

# Problem Set III

## Quant Macro II

February 7, 2024

**Deadline: Feb 16.** Please send the Matlab codes plus a PDF (made with Latex) to both Eustache and me by Feb 16, 12pm. One submission per team of 2.

**Exercise 6.** We start from Exercise 5 of Problem Set 2.

1. Start from Exercise 5, question 2. Use Golden on  $a'$  and VFI to compute value functions, but compute the function  $\hat{n}(a, \varepsilon, a')$  using Newton rather than a bisection method. Which one is faster?
2. Now, replace the Golden on  $a'$  by a Golden on  $c$ . Be very careful with the bounds on  $a'$ . Does it work better? Faster? Do you still need a root finder to find  $n$ ?

**Exercise 7: Deterministic transition.** Take the most efficient of your codes with exogenous labor supply. We now assume that there is aggregate productivity  $A_t$  such that the production function is

$$F(K_t, N_t) = A_t N_t^\alpha K_t^{1-\alpha}.$$

1. In  $t = 0$ , we are in steady-state, where  $A_t$  is fixed equal to 1 and the measure is stationary. Suddenly and unexpectedly, in  $t = 1$ , productivity jumps up to  $A_t = 1.01$ . Then, productivity goes back to steady-state with some persistence  $\rho_a = 0.9$ :

$$A_t = (1 - \rho_a) + \rho_a A_{t-1} \quad \forall t \geq 2.$$

Compute the equilibrium path for the interest rate. Plot the aggregate paths for capital, wages, and interest rates.

2. Compute the welfare gains for each household, using consumption equivalents. Who are the households who benefit most?
3. Can you compute the gains in terms of wealth?

**Exercise 8.** Take the most efficient of your codes with endogenous labor supply of Exercise 6. We add a government, which finances some spending  $G$  with a HSV tax function on labor income *only*. We assume  $\tau = 0.18$ .

1. Find  $\lambda$  and  $B$  such that in equilibrium,  $G/Y$  and average hours are **around** 10% and 0.3, respectively. What is the implied value of  $r$ ? How large is aggregate welfare?
2. Now, fix  $G$  and preference parameters, and find the new  $\lambda$  when  $\tau = 0.30$ . What is the implied value of  $r$ ? How large is aggregate welfare? How large is capital?

**Exercise 9: Deterministic transition.** Take the code of exercise 8. We want to analyze the transition from question 8a to 8b.

- **[This question is very hard, don't be surprised if the code is very slow, reduce the number of points and loosen convergence criteria if needed; do your best!]** Compute the transition from question 8a to 8b. Why is this exercise significantly harder than exercise 7? What could we do to speed up the codes?
- **[If you have done some progress with question 9a]** How large is aggregate welfare at the moment of the tax reform, compared to the long-run steady state? Why?