Quand Macro II: Final Exam

May 2023

You have 2 hours for this exam. Neither books, nor class notes are permitted. Please, provide **concise** *and* **legible/readable** *answers. If you think you need to make additional assumptions to answer some questions, please go ahead, and state them clearly in your answer.*

In this exam, we want to design a model which would be appropriate to evaluate welfare gains and losses of the pension reform which has been recently voted in France. For that purpose, we will proceed in three steps. First, we will build together an overlapping generations (OLG) model without any pension system. Second, we will model the current pension system. Third, we will model the reform.

1 An OLG model [10 pts]

Consider the following economic environment. Time is discrete and indexed by $t = 0, 1, \ldots$ and the economy is stationary (for now).

Demographics. The economy is populated by J overlapping cohorts of households. Each cohort is born at age $j = 1$ (think, at age 20) with measure one. Households die with probability one at the end of period J (that is, when about to turn $J + 1$), where $J = 80$. We define as "newborns" the households at the point in time when they enter the model, at $j = 1$.

Preferences. Households' intratemporal utility is a function of consumption and hours:

$$u(c, h) = \frac{c^{1-\gamma}}{1-\gamma} - B\frac{h^{1+\varphi}}{1+\varphi}$$

where $\gamma \geq 0$, $\varphi \geq 0$ and $B \geq 0$.

Idiosyncratic productivity and asset market. Newborns start with zero wealth, $a = 0$, and draw their initial productivity level x from the stationary distribution π_x (defined next). After the initial draw, productivity x subsequently follows a standard AR(1) process, with a transition matrix Π_x, which admits a unique stationary distribution π_x.

From $j = 43$ onwards, productivity x drops to 0. In other words, households supply labor elastically until $j = 42$ (that is, until age 61 included) and receive labor income wxh, where x is the wage provided on the labor market for one unit of efficient labor. Then they *retire* at the end of period $j = 42$ (when turning 62).

Households can save using risk-free bonds with interest rate r, subject to the following borrowing constraint: $a' \geq 0$.

Technology. Competitive firms use labor and capital to produce the final good, using a Cobb-Douglas production function. Capital depreciates at rate δ.

a. The state of a working-age household is (a, x, j), where j is the age of the household. Write the state of a retired household. Write the household's maximization problem recursively. You may have to distinguish two cases: working-age and retired household. **[3 pts]**

b. Define the measure as carefully as you can. Define a recursive competitive equilibrium. **[2 pts]**

c. What can you say about the savings decisions in the very last period of life, at $j = J$? What can you say about the typical lifecycle profile of savings in this economy? (That is, how does wealth look like as a function of age?) **[2 pts]**

d. Explain how you would solve this model in a computer. What would you guess? What would you iterate on? Explain carefully how you would compute value functions; and measure. Do you think this model is harder or simpler to solve than the basic model that we have seen in class? Why? **[3 pts]**

2 A Pension System [4 pts]

Now, we want to model the current pension system, where a government runs a social security as follows. Every period, labor income is taxed at a flat rate τ. The fiscal revenues raised from this labor tax are redistributed to all retired households, in the form of pensions. Pensions are proportional to the household's productivity in the very last period of working life, at $j = 42$. That is, a retired household who experienced a given productivity $\hat{x}$ at age $j = 42$ will receive a constant pension $\chi\hat{x}$ during his entire retirement period, where χ is a positive real number chosen such that the social security's budget constraint holds.

a. Update the household's maximization problem. **[1 pt]**

b. Write the government's budget constraint. Explain how you would update the algorithm. **[1 pt]**

c. You want to calibrate your model, that is, to discipline your model to ensure that the parameters that you choose are doing a good job at describing the real world. What moments do you particularly look at in the data? In your model? **[2 pts]**

3 A Pension Reform [6 pts]

Suddenly, the government decides to change the retirement age, from 62 to 64. We assume that the tax rate τ is kept constant, and the rate χ is adjusted so that the social security's budget constraint still holds.

a. Who will gain and loose from the reform? In steady state? During the transition? Explain how you would measure welfare gains and losses in this economy. **[2 pts]**

b. What are the key ingredients which are missing in this environment to make the model more insightful to the policy maker? Report what you perceive as the three most important elements that should be added in the model. **[4 pts]**