid: pay K

$R_b > R_m$

$h \geq 0$

$$v(m,h,\varepsilon) = \max\{v^{A}(m,h,\varepsilon), v^{NA}(m,h,\varepsilon)\}$$

$$v^{NA}(m,h,\varepsilon) = \max_{m',c} u(c) + \beta \mathbb{E}_{\varepsilon'} v(m',h',\varepsilon')$$

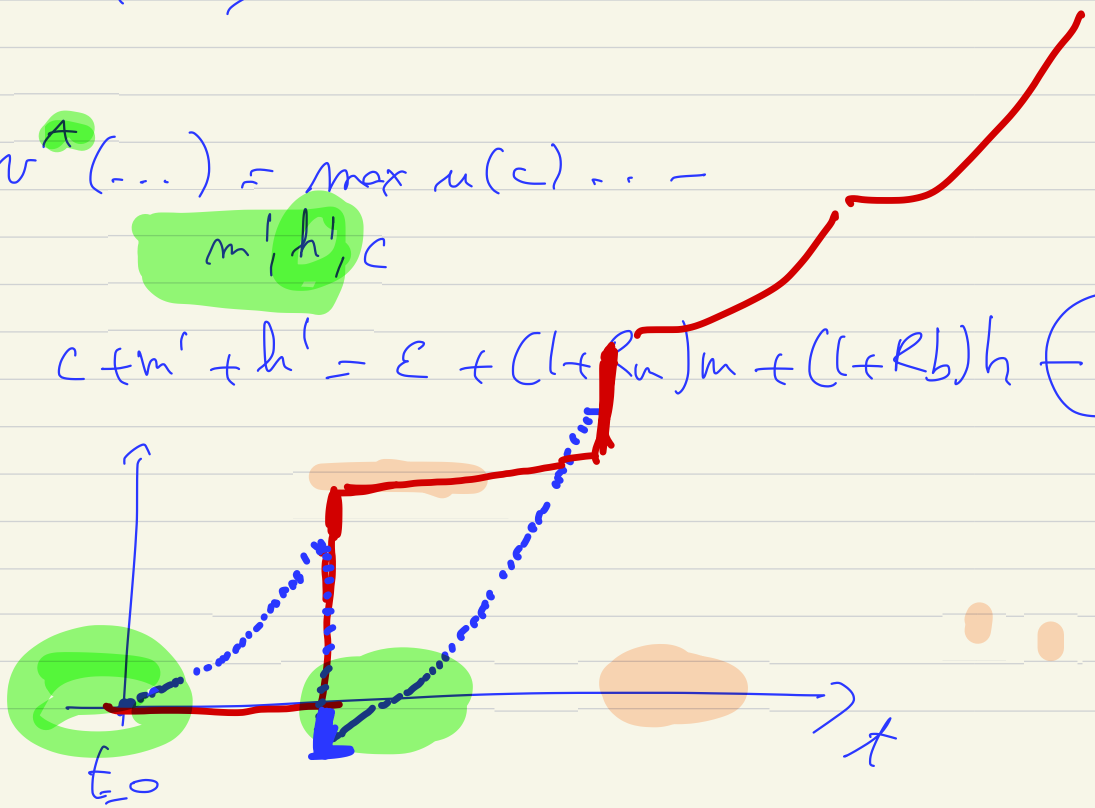
$$c + m' = \varepsilon + (1+R_m)m$$

$$h' = (1+R_b)h$$

$$m' \geq \underline{b}$$

$$v^{A}(\ldots) = \max_{m',h',c} u(c) \ldots -$$

$$c + m' + h' = \varepsilon + (1+R_m)m + (1+R_b)h - K$$



wealthy hand-to-mouth

large fraction of HH high MPC

... consume

DATA... MPC / liquid wealth

MPC / mortgage / house

test

HANK