

HWI out
 Proj I Part 1 due next Monday
 Proj I Part 2 out next Monday!

L7 Relational Algebra

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 Fall 2016

Supplemental Materials

Ramakrishnan
 Sections 4.1 and 4.2

Helpful References

https://en.wikipedia.org/wiki/Relational_algebra

Overview

Last time, learned about
 pre-relational models
 an informal introduction to relational model
 an introduction to the SQL query language.

Learn about formal relational query languages
 Relational Algebra (algebra: perform operations)
 Relational Calculus (logic: are statements true?)

Keys to understanding SQL and query processing

Who Cares?

Clean query semantics & rich program analysis
 Helps/enables optimization
 Opens up rich set of topics
 Materialized views
 Data lineage/provenance
 Query by example
 Distributed query execution
 ...

What's a Query Language?

Allows manipulation and **retrieval of data** from a database.

Traditionally: QL != programming language
 Doesn't need to be turing complete
 Not designed for computation
 Supports easy, efficient access to large databases

Recent Years

Scaling to large datasets is a reality
 Query languages are a powerful way to
 think about data algorithms scale
 think about asynchronous/parallel programming

What's a Query Language?

4 MapReduce is **not** the answer

- MapReduce is a powerful primitive to do many kinds of parallel data processing
- BUT
 - Little control of data flow
 - Fault tolerance guarantees not always necessary
 - Simplicity leads to inefficiencies
 - Does not interface with existing analysis software
 - Industry has existing training in SQL

➔ SQL interface for Hadoop critical for mass adoption

Tutorial at VLDB 2015 Abadi et al.
<http://www.slideshare.net/abadi/sqlonhadoop-tutorial>

Formal Relational Query Languages

Formal basis for real languages e.g., SQL

Relational Algebra

Operational, used to represent execution plans

Relational Calculus

Logical, describes what data users want
 (not operational, fully declarative)

Relational Algebra

1+2

2+1

0+3

...

{ (a,b) | a, b are integers and a+b=3 }

Prelims

Query is a function over **relation instances**

$$Q(R_1, \dots, R_n) = R_{\text{result}}$$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation

Position easier for formal defs

one-indexed (not 0-indexed!!!)

Named more readable

Both used in SQL

Prelims

Relation (for this lecture)

Instance is a set of tuples

Schema defines field names and types (domains)

Students(sid int, name text, major text, gpa int)

How are relations different than generic sets (\mathbb{R})?

Can assume item structure due to schema

Some algebra operations (x) need to be modified

Will use this later

Relational Algebra Overview

Core 5 operations

PROJECT (π)

SELECT (σ)

UNION (\cup)

SET DIFFERENCE ($-$)

CROSSPRODUCT (\times)

Additional operations

RENAME (ρ)

INTERSECT (\cap)

JOIN (\bowtie)

DIVIDE ($/$)

Instances Used Today: Library

Students, Reservations

R1

sid	rid	day
1	101	10/10
2	102	11/11

Use positional or named field notation

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

Fields in query results are inherited from input relations (unless specified)

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

Project

$$\pi_{\langle attr1, \dots \rangle}(A) = R_{result}$$

Pick out desired attributes (subset of columns)

Schema is subset of input schema in the projection list

$\pi_{\langle a, b, c \rangle}(A)$ has output schema (a, b, c) w/ types carried over

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$$\pi_{name, age}(S2) =$$

name	age
aziz	21
barak	21
trump	88
rusty	21

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$$\pi_{age}(S2) =$$

age
21
88

Where did all the rows go?

Real systems typically don't remove duplicates. Why?

Select

$$\sigma_{\langle p \rangle}(A) = R_{result}$$

Select subset of rows that satisfy condition p

Won't have duplicates in result. Why?

Result schema same as input

Select

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

$$\sigma_{age < 30}(S1) =$$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21

$$\pi_{name}(\sigma_{age < 30}(S1)) =$$

name
eugene
barak

Commutatively

$$\begin{aligned}
 A + B &= B + A \\
 A * B &= B * A \\
 A + (B * C) &= (B * C) + A \\
 A + (B + C) &= (A + B) + C \\
 A + (B * C) &= (A + B) * C
 \end{aligned}$$

Commutatively

$$\begin{aligned}
 A + B &= B + A \\
 A * B &= B * A \\
 A + (B * C) &= (B * C) + A \\
 A + (B + C) &= (A + B) + C \\
 \del{A + (B * C)} &= \del{(A + B) * C}
 \end{aligned}$$

Commutatively

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S))$$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21

Commutatively

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S))$$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

$$\pi_{\text{age}} \left(\begin{array}{|c|c|c|c|} \hline \text{sid} & \text{name} & \text{gpa} & \text{age} \\ \hline 1 & \text{eugene} & 4 & 20 \\ 2 & \text{barak} & 3 & 21 \\ 3 & \text{trump} & 2 & 88 \\ \hline \end{array} \right) = \begin{array}{|c|} \hline \text{age} \\ \hline 20 \\ 21 \\ 88 \\ \hline \end{array}$$

Commutatively

$$\sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

age
20
21
88

Commutatively

$$\sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

$$\sigma_{\text{age} < 30} \left(\begin{array}{|c|} \hline \text{age} \\ \hline 20 \\ 21 \\ 88 \\ \hline \end{array} \right) = \begin{array}{|c|} \hline \text{age} \\ \hline 20 \\ 21 \\ \hline \end{array}$$

Commutatively

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30}(S))?$$

Commutatively

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30}(S)) \neq \sigma_{\text{age} < 30}(\pi_{\text{name}}(S))$$

Commutatively

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30}(S)) \neq \sigma_{\text{age} < 30}(\pi_{\text{name, age}}(S))$$

Commutatively

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30}(S)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(S))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30}(S)) = \pi_{\text{name}}(\sigma_{\text{age} < 30}(\pi_{\text{name, age}}(S)))$$

OK!

Union, Set-Difference

$$A \text{ op } B = R_{\text{result}}$$

A, B must be *union-compatible*

Same number of fields

Field i in each schema have same type

Result Schema borrowed from first arg (A)

$$A(\text{big int, poppa int}) \cup B(\text{thug int, life int}) = ?$$

Union, Set-Difference

$$A \text{ op } B = R_{\text{result}}$$

A, B must be *union-compatible*

Same number of fields

Field i in each schema have same type

Result Schema borrowed from first arg (A)

$$A(\text{big int, poppa int}) \cup B(\text{thug int, life int}) = R_{\text{result}}(\text{big int, poppa int})$$

Union, Intersect, Set-Difference

S1				S2			
sid	name	gpa	age	sid	name	gpa	age
1	eugene	4	20	4	aziz	3.2	21
2	barak	3	21	2	barak	3	21
3	trump	2	88	3	trump	2	88
				5	rusty	3.5	21

sid	name	gpa	age
1	eugene	4	20
4	aziz	3.2	21
5	rusty	3.5	21
3	trump	2	88
2	barak	3	21

$S1 \cup S2 =$

Union, Intersect, Set-Difference

S1				S2			
sid	name	gpa	age	sid	name	gpa	age
1	eugene	4	20	4	aziz	3.2	21
2	barak	3	21	2	barak	3	21
3	trump	2	88	3	trump	2	88
				5	rusty	3.5	21

sid	name	gpa	age
1	eugene	4	20

$S1 - S2 =$

Note on Set Difference & Performance

Notice that most operators are monotonic
 increasing size of inputs \rightarrow outputs grow
 if $A \supseteq B \rightarrow Q(A, T) \supseteq Q(B, T)$
 can compute *incrementally*

Set Difference is *not monotonic*

if $A \supseteq B \rightarrow T - A \subseteq T - B$
 e.g., $5 > 1 \rightarrow 9 - 5 < 9 - 1$

Set difference is *blocking*:

For $T - S$, must wait for all S tuples before any results

Cross-Product

$$A(a_1, \dots, a_n) \times B(a_{n+1}, \dots, a_m) = R_{\text{result}}(a_1, \dots, a_m)$$

Each row of A paired with each row of B

Result schema concatenates A and B 's fields, inherit if possible

Conflict: students and reservations have *sid* field

Different than mathematical "X" by flattening results:

math $A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$
 e.g., $\{1, 2\} \times \{3, 4\} = \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$
 what is $\{1, 2\} \times \{3, 4\} \times \{5, 6\}$?

Cross-Product

S1				R1		
sid	name	gpa	age	sid	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	2	102	11/11
3	trump	2	88			

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	1	101	10/10
3	trump	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11
3	trump	2	88	2	102	11/11

$S1 \times R1 =$

Rename

$$\rho(<\text{new_name}>(<\text{mappings}>), Q)$$

Explicitly defines/changes field names of schema

$$\rho(C(I \rightarrow \text{sid1}, 5 \rightarrow \text{sid2}), S1 \times R1)$$

sid1	name	gpa	age	sid2	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	1	101	10/10
3	trump	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11
3	trump	2	88	2	102	11/11

$C =$

Administrivia

Azure codes – should have gotten google invite
What kind of transactional guarantees do you think google spreadsheets provides?

Redeem: <https://www.microsoftazurepass.com/>
(on course website)

You are not required to understand lec3.md

They were my lecture notes

Proj1 PartI returned today

PostgreSQL passwords written on them

HW2 out on Friday

Project $\pi(\text{blue} \cup \text{orange}) = \text{blue}$

Select $\sigma(\text{blue} \cup \text{orange}) = \text{blue} \cup \text{orange}$

Cross product $\text{blue} \times \text{orange} = \text{blue} \cup \text{orange}$

Difference $\text{blue} - \text{orange} = \text{blue}$

Union $\text{blue} \cup \text{orange} = \text{blue} \cup \text{orange}$

Intersect $\text{blue} \cap \text{orange} = \text{orange}$

Compound/Convenience Operators

INTERSECT (\cap)
JOIN (\bowtie)
DIVIDE ($/$)

Intersect

$$A \cap B = R_{\text{result}}$$

A,B must be *union-compatible*

Intersect

S1				S2			
sid	name	gpa	age	sid	name	gpa	age
1	eugene	4	20	4	aziz	3.2	21
2	barak	3	21	2	barak	3	21
3	trump	2	88	3	trump	2	88
				5	rusty	3.5	21

$$S1 \cap S2 =$$

sid	name	gpa	age
2	barak	3	21
3	trump	2	88

Intersect

$$A \cap B = R_{\text{result}}$$

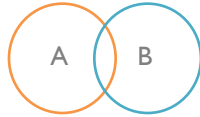
A,B must be *union-compatible*

Can we express using core operators?

$$A \cap B = ?$$

Intersect

$$A \cap B = R_{\text{result}}$$

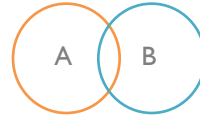


Can we express using core operators?

$$A \cap B = A - (A - B)$$

Intersect

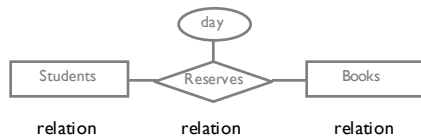
$$A \cap B = R_{\text{result}}$$



Can we express using core operators?

$$A \cap B = A - (A - B)$$

Joins (high level)



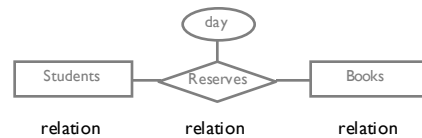
What if you want to query across all three tables?

e.g., all names of students that reserved "The Purple Crayon"

Need to combine these tables

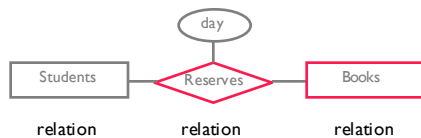
Cross product? But that ignores foreign key references

Joins (high level)



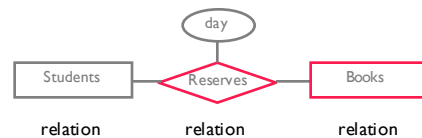
SI				RI			BI	
sid	name	gpa	age	sid	rid	day	rid	name
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barak	3	21	2	102	11/11	102	1984
3	trump	2	88					

Joins (high level)



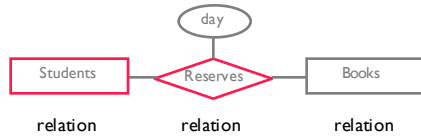
SI				RI			BI	
sid	name	gpa	age	sid	rid	day	rid	name
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barak	3	21	2	102	11/11	102	1984
3	trump	2	88					

Joins (high level)



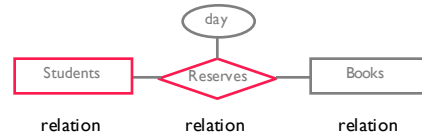
SI				RBI				
sid	name	gpa	age	sid	(rid)	day	(rid)	name
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barak	3	21	2	102	11/11	102	1984
3	trump	2	88					

Joins (high level)



SI				RBI				
sid	name	gpa	age	sid	(rid)	day	(rid)	name
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barak	3	21	2	102	11/11	102	1984
3	trump	2	88					

Joins (high level)



SRBI								
(sid)	(name)	gpa	age	(sid)	(rid)	day	(rid)	(name)
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barak	3	21	2	102	11/11	102	1984

theta (θ) Join

$$A \bowtie_c B = \sigma_c(A \times B)$$

Most general form

Result schema same as cross product

Often *far* more efficient to compute than cross product

Commutative

$$(A \bowtie_c B) \bowtie_c C = A \bowtie_c (B \bowtie_c C)$$

theta (θ) Join

SI				RI		
sid	name	gpa	age	sid	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	2	102	11/11
3	trump	2	88			

$$SI \bowtie_{SI.sid \leq RI.sid} RI =$$

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11

Equi-Join

$$A \bowtie_{attr} B = \pi_{all\ attr\ except\ B_{attr}}(A \bowtie_{A_{attr} = B_{attr}} B)$$

Special case where the condition is attribute equality

Result schema only keeps *one copy* of equality fields

Natural Join ($A \bowtie B$):

Equijoin on *all* shared fields (fields w/ same name)