Quantitative Macroeconomics II

Aggregate dynamics in economies with incomplete markets and idiosyncratic risk

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Recap

QM2 so far

- Theories of consumption and wealth inequality with exogenous income risk
  1. Idiosyncratic risk in complete markets
  2. Aggregation across consumers with heterogeneous wealth levels
  3. Idiosyncratic risk in incomplete markets

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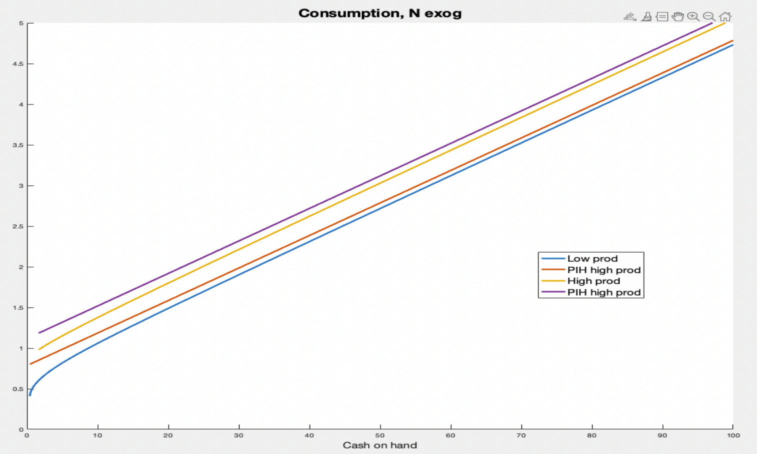
- Theories of consumption and wealth inequality with exogenous income risk
  1. Idiosyncratic risk in complete markets
  2. Aggregation across consumers with heterogeneous wealth levels
  3. Idiosyncratic risk in incomplete markets
     - The income fluctuation problem and the PIH
     - Stationary Recursive Competitive Equilibrium in an economy with idiosyncratic risk
  4. This session: **Aggregate dynamics in economies with idiosyncratic risk and incomplete markets**

Recap: Income fluctuations problem

- $log(y_t) = 0.9 * log(y_{it-1}) + \epsilon_t$, $y \in [y_h, y_m, y_l]$, $\bar{y} = 1$, $\sigma_\epsilon = 0.2$
- $R \approx 1/\beta$
- CRRA=2

Recap IFP: Policy functions, exogenous labor supply



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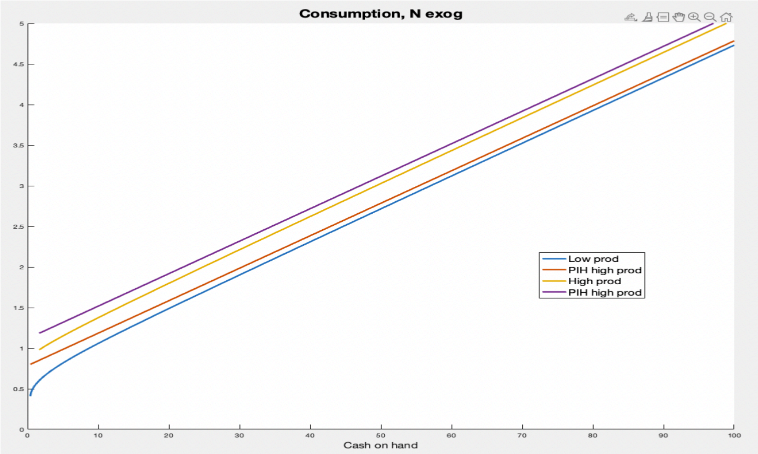
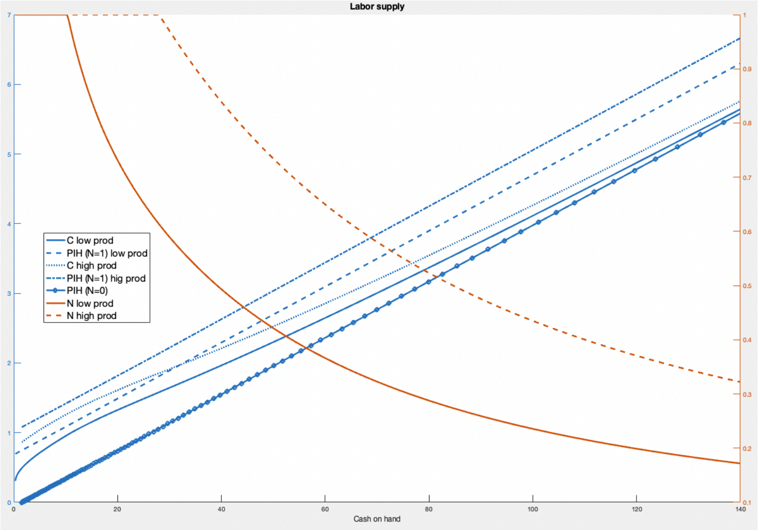


- Substantial precautionary savings at low a
- $c \approx c^{PIH}$ at high a

$\Rightarrow$ higher MPC / income effects (& lower subs. effects) at low a

Recap IFP: Policy functions, endogenous labor supply



Recap: Stationary recursive competitive equilibrium

A neoclassical economy with idiosyncratic earnings risk(Aiyagari 1994)

Recap SRCE: The Aiyagari (1994) economy

- $t = 1, 2, ...$
- 1 perishable good, used for consumption and investment
- **Agents**: representative firm, continuum of measure 1 of inf.-lived, ex-ante identical consumers
- **Preferences** $U(c_0, c_1, c_2, ...) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
- **Endowment** of Labour efficiency units:
  - $\varepsilon_t \in E \equiv \{\varepsilon^1, \varepsilon^2, ..., \varepsilon^{N-1}, \varepsilon^N\}$
  - ε_t follows i.i.d Markov process: $\pi(\varepsilon', \varepsilon) = \Pr(\varepsilon_{t+1} = \varepsilon' \mid \varepsilon_t = \varepsilon)$ with unique ergodic distribution $\Pi_i, i = 1, .., N$
  - LLN applies and π well-behaved: $H_t = \sum_{i=1}^{N} \varepsilon_i \Pi^* (\varepsilon_i)$, for all t

The economy (cont.)

- **Incomplete markets** imply BC: $c_t + a_{t+1} = (1 + r_t)\, a_t + w_t \varepsilon_t$
- **Borrowing constraint**: $a_{t+1} \geq -b$
- CRS **Technology** $Y_t = F(K_t, H_t)$, depreciation δ - No aggregate risk
- **Market Structure**: All markets (for goods, capital, labour) competitive

Recap: Stationary recursive competitive equilibrium

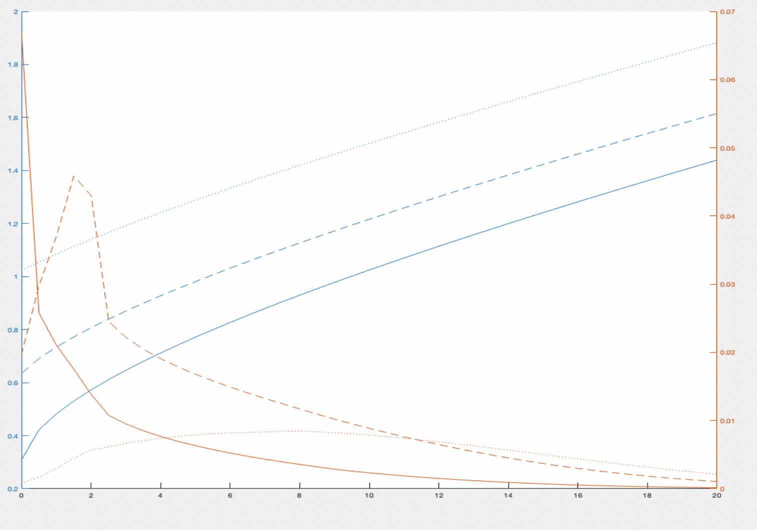
Recap: Stationary recursive competitive equilibrium

A SRCE is a value function $v : S \to \mathbb{R}$; policy functions for the household $a' : S \to \mathbb{R}$, and $c : S \to \mathbb{R}_+$; firm's choices H and K; prices r and w; & a stationary measure λ^* s.t.:

- given r and w, a' and c solve the HH problem and v is the associated value function,
- given r and w, the firm chooses optimally its capital K and its labor H, i.e. $r + \delta = F_K(K, H)$ and $w = F_H(K, H)$,
- the labor market clears: $H = \int_{A\times E} \varepsilon d\lambda^*$,
- the asset market clears: $K = \int_{A\times E} a'(a, \varepsilon) d\lambda^*$,
- the goods market clears:[1] $\int_{A\times E} c(a, \varepsilon) d\lambda^* + \delta K = F(K, H)$,
- $\forall (\mathcal{A} \times \mathcal{E}) \in \Sigma_s$, the invariant prob. measure λ^* satisfies $\lambda^* (\mathcal{A} \times \mathcal{E}) = \int_{A\times E} Q((a, \varepsilon), \mathcal{A} \times \mathcal{E}) d\lambda^*$ with

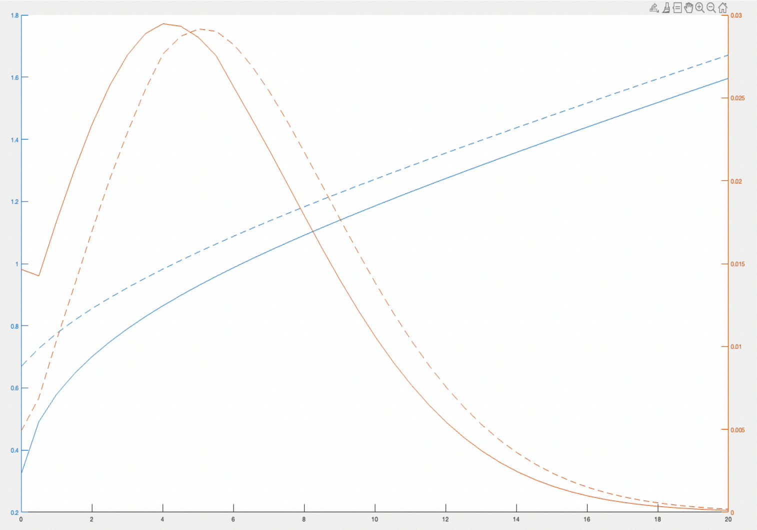
$$Q((a, \varepsilon), \mathcal{A} \times \mathcal{E}) = I_{\{a'(a,\varepsilon)\in\mathcal{A}\}} \sum_{\varepsilon'\in\mathcal{E}} \pi(\varepsilon', \varepsilon)$$

Recap: Wealth distribution in a Huggett (1993) economy



$$log(y_t) = 0.9 * log(y_{it-1}) + \epsilon_t,\ \bar{y} = 1,\ y \in [y_h, y_m, y_l],\ \sigma_\epsilon = 0.22,$$
$$A = 6 * \bar{y},\ \text{CRRA}=2$$

Recap: Wealth distribution in a Huggett economy



2 states with $y_h = 3 * y_l$, $\bar{y} = 1$, $A = 6\bar{y}$, CRRA=2

This session

- Aim: Study the interaction of inequality with aggregate fluctuations and macro-policy
    1. How do aggregate fluctuations affect inequality and welfare?
    2. How does inequality in wealth, income, consumption change business cycles and their welfare costs?
    3. How do structural / insurance / redistribution policies affect aggregate demand and business cycles?
- First step: Combine Aiyagari (1994) with RBC model: Krusell and Smith (KS 98)

Learning points

- Understand the complications caused by aggregate fluctuations
- Understand the Krusell and Smith (1998) method
- Understand why, in their environment, idiosyncratic risk does not change aggregate fluctuations much
- Understand an alternative, simpler solution method: Boppart, Krusell and Mitman (BKM 19)

Outline of this session

1. The economy: general setup
2. Dynamic recursive competitive equilibrium
3. An approximate equilibrium
4. KS 98: Near-Aggregation
5. An alternative, simpler, method: BKM (2019)
6. Applications:
    - Accounting for wealth dispersion (KS 98)
    - The welfare costs of business cycles (Krusell et al 2009)
    - Idiosyncratic risk and the dynamics of aggregate demand and output (KMP 2016)

The economy

- $t = 1, 2, ...$
- 1 perishable good, used for consumption and investment
- **Agents**: representative firm, continuum of measure 1 of inf.-lived, ex-ante identical consumers
- **Preferences** $U(c_0, c_1, c_2, ...) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
- **Endowment**: initial capital k_{i0}, and of labour efficiency units: $\varepsilon_t \in E \equiv \left\{\varepsilon^1, \varepsilon^2, ..., \varepsilon^{N-1}, \varepsilon^N\right\}$
- **Technology** CRS with TFP shocks $z_t \in \mathbb{Z} = \{z_1, ..., z_n\}$: $Y_t = z_t F(K_t, H_t)$, depreciation δ
- ε_t, z_t follow a joint Markov process

$$\pi\left(z', \varepsilon' | z, \varepsilon\right) = \Pr\left(z_{t+1} = z', \varepsilon_{t+1} = \varepsilon' | z_t = z, \varepsilon_t = \varepsilon\right) \quad (1)$$

The economy (cont.)

- **Incomplete markets** imply BC: $c_t + a_{t+1} = (1 + r_t)\, a_t + w_t \varepsilon_t$
- **Borrowing constraint**: $a_{t+1} \geq 0$
- **Structure of markets** (for goods, capital, labour): competitive

Recap: Stationary equilibrium

- Stationary equilibrium w. constant $z_t = z$
  1. By definition: constant distribution of a_{it}
  2. $\Rightarrow$ Constant $A_t = K_t$
  3. $\Rightarrow$ Constant $1 + r_t = f'(K_t) + 1 - \delta$
  4. $\Rightarrow$ HH decisions independent of other HHs' decision rules!
  5. Solve recursive problem with $1 + r$ at all $a, \varepsilon \in S$
  6. Solution plus Markov-structure of ϵ yields transition law for distributions λ_t
  7. Can show that this transition law has a fixed point
  8. So can iterate on $1 + r_t$ until market-clearing - done!

The complication of time-varying z_t

1. z_t, via distribution of a_{it} and correlation with ϵ_{it}, directly affects distribution of current cash-on-hand, and thus of savings and λ_{at+1}, implies time variation in $\{\lambda_{at+1}\}$
2. To make optimal savings choices, agents need to forecast $r_{t+1} = F_K(K_{t+1}, L) + 1 - \delta$, $w_{t+1} = F_L(K_{t+1}, L)$
3. Current λ_{at} determines savings, thus $\{\lambda_{at+1}\}$ and K_{t+1}
4. HHs need to use a law of motion for λ_t to make decisions
5. Implications
   - Decision rules contain λ_t, the joint distribution of a_{it} and ε_{it}, an infinite dimensional object
   - Equilibrium includes a law of motion for λ_t
   - Impossible to solve generally: need approximation

HH Problem

$$
\begin{aligned}
v(a,\varepsilon;z,\lambda) &= \max_{c,a'}\left\{u(c)+\beta\sum_{\varepsilon'\in E,z'\in Z} v(a',\varepsilon';z',\lambda')\pi(z',\varepsilon'|z,\varepsilon)\right\} \\
& \quad s.t. \\
c+a' &= w\left(z,K\left(\lambda\right)\right)\varepsilon + R\left(z,K\left(\lambda\right)\right)a \\
a' &\geq 0 \\
\lambda' &= \Psi\left(z,\lambda,z'\right)
\end{aligned}
$$

- State variables: individual: a,ε; aggregate: z,λ
- $\Psi\left(z,\lambda,z'\right)$ is the LOM for λ, depends on z'

A Dynamic recursive competitive equilibrium

... is a value function v; HH decision rules a', and c; firm choice functions H and K; pricing functions r and w; and, a law of motion Ψ such that:

- $r(z,K)$ and $w(z,K)$,a' and c solve the HH's problem (**??**) and v is the associated value function,
- given prices, the firm chooses optimally K and H, i.e.

$$\begin{aligned} r(z,K)+\delta &= zF_K(K,H), \\ w(z,K) &= zF_H(K,H), \end{aligned} \tag{2}$$

- the labor market clears $H = \int_{A\times E} \varepsilon d\lambda$,
- the asset market clears: $K = \int_{A\times E} a d\lambda$
- the goods market clears:
  $\int_{A\times E} c(a,\varepsilon;z,\lambda)d\lambda + \int_{A\times E} a'(a,\varepsilon;z,\lambda)d\lambda =$
  $zF(K,H) + (1-\delta)K$

A Dynamic recursive competitive equilibrium cont.

- For every pair (z, z'), the aggregate law of motion Ψ is generated by the exogenous Markov chain π and the policy function a' as follows:

$$\lambda'(\mathcal{A}\times\mathcal{E}) = \Psi_{(\mathcal{A}\times\mathcal{E})}\left(z,\lambda,z'\right) = \int_{A\times E} Q_{z,z'}\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right)d\lambda, \tag{3}$$

where $Q_{z,z'}$ is the transition function between two periods where the aggregate shock goes from z to z' and is defined by

$$Q_{z,z'}\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right) = I_{\{g(a,\varepsilon;z,\lambda)\in\mathcal{A}\}}\sum_{\varepsilon'\in\mathcal{E}}\pi_{\varepsilon}(\varepsilon'|z,\varepsilon,z'), \tag{4}$$

where I is the indicator function, $g(a,\varepsilon;z,\lambda)$ is the optimal saving policy, and $\pi_{\varepsilon}(\varepsilon'|z,\varepsilon,z')$ is the conditional transition probability for ε which can be easily derived from π.

KS 98: a limited-information approximation to the LOM Ψ

1. Focus on λ_a, the marginal distribution of wealth
2. Assume HHs only know first M moments of λ_a, i.e. $m_i, i = 1, ..., M$: mean, variance, ...
3. Forecast K' as a linear function of $m_i, i = 1, ..., M$
4. Result (KS 1998): With standard parameters in the NCGM, $M = 1$ gives excellent forecasts of K' (i.e. close to those with $M > 1$) in the forecasting rule

$$\ln K' = b_z^0 + b_z^1 \ln K, \tag{5}$$

Limited information HH Problem

$$
\begin{aligned}
v(a,\varepsilon;z,K) &= \max_{c,a'}\left\{u(c)+\beta\sum_{\varepsilon',z'}v(a',\varepsilon';z',K')\pi(z',\varepsilon'|z,\varepsilon)\right\} \qquad (6)\\
& s.t.\\
c+a' &= w\left(z,K\right)\varepsilon+R\left(z,K\right)a\\
a' &\geq 0\\
\ln K' &= b_z^0+b_z^1\ln K.
\end{aligned}
$$

- NB: Only two more state var.s w.r.t. stationary case: z_t, K_t

Equilibrium

Equilibrium is a fixed point to the law of motion s.t. the perceived LOM of the agents is consistent with the actual LOM that results from aggregating individual decisions

$$\ln K' \quad = \quad b_z^0 + b_z^1 \ln K \quad \forall z \in \mathbb{Z} \tag{7}$$

KS 98 Algorithm

1. Guess the coefficients of the law of motion $\left\{b_z^0, b_z^1\right\}$
2. Solve the household problem and obtain the decision rules $a'\left(a, \varepsilon; z, K\right)$, $c\left(a, \varepsilon; z, K\right)$
3. Simulate the economy for N individuals and T periods by drawing a sequence for $z_t, t = 1, .., T$, then for $\varepsilon_{it}, i = 1, ..N, t = 1, ..T$ conditional on the time-path for the aggregate shocks. Use decision rules to generate $\left\{a_t^i\right\}_{t=1,i=1}^{T,N}$ and $A_t = \frac{1}{N}\sum_{i=1}^{N} a_t^i$.
4. Discard T^0 periods, run the regression $\ln A_{t+1} = \beta_z^0 + \beta_z^1 \ln A_t$ to estimate$\left(\beta_z^0, \beta_z^1\right)$
5. If $\left(\beta_z^0, \beta_z^1\right) \neq \left(b_z^0, b_z^1\right)$, then try a new guess and go back to step 1.
6. (Calculate $R2$ in 4., add moments, compare.)

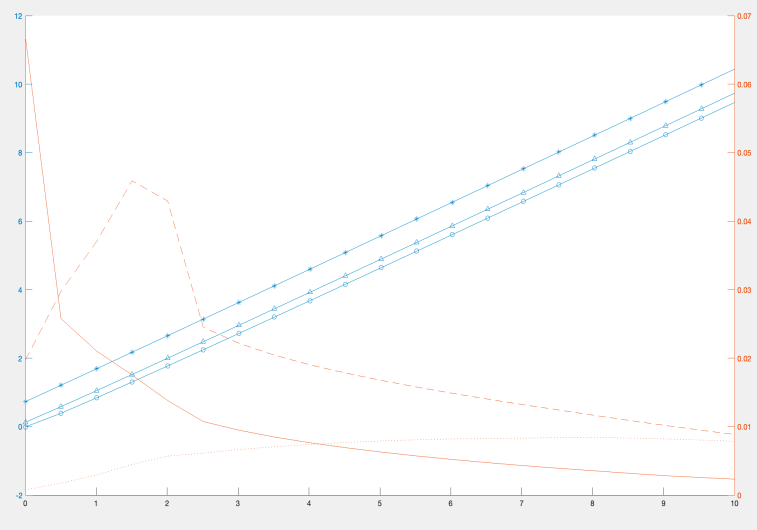
KS 98: A Near-Aggregation Result

Interpret $\varepsilon = 1(0)$ as (un)employment, add $z_t \in \{z_b, z_g\}$ with z_g expansion, z_b recession. Find the following LOM for K

$$\ln K' = \begin{cases} 0.095 + 0.962 \ln K, & \text{for } z = z_g \\ 0.085 + 0.965 \ln K, & \text{for } z = z_b \end{cases} \tag{8}$$

The approximation is extremely accurate: delivers an $R^2 = 0.999998$!

Savings in a Huggett economy



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$$A = 6 * \bar{y},\ \text{CRRA}=2$$

Intuition for Near-Aggregation

Agg. equilibrium independent of distribution of a_{it} around $\bar{a}$-why?

1. Transitory ϵ, elastic cap supply: $(1+r) \approx 1/\beta$, self-insurance effective (savings motive intertemporal smoothing, not precaution against shocks)
2. $\Rightarrow$ Decision rules $a'(a, \epsilon, \Psi)$ are very linear in a unless close to the borrowing limit
3. But decisions of the asset poor have very little effect on aggregate assets - the rich matter more for wealth.
4. Linear pol. fcts $a'(a)$ aggregate into linear LOM $K'(K)$

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4. Linear pol. fcts $a'(a)$ aggregate into linear LOM $K'(K)$
5. But:
   - $K'_{HA}(K)$ may $\neq K'_{RA}(K)$ (But KS 98: very similar)
   - And consumption at low a matters more for C
   - No near-aggregation e.g. in bond-economy ($B_t = B, \forall t$)

Alternative solution procedures

- KS98 works well for few exogenous shocks, few moments of λ_t. O/w need alternatives (JEDC 2010 issue)
- E.g. Reiter (09): Perturbation of individual policy functions around aggregate variables in stationary equilibrium with distribution λ.
- Intuition: Linearise stationary-equilibrium pol. fcts in aggregate states z_t, K_t
- But: if $\lambda_0 = \lambda$ and PFs linear in z_t, K_t around Stat Equi, then $K_{s+1} = \int_{a,\epsilon} a'(a, \epsilon; z, K) d\lambda_s$ is linear function of $z_t, t = 1, ..., s$
- So one impulse response function (IRF) suffices to simulate economy - can add more shocks by "stacking" responses, larger shocks by scaling

Boppart, Krusell, Mitman (2018) algorithm: IRF as numerical derivative

1. Compute stationary equilibrium distribution: (easy)
2. Perturbation by "MIT" shock: Compute *transition* after a one-time (persistent) shock to z_1 (easy)
3. Yields nonlinear IRF $y_t(z_1), t = 1, 2, ...,$ for any y_t
4. Interpret $\{\frac{y_t}{z_1}\}$ as a numerical derivative $\frac{\delta y_s}{\delta z_{s-(t-1)}}$
5. Assume reaction of economy to shocks is linear
   5.1 $\frac{\delta y_s}{\delta z_{s-t+1}}$ is constant independent of history
   5.2 y_t after a history of shocks $z_1, ..., z_t$ is given by sum $y_t \frac{\delta y_t}{\delta z_{-t+1}} z_1 + ... + \frac{\delta y_t}{\delta z_0} z_t$ (1)
6. Check linearity (need to compute two more transitions):
   - Scaleability: $y_t(\alpha z_1), t = 1, 2, ..., T = \alpha y_t(z_1), t = 1, 2, ..., T$?
   - Additivity: $y_t(z_1, z_s) = y_t(z_1) + y_t(z_s), t = s, 2, ..., T$?
7. If check passed, simulate using (1)!

Applications

1. Krusell and Smith (1998): Accounting for wealth dispersion
2. Krusell et al (2009): Revisiting the welfare costs of business cycles
3. Krueger, Mitman and Perri (2016): Macroeconomics and Household Heterogeneity, Handbook of Macroeconomics.

Krusell and Smith (1998)

- Wealth distribution in the benchmark economy: too much wealth at the bottom, too little at the top
- Unemployment insurance benefits reduce wealth at bottom, but do not increase concentration at the top
- Introduce stochastic variation in discount factors $\beta \in [\beta_1, \beta_2, \beta_3]$ with Markov-transitions calibrated to match expected duration with average life-expectancy
- Leads to some patient individuals holding most wealth - "The poor are poor because they are impatient"

Krusell et al (2009): Revisiting the welfare effects of business cycles

- Lucas (1987): Costs of eliminating business cycles (=replacing observed aggregate consumption with its trend) is extremely small ($< 1/100\%$ of permanent consumption) with CRRA utility and reasonable risk aversion
- Krusell et al (2009): What if there is individual-specific and aggregate uncertainty, and agents differ in wealth, employment / unemployment, etc.

Krusell et al (2009): Revisiting the welfare effects of business cycles

- Model Economy: Same as in Krusell and Smith (1998) (but with extension to short-/long-term unemployment)
- Since ε, z_t are correlated (more likely to become unemployed in recessions), need to be careful how to eliminate "aggregate" fluctuations:

  Setting z_t to a constant not enough: need to identify i_t such that $\varepsilon_t = \rho z_t + i_t$ and i_t uncorrelated with z_t (linear projection)

Krusell et al (2009): Results

Welfare benefits of eliminating bus cycles that are larger on average (0.1 percent of aggregate consumption) and heterogeneous: U-shape across wealth distribution

- Wealth-poor (low β) are basically hand-to-mouth consumers, so $\Delta Var(c) \approx \Delta Var(y)$: large reduction in consumption volatility (corresponding to +0.15 % of average consumption for pctiles 1-5)
- Wealth-rich are (self-)insured, so no change in consumption volatility. But: fall in aggregate precautionary savings increases return on capital, equivalent to +1.15 % pctile 95-99
- Largest gain for long-term unemployed, low-β, borrowing-constrained agents +35% of permanent consumption

Krueger, Mitman and Perri (2016): Macroeconomics and Household Heterogeneity

- Post-2007 Great Recession: Renewed interest in interplay of macroeconomics and inequality
- Documents
  - Structure of US inequality before 2007
  - Unequal impact of GR
- Structural KS (98)-style model with additional features to understand this
- Investigates effects of different model features

Krueger, Mitman and Perri (2016): Facts

	Variable						
	Earn.	Disp Y		Cons. Exp		Net Worth	
Source	PSID	PSID	CPS	PSID	CE	PSID	SCF(2007)
Mean (2006$)	54,349	64,834	60,032	42,787	47,563	324,951	538,265
% Share by:							
Q1	3.6	4.5	4.4	5.6	6.5	-0.9	-0.2
Q2	9.9	9.9	10.5	10.7	11.4	0.8	1.2
Q3	15.3	15.3	15.9	15.6	16.4	4.4	4.6
Q4	22.7	22.8	23.1	22.4	23.3	13.0	11.9
Q5	48.5	47.5	46.0	45.6	42.4	82.7	82.5
90 – 95	10.9	10.8	10.1	10.3	10.2	13.7	11.1
95 – 99	13.1	12.8	12.8	11.3	11.1	22.8	25.3
Top 1%	8.0	8.0	7.2	8.2	5.1	30.9	33.5
Gini	0.43	0.42	0.40	0.40	0.36	0.77	0.78
Sample Size	6,232	6,232	54,518	6,232	4,908	6,232	2,910

Source: Krueger et al (2016)

Krueger, Mitman and Perri (2016): Facts

Table 2: PSID Households across the net worth distribution: 2006

	% Share of:			% Expend. Rate		Head's	
NW Q	Earn.	Disp Y	Expend.	Earn.	Disp Y	Age	Edu (yrs)
Q1	9.8	8.7	11.3	95.1	90.0	39.2	12
Q2	12.9	11.2	12.4	79.3	76.4	40.3	12
Q3	18.0	16.7	16.8	77.5	69.8	42.3	12.4
Q4	22.3	22.1	22.4	82.3	69.6	46.2	12.7
Q5	37.0	41.2	37.2	83.0	62.5	48.8	13.9
	Correlation with net worth						
	0.26	0.42	0.20				

Source: Krueger et al (2016)

Krueger, Mitman and Perri (2016): Facts

Table 3: Annualized Changes in Selected Variables across PSID Net Worth

	Net Worth*				Disp Y (%)		Cons. Exp.(%)		Exp. Rate (pp)	
		(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
		04-06		06-10	04-06	06-10	04-06	06-10	04-06	06-10
All	15.7	44.6	-3.0	-10	4.1	1.2	5.6	-1.3	0.9	-1.6
NW Q										
Q1	NA	12.9	NA	6.6	7.4	6.7	7.1	0.6	-0.2	-4.2
Q2	121.9	19.5	24.4	3.7	6.7	4.1	7.2	2	0.3	-1.3
Q3	32.9	23.6	4.3	3.3	5.1	1.8	9	0	2.3	-1.1
Q4	17.0	34.7	1.7	3.8	5.0	1.7	5.9	-1.5	0.5	-2
Q5	11.6	132.2	-4.9	-68.4	1.8	-1.2	2.7	-3.5	0.5	-1.4

∗The first figure is the percentage change (growth rate), the second is the change in 000's of dollars

Source: Krueger et al (2016)

Krueger, Mitman and Perri (2016): Facts

Table 4: Decomposing changes in expenditures growth

	Change C Growth	Change Y Growth	Change C/Y Growth
	$g_{c,t} - g_{c,t-1}$	$g_{y,t} - g_{y,t-1}$	$\frac{\rho_{it}-\rho_{it-1}}{\rho_{it-1}} - \frac{\rho_{it-1}-\rho_{it-2}}{\rho_{it-2}}$
All	-6.9	-2.9 (42%)	-3.8 (55%)
NW Q			
Q1	-6.5	-0.7 (11%)	-4.5 (69%)
Q2	-5.2	-2.6 (50%)	-2.3 (44%)
Q3	-9.0	-3.3 (37%)	-5.2 (58%)
Q4	-7.4	-3.3 (48%)	-3.8 (55%)
Q5	-6.2	-3.0 (42%)	-3.4 (55%)

Source: Krueger et al (2016)

Krueger, Mitman and Perri (2016): Summary Facts

- Wealth ineqality$\gg$ income inequality $>$ cons inequality
- W-quintiles 1 and 2 account for 0 wealth, but $\approx 25\%$ of consumption
- Cons. rates decline with wealth
- Decline in cons during GR much larger than in income at low wealth

Krueger, Mitman and Perri (2016): Model

- Recalibration of KS 98
- Plus additional features
  1. "Intensive" labor market risk: persistent shocks to labor earnings ("$\sigma(y)$")
  2. Heterogeneous discount factors ("$\sigma(\beta)$â ")
  3. Stylised life cycle: Constant probability to retire, and die ("LC")
  4. Realistic unemployment insurance (rep rate of 10/50 %, vs. 1 % in KS 98)

Krueger, Mitman and Perri (2016): Model vs. Data

Table 6: Net Worth Distributions: Data v/s Models

	Data		Models	
% Share held by:	PSID, 06	SCF, 07	Bench	KS
$Q1$	-0.9	-0.2	0.3	6.9
$Q2$	0.8	1.2	1.2	11.7
$Q3$	4.4	4.6	4.7	16.0
$Q4$	13.0	11.9	16.0	22.3
$Q5$	82.7	82.5	77.8	43.0
$90-95$	13.7	11.1	17.9	10.5
$95-99$	22.8	25.3	26.0	11.8
$T1\%$	30.9	33.5	14.2	5.0
Gini	0.77	0.78	0.77	0.35

Source: Krueger et al (2016)

- Benchmark replicates wealth distribution, original KS doesn't

Krueger, Mitman and Perri (2016): Model features & wealth distr.

Table 7: Net Worth Distributions and Consumption Decline: Different Versions of the Model

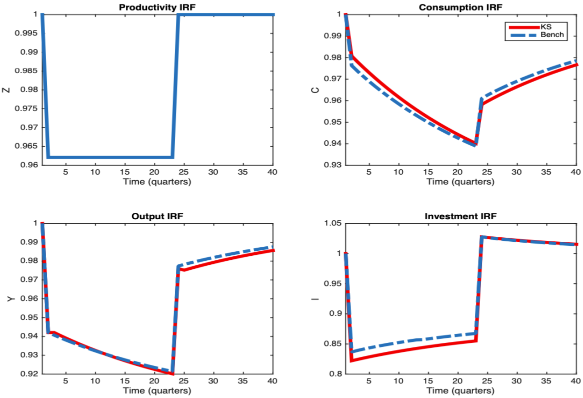
	Models*				
% Share:	KS	$+\sigma(y)$	+Ret.	$+\sigma(\beta)$	+UI
$Q1$	6.9	0.7	0.7	0.7	0.3
$Q2$	11.7	2.2	2.4	2.0	1.2
$Q3$	16.0	6.1	6.7	5.3	4.7
$Q4$	22.3	17.8	19.0	15.9	16.0
$Q5$	43.0	73.3	71.1	76.1	77.8
90 – 95	10.5	17.5	17.1	17.5	17.9
95 – 99	11.8	23.7	22.6	25.4	26.0
$T1\%$	5.0	11.2	10.7	13.9	14.2
Wealth Gini	0.350	0.699	0.703	0.745	0.767

Source: Krueger et al (2016)

Krueger, Mitman and Perri (2016): Model features and implications

- Highly persistent earnings risk $\rightarrow$ high wealth dispersion
- Life-cycle structure $\rightarrow$ low cons share of the rich
- Disc-factor heterogeneity $\rightarrow$ right-tail of w distribution
- UI insurance $\rightarrow$ low wealth of the poor

Krueger, Mitman and Perri (2016): A "great" recession



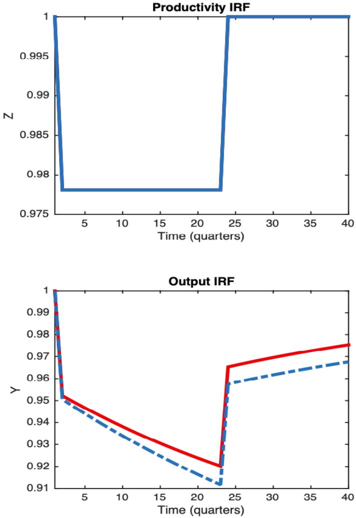
Source: Krueger et al (2016)

- Higher share of low-wealth / high-MPC HHs amplifies recession
- But: propagation only through I, K

Why inequality matters: Additional propagation relative to KS 98

- More low-wealth / high-MPC HHs: amplifies recession
- But: propagation only through I, K
- Missing:
  1. Endogenous labor supply: dampens role of heterogeneity
  2. Demand effects on output: amplifies it
- Demand-determined output: 2 ways
  1. Production externality from consumption demand:
     $Y = Z^{\star} F(K, L) = Z C^{\omega} F(K, L)$
  2. Monop. competition and price rigidities: HANK! - next lecture

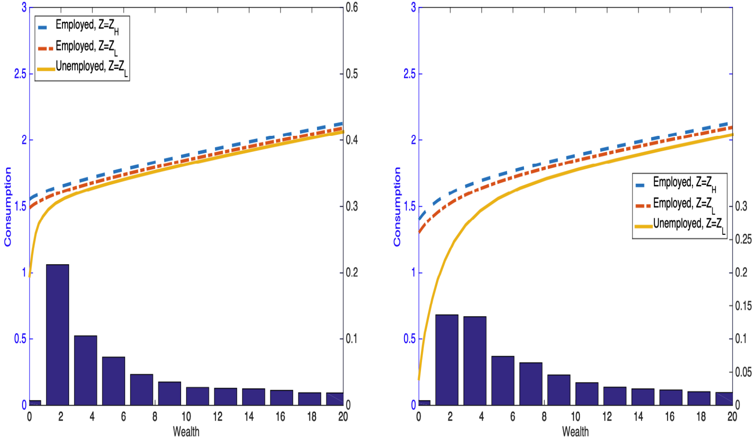
KMP: Recession with consumption externality



Source: Krueger et al (2016)

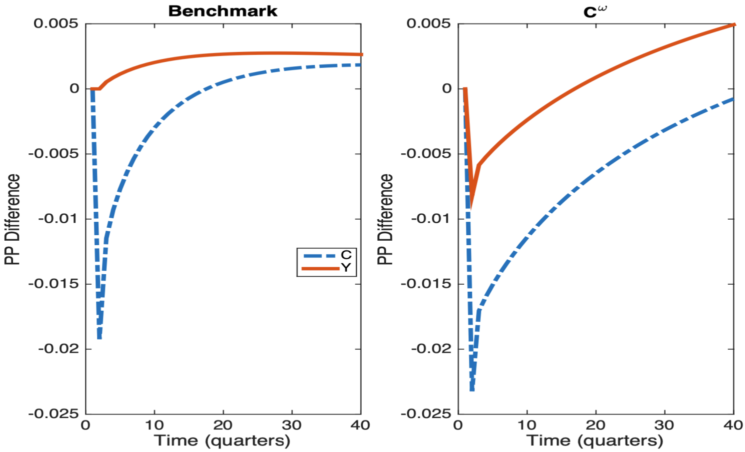
KMP: The stabilising role of unemployment insurance, benchmark calibration

KMP: pol functions & distributions w. 50 / 10 % replacement



Source: Krueger et al (2016)

KMP: Recessions w. 50 / 10 % replacement



Source: Krueger et al (2016)

- Unemployment insurance dampens business cycles

KMP: Near-aggregation

- Wealth distribution matters for C and I responses to TFP shocks
- ... and fluctuates: coef of var(share of constrained HH)=7 percent
- But near-aggregation still holds: Z, K predict well fluctuations in the wealth distribution

Summary

- Aggregate dynamics in exogenous variables cause endogenous fluctuations in wealth distribution and thus prices
- This introduces the wealth distribution as a state variable in the HH problem - impossible.
- Krusell and Smith (1998): RBC model with uninsured unemployment risk implies “near-aggregation"
- Heterogeneity in discount factors can generate more dispersed wealth distribution
- Pers. earnings risk, life-cycle, etc (KMP 16): heterogeneity matters for bus cycles
- With heterogeneous agents, welfare costs of business cycles are much larger and U shaped across the wealth distribution

Quantitative Macroeconomics II

Aggregate dynamics in economies with incomplete markets and idiosyncratic risk

Tobias Broer