Quantitative Macroeconomics II, March 2024

Problem Set

Solving a simple HANK economy

Due Date: Friday 29 March

Please hand in your answers and the code solving the model by email using file names that contain all group members' last names (e.g. BROER_ELINA_PS_1.pdf). Hand-written answers are also fine.

Consider a heterogenous-agent New-Keynesian economy, for $t = 0, 1, ..., T$ that features a continuum of households i located on the unit interval ($i \in [0, 1]$) with preferences

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

$$c_t + b_{t+1} \leq (1 - \tau_t) * w_t * l_t * s_t + (1 + r_t) b_t + D_t + T_t \ \forall t$$

$$s_t \in \left\{ s^1, \ldots, s^N \right\}, \ \Pr\left(s_{t+1} = s^i | s_t = s^j \right) = \Pi_{ji} \ \forall t$$

$$b_0, s_0 \text{ given and } c_t \geq 0 \text{ and } b_{t+1} \geq \underline{b} \forall t$$

where c_t is consumption, b_t are holdings of one-period bonds denominated in consumption units, τ_t is a labor tax, w_t is the going wage rate per unit of effective labor, s_t is an idiosyncratic, time-varying individual endowment of effective labor units, l_t is labor supply, r_t is the net real interest rate, D_T is a payment of firm dividends that (for simplicity) is homogeneous, i.e. the same for all households, T_t is a homogeneous lump-sum transfer from the government, and $\underline{b} \leq 0$ is a borrowing limit (lower bound on bond holdings). All elements of the household budget constraint are denominated in consumption units (therefore in real, as opposed to nominal, terms).

$log(s_t)$ follows an exogenous AR(1) process

$$log(s_{t+1}) = \rho log(s_t) + \varepsilon_{t+1}$$

with $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$

Assume that the utility function takes the following form

$$u = \left[\frac{C^{hh,1-\gamma}}{1-\gamma} - \phi \frac{n^{1+\psi}}{1+\psi} \right] \tag{1}$$

A competitive representative firm produces final goods Y_t using a linear production function

$$Y_t = \Gamma_t L_t \tag{2}$$

where L_t is labor demand by firms and Γ_t is an aggregate productivity shifter that follows an AR(1) process

$$\Gamma_{t+1} = \rho^{\Gamma}\Gamma_t + \zeta_{t+1}$$

with $\zeta_{t+1} \sim N(0, \sigma^2_\zeta)$.

A representative union sets nominal wages w_t for all workers subject to (unspecified) adjustment costs, giving rise the a New Keynesian Wage Phillips Curve

$$\pi^w_t = \kappa\left(\varphi\left(L^{hh}_t\right)^{\psi} - \frac{1}{\mu}\left(1-\tau_t\right)w_t\left(C^{hh}_t\right)^{-\sigma}\right) + \beta\pi^w_{t+1}$$

where π^w_t is nominal wage inflation, L^{hh}_t is household labor supply, μ is a steady state wage markup.

There is a Central bank that sets the nominal interest rate i_t according to a simple Taylor rule

$$i_t = R\pi_t^{\psi_\pi}\nu_t \tag{3}$$

where R is the steady state interest rate, $\pi_t = \frac{P_t}{P_{t-1}}$, $\psi_\pi > 1$ is a parameter, and ν_t is an exogenous monetary shock that follows an AR(1) process

$$\nu_{t+1} = \rho^{\nu}\nu_t + \epsilon_{t+1}$$

with $\epsilon_{t+1} \sim N(0, \sigma^2_\epsilon)$.

Assume also, that there is a constant stock of government debt B, whose interest payments are financed every period by lump-sum transfers $T_t = (1 + r_t)B_t$.

Consider the following parameter values:

β	γ	ψ	κ	σ^2_ϵ	ρ	$\underline{b}$	ρ^A	σ^2_A	μ	ψ_π	ρ^ν	σ^2_ν	B
0.99	2	1	0.1	0.1	0.96	0	0.95	0.0004	2	1.5	0.5	0.0001	2

Question 1: Solving for the stationary distribution and calibrating preferences

Use Rouwenhorst's method or Tauchen's method, to transform the AR(1) stochastic process for $w_t = log(w_t)$ to an N state Markov chain, defined by a support $Z_1 < Z_2 < ... < Z_N$ and a $N \times N$ Matrix of transition probabilities P. Choose $N = 5$. Note that in steady state goods and wage inflation are both zero. Assume a unit labor supply $L_t = 1$. Write a program that solves for the interest rate r that clears the asset market in this steady state of the economy. Note that the government budget constraint implies that labor taxes increase with the interest rate, which affects labor supply. But setting $\phi = 1/\mu * (1 - tax) * w * C^(-\gamma)$ makes the NKWPC hold in steady state given a labor supply of 1. Plot the the distribution of assets and comment.

Question 2: A monetary-policy shock.

Consider the response of the economy in steady state to a 25-basis-point expansionary monetary-policy shock $\epsilon_t < 0$ that is unanticipated by agents. Set $T = 100$. Suppose agents do not anticipate any other shock after that. Solve for the response of endogenous variables to this "MIT" shock. In particular,

- Write a program that maps $\{\pi_t\}$ into $\{i_t\}$ via the Taylor rule.
- Write a program that maps $\{i_t\}$ into $\{r_t\}$ via the Fisher equation.
- Note that final-goods-firms' optimality maps $\{\pi_t, \Gamma_t\}$ to $\{\pi_t^w\}$ where $\pi_t^w \equiv W_t/W_{t-1} - 1$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t/\Gamma_t}{W_{t-1}/\Gamma_{t-1}} - 1 = \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} - 1$$

- Write a program that solves for $\{L_t\}$ using the NKPC .

$$\pi_t^w = \left[\kappa\left(\varphi\left(L_t^{hh}\right)^{\nu} - \frac{1}{\mu}\left(1-\tau_t\right)w_t\left(C_t^{hh}{}_t\right)^{-\sigma}\right) + \beta\pi_{t+1}^W\right]$$

- Write a program that solves for the path of household asset demand $\{A_t^{hh}\}$ given $\{r_t\}, \{L_t\}$

Then do the following, after choosing a convergence criterion $crit$.

1. Guess a path of $\{\pi_t\}^i$.
2. Use the DAG structure of the economy to calculate $\{A_t^{hh}\}^i$.
3. If $norm(\{A_t^{hh}\}^i - B) > crit$, choose new $\{\pi_t\}^{i+1}$ and go back to 1. I suggest you do this using a quasi-Newton method: First calculate the Jacobian $\mathbb{H}$ with elements $\mathbb{H}_{ts} = \frac{\partial A_t^{hh}}{\partial \pi_s}$ using direct computation (by solving the household policies backward, and simulating the distribution forward, for each small change $d\pi_s, s = 1, ..., T$). Then use Newton's method with this steady-state Jacobian. If this is too difficult, just use any other solver.